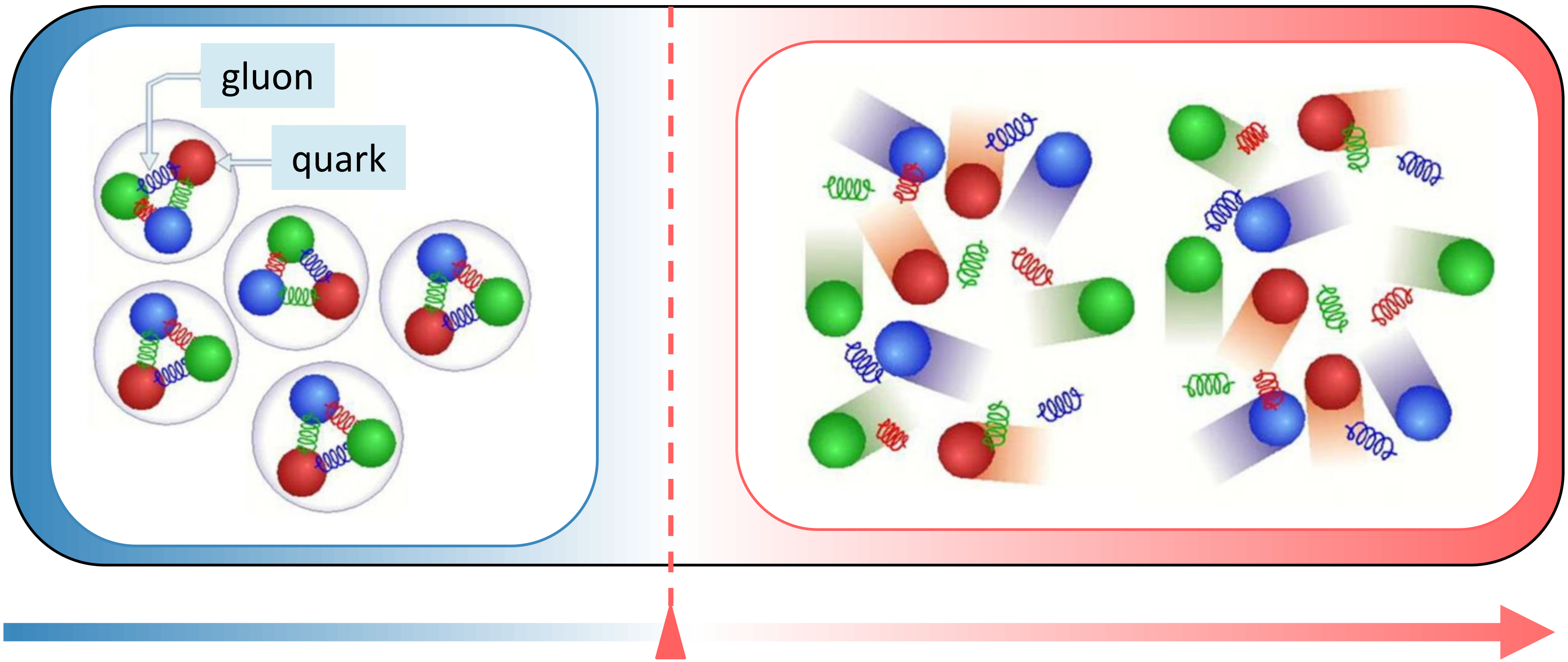
Stress tensor distribution around static quarks in hot medium

Ryosuke Yanagihara (Osaka University)

For FlowQCD collaboration :
Takumi Iritani, Masakiyo Kitazawa, Masayuki Asakawa, Tetsuo Hatsuda

Confined vs. Deconfined



0

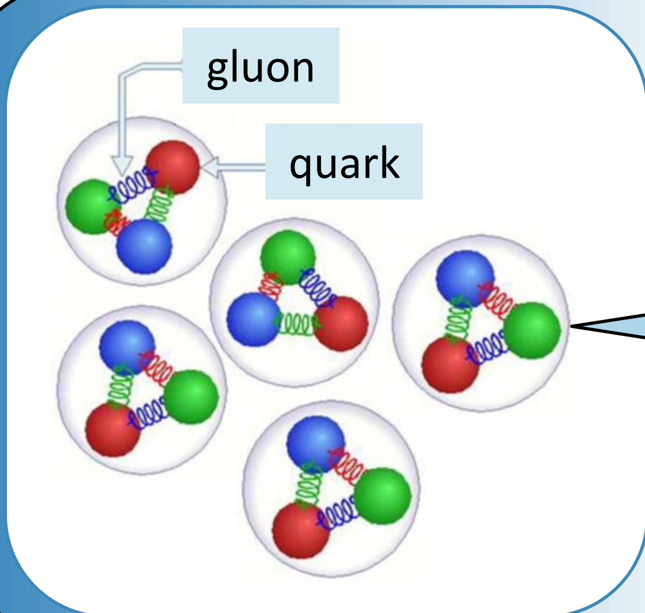
Critical temperature T_c

T

Confined

Deconfined

Confined vs. Deconfined



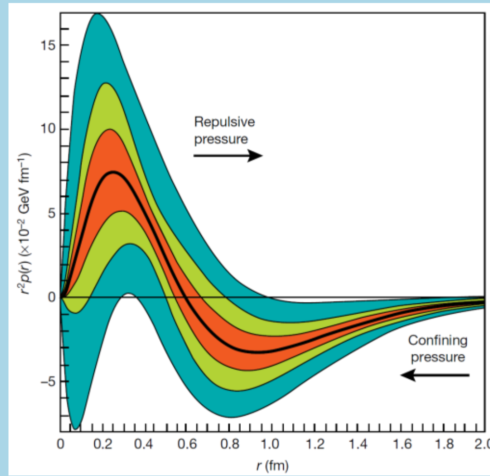
0

T_c

Confined

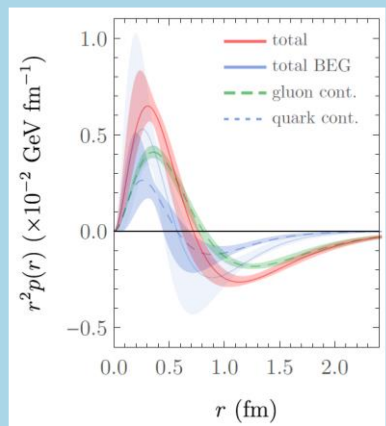
Pressure distribution inside Hadrons

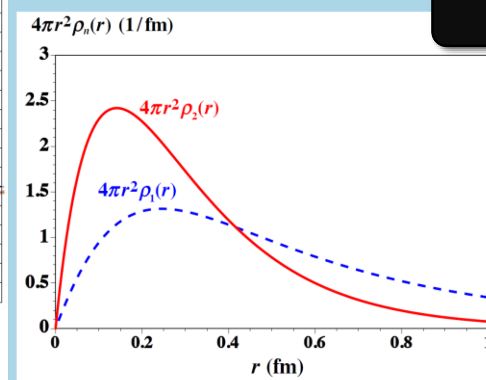
Exp.



Burkert *et al.*, Nature **557** (2018) 396.

Th.



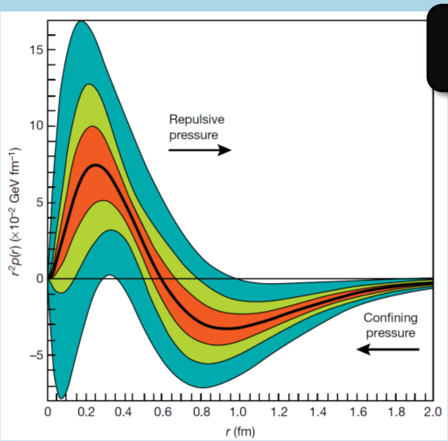


Shanahan *et al.*, PRL **122** (2019) no7, 072003.
Kumano *et al.*, PRD **97** (2018) 014020.

Pressure distribution inside hadrons vs. Our study

Pressure distribution inside Hadrons

Exp.



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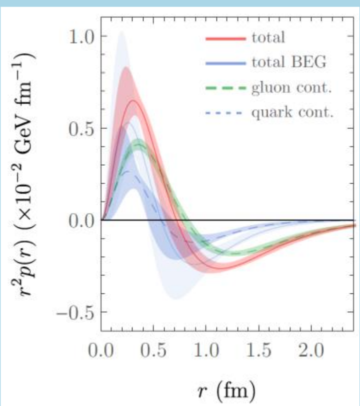

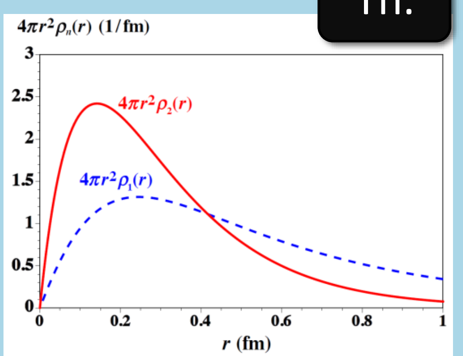

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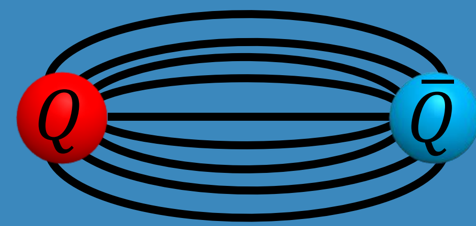
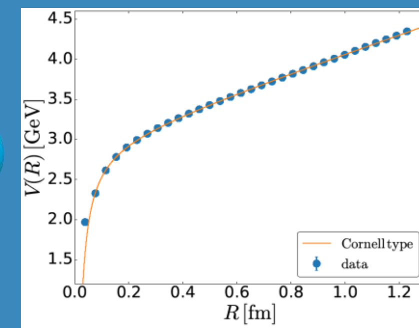
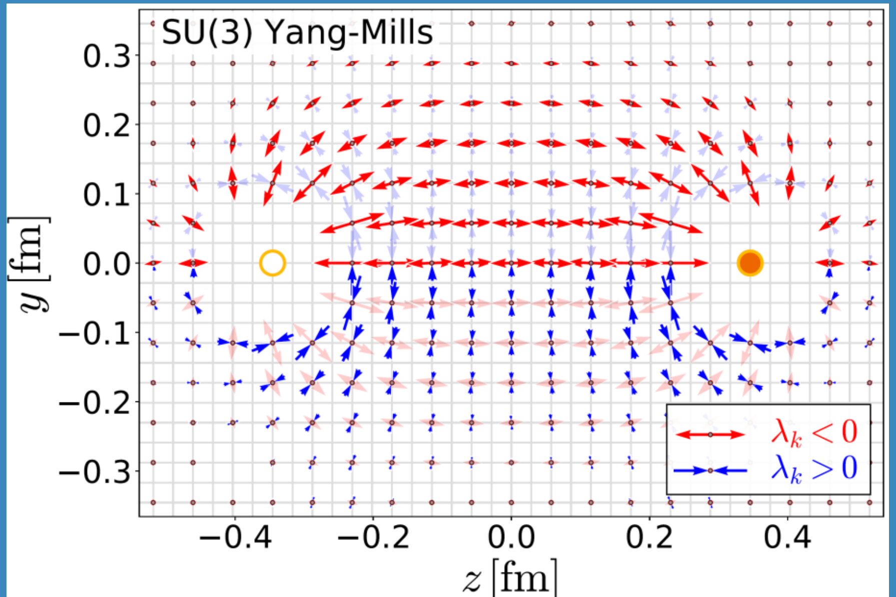
Our study

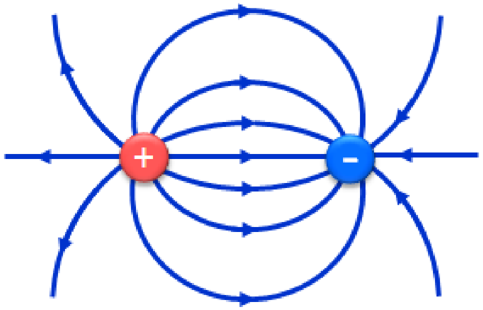


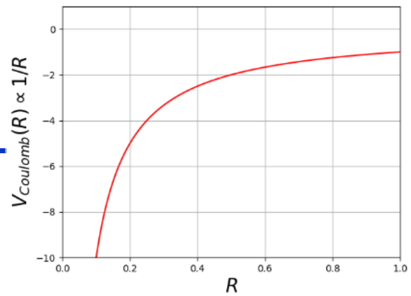




Flux tube

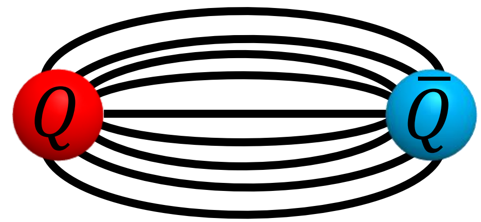
QED

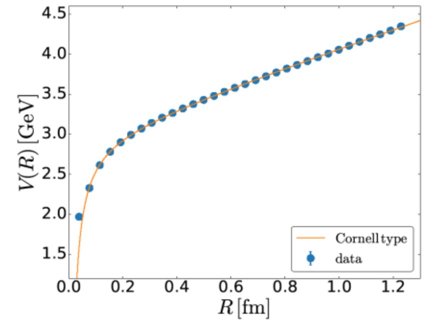




- ✓ Electric field spreads all over the space
- ✓ Coulomb potential

QCD

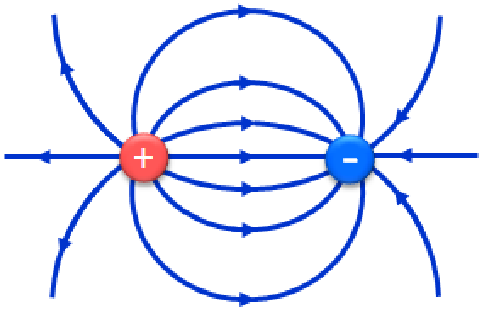


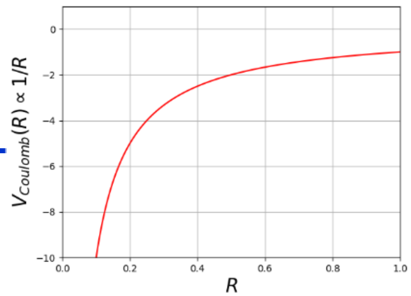


- ✓ Flux tube, squeezed one-dimensionally
- ✓ Confinement potential

Flux tube

QED





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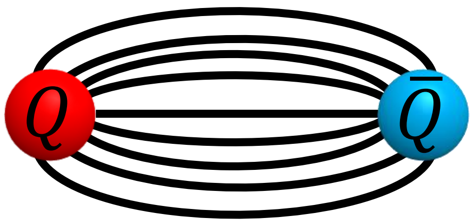

