Interglueball potential in lattice gauge theory

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YITP
Kyoto

What is dark matter ??

Dark ≠ black ✕

Dark = invisible ✓

Something invisible around us!

- What is it?

Nobody knows!

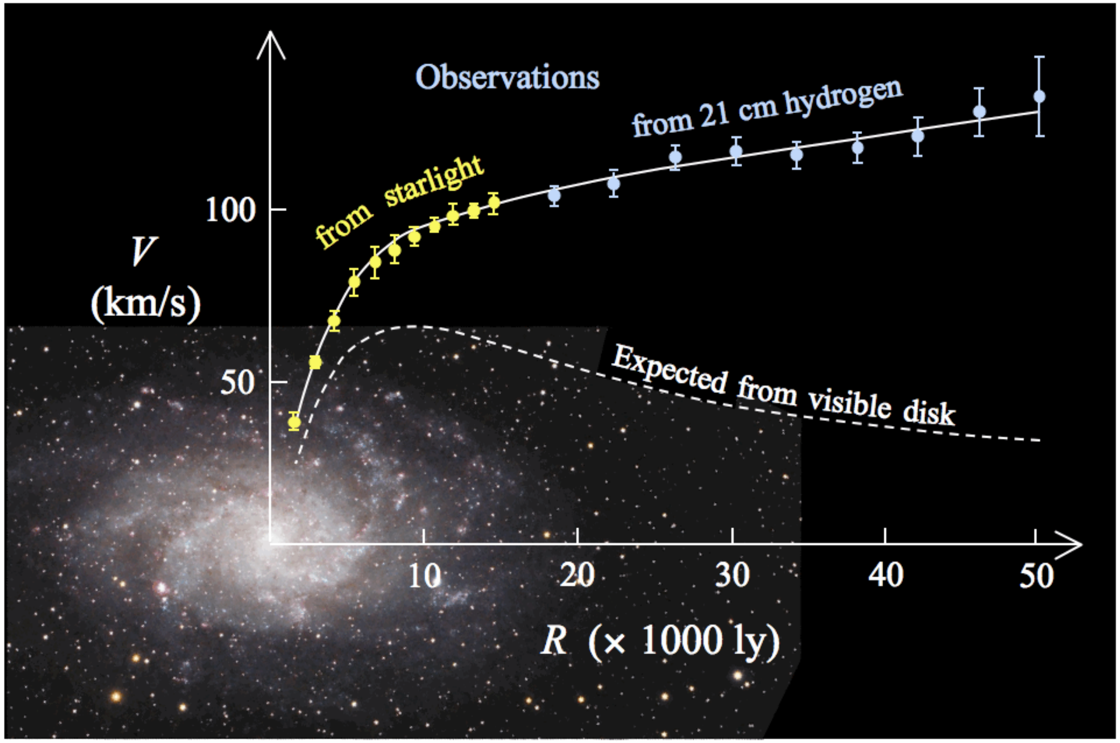
Unveiling it is one of the most important goal of cosmology, astrophysics, particle physics.

- Why do we know it exists?

Let us see…

Galactic rotation curve

Velocity of the disc cannot be explained by visible stars

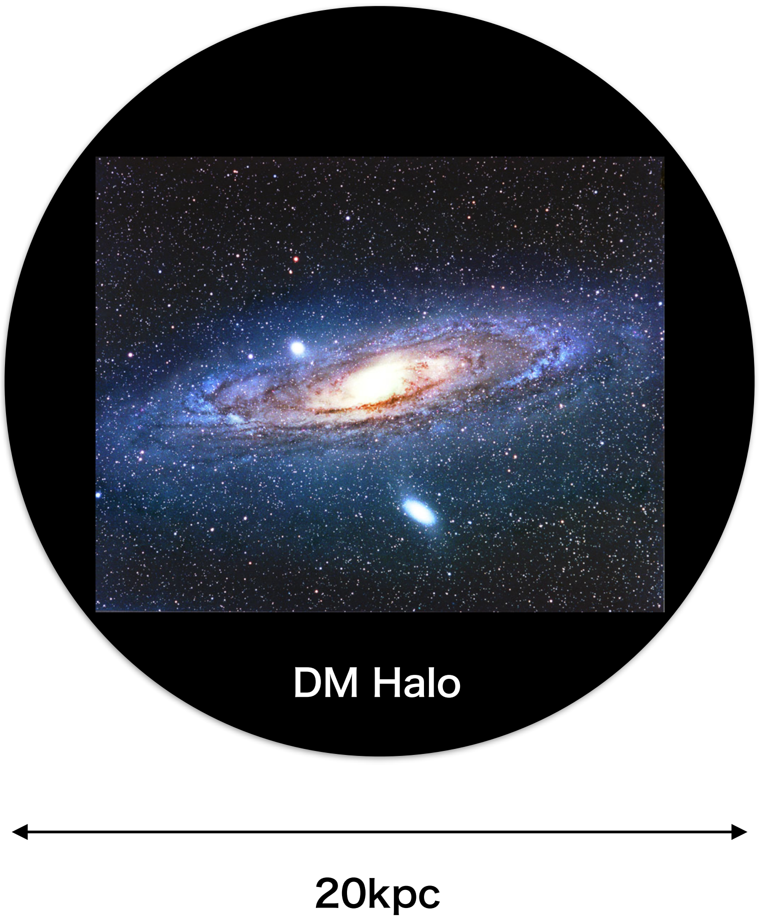


Suggesting additional something invisible surrounding the Milky way.

