Understanding the axial structure of the nucleon from QCD André Walker-Loud

**Frontiers in Lattice QCD and related topics Yukawa Institute for Theoretical Physics April 2019** 



# Neutron lifetime and the axial coupling



Czarnecki, Marciano, Sirlin Phys.Rev.Lett. 120 (2018) [arXiv:1802.01804]

□ It is also important to have a Standard Model prediction for g<sub>A</sub> - LQCD





**□** There is currently a 4-sigma discrepancy between the **beam** and **bottle** measurements of the neutron lifetime

 $\tau_n^{\text{beam}} = 888.0(2.0)s \quad \tau_n^{\text{bottle}} = 879.4(0.6)s$ 

 $\tau_n = \frac{5172.0 \pm 1.1 \text{ seconds}}{(1 + 3g_A^2)}$ 

□ This has generated a lot of interest that the discrepancy could be caused by new physics (hidden decay mode)

 $\Box$  The numerator, 5172.0(1.1), depends upon several quantities which must be measured experimentally - almost all of which are receiving new scruting given this discrepancy  $\frac{1}{\tau_{\pi}} = \frac{G_{\mu}^2 |V_{ud}|^2}{2\pi^3} m_e^5 (1+3g_A^2)(1+RC)f$ □ In particular, the radiative corrections (RC) have been updated with a

multi-sigma shift Seng, Gorchtein, Patel, Ramsey-Musolf: PRL 121 (2018) [1807.10197] Seng, Gorchtein, Ramsey-Musolf: 1812.03352



# Neutron lifetime and the axial coupling



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In particul multi-sign We need a 0.13% uncertainty to be comparable to experimental uncertainty



# nucleon axial coupling from LQCD





- To gain confidence in the application of Lattice QCD to nuclear physics, we must benchmark (calibrate) our calculations against well known quantities of interest
- □ g<sub>A</sub> was supposed to be a good benchmark calculation for single nucleon structure - but it proved to have significant systematic challenges, preventing results with the precision anticipated
- □ FLAG 2019 has included single nucleon quantities in their averaging for the first time



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   calculation for single nucleon structure but it
   proved to have significant systematic challenges,
   preventing results with the precision anticipated
- □ FLAG 2019 has included single nucleon quantities in their averaging for the first time
- Notice one result is significantly more precise than the others









model average

1.35

1.30

1.25 -

1.20 -

1.15 -

1.10 -

0.00

ga

Bart van Lith

E. Berkowitz



-	$ g_A(\epsilon_{\pi}, a \simeq 0.09 \text{ fm})$			$\mathbf{I}$ $\mathbf{a} \simeq$	0.09
)5	0.10	0.15	0.20	0.25	(

0.05





# Standard Systematics

continuum limit need 3 or more lattice spacings

### infinite volume limit

 $t_{comp} \propto V^{5/4}$ 





 $t_{comp} \propto \frac{1}{a^6}$ 

#### physical pion masses

exponentially bad signal-to-noise problem



Slide adapted from E. Berkowitz 5



### Most difficult challenge: an exponentially bad signal-to-noise problem

Each quark propagator carries information about pions and nucleons (conversations with David Kaplan)





Parisi, Phys. Rep. 103 (1984) 203  $\sim e^{-\frac{1}{2}m_{\pi}t} + e^{-\frac{1}{3}m_{N}t} + \cdots$ Lepage, TASI 1989  $\lambda_{\pi}(t) \gg \lambda_N(t)$  $\lambda_i(t) \sim e^{-E_i t}$ 

$$\bar{d}\gamma_5 u: C(t) = A_\pi e^{-m_\pi t} + \cdots$$

#### Large pion eigenvalues must cancel to expose small nucleon eigenvalues

$$(u^T C \gamma_5 d)u: C(t) = A_N e^{-m_N t} + \cdots$$





2-point correlation function





Effective mass of Pion 2-point correlation function red and black "data" are from different choices of *interpolating* operators

Noise is constant in time - can determine very clean ground state (blue band)

For pions, need to consider leading finite temperature effects

$$C(t) = \sum_{n} z_n z_n^{\dagger} \left( e^{-E_n t} + e^{-E_n (T-t)} \right)$$
$$m_{eff}^{\cosh}(t,\tau) = \frac{1}{\tau} \cosh^{-1} \left( \frac{C(t+\tau) + C(t-\tau)}{2C(t)} \right)$$



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# 2-point correlation function



Two examples of nucleon effective mass

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{Signal}{Noise} \to \sqrt{N_{stat}}e^{-(m_N - \frac{3}{2}m_\pi)t}$$

# LQCD challenges for NP



Correlated late-time fluctuations... what is the ground state?

Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

This problem is exacerbated with form-factor calculations ( $g_A$ ) and 2+ nucleons

- quark contraction cost becomes dominant
- density of excited states grows significantly and gap becomes small (nuclear interaction energies instead of pion mass gap)









### **Nucleon axial charge calculation** fixed source-sink separation time, t<sub>sep</sub>





$$R_3 = g_{\lambda} + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \cdots$$

in long-time (**t**<sub>sep</sub>) limit - should be flat

6

t<sub>sep</sub>

14

12

10

Repeat for multiple values of **t**<sub>sep</sub> Extrapolate to  $\mathbf{t_{sep}} \rightarrow \infty$ 

> fm typical calculation 1.22 Bhattacharya, Cirigliano, 1.05 Cohen, Gupta, Lin, Yoon 0.875 Phys.Rev.D 94 (2016)



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arXiv.org > hep-lat > arXiv:1612.06963 Phys. Rev. D96 (2017)

**High Energy Physics – Lattice** 

#### On the Feynman-Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud (Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

Feynman-Hellmann theorem

 $\partial_{\lambda} E_n |_{\lambda}$ 

"Follow your nose" in QFT

"Feynman-Hellmann" correlation function

 $rac{\partial m_{\lambda}^{eff}(t, au)}{\partial \lambda}$ 

$$_{=0} = \langle n | H_{\lambda} | n \rangle$$

$$\frac{1}{\tau}\Big|_{\lambda=0} = \frac{1}{\tau} \left[ \frac{-\partial_{\lambda}C_{\lambda}(t+\tau)}{C_{\lambda}(t+\tau)} - \frac{-\partial_{\lambda}C_{\lambda}(t)}{C_{\lambda}(t)} \right]_{\lambda=0}$$
$$= g_{\lambda} + z \left( e^{-(t+1)\Delta_{10}} - e^{-t\Delta_{10}} \right) + \cdots$$

 $\Delta_{10} = E_1 - E_0$  more than exponentially suppressed



arXiv.org > hep-lat > arXiv:1612.06963 Phys. Rev. D96 (2017)

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Feynman-Hellmann theorem

"Follow your nose" in QFT

"Feynman-Hellmann" correlation function

"Feynman-Hellmann" propagator

 $\partial_{\lambda} E_n |_{\lambda=0} = \langle n | H_\lambda | n \rangle$ 

derivative correlation function



standard 2-point function



### On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

our unconventional method



- The correlation function is given by
- excited state contamination is demonstrably controlled
- we can access very early Euclidean time, allowing the use of exponentially more precise numerical points
- No background field is used the FH-theorem is used to "derive" our "Feynman-Hellmann Correlation function" analytically

#### Phys. Rev. D96 (2017)

arXiv:1612.06963

"raw" correlation functions





### On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements

our unconventional method



**Our unconventional method is similar too** Otraced back to Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987): Güsken, Low, Mutter, Sommer, Patel, Schilling PLB227 (1989) first computed  $-\partial_{\lambda}C_{\lambda}(t)$ 

- **OBulava, Donnellan, Sommer, JHEP 1201 (2012):** combined above with GEVP
- Ode Divitiis, Petronzio, Tantalo, PLB718 (2012): computed derivatives of form factors
- with background field ( $\lambda$ ) varying strength of field to extract derivative

#### **Our method:**

Ouses analytic representation of derivative correlator instead of background field (cheaper) Ouses complete spectral decomposition of correlator, including contact operators **O**analysis was pushed to greater detail, showing stability of analysis (PRD96 [1612.06963], [1704.01114], Nature 558 [1805.12130])