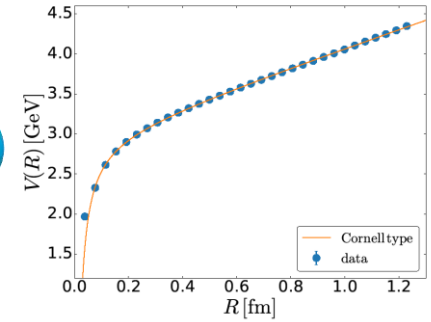

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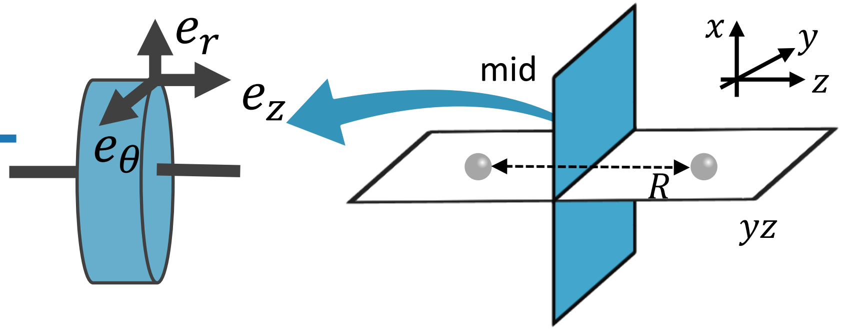
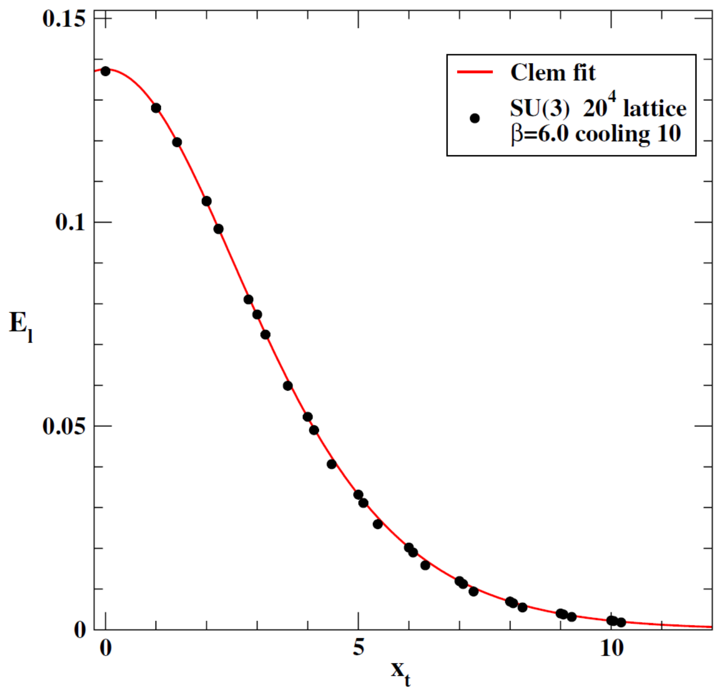
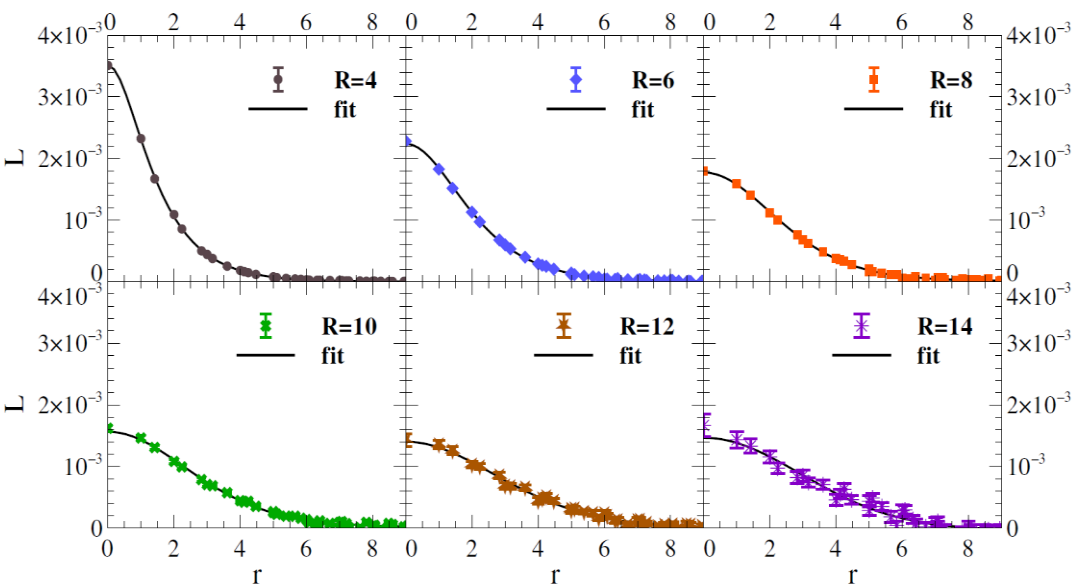
Local interaction

Maxwell stress

A lot of previous studies







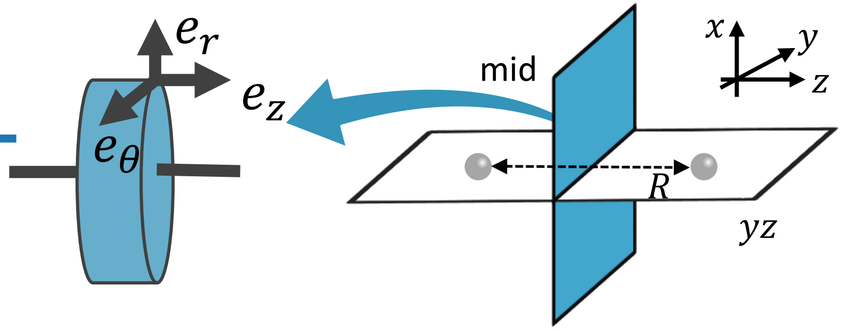
Color electric field

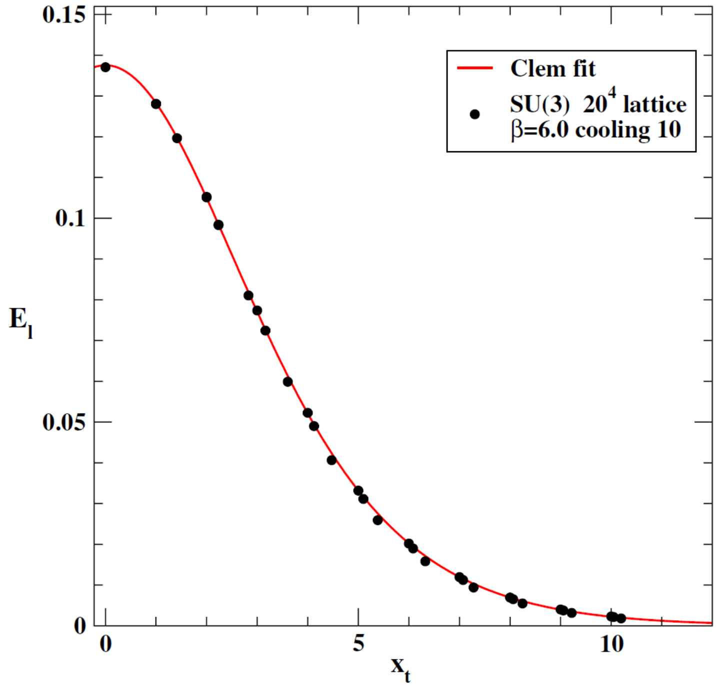
Cea *et al.*, PRD**88** (2012) 054504.

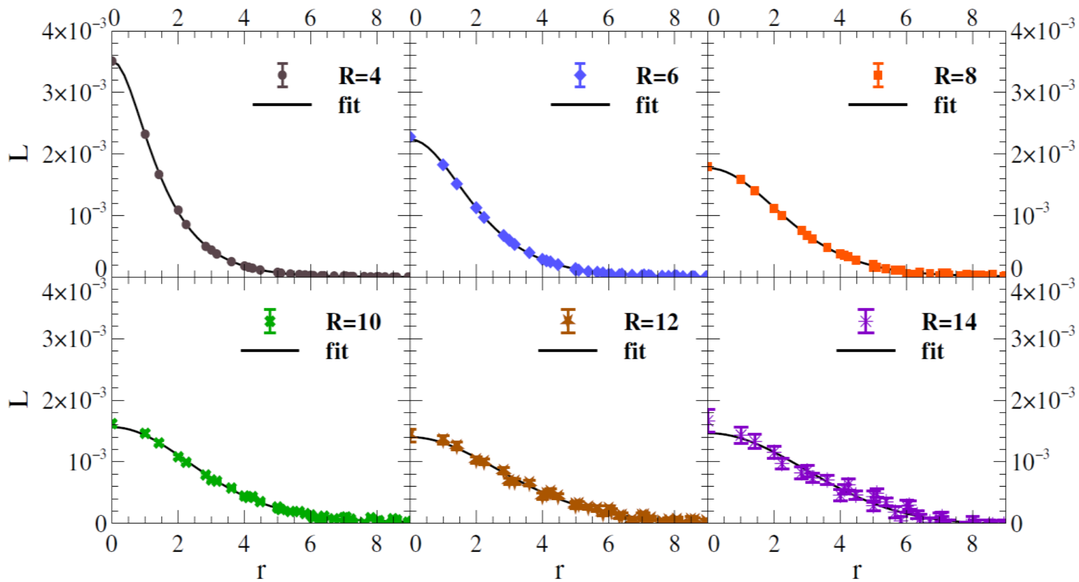
Action density

Cardoso *et al.*, PRD**86** (2013) 054501.

A lot of previous studies







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More direct physical quantity : Stress tensor !!

Energy momentum tensor (EMT)

Energy density

Momentum density

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Pressure

Stress tensor

✓ **Stress is force per unit area**

$$f_i = \sigma_{ij} n_j \;\; ; \;\; \sigma_{ij} = -T_{ij}$$

Landau and Lifshitz

Energy momentum tensor (EMT)

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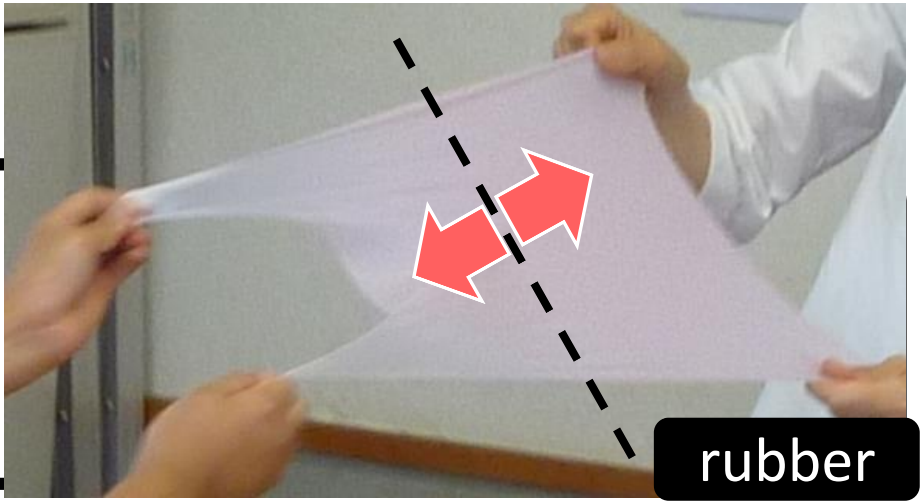
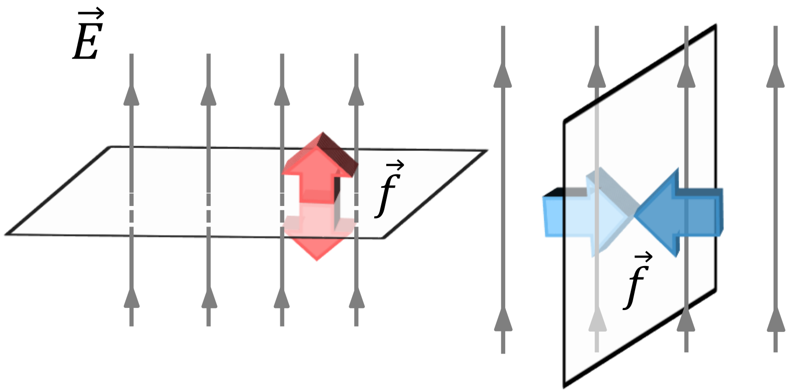


Maxwell stress

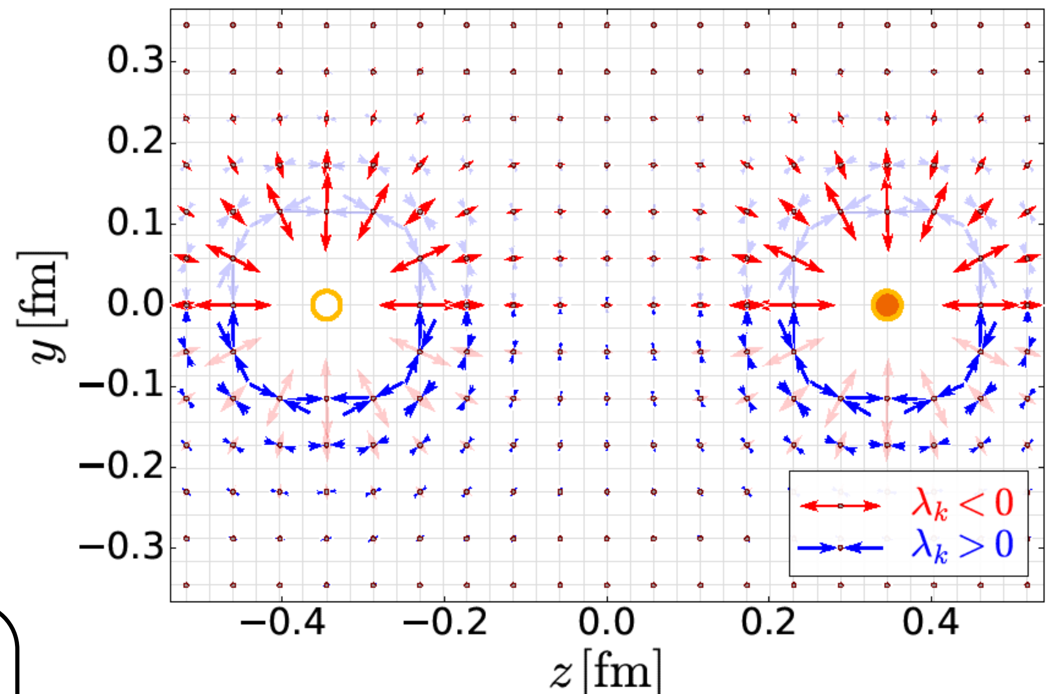
$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{\delta_{ij}}{2} B^2 \right)$$