(Zwicky, 1930's)

Dark matter halo

Our galaxy is surrounded by a halo of dark matter



DM density at the Earth: 0.3GeV/cm^3

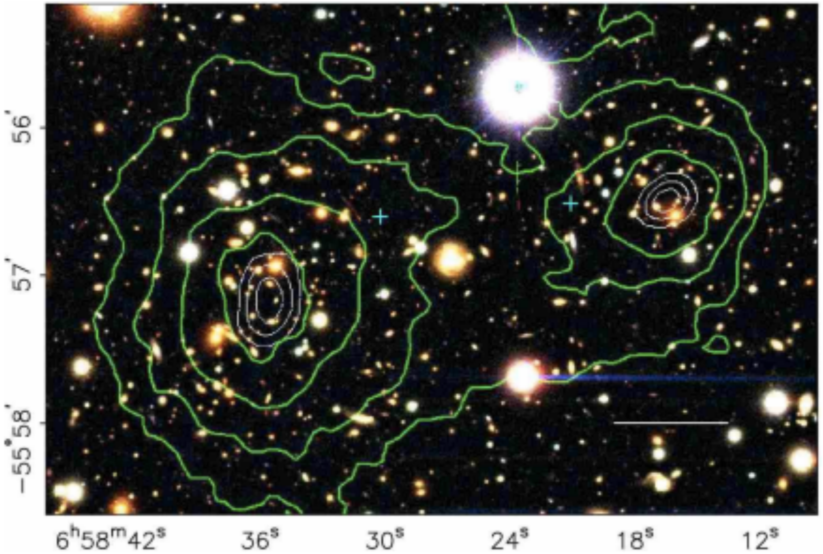
DM halo :

⇒ Weakly interacting
with star, gas, and each other

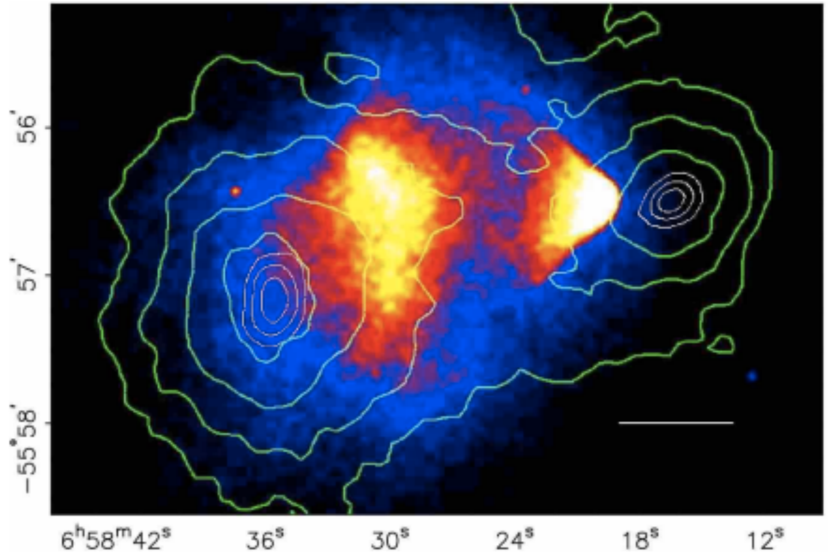
⇒ Nonrelativistic

A more powerful proof : Galactic collision

Bullet cluster



Magellan telescope

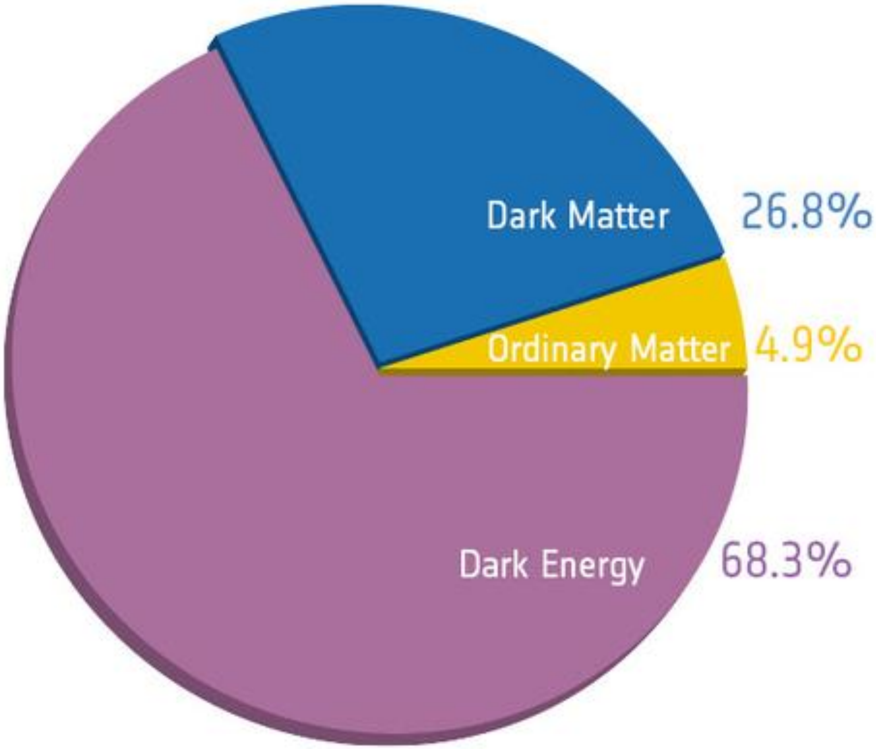


Chandra Xray image

Difference between luminous (baryonic)
and total mass distributions!