#### Phys. Rev. D96 (2017)

arXiv:1612.06963

"Feynman-Hellmann" propagator

$$= \int dt_{\mathcal{O}} \qquad \square$$
$$= S_{FH}(y, x) = \sum_{z} S(y, z) \Gamma(z) S(z, x)$$

OChambers et al. PRD90 (2014), PRD92 (2015) Savage et al. PRL199 (2017); used unconventional method



# On the Feynman-Hellmann Theorem in QFT and the calculation of matrix elements



fixed source-sink separation time,  $t_{sep}$  repeat for a few different  $t_{sep}$ 

$$R_3 = g_{\lambda} + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)}$$

our unconventional method



#### Phys. Rev. D96 (2017)

arXiv:1612.06963

PNDME arXiv:1606.07049





#### Berkowitz et al. Some Lattice QCD Details PRD96 (2017) [1701.07559]

- PRD96 (2017) [1701.07559]
  - Approximate chiral symmetry, many finite lattice spacing operators not allowed  $\Box$  Leading discretization errors begin at O(a<sup>2</sup>)
- □ To control the three standard systematics for LQCD calculations, need **u** multiple lattice spacings
  - **u** multiple volumes
  - □ pion masses at/near the physical pion mass
- PRD87 (2013) [1212.4768]
- used very successfully: LHPC; NPLQCD; Aubin, Laiho, Van de Water; ... □ Fully developed Mixed-Action EFT: Bar, Bernard, Rupak, Shoresh; Tiburzi; Chen, O'Connell, Van de Water, Walker-Loud; ... □ This motivated us to use an improved version of this action

□ Möbius Domain Wall Fermions on gradient flowed 2+1+1 HISQ ensembles Berkowitz et al.

 $\Box$  The only set of publicly available ensembles which satisfy these criteria are the Nf=2+1+1 Highly Improved Staggered Quark (HISQ: Follana et al. PRD75 (2007) [hep-lat/0610092]) ensembles generated by the MILC Collaboration Bazavov et al. PRD82 (2010) [1004.0342],

The DWF on asquad action (Renner et al. [LHPC] NPPS 140 (2005) [hep-lat/0409130]) was



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#### Berkowitz et al. Some Lattice QCD Details PRD96 (2017) [1701.07559]

□ Möbius Domain Wall Fermions on gradient flowed 2+1+1 HISQ ensembles Berkowitz et al. PRD96 (2017) [1701.07559]

breaking than the HYP smearing used in DWF on asqtad





# Gradient Flow smearing of HISQ cfgs more effective at reducing residual chiral symmetry

 $m_{res} < 0.1 m_1$  on all ensembles for small-to-moderate L<sub>5</sub> and M<sub>5</sub>  $\leq 1.3$ 



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### Some Lattice QCD Details PRD96 (2017) [1701.07559]

#### Renormalization

- The currents used in the calculation must be renormalized to match physical currents
- By definition

$$Z_V \mathring{g}_V = 1$$

• The renormalized value of the axial coupling

$$g_A = \frac{Z_A}{Z_V} \frac{\mathring{g}_A}{\mathring{g}_V}$$

- Our action uses (Möbius) Domain-Wall fermions, which have very good chiral symmetry properties
- Using the RI/SMOM non-perturbative renormalization scheme, we find

$$rac{Z_A}{Z_V}=1~$$
 to 1 part in 104

a confirmation that our action respects chiral symmetry to a good degree.







# Some Lattice QCD Details

### HISQ gauge configurations and mixed action



#### Möbius domain-wall valence quarks

- chiral symmetry at finite lattice spacing
- Ludicrously fast GPU solver (QUDA)

Berkowitz et al. PRD96 (2017) [1701.07559]



unofficial MILC cow **MILC** = MIMD Lattice Computation (the acronym has an acronym in the acronym)

# MILC configurations are the only publicly available dataset capable of

- chiral extrapolation to physical pion mass
- continuum extrapolation
- infinite volume extrapolation

HISQ action Errors starting at  $O(\alpha_s a^2, a^4)$ Lüscher-Weisz action Errors starting at  $O(\alpha_s^2 a^2, a^4)$ 