- ✓ Perpendicular plane: $\lambda_k < 0$
- ✓ Parallel plane: $\lambda_k > 0$

✓ Stress tensor

$$T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)}$$
$$(i, j = 1,2,3 \; ; \; k = 1,2,3)$$



(Length of arrows= $\sqrt{|\lambda_k|}$)

Measurement on the lattice

To do

①Prepare $Q\bar{Q}$ on the lattice ②Measure EMT around $Q\bar{Q}$

Measurement on the lattice

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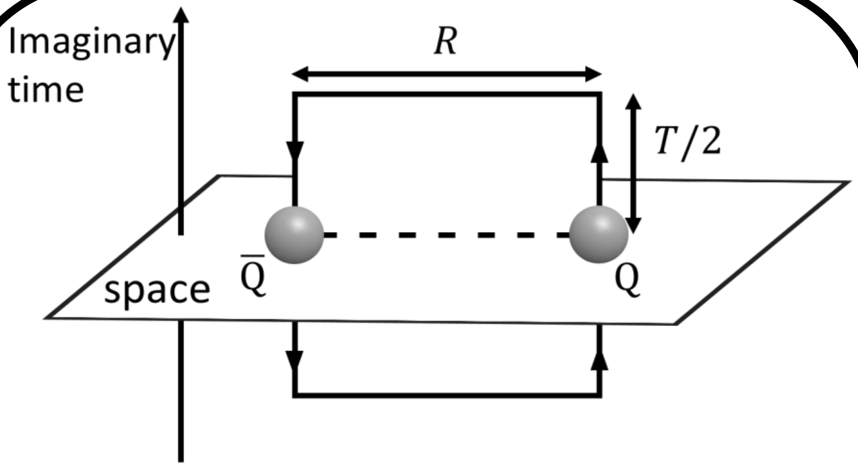
Wilson Loop

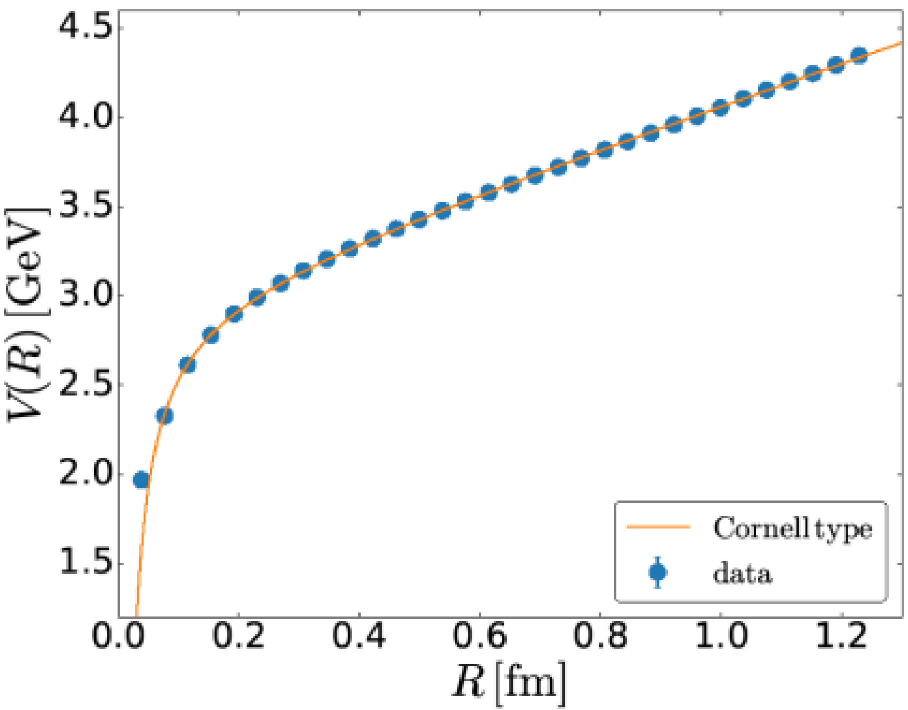


$\langle W(R,T)\rangle$
$= C_0 \exp[-V(R)T] + C_1 \exp[-V_1(R)T] + \cdots$

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log\langle W(R,T)\rangle$$

Ground state potential

Confinement potential



✓ quenched SU(3) Yang-Mills
✓ $\beta = 6.600$ ($a = 0.038$ fm)

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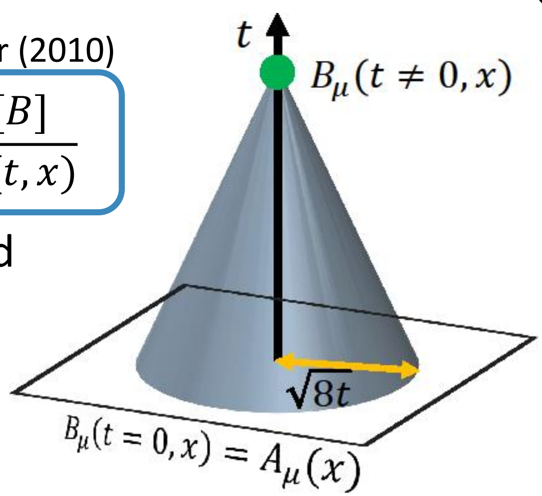
Gradient flow

Flow eq. Lüscher (2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

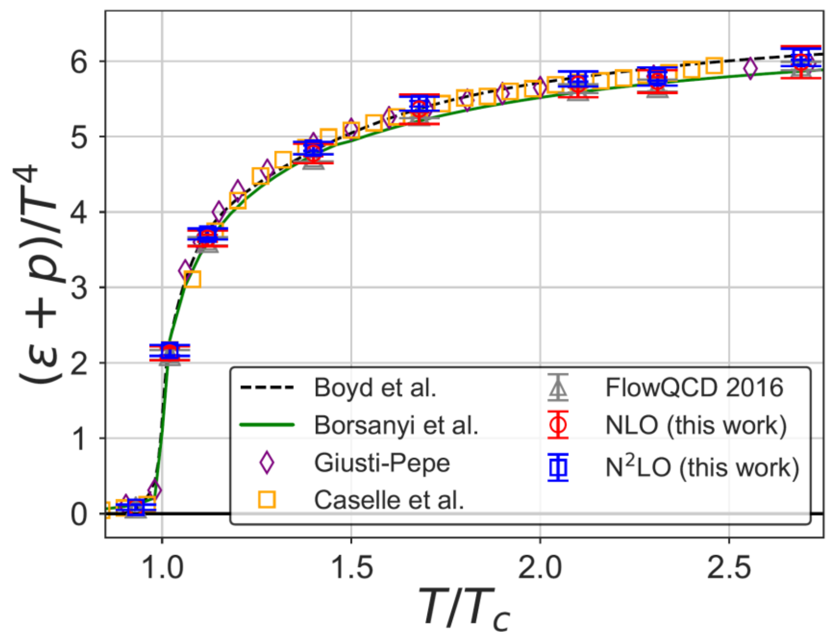
B_μ : smeared field



EMT defined via gradient flow Suzuki (2013)

$$T_{\mu\nu}(t,x) = \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle] + O(t)$$

Iritani *et al*. (2018)



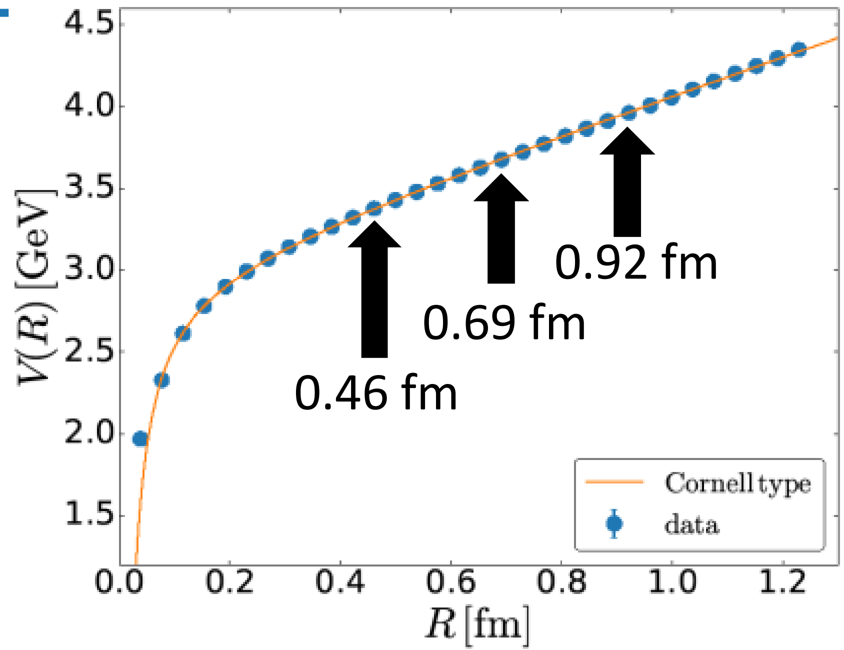
Entropy density vs. temperature

✓ 2-loop coefficient is now available !

Harlander *et al*. (2018)

Set up

- ✓ Quenched SU(3) Yang-Mills gauge theory
- ✓ Wilson gauge action
- ✓ Clover operator
- ✓ Continuum limit
- ✓ APE smearing for spatial links
- ✓ Multihit improvement in temporal links
- ✓ Simulation using BlueGene/Q @ KEK

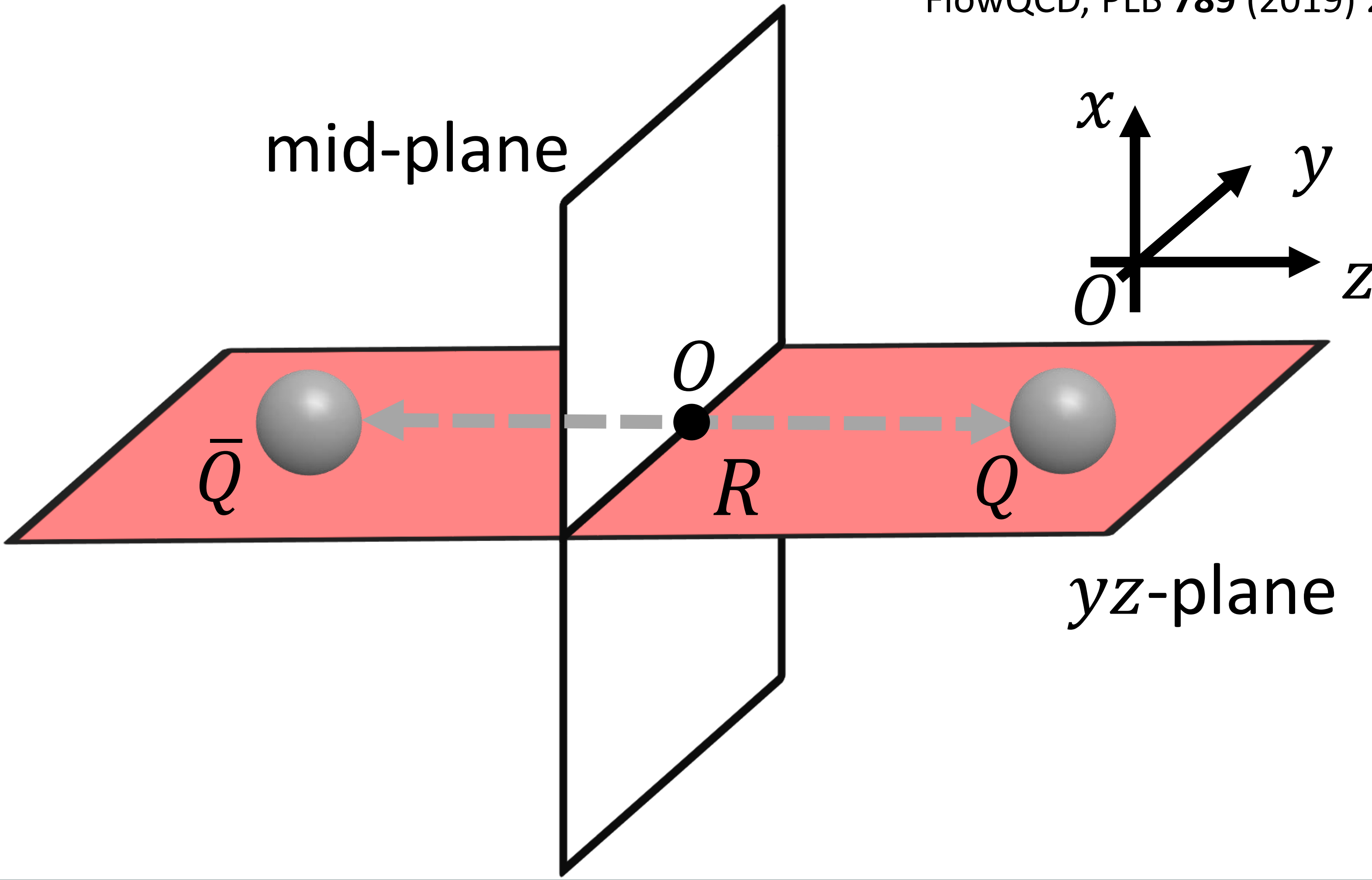


β	Lattice spacing	Lattice size	# of statistics
6.304	0.057 fm	48^4	140
6.465	0.046 fm	48^4	440
6.513	0.043 fm	48^4	600
6.600	0.038 fm	48^4	1500
6.819	0.029 fm	64^4	1000

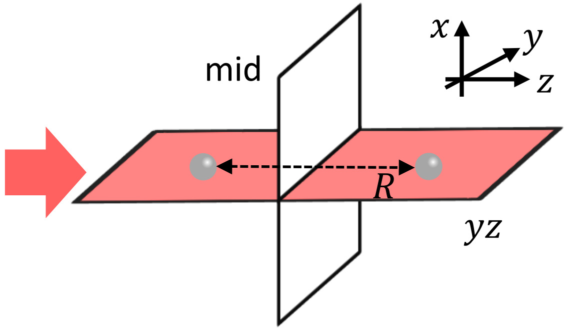
A lattice study of stress distribution around $Q\bar{Q}$ in vacuum

Stress distribution in terms of local interaction

FlowQCD, PLB **789** (2019) 210.