Dark matter : 27% of the energy component of the Universe

From the cosmic microwave background analysis (Planck), fraction of dark matter can be derived

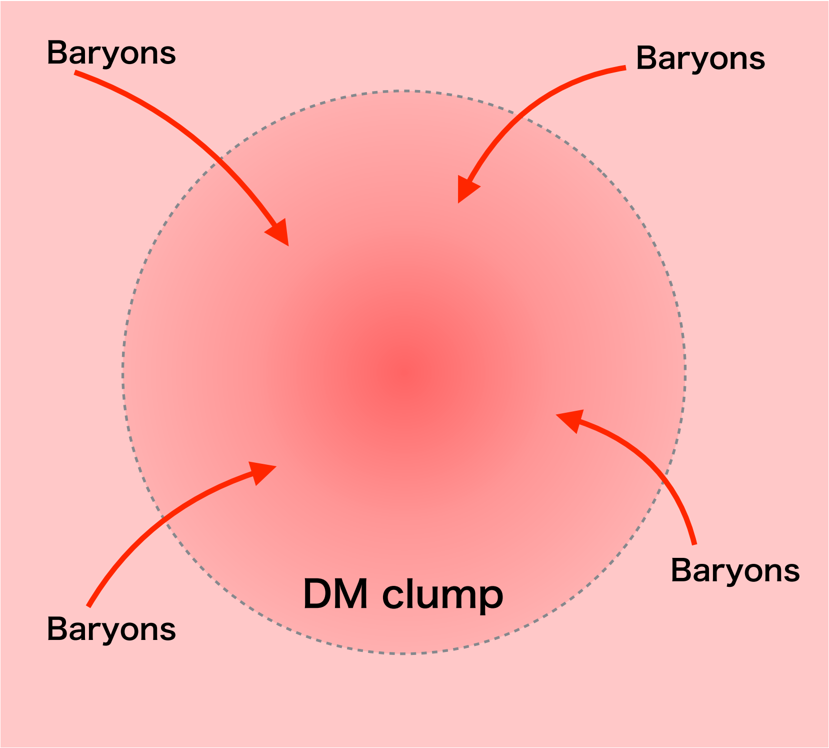


Cosmic "makeup". Credit: ESA/Planck

⇒ Most of matter in our Universe is dark.

Formation of galaxy

Dark matter is required to speed up the formation of galaxies



Baryon concentration catalyzed
by dark matter clumps
during the cooling
(Early Universe, high temperature)

If no dark matters, galaxy formation is much slower.

⇒ Dark matter absolutely required in our existence!

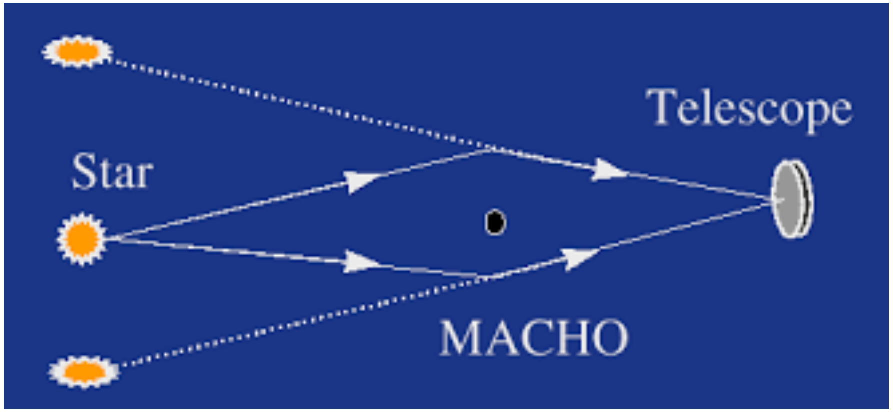
Is the dark matter a MACHO?

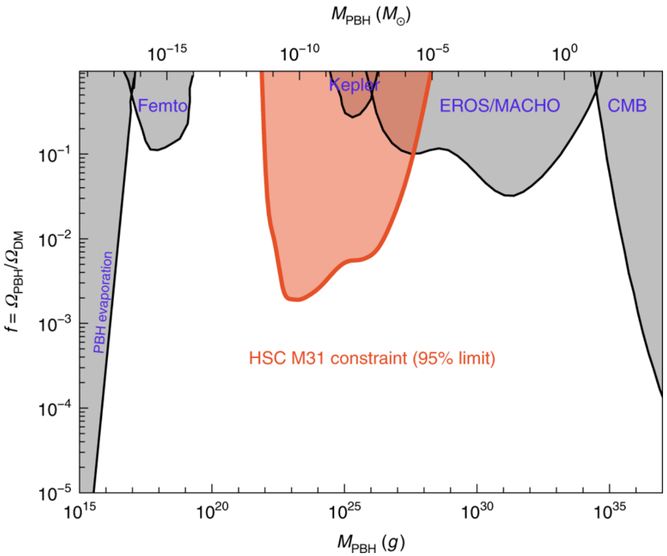
MACHO : Massive Compact Halo Object

Almost non luminous astronomical body

Example : primordial blackholes, brown dwarfs

Can be probed with gravitational lensing





H. Niikura et al., Nature Astronomy (2019) (arXiv:1701.02151 [astro-ph.CO])

MACHOs are not favored by observations,
even if a window (around $M_{PBH}/M_\odot \sim 10^{-12}$) is still left

⇒ Dark matter is likely to be particles?