Slide adapted from CC. Chang





 $g_V^{\text{eff}} t_{\text{max}}$ 

#### **Correlation function analysis**

- Simultaneous fit to 6 correlation functions, 2-point, g<sub>A</sub>, 0  $g_V(SS \text{ and } PS)$
- Complete 2-state fit (g.s. and 1 excited state, including 0 transitions)
- Bayesian constrained fit as a pre-conditioner for 0 unconstrained non-linear regression
- Stability analysis is performed varying min and max 0 time in the correlator analysis





- early time has excited state contamination О
- late time susceptible to correlated 0 fluctuations
- excited state subtracted results are constant О in fit region
- resulting bootstrap distributions ( $N_{bs}=5000$ ) 0 are Gaussian and show no outliers







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Nature 558 (2018) no.7708, 91-94 **Download the arXiv version or supplemental** material - 36 pages of detail



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### **Renormalized LQCD results**





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# Extrapolations $\epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{w_0^2}$

- ChiPT: EFT expanding around  $m_{\pi}$ = 0
  - best hope for model-independent extrapolation
  - not guaranteed to converge around  $m_{\pi}$  135 MeV
- Mild  $m_{\pi}$  a dependence
  - Taylor expansion works well for extrapolation/interpolation

Dimensionless parameters: lattice spacing, volume, pion mass

$$m_{\pi}L \quad \epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$



# ExtrapolationsDimensionlattice spacing $\epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{w_0^2}$

- NNLO  $\chi PT$ : E
- NNLO+ct  $\chi$ PT : E
- NLO Taylor  $\epsilon_{\pi}^2$ :  $c_0$
- NNLO Taylor  $\epsilon_{\pi}^2$ :  $c_0$ 
  - NLO Taylor  $\epsilon_{\pi}$ :  $c_0$
- NNLO Taylor  $\epsilon_{\pi}$ :  $c_0$

Dimensionless parameters: lattice spacing, volume, pion mass

$$m_{\pi}L \quad \epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

Eq. (S8) + 
$$\delta_a$$
 +  $\delta_L$   
Eq. (S8) +  $c_4 \epsilon_{\pi}^4 + \delta_a + \delta_L$   
 $c_0 + c_2 \epsilon_{\pi}^2 + \delta_a + \delta_L$   
 $c_0 + c_2 \epsilon_{\pi}^2 + c_4 \epsilon_{\pi}^4 + \delta_a + \delta_L$   
 $c_0 + c_1 \epsilon_{\pi} + \delta_a + \delta_L$   
 $c_0 + c_1 \epsilon_{\pi} + c_2 \epsilon_{\pi}^2 + \delta_a + \delta_L$ 



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- NNLO+ct  $\chi$ PT : Eq. (S

Dimensionless parameters: lattice spacing, volume, pion mass

$$m_{\pi}L \quad \epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

$$g_{A} = g_{0} + c_{2}\epsilon_{\pi}^{2} - \epsilon_{\pi}^{2} \left(g_{0} + 2g_{0}^{3}\right) \ln\left(\epsilon_{\pi}^{2}\right) + g_{0}c_{3}\epsilon_{\pi}^{3}$$

$$NNLO XPT$$

$$S(8) + \delta_{a} + \delta_{L}$$

$$S(8) + c_{4}\epsilon_{\pi}^{4} + \delta_{a} + \delta_{L}$$

NLO Taylor  $\epsilon_{\pi}^2$ :  $c_0 + c_2 \epsilon_{\pi}^2 + \delta_a + \delta_L$ NNLO Taylor  $\epsilon_{\pi}^2$ :  $c_0 + c_2 \epsilon_{\pi}^2 + c_4 \epsilon_{\pi}^4 + \delta_a + \delta_L$ NLO Taylor  $\epsilon_{\pi}$ :  $c_0 + c_1 \epsilon_{\pi} + \delta_a + \delta_L$ NNLO Taylor  $\epsilon_{\pi}$ :  $c_0 + c_1\epsilon_{\pi} + c_2\epsilon_{\pi}^2 + \delta_a + \delta_L$ 