Stress distribution around $Q\bar{Q}$



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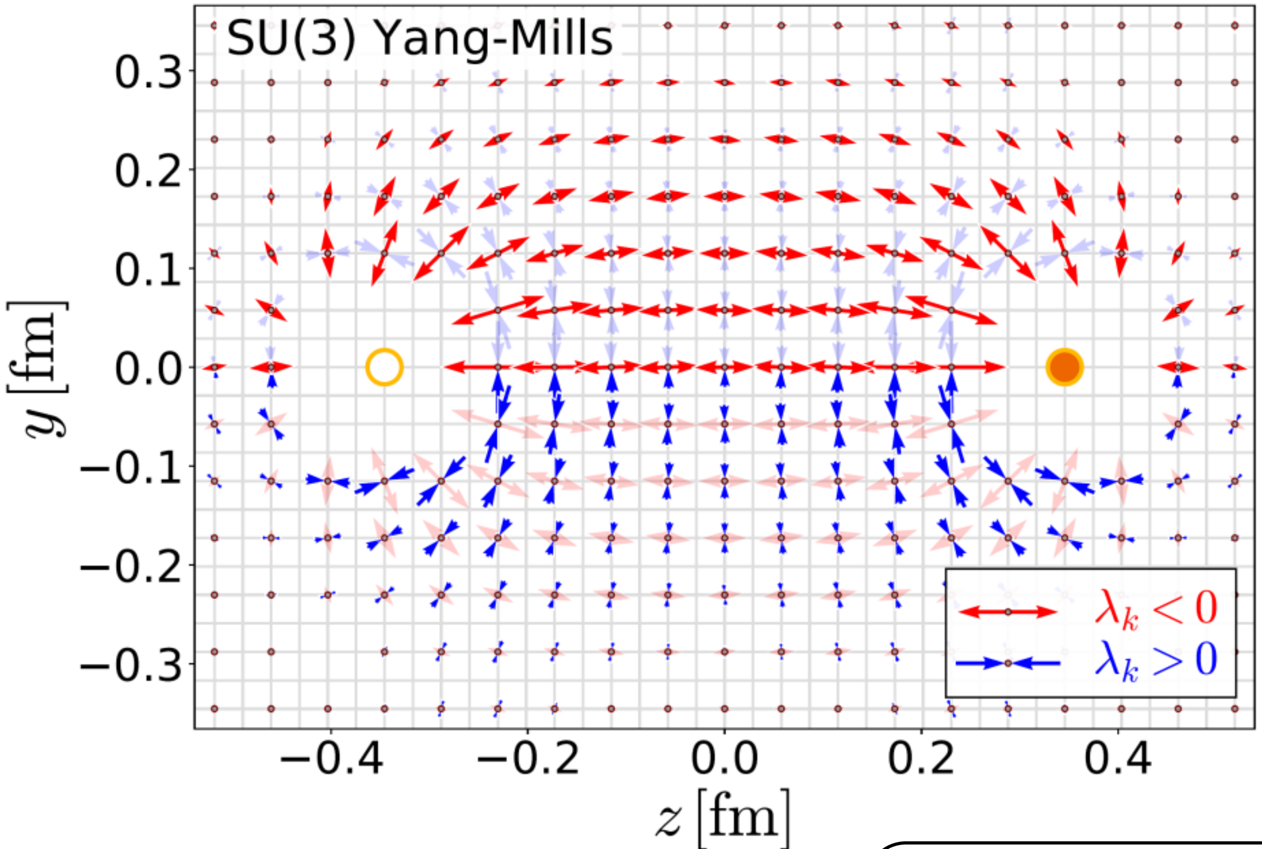


- ✓ $a = 0.029$ fm
- ✓ $t/a^2 = 2.0$
- ✓ $R = 0.69$ fm
- ✓ Length of arrows = $\sqrt{|\lambda_k|}$

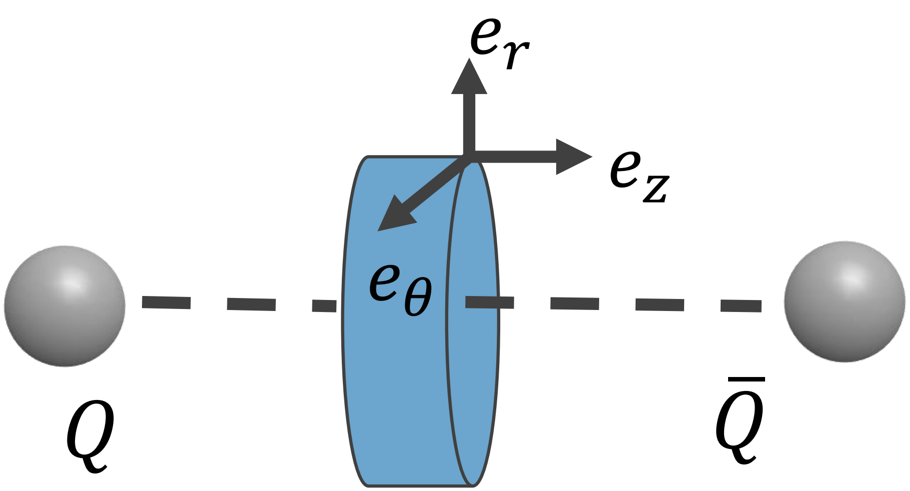
- ✓ Gauge invariant
- ✓ Local interaction
- ✓ squeezed

Stress distribution around $Q\bar{Q}$: Cylindrical coordinates

$$T_{\mu\nu} = \begin{pmatrix} T_{44} & & & O \\ & T_{zz} & & \\ & & T_{rr} & \\ O & & & T_{\theta\theta} \end{pmatrix}$$

Diagonalized EMT

(Cylindrical / Parity symmetry)



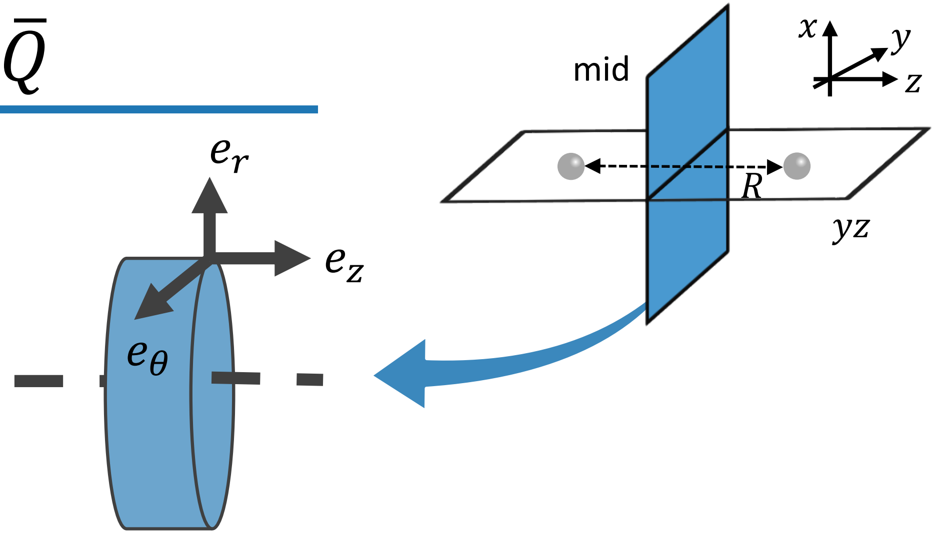
Degeneracy (Maxwell Theory)

$$|T_{44}| = |T_{zz}| = |T_{rr}| = |T_{\theta\theta}|$$

Stress distribution around $Q\bar{Q}$

Properties in non-Abelian theory

- ✓ $T_{44} \approx T_{zz}, T_{rr} \approx T_{\theta\theta}$ (Degeneracy)
- ✓ $T_{44} \neq T_{rr}$ (Separation)
- ✓ $\sum_\mu T_{\mu\mu} \neq 0$ (Trace anomaly$\neq$ 0)



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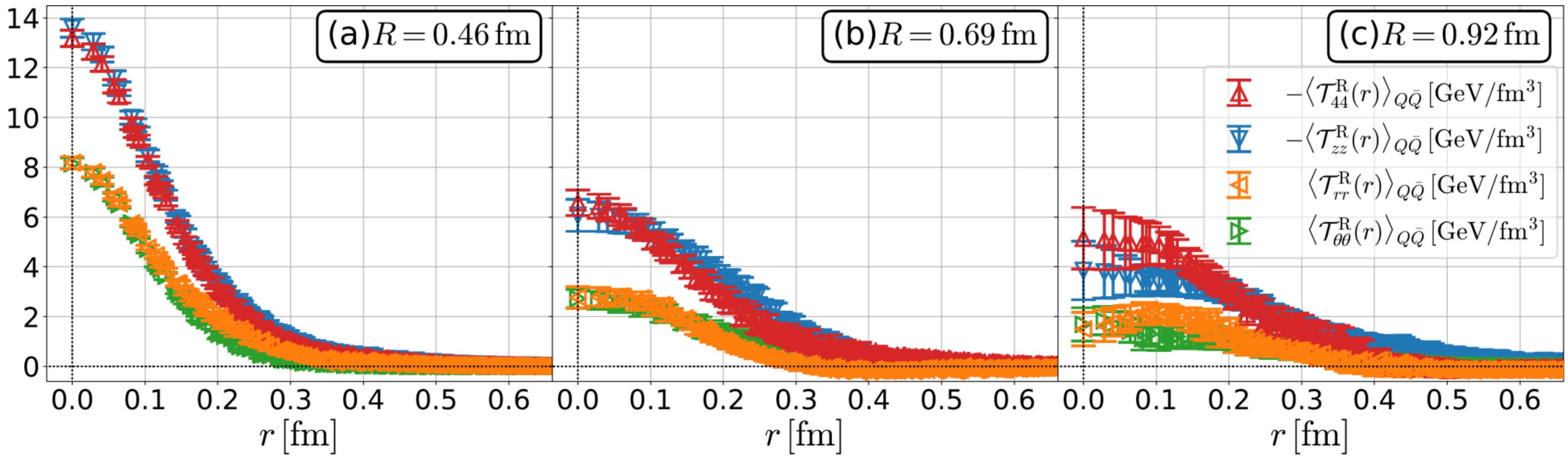
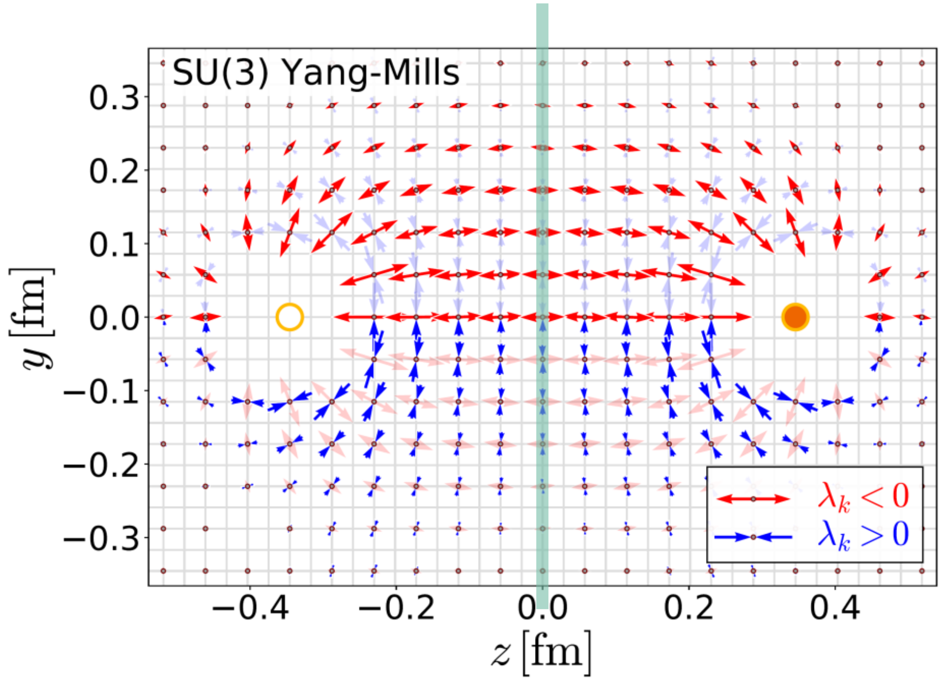


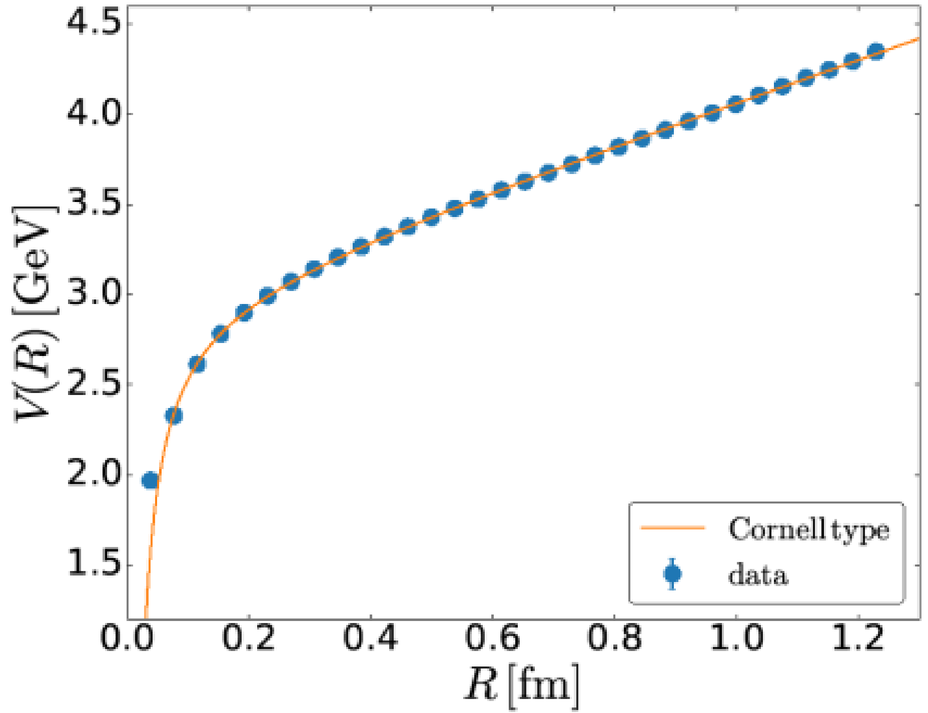
(Note : after double limit)