WIMP dark matter

WIMP : weakly interacting massive particle

WIMP = particle physics

Property of WIMPs:

- No charge, no color
- Not neutrino (ruled out by Bigbang nucleosynthesis)
- No candidates in standard model of particle physics

Challenge in particle physics:

⇒ Find theory explaining dark matter!

SU(N) Yang-Mills theory

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$$

(a =1,…,N_c^2−1)

⇒ The simplest interacting theory

Important properties:

$\mathscr{L}_{\rm YM}$ does not have apparent scale, but scale is dynamically generated (dimensional transmutation)

Renormalizable theory, running coupling has logarithmic scale variation,

difference of N_c can generate $\Lambda_{\rm YM}$'s which differ by orders of magnitude

No scalars and massive fermions ⇒ Free from quadratic divergences

⇒ No important fine-tuning problem in the choice of $\Lambda_{\rm YM}$!

(Suppose a GUT which generates SM and DM, the difference of mass scales between SM and DM is not serious)

⇒ Theory with very high naturalness

Dark matter in hidden YM theory:

Lightest particles are glueballs ! ⇒ SU(N) glueballs are candidate of DM

(summarized in the report of USQCD Collaboration : arXiv:1904.09964 [hep-lat])

Self-interacting dark matter

The DM distribution can be predicted in N-body simulation with gravity only

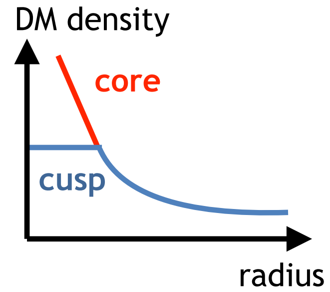
⇒ Successful in describing the large scale structure (scale > Mpc)

Introducing DM self-interaction changes its distribution smaller than Mpc

There are (were?) several problems in the galactic DM distribution:

- **Core vs Cusp problem:**

  N-body simulation predicts cuspy DM distribution near the galactic center, whereas observations suggest flat ones.



- **Too-big-to-fail problem:**

  Satellite galaxies are less dense than those predicted by the N-body simulation.

- **Missing satellite problem:**

  More satellite galaxies than those predicted by the N-body simulation are observed (resolved?).

DM-DM self-interaction ↔ DM-DM scattering ↔ DM-DM potential

must be studied

Object of study

In this work, we study the interglueball interaction on lattice which is the only way to quantify nonperturbative physics of nonabelian gauge theory.

Object:

In this work, we study the interglueball interaction of SU(N) Yang-Mills theory on lattice.

(Please be careful, SU(2), SU(3), and SU(4) may alternate, but the global feature is the same).

Setup

We consider the SU(2), SU(3), and SU(4) pure Yang-Mills theory

- Standard SU(N) plaquette action :

  Lattice spacings : β = 2.5 (N_c=2), 5.7 (N_c=3), 10.789 (N_c=4),
  10.9 (N_c=4)

  Volume : 16^3x24

  Confs. generated with pseudo-heat-bath method

- Improvement of glueball operator : APE smearing

- We use all space-time translational and cubic rotational symmetries
  to effectively increase the statistics
  (like the all-mode average for meson and baryon observables)

Scale determination (example of SU(4))

We do not know the scale of the YM theory, so we leave it as a free parameter Λ

Nevertheless, all quantities calculated on lattice depends on Λ

⇒ We express all quantities in unit of Λ.

Relation between Λ and string tension:

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

$$= 0.524(40) \qquad \text{(for SU(4))}$$

Fitted from the analysis of the running coupling

C. Allton et al., JHEP **0807** (2008) 021
M. Teper, Acta Phys. Polon. B **40** (2009) 3249

String tension for several β in SU(4) YM :

β	$a\sqrt{\sigma}$	
10.789	0.2706(8)	B. Lucini et al., JHEP 0406 (2004) 012
10.9	0.228(7)	M. Teper, Phys. Lett. B **397** (1997) 223; hep-th/9812187
11.1	0.197(8)	M. Teper, Phys. Lett. B **397** (1997) 223; hep-th/9812187
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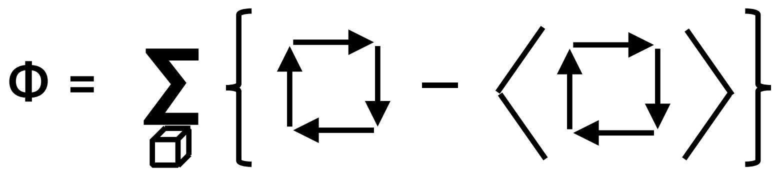
String tension for several β in SU(4) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
10.789	0.2706(8)	0.142(11)
10.9	0.228(7)	0.119(10)
11.1	0.197(8)	0.103(9)
11.4	0.14277(72)	0.075(6)