Extrapolations Dimensionless parameters: lattice spacing, volume, pion mass  $\epsilon_a^2 = \frac{1}{4\pi} \frac{a^2}{w_0^2}$ 

 $\delta_a = a_2 \varepsilon_a^2 + b_4 \varepsilon_a^2 \varepsilon_\pi^2 + a_4 \varepsilon_a^4 + \left[a_1 \sqrt{4\pi} \varepsilon_a + s_2 \alpha_S \alpha_a^2\right]$ 

- NNLO  $\chi PT$ : Eq.
- NNLO+ct  $\chi$ PT : Eq.
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$$m_{\pi}L \quad \epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

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$$NNLO XPT$$

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$$+ c_{2}\epsilon_{\pi}^{2} + \delta_{a} + \delta_{L}$$

$$+ c_{2}\epsilon_{\pi}^{2} + c_{4}\epsilon_{\pi}^{4} + \delta_{a} + \delta_{L}$$

$$+ c_{1}\epsilon_{\pi} + \delta_{a} + \delta_{L}$$

$$+ c_{1}\epsilon_{\pi} + c_{2}\epsilon_{\pi}^{2} + \delta_{a} + \delta_{L}$$





Dimensionless parameters: lattice spacing, volume, pion mass

$$m_{\pi}L \quad \epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

$$F_{1}(x) = \sum_{n \neq 0} \left[ K_{0}(x|\mathbf{n}|) - \frac{K_{1}(x|\mathbf{n}|)}{|x|\mathbf{n}|} \right]$$

$$g_{0}F_{3}(m_{\pi}L) + f_{3}\varepsilon_{\pi}^{3}F_{1}(m_{\pi}L) \qquad F_{3}(x) = -\frac{3}{2}\sum_{n \neq 0} \frac{K_{1}(x|\mathbf{n}|)}{|x|\mathbf{n}|}$$

$$g_{A} = g_{0} + c_{2}\varepsilon_{\pi}^{2} - \varepsilon_{\pi}^{2} \left( g_{0} + 2g_{0}^{3} \right) \ln \left( \varepsilon_{\pi}^{2} \right) + g_{0}c_{3}\varepsilon_{\pi}^{3}$$

$$\mathbf{NNLO XPT}$$
Eq. (S8) +  $\delta_{a} + \delta_{L}$ 
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$$c_{0} + c_{1}\varepsilon_{\pi} + \delta_{a} + \delta_{L}$$

$$c_{0} + c_{1}\varepsilon_{\pi} + c_{2}\varepsilon_{\pi}^{2} + \delta_{a} + \delta_{L}$$









f

0.00

0.05









NNLO  $\chi PT$ : Eq. (S8) +  $\delta_a$  +  $\delta_L$ NNLO+ct  $\chi$ PT: Eq. (S8) +  $c_4\epsilon_{\pi}^4 + \delta_a + \delta_L$ NLO Taylor  $\epsilon_{\pi}^2$ :  $c_0 + c_2 \epsilon_{\pi}^2 + \delta_a + \delta_L$ NNLO Taylor  $\epsilon_{\pi}^2$ :  $c_0 + c_2 \epsilon_{\pi}^2 + c_4 \epsilon_{\pi}^4 + \delta_a + \delta_L$ NLO Taylor  $\epsilon_{\pi}$ :  $c_0 + c_1 \epsilon_{\pi} + \delta_a + \delta_L$ NNLO Taylor  $\epsilon_{\pi}$ :  $c_0 + c_1\epsilon_{\pi} + c_2\epsilon_{\pi}^2 + \delta_a + \delta_L$ 



### convergence of the chiral expansion...



$$\epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4 \epsilon_\pi^4$$

$$g_{A} = g_{0} - \epsilon_{\pi}^{2} (g_{0} + 2g_{0}^{3}) \ln(\epsilon_{\pi}^{2}) + c_{2}\epsilon_{\pi}^{2} + g_{0}c_{3}\epsilon_{\pi}^{3} + \epsilon_{\pi}^{4} \left[ c_{4} + \tilde{\gamma}_{4} \ln(\epsilon_{\pi}^{2}) + \left( \frac{2}{3}g_{0} + \frac{37}{12}g_{0}^{3} + 4g_{0}^{5} \right) \ln^{2}(\epsilon_{\pi}^{2}) \right]$$
Bernard and Meissner (CD06)  
Phys.Lett.B639 [hep-lat/0605010]  
 $F \longrightarrow F_{\pi}$ 