EMT and confinement potential

From EMT



confinement potential



$$F_{\text{stress}} \coloneqq \int_{\text{mid}} \langle T_{zz} \rangle_{Q\bar{Q}}\, d^2x$$

$$V(R) = a + bR + c/R$$

$$F_{\text{pot}} \coloneqq -\frac{dV(R)}{dR}$$

EMT and confinement potential

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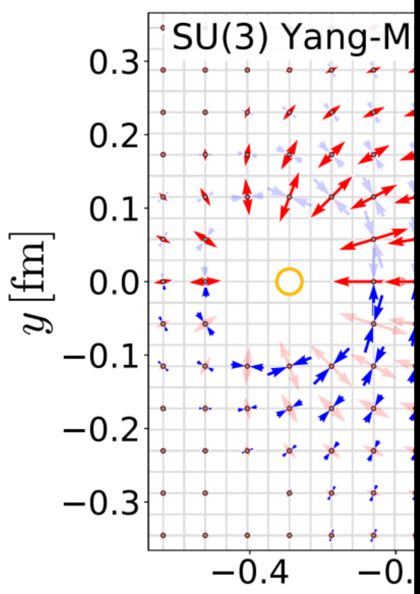
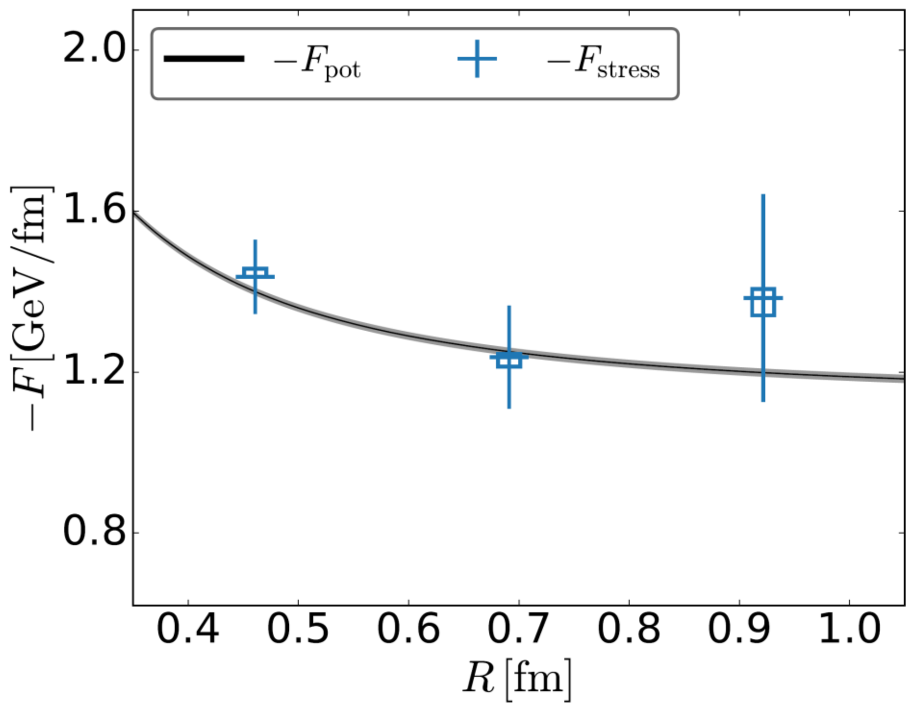
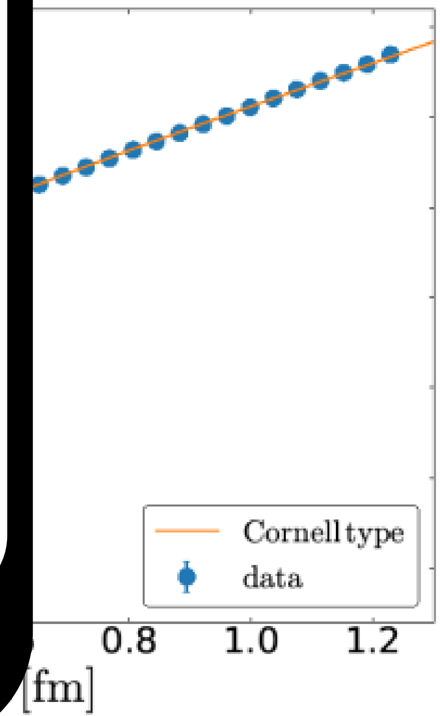




Good agreement !!

ntial



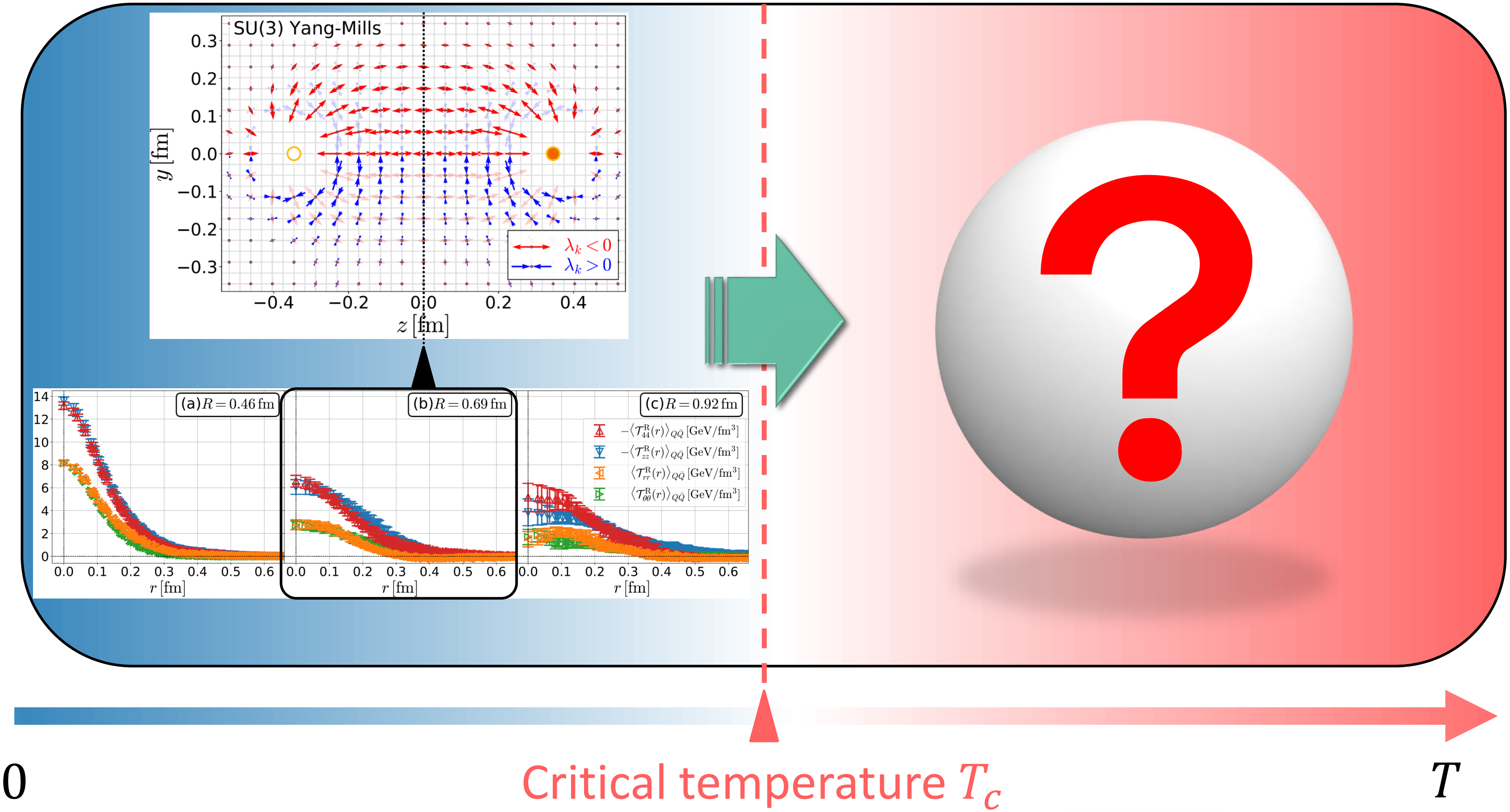
$$V(R) = a + bR + c/R$$

$$F_{\mathrm{pot}} \coloneqq -\frac{dV(R)}{dR}$$

$$F_{\mathrm{stress}} \coloneqq \int_{\mathrm{mid}} \langle T_{zz} \rangle_{Q\bar{Q}}\, d^2x$$

Toward analysis at nonzero temperature

Stress distribution around $Q\bar{Q}/Q$ at nonzero temperature



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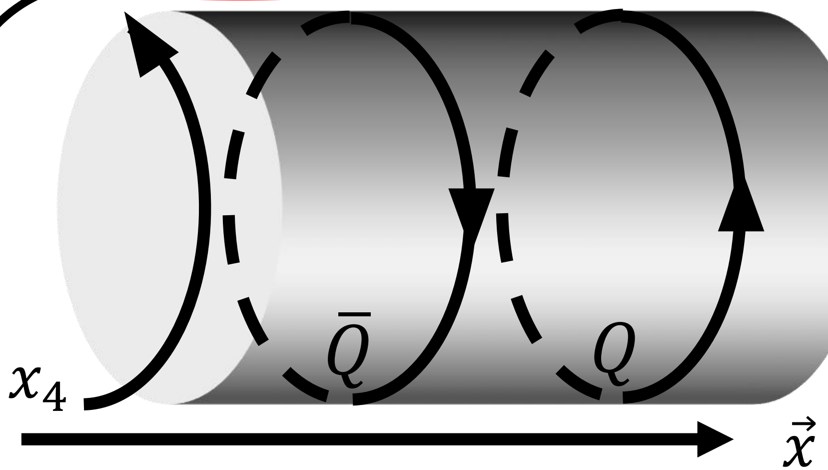
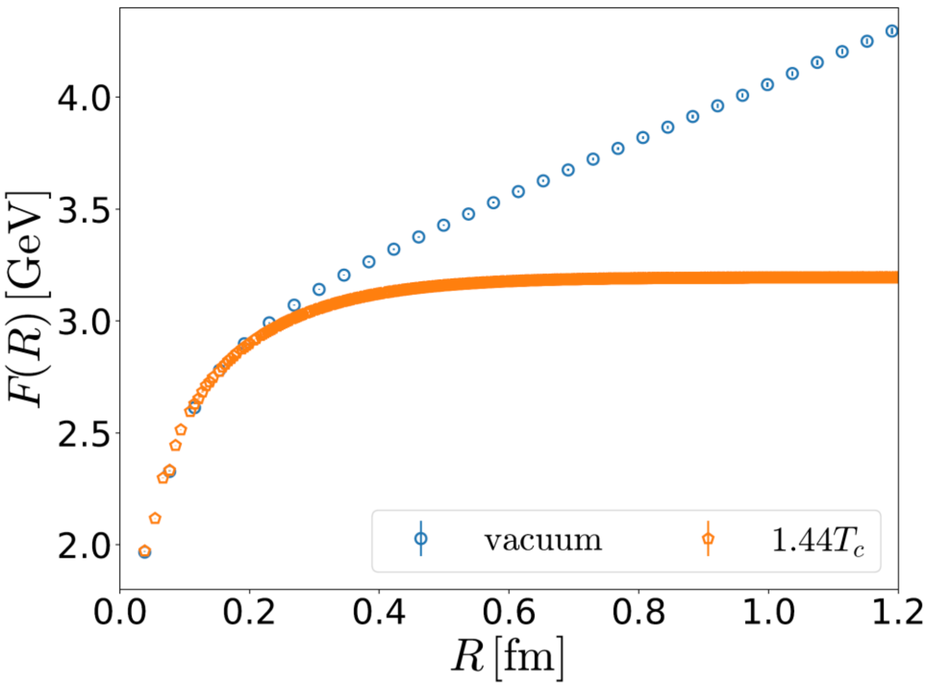
Polyakov Loop



$$e^{-F(R)/T} = \frac{1}{3}\langle \mathrm{Tr}\, \Omega^\dagger(\vec{x})\Omega\,(\vec{y})\rangle$$

Color singlet free energy
(We use Coulomb gauge fixing)

Free energy



- ✓ quenched SU(3) Yang-Mills
- ✓ $\beta = 6.600$ ($a = 0.038$ fm)

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Gradient flow

Flow eq. Lüscher (2010)

$$\frac{\partial B_\mu(t,x)}{\partial t} = -g_0^2 \frac{\delta S[B]}{\delta B_\mu(t,x)}$$

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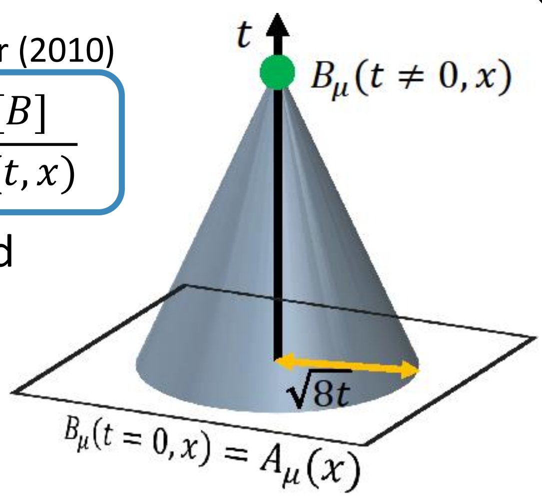

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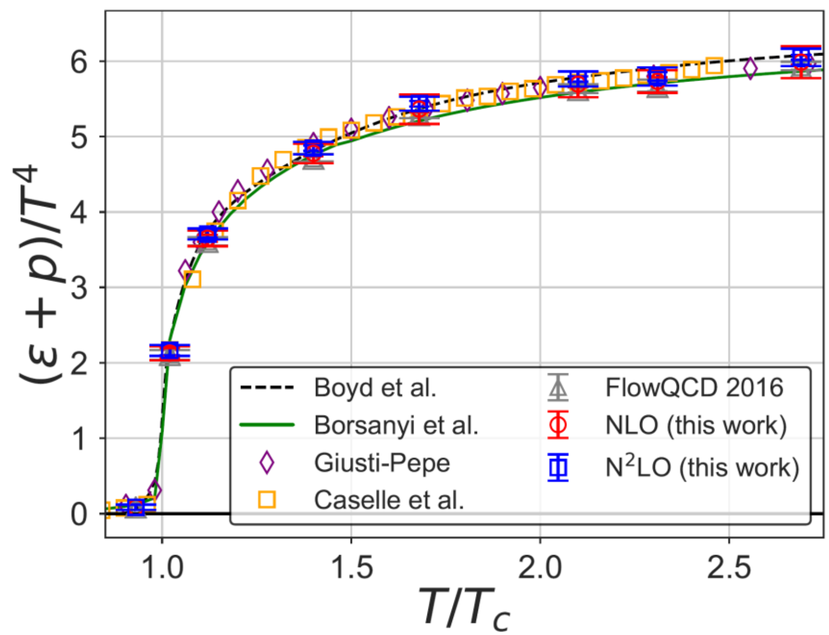
Iritani *et al.* (2018)



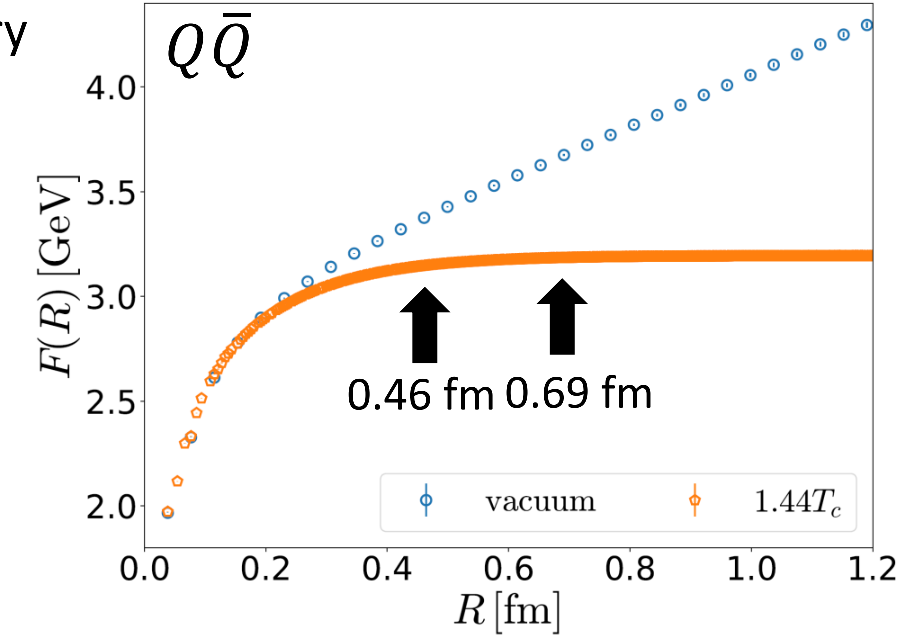
Entropy density vs. temperature

✓ 2-loop coefficient is now available !