⇒ Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

0^{++} glueball operator:



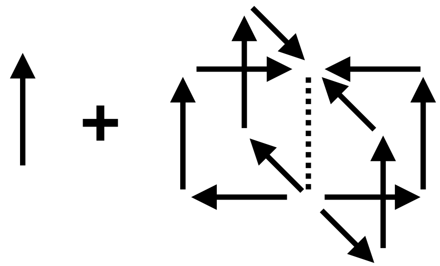
Glueball has expectation value → subtract

Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize $\mathrm{Re\,Tr}\,[\,U^{(n+1)}\,V^{(n)\dagger}\,]$

where $V^{(n)} = \alpha \times$



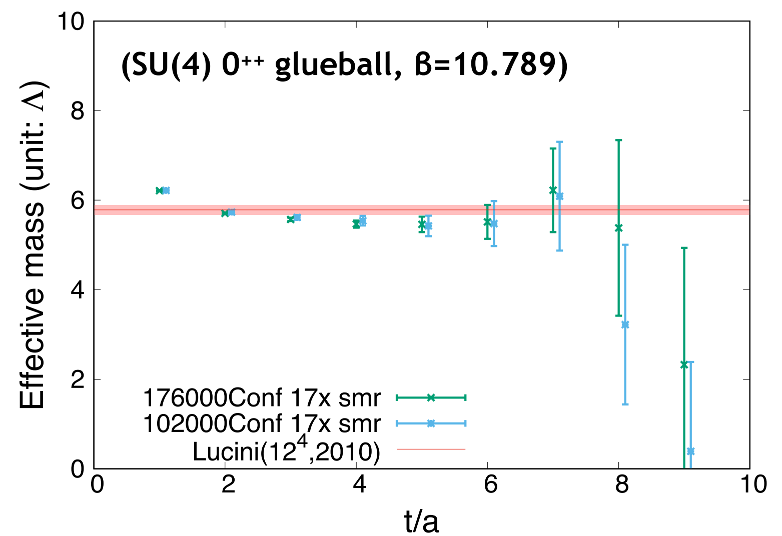
⇒ Gaussian spread: $2\sqrt{\dfrac{n}{4+\alpha}}$

(in lattice unit)

Ape Collaboration, PLB **192** (1987) 163
N. Ishii et al., PRD **66**, 094506 (2002)

Optimal parameters:

	n	α
SU(4),β=10.789	17	2.3
SU(4),β=10.9	21	2.3
SU(4),β=11.1	37	2.3



Nambu-Bethe-Salpeter amplitude

$$C_{\phi\phi}(t, \mathbf{x}-\mathbf{y}) \equiv \frac{1}{V}\sum_{\mathbf{r}} \langle 0 | T[\phi(\mathbf{x}+\mathbf{r}, t)\phi(\mathbf{y}+\mathbf{r}, t) \cdot \mathcal{J}(0)] | 0 \rangle$$

$\mathcal{J}(0)$: source op.

The source is smeared, but the sinks are not

For the glueball, caution is needed :

- 2-glueball (0^{++}) state mixes with all other multi-glueball states:

  ⇒ The source may be chosen as 1-body, 2-body, etc, on convenience

- Multi-glueball operators also have expectation value!

  (often called “VEV”, but it corresponds to the divergence caused by the mixing with the identity operator)

  ⇒ We then have to subtract the “VEV” of both source and sink

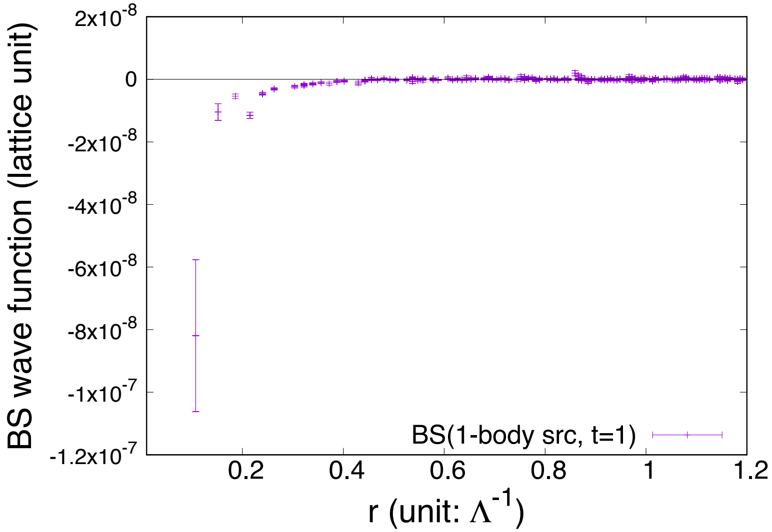
  (removing the source “VEV” will automatically remove sink “VEV”:

  $\langle(\varphi_{src}\varphi_{src}-\langle\varphi_{src}\varphi_{src}\rangle)(\varphi_{snk}\varphi_{snk}-\langle\varphi_{snk}\varphi_{snk}\rangle)\rangle=\langle(\varphi_{src}\varphi_{src}-\langle\varphi_{src}\varphi_{src}\rangle)\varphi_{snk}\varphi_{snk}\rangle$)

  ⇒ Important consequence : fulfills the cluster decomposition!