### convergence of the chiral expansion...



can we trust extrapolation of quantities with chiraly-enhanced behavior? if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?



$$g_A = g_0 - \epsilon_{\pi}^2 (g_0 + 2g_0^3) \ln(\epsilon_{\pi}^2) + c_2 \epsilon_{\pi}^2 + g_0 c_3 \epsilon_{\pi}^3$$

$$\epsilon_{\pi} = \frac{m_{\pi}}{4\pi F_{\pi}}$$

$$g_{A} = g_{0} - \epsilon_{\pi}^{2} (g_{0} + 2g_{0}^{3}) \ln(\epsilon_{\pi}^{2}) + c_{2} \epsilon_{\pi}^{2} + g_{0} c_{3} \epsilon_{\pi}^{3} + \epsilon_{\pi}^{4} \left[ c_{4} + \tilde{\gamma}_{4} \ln(\epsilon_{\pi}^{2}) + \left( \frac{2}{3} g_{0} + \frac{37}{12} g_{0}^{3} + 4g_{0}^{5} \right) \ln^{2}(\epsilon_{\pi}^{2}) \right]$$
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 $F \longrightarrow F_{\pi}$ 



### **Continuum and infinite volume extrapolation**





### **Continuum and infinite volume extrapolation**





### Model average extrapolation



Fit	$v^2/dof$	$\mathcal{L}(D M_{t})$	P(M,  D)	$P(a \mid M_1)$
<b>I</b> . IC	$\chi$ /uor	$\mathcal{L}(\mathcal{D} \mathcal{M}_k)$	$I(M_k D)$	I(gA Wk)
NNLO $\chi PT$	0.727	22.734	0.033	1.273(19)
NNLO+ct $\chi PT$	0.726	22.729	0.033	1.273(19)
NLO Taylor $\epsilon_{\pi}^2$	0.792	24.887	0.287	1.266(09)
NNLO Taylor $\epsilon_\pi^2$	0.787	24.897	0.284	1.267(10)
NLO Taylor $\epsilon_{\pi}$	0.700	24.855	0.191	1.276(10)
NNLO Taylor $\epsilon_\pi$	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

NNLO $\chi PT$ :	Eq. (S8) + $\delta_a + \delta_L$
NNLO+ct $\chi$ PT :	Eq. (S8) + $c_4\epsilon_{\pi}^4 + \delta_a + \delta_L$
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NNLO Taylor $\epsilon_{\pi}$ :	$c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$













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# https://github.com/callat-qcd/project\_gA

# raw correlation functions, correlation function analysis results, extrapolation analysis

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correlation_functions	updated README; moved correlation fur	nction data to correlation_functi		23 days ago
🗖 data	added logo's to README			7 days ago
plots	moved plotting scripts to plots folder updated README; moved correlation function data to correlation_functi			2 months ago
sample_corr_fit				23 days ago
.gitignore	loop through models and model average			7 months ago
README.md	final image width tweak?			7 days ago
callat_ga_lib.py	added ability to control linspace for plots	s; created sample fitter to		26 days ago
ga_workbook.ipynb	moved plot scripts to plot folder			27 days ago
https://github.com				



# What are the implications now?





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### **Final result**

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%

$$g_A^{\rm QCD} = 1.2711(103)^s$$

Isospin corrections

□ The leading radiative corrections are subtracted from the experimental measurement leaving corrections of  $\mathcal{O}\left(\frac{\alpha_{EM}^2}{\pi^2}\right) \sim 0.0005\%$  $\Box \text{ There are } (\mathbf{m_d}-\mathbf{m_u})^2 \text{ corrections } \mathcal{O}\left(\frac{(m_d-m_u)^2}{(m_d+m_u)^2}\epsilon_{\pi}^4\right) \sim 0.002\%$  $\Box \text{ There are mixed corrections of } \mathcal{O}\left(\alpha_{EM}\frac{m_d-m_u}{m_d+m_u}\epsilon_{\pi}^2\right) \sim 0.004\%$  $\Box \text{ The largest isospin correction comes from the extrapolation to} \quad \epsilon_{\pi^-} = \frac{m_{\pi^-}}{4\pi F_{\pi^-}} \quad \epsilon_{\pi^0} =$ O(10-3) - Ando et al. Phys. Lett. B595 (2004) [nucl-th/0402100]