Harlander *et al.* (2018)

Set up (quark—anti-quark, single quark)

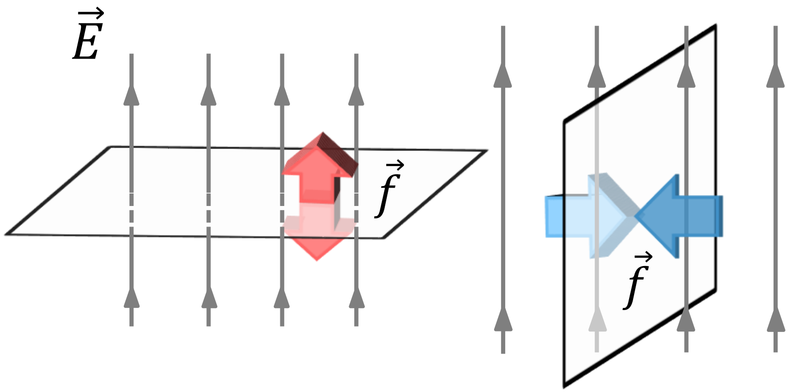
- ✓ Quenched SU(3) Yang-Mills gauge theory
- ✓ Wilson gauge action
- ✓ Clover operator
- ✓ Fixed a, t
- ✓ Multihit improvement in temporal links
- ✓ Simulation using OCTOPUS, Reedbush



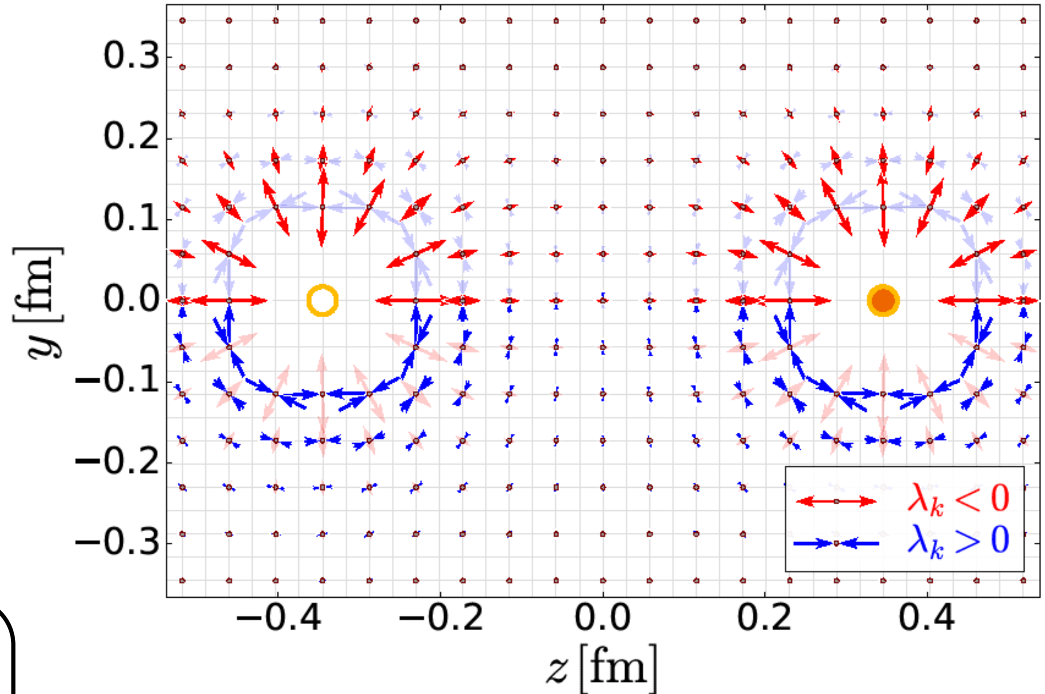
β	Lattice spacing	Spatial size	Temporal size	T/T_c	# of statistics
6.600	0.038 fm	48^3	12	1.44	640

Maxwell stress (revisit)

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{\delta_{ij}}{2} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{\delta_{ij}}{2} B^2 \right)$$



- ✓ Perpendicular plane: $\lambda_k < 0$
- ✓ Parallel plane: $\lambda_k > 0$

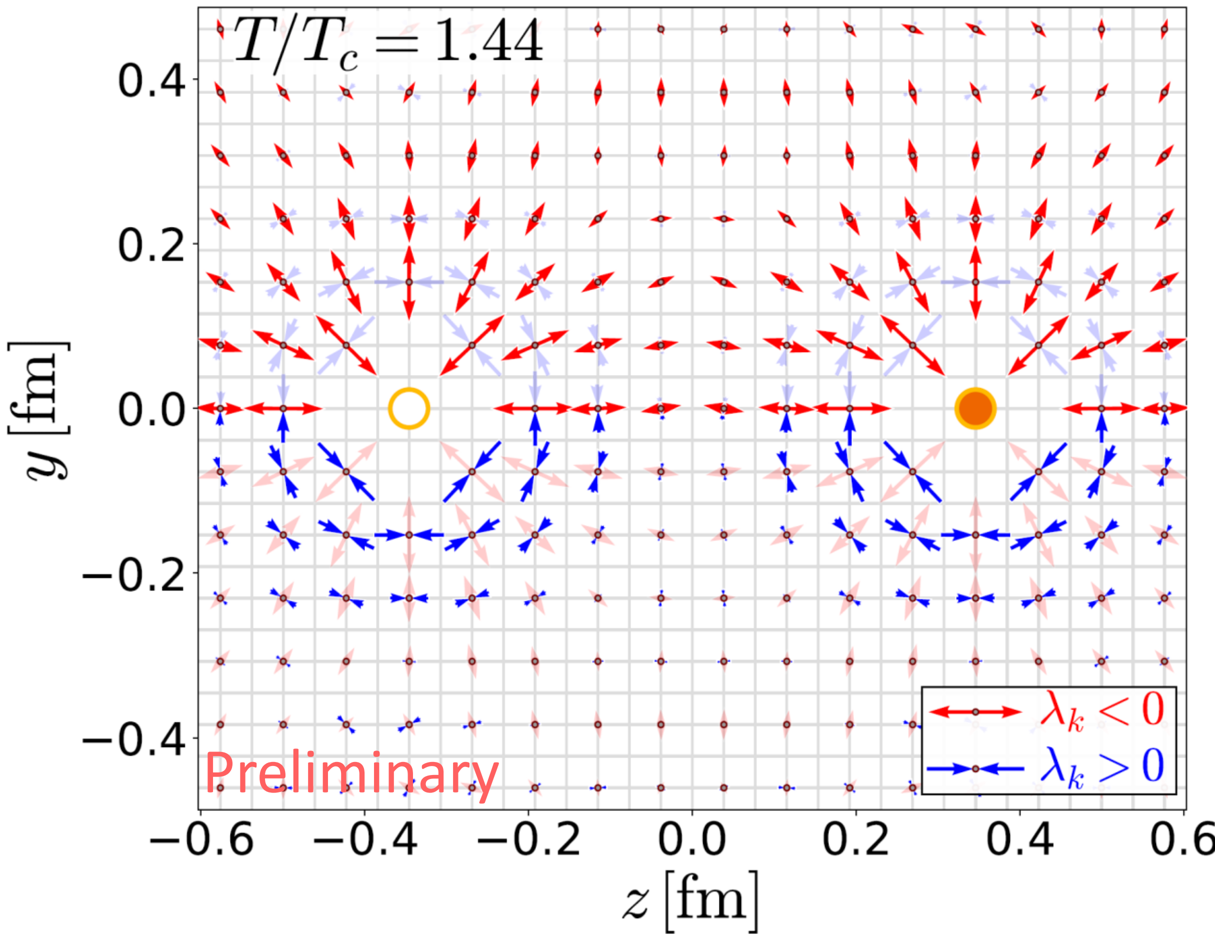


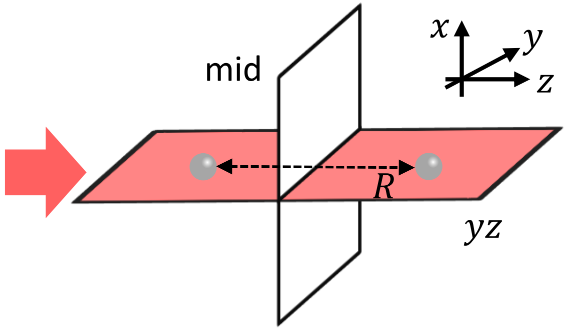
✓ Stress tensor

$$T_{ij} n_j^{(k)} = \lambda_k n_i^{(k)}$$
$$(i,j = 1,2,3\ ;\ k = 1,2,3)$$

(Length of arrows= $\sqrt{|\lambda_k|}$)

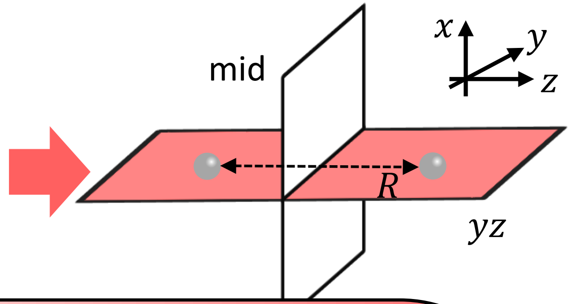
Stress distribution around $Q\bar{Q}$

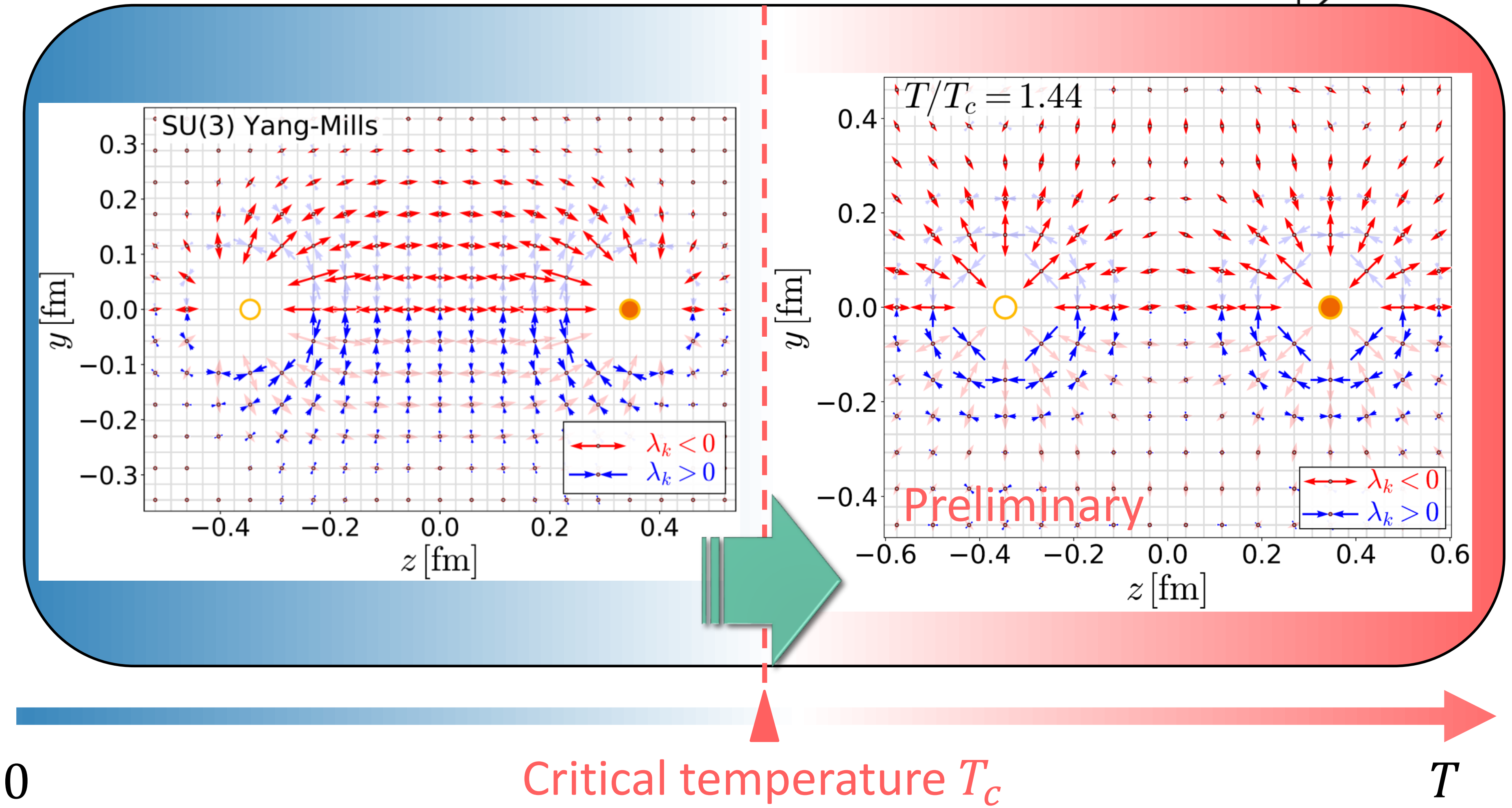




- ✓ singlet
- ✓ $a = 0.038$ fm (fixed)
- ✓ $t/a^2 = 2.0$ (fixed)
- ✓ $R = 0.69$ fm
- ✓ Length of arrows $= \sqrt{|\lambda_k|}$