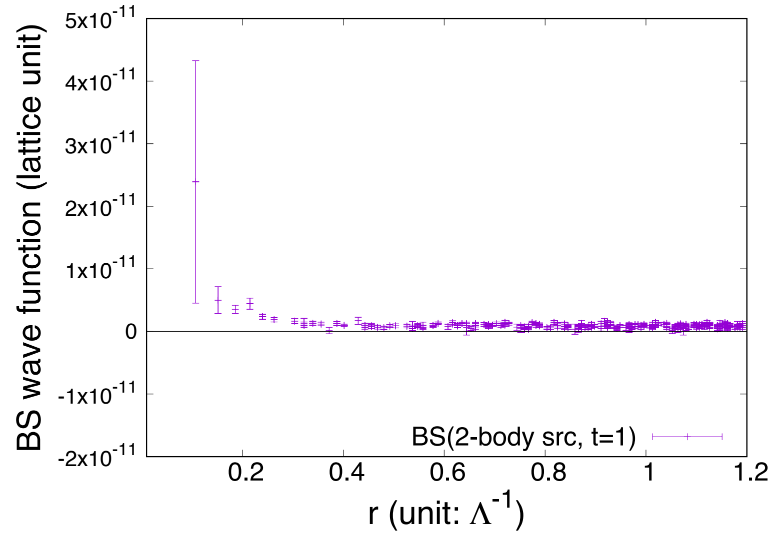
Glueball NBS wave function plot

1-body source:

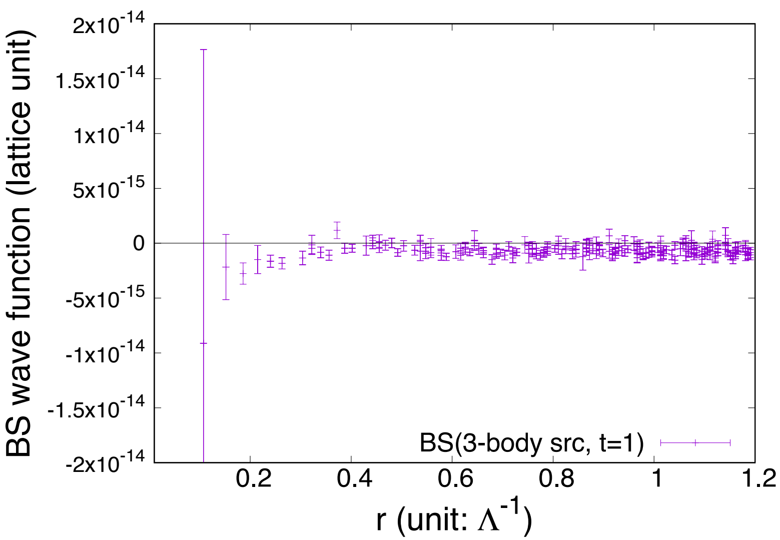


2-body source:

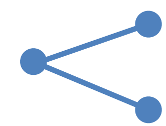
(case of SU(2), β=2.5)



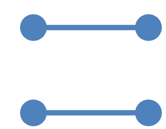
3-body source:



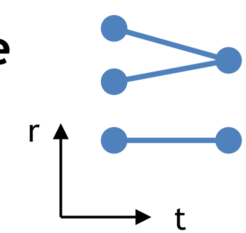
- **1-body src BS is 0 at large r due to cluster decomposition**



- **2-body src BS is finite at large r ⇒ Two free glueballs**



- **3-body src BS should be finite at large r, but large error**



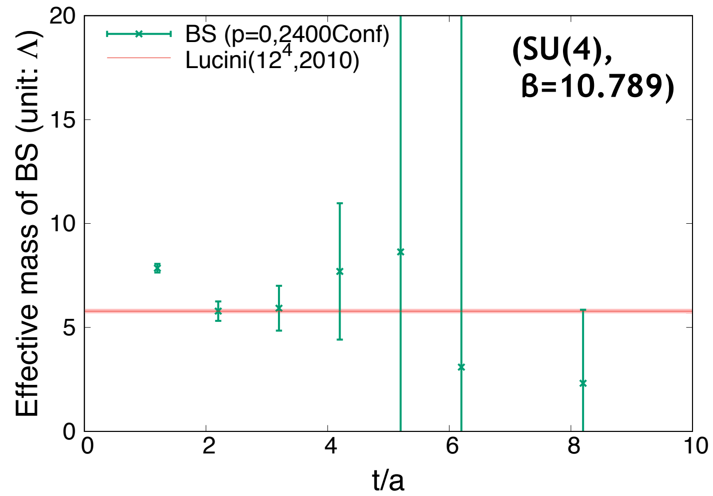
Luescher's method

Calculate the scattering phase shift : need the modulation of the energy of NBS wavefunction in momentum

Problem for the interglueball scattering :

⇒ The glueball 2-body state mixes with 1-body state (at least for 0^{++})

⇒ GS saturation of 2-body scattering dominated by 1-glueball state !