 $m_{\pi^0}$  $4\pi F_{\pi^0}$ □ There are non-universal structure corrections which have not been determined but are expected to be





### The Neutron Lifetime on Sierra Early Science

1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



Nature 558 (2018) no. 7708, 91-94

The vertical gray band denotes the physical value of pion mass - these points are significantly more expensive than the rest to compute - but the most valuable for the final predictions
 The green point in our publication cost as much computing time as all the other points combined
 The green point from Sierra has 10x more statistics than our publication
 The red point from our publication was not useful
 The red point from Sierra came from an entirely new calculation and is now very useful
 The blue point from Sierra was entirely unattainable from previous computers (it still needs more statistics to be useful)



#### Sierra Early Science





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How precise can we get  $g_A$ ?

- physical pion mass points likely this year with our INCITE allocation on Summit at OLCF
- masses. This can still be achieved with Summit (6 Volta GPUs/IBM Power9 node)
- puzzle is still a puzzle (i.e. it is being used to search for new physics)



□ Without changing strategy (just 3 lattice spacings), we should be able to get 0.5% by improving the three

□ To achieve 0.2% precision, at least a 4th lattice spacing will be required, ideally at two or more pion

 $\Box$  To convince the broad community of a 0.2% or better uncertainty, we will have to incorporate isospin violating corrections to ensure they behave as expected theoretically - particularly if the neutron lifetime



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- Argonne target in South Dakota.

  - Leptogenesis
- □ The T2K and NOVA experiments are also conducting oscillation experiments
  - and K. Mcfarland]
- nucleon cross sections

  - true uncertainty of our understanding)



DUNE is a future neutrino oscillation experiment that will fire a beam of neutrinos from FNAL into an

□ A determination of the CP-violating phase in the neutrino-mixing (PMNS) matrix is one of the goals • enough CP violation could explain the matter/anti-matter asymmetry of the universe through

□ "A determination of the nucleon axial form factor at the 5% level would be very helpful, possibly allowing for the isolation of nuclear effects" [private communications with T2K members, Y. Hayato

Ultimately, we need to understand neutrino-NUCLEUS cross sections which begins with neutrino-

 $\Box$  The experimental data on  $g_A(Q^2)$  is sufficiently limited that a simple dipole-formfactor is assumed <sup>□</sup> The dipole model is too simplistic and overly constraining (the quoted uncertainties do not reflect the





that determined from the phenomenological determination - two recent examples here  $\Box$  A few years ago - this was the same situation with  $g_A$  (no one understood why gA results were consistently low compared to the experimental value)

seems warranted



- □ All lattice QCD results determine an axial form factor with a significantly different slope (30%) than
- □ Examining the LQCD results, it is difficult to understand/guess where this discrepancy is coming from
- □ For g<sub>A</sub>, we made progress by pushing to the extreme the LQCD calculations a similar strategy here



□ Inherent to our gA calculation was the "Feynman-Hellmann" Propagator

$$- = S_{FH}(y, x) = \sum_{z} S(y)$$

For each choice of current and momentum, a new FH propagator is required
 We have tried several variants of stochastic methods to relax this constraint, but the noise is too large
 We have resorted to the standard fixed source-sink separation method (with our tail between our legs a little)
 O(t<sub>0</sub>)
 repeat for multiple values of t<sub>sep</sub>

However, if there was a lesson to be learned from our g<sub>A</sub> calculation when applying the fixed source-sink separation method - it is imperative to use many values of t<sub>sep</sub> and also small values
 See also S. Meinel, Chiral Dynamics 2012 and Hasan et al. (LHPC) 1903.06487