Stress distribution around $Q\bar{Q}$



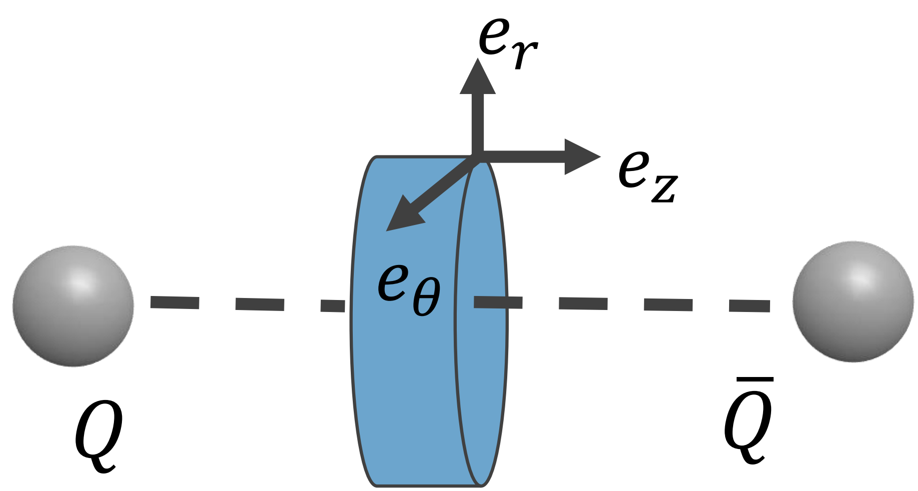


Stress distribution around $Q\bar{Q}$: Cylindrical coordinates

$$T_{\mu\nu} = \begin{pmatrix} T_{44} & & & O \\ & T_{zz} & & \\ & & T_{rr} & \\ O & & & T_{\theta\theta} \end{pmatrix}$$

Diagonalized EMT

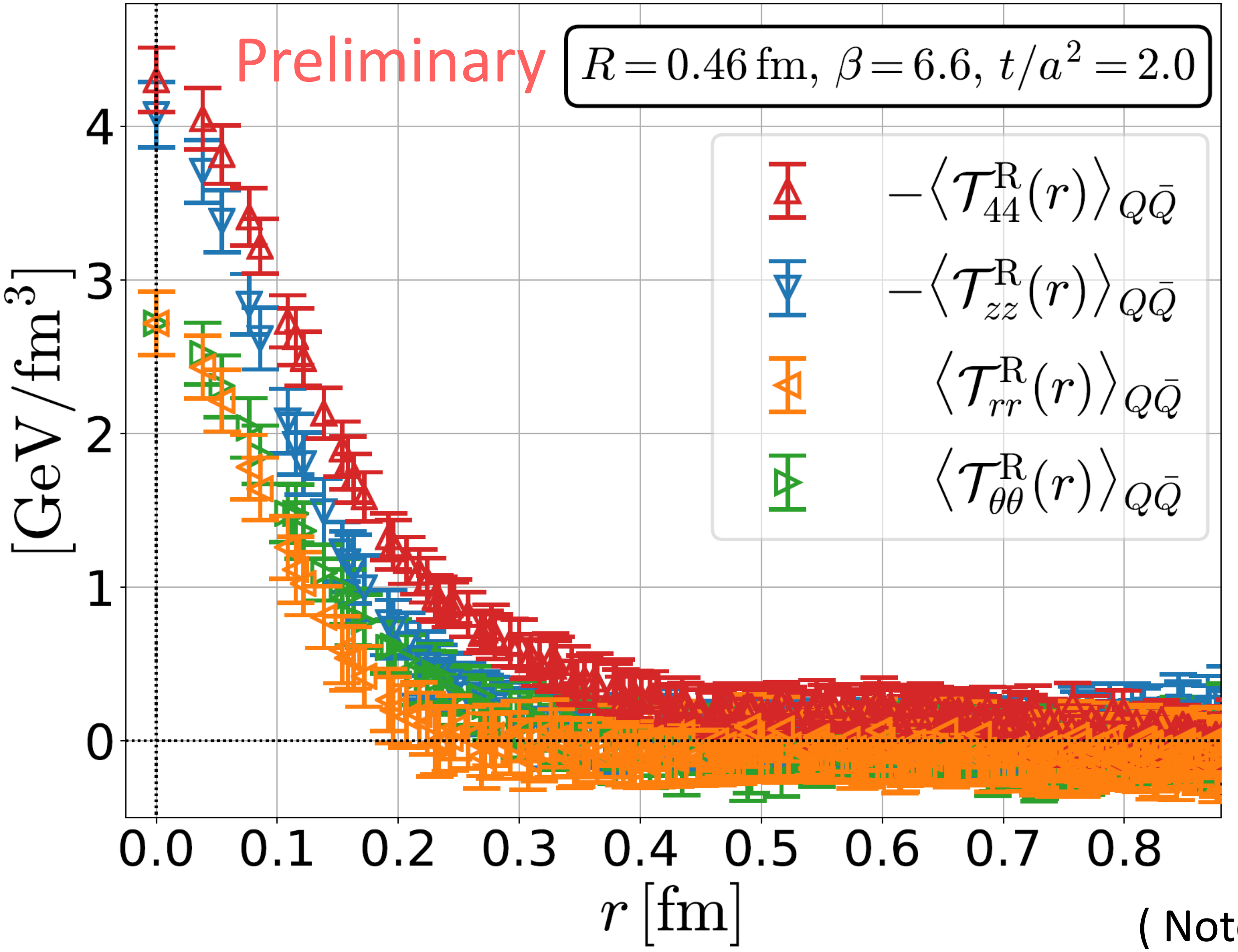
(Cylindrical / Parity symmetry)

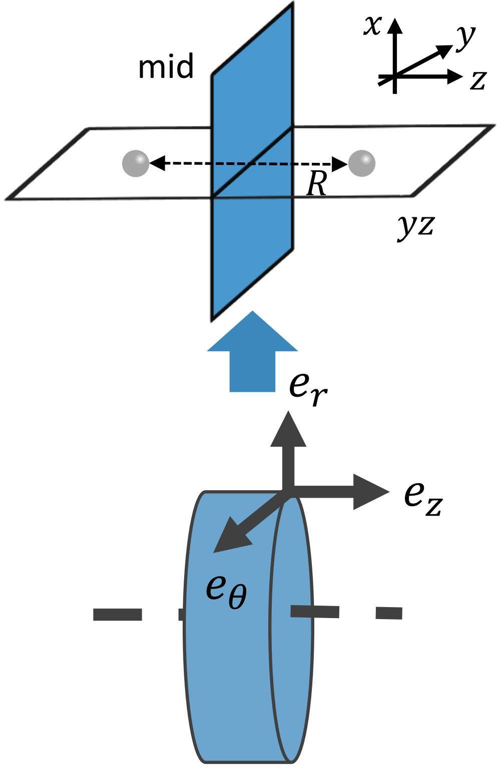


Degeneracy (Maxwell Theory)

$$|T_{44}| = |T_{zz}| = |T_{rr}| = |T_{\theta\theta}|$$

Stress distribution around $Q\bar{Q}$

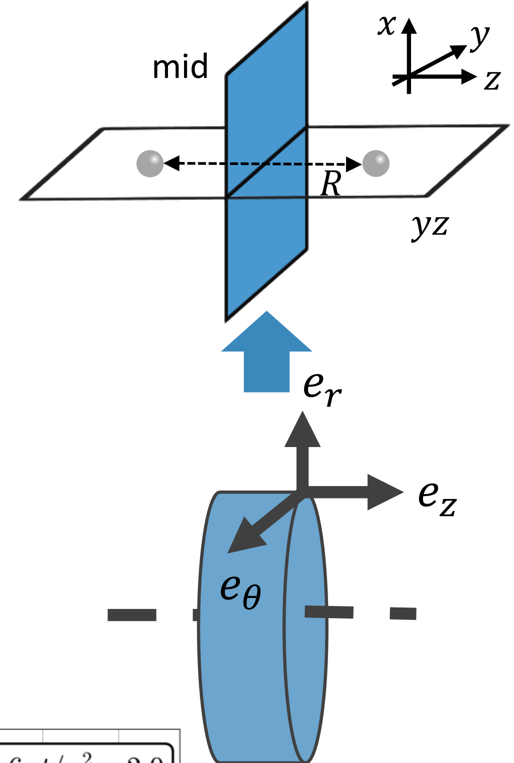


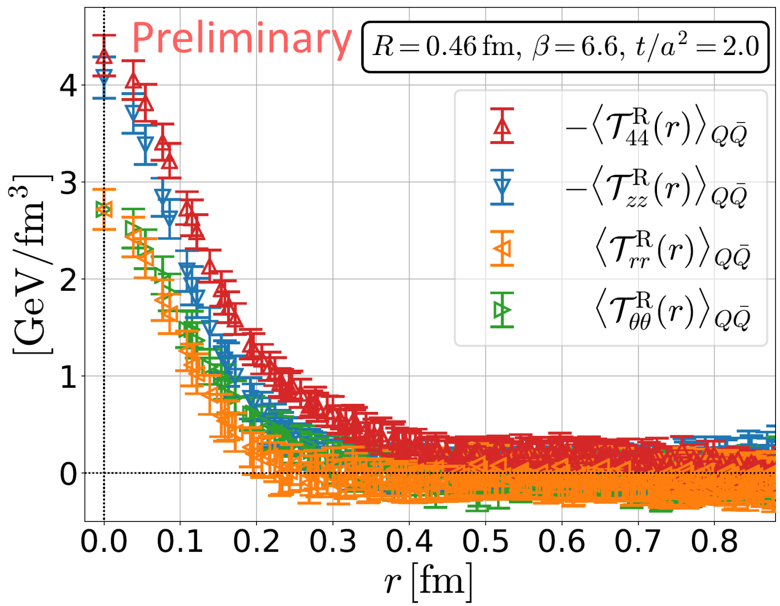


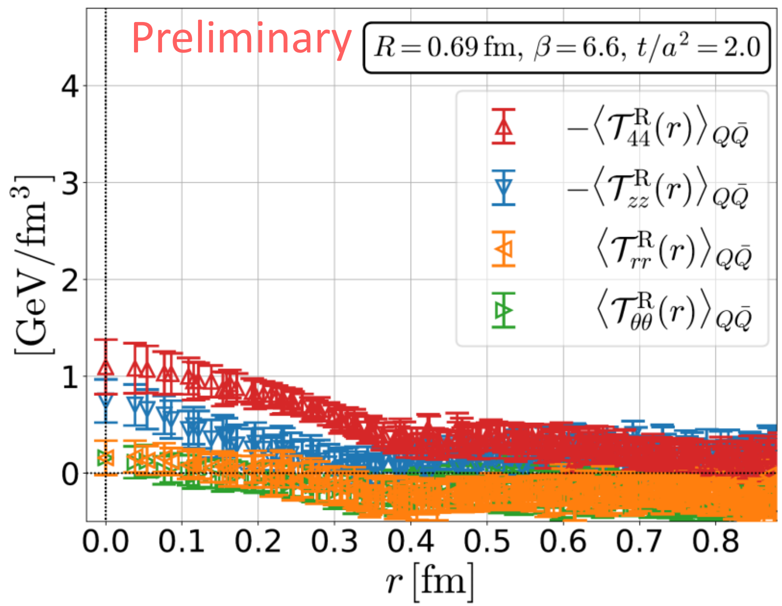
(Note : singlet)

Stress distribution around $Q\bar{Q}$

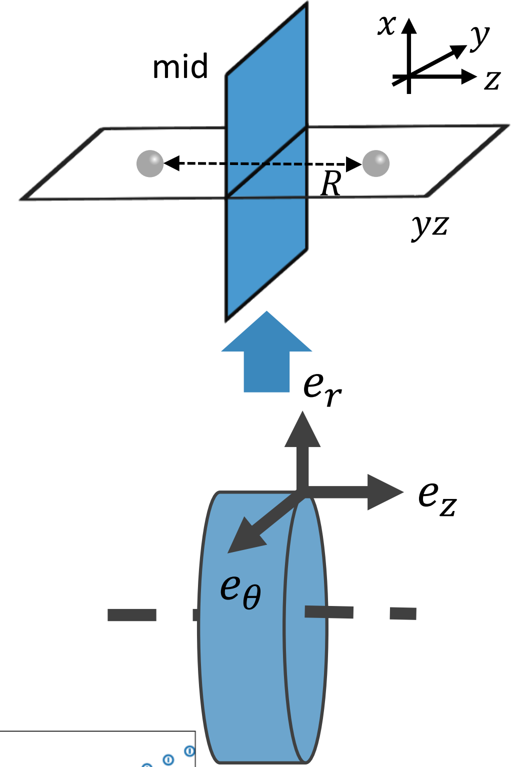
- ✓ $T_{rr} \approx T_{\theta\theta}$ (Degeneracy)
- ✓ $T_{44} \neq T_{rr}$ (Separation)
- ✓ $\sum_\mu T_{\mu\mu} \neq 0$ (Trace anomaly$\neq 0$)
- ✓ Damping $\leftrightarrow$ Debye mass
- ✓ $dF/dR \leftrightarrow \int T_{zz}\, d^2x$