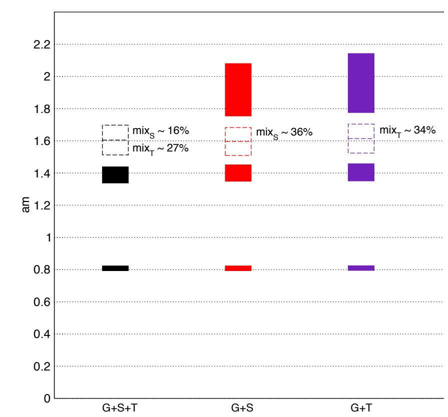


What about diagonalization? (remove 1-body state)

⇒ Many glueball states with energy close to $2m_{GB}$...?

⇒ Maybe difficult to distinguish the $2m_{GB}+\Delta E$ level from other glueball states

(momentum modulation may be visible, but challenging)



B. Lucini et al., JHEP **1008** (2010) 119

Difficult to calculate interglueball scattering with Luescher's method

Time-dependent HALQCD method

Extract the potential from the NBS wave function

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}+\frac{1}{m_\phi}\nabla^2\right]R(t,\mathbf{r})=\int d^3r'U(\mathbf{r},\mathbf{r}')R(t,\mathbf{r}')$$

$$R(t,\mathbf{r})\equiv\frac{C_{\phi\phi}(t,\mathbf{r})}{e^{-2m_\phi t}}$$

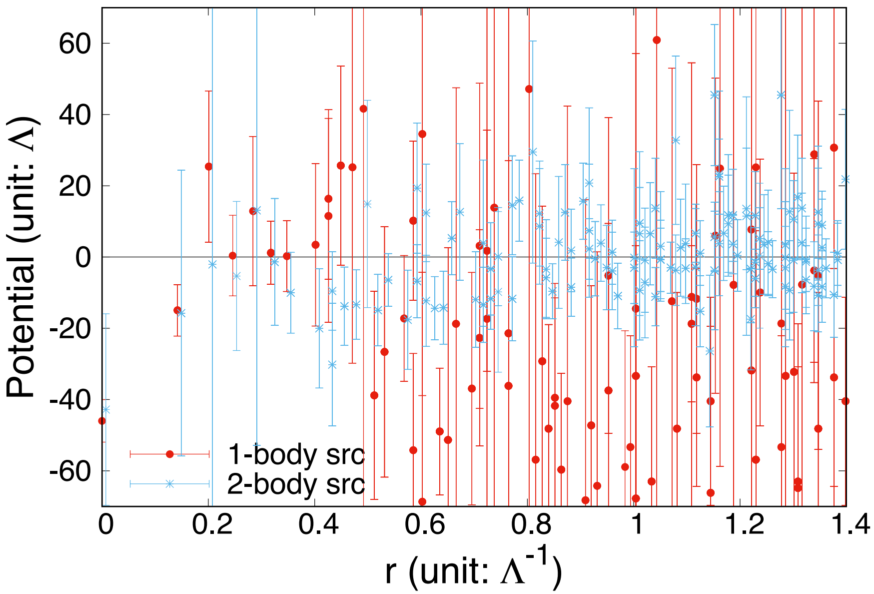
N. Ishii et al., PLB **712** (2012) 437.

- Crucial advantage : do not need ground state saturation

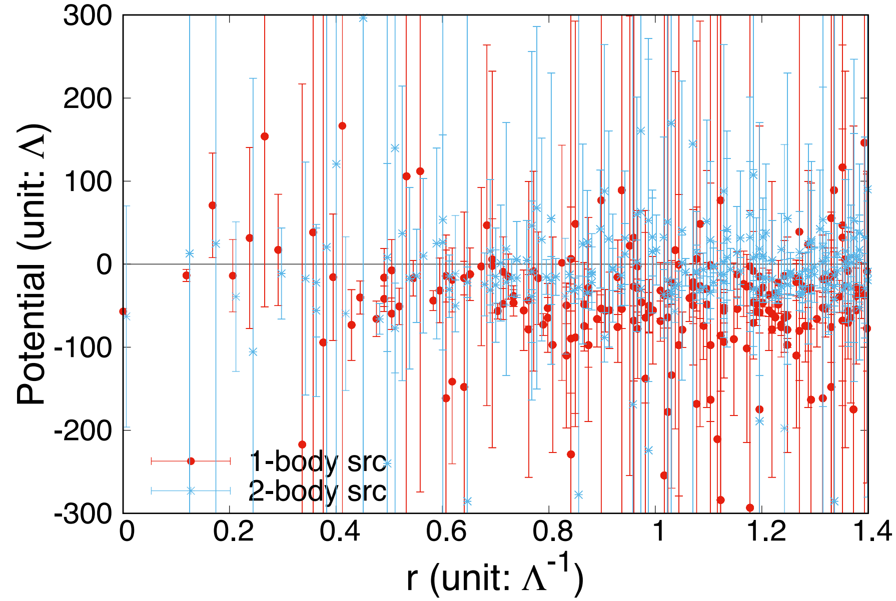
➡ Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation

- Inelastic threshold for glueball = $3m_\varphi$: high enough to consider t=2,3

SU(4) result : potential plot (local central only)



(ß = 10.789,
176000 confs)



(ß = 10.9,
210000 confs)