 $(z, z)\Gamma(z)S(z, x)$ 



### a09m310 $t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]$



![](_page_57_Figure_3.jpeg)

# PRELIMINARY

![](_page_57_Picture_5.jpeg)

### a09m310

### $t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]$

![](_page_58_Figure_3.jpeg)

![](_page_58_Figure_4.jpeg)

![](_page_58_Figure_5.jpeg)

![](_page_58_Figure_6.jpeg)

# PRELIMINARY

![](_page_58_Picture_8.jpeg)

a09m310

### $t_{sep} = [3,4,5,6,7,8,9,10,11,12,13,14]$

![](_page_59_Figure_3.jpeg)

![](_page_59_Figure_4.jpeg)

![](_page_59_Figure_5.jpeg)

![](_page_59_Figure_6.jpeg)

# PRELIMINARY

![](_page_59_Figure_8.jpeg)

Nature 558 (2018) no.7708, 91-94

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

**D** The success of this result was enabled through several key features:

- $\Box$  an unconventional strategy that ean exploit exploit to a normalized by the second strategy that the second strategy the second strategy the second strategy that the second strategy the second strategy the second strategy the second strategy that the second strategy term strateg early time and has **demonstrable control of excited state** contributions
- $\Box$  access to a set of ensembles (HISQ 2+1+1 from MILC) that allowed for control over all standard lattice systematics,
- *Qudicrously fast* GPU code QUDA
- **D** an action with **improved stochastic behavior** and a **mild continuum** extrapolation  $m_{\pi} \to m_{\pi}^{phys}, a \to 0, L \to \infty$

AMMAN

**D** access to Leadership Computing

 $\Box$  Making progress in understanding  $g_A(Q^2)$  - it seems essential to have enough  $t_{sep}$ values to control the infinite separation extrapolation - more than is common

arXiv:1805.12130 https://github.com/callat-qcd/project\_gA

![](_page_60_Figure_15.jpeg)

#### Nature 558 (2018) no.7708, 91-94 A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

Lattice QCD Team

(postdoc, grad student)

![](_page_61_Picture_4.jpeg)

Chia Cheng (Jason) Chang Amy Nicholson Enrico Rinaldi Evan Berkowitz Nicolas Garron **David Brantley** Henry Monge-Camacho Chris Monahan Chris Bouchard Kate Clark Bálint Joó Thorsten Kurth Kostas Orginos Pavlos Vranas André V er-Loud U.S. DEPARTMENT OF ENERGY CAK RIDC

![](_page_61_Picture_6.jpeg)

LBNL, RIKEN-iTHEMS Berkeley —> UNC, Chapel Hill **RIKEN-BNL** Forschungszentrum Jülich Liverpool W&M, LBNL  $\longrightarrow$  LLNL W&M, LBNL -> UNC  $INT \longrightarrow W\&M$ Glasgow NVIDIA JLab NERSC, LBNL ⊾W&M, JLab LING AAAA NL

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Science

![](_page_61_Picture_8.jpeg)

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https://doi.org/10.1038/s41586-018-0161-8

arXiv:1805.12130 https://github.com/callat-qcd/project\_gA

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![](_page_61_Picture_12.jpeg)

**DOE** Topical Collaboration Double Beta Decay

#### Nature 558 (2018) no.7708, 91-94 A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

Lattice QCD Team

(postdoc, grad student)

![](_page_62_Picture_4.jpeg)

Chia Cheng (Jason) Chang Amy Nicholson Enrico Rinaldi Evan Berkowitz Nicolas Garron **David Brantley** Henry Monge-Camacho Chris Monahan Chris Bouchard Kate Clark Bálint Joó Thorsten Kurth Kostas Orginos Pavlos Vranas André V er-Loud

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![](_page_62_Picture_6.jpeg)

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![](_page_62_Picture_8.jpeg)

ADERSHIP MPUTING

https://doi.org/10.1038/s41586-018-0161-8

arXiv:1805.12130 https://github.com/callat-qcd/project\_gA

![](_page_62_Picture_11.jpeg)

**DOE** Topical Collaboration Double Beta Decay

![](_page_62_Picture_13.jpeg)

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