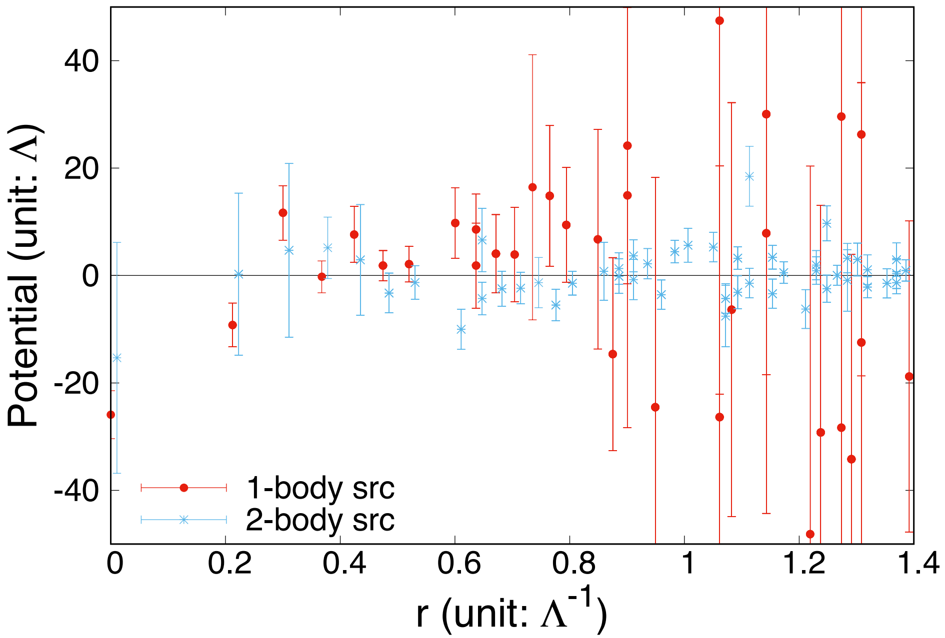
3 regions :

- Very short range (lattice unit 0 and 1) : artifact due to ?

  (also appeared in the SU(3) case, maybe related w/ the failure of Luescher's method)

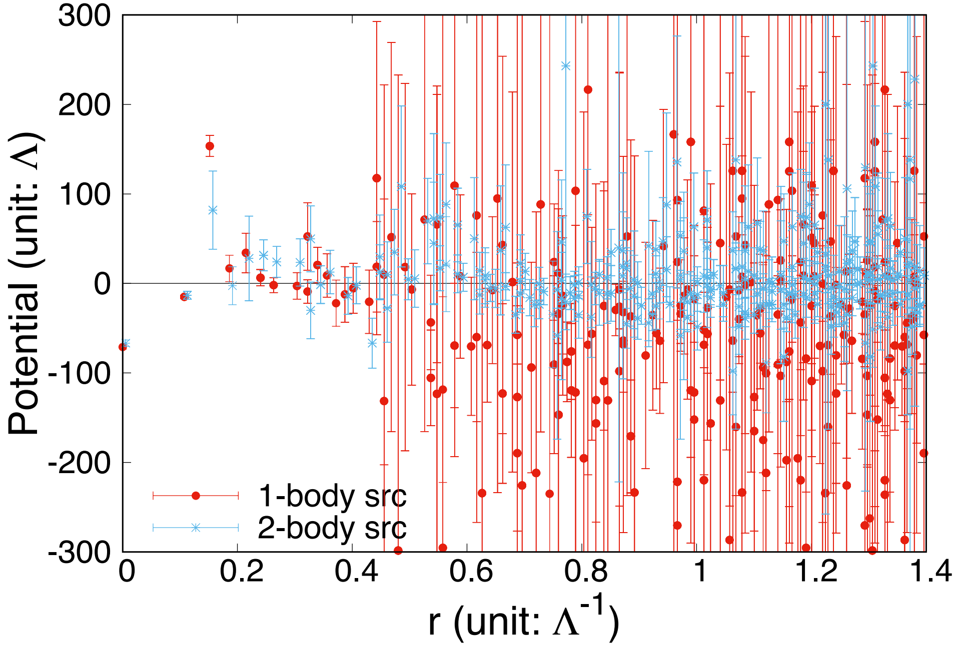
- Short range ($r < 0.4\ \Lambda^{-1}$) : looks repulsive (determined from 1-body src)
- Long range ($r > 0.4\ \Lambda^{-1}$) : flat (determined from 2-body src)

SU(3) result



(β = 5.7,
158641 confs)

SU(2) result



(β = 2.5,
1045000 confs)

Summary

- Glueballs of the SU(N) Yang-Mills theory are good candidates of dark matter : study of self-interaction is important.
- We studied the interglueball potential in the SU(2), SU(3), and SU(4) Yang-Mills theory.
- Luescher’s method has difficulty in the calculation of glueball scattering due to the mixing between 1-body and 2-body states.
- HALQCD method probes the spatial modulation of the correlator: we think it is OK for the glueball potential calculation.
- Time-dependent HALQCD method is important for the interglueball potential because the signal becomes noisy before the ground state saturation.
- Interglueball potential repulsive for $r < 0.4\Lambda^{-1}$? flat at $r > 0.4\Lambda^{-1}$.

Homeworks:

- Extraction of the scattering cross section.
- Reduce statistical error with cluster decomposition principle.
- Operator dependence (artifact) at the short distance to be discussed.

Backup