Stress distribution around $Q\bar{Q}$



- ✓ $T_{rr} \approx T_{\theta\theta}$ (Degeneracy)
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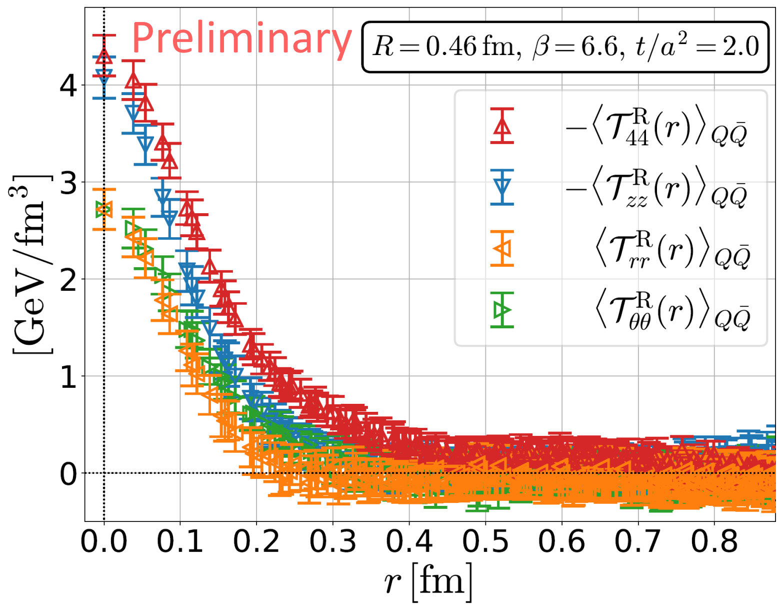
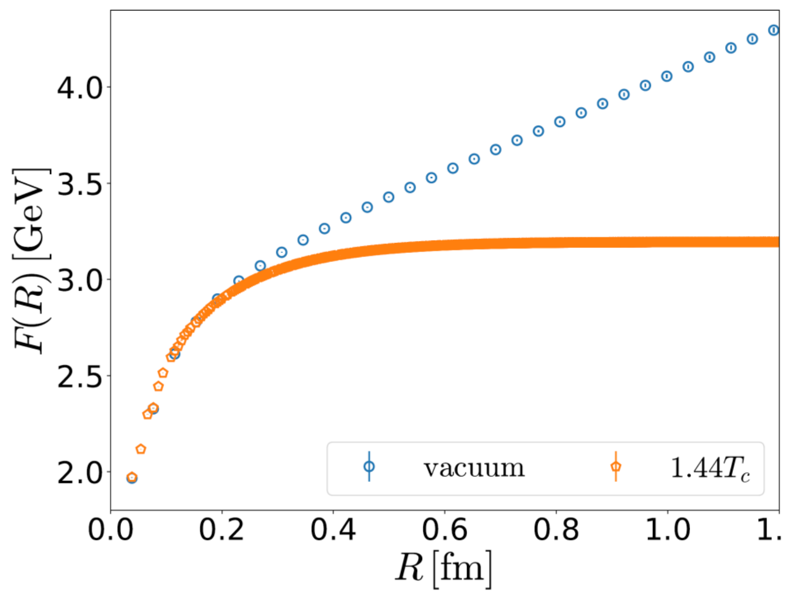
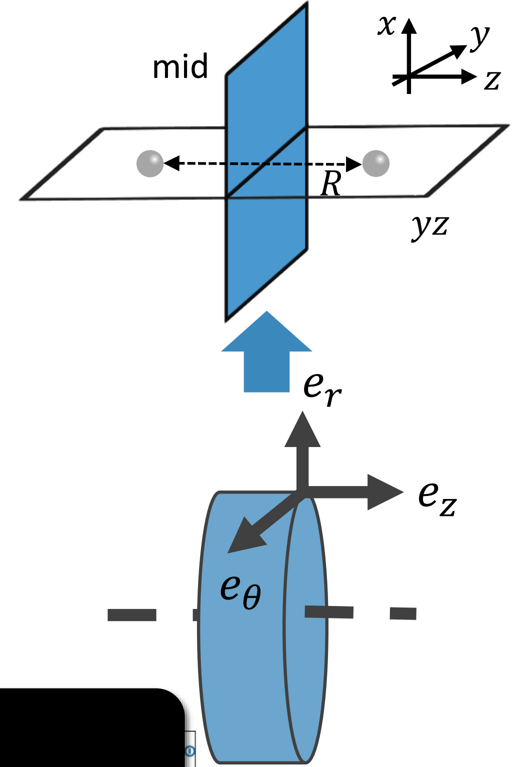
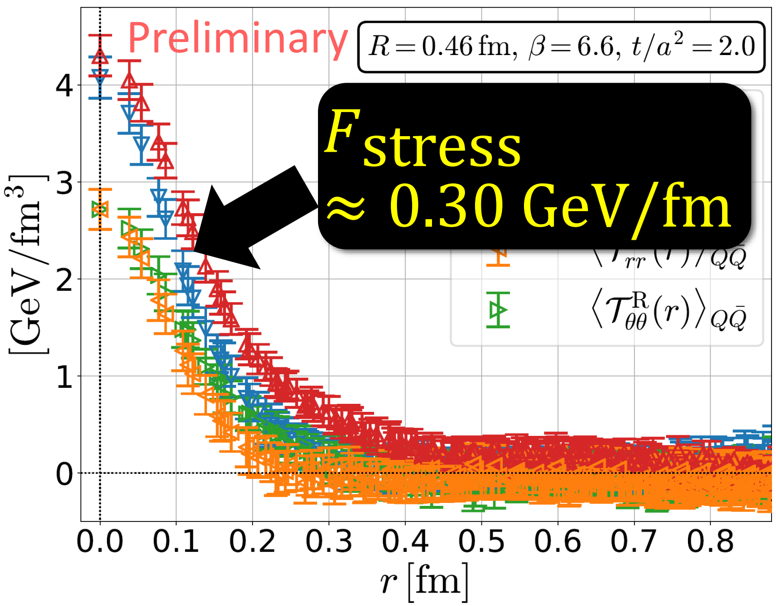


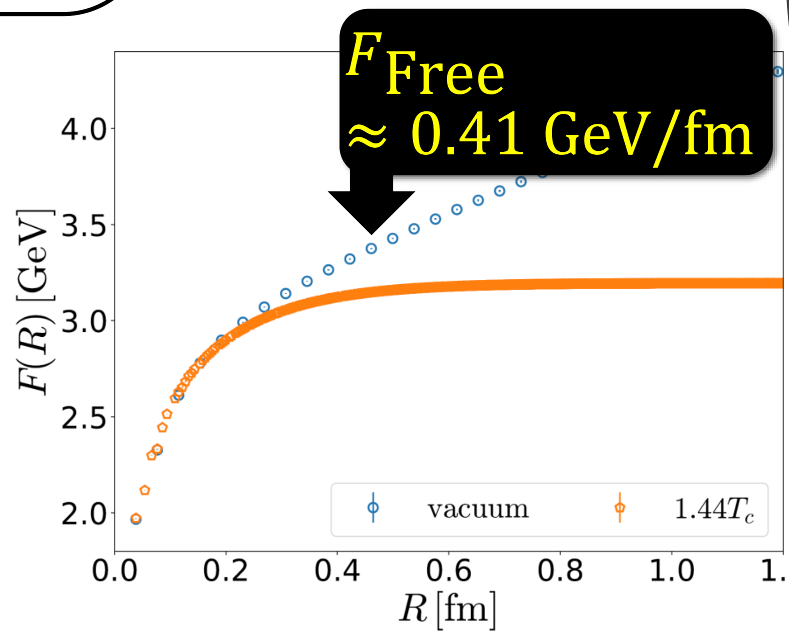


Stress distribution around $Q\bar{Q}$



- ✓ $T_{rr} \approx T_{\theta\theta}$ (Degeneracy)
- ✓ $T_{44} \neq T_{rr}$ (Separation)
- ✓ $\sum_\mu T_{\mu\mu} \neq 0$ (Trace anomaly$\neq 0$)
- ✓ Damping ↔ Debye mass
- ✓ $dF/dR \leftrightarrow \int T_{zz}\, d^2x$



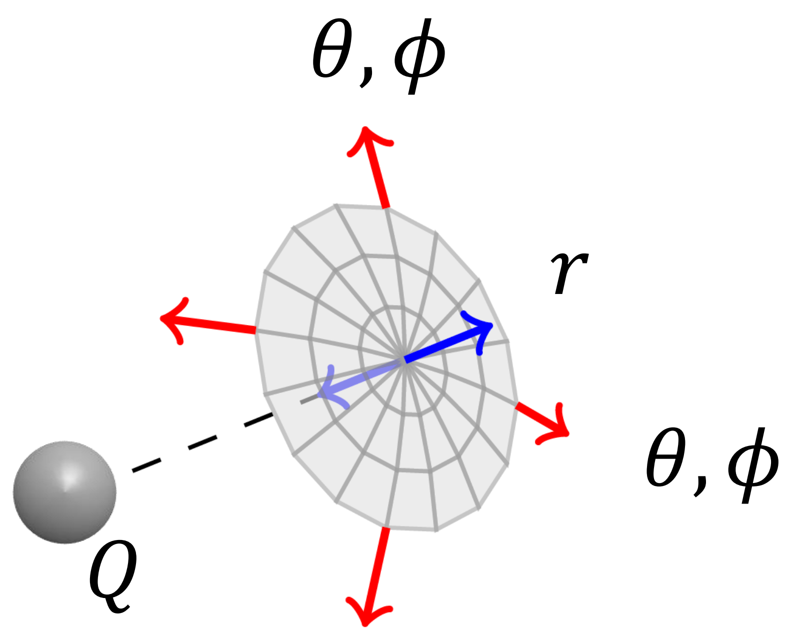


Stress distribution around Q : Spherical coordinates

$$T_{\mu\nu} = \begin{pmatrix} T_{44} & & & O \\ & T_{rr} & & \\ & & T_{tt} & \\ O & & & T_{tt} \end{pmatrix}$$

Diagonalized EMT

(Spherical symmetry)

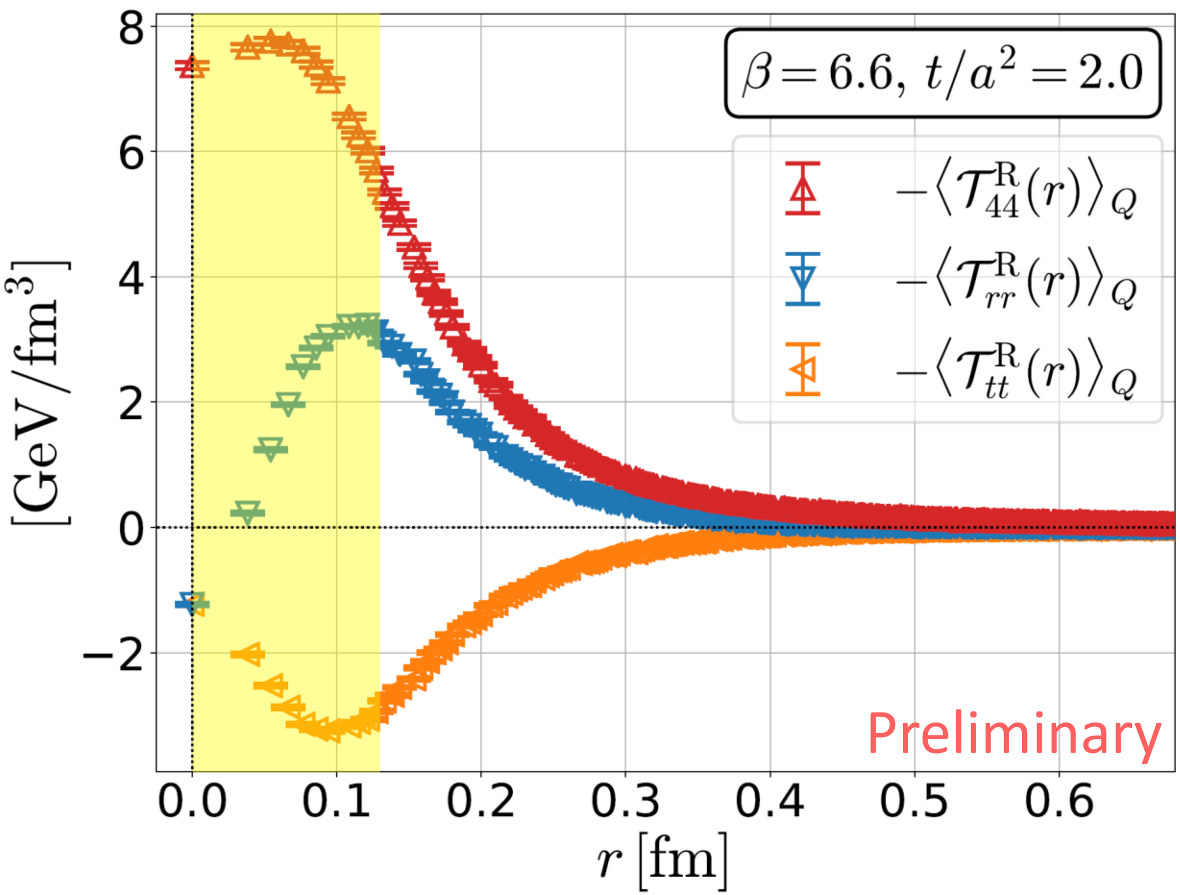


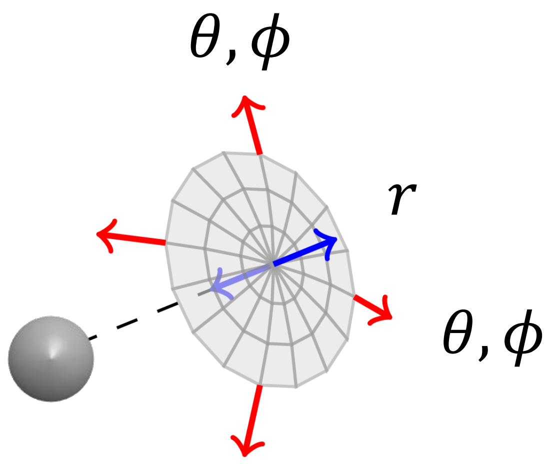
Degeneracy (Maxwell Theory)

$$|T_{44}| = |T_{rr}| = |T_{tt}|$$

$(t: \theta, \phi)$

Stress distribution around Q





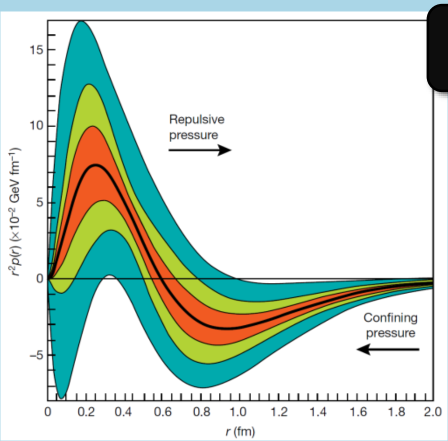
- ✓ Overlap : $r \lesssim \sqrt{2t}$
- ✓ Separation
- ✓ Around origin $\sim \alpha_s(r)/r^4$
- ✓ Damping $\sim e^{-m_D r}$
- ✓ Conservation law :

$$\partial_r (r^2 T_{rr}) = r T_{tt}$$

Pressure distribution inside hadrons vs. Our study

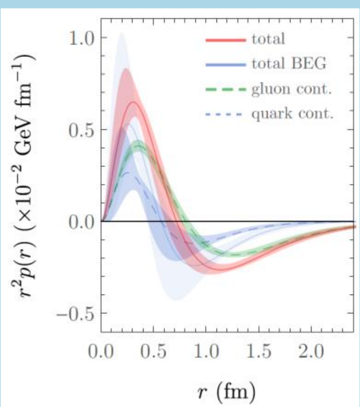
Pressure distribution inside Hadrons

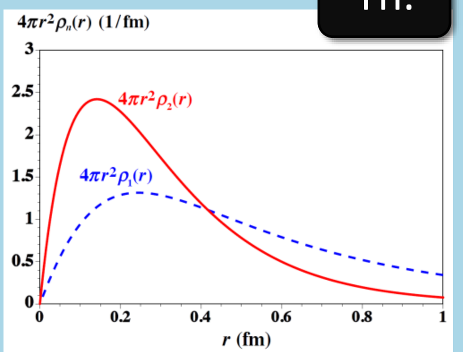
Exp.



Burkert *et al.*, Nature **557** (2018) 396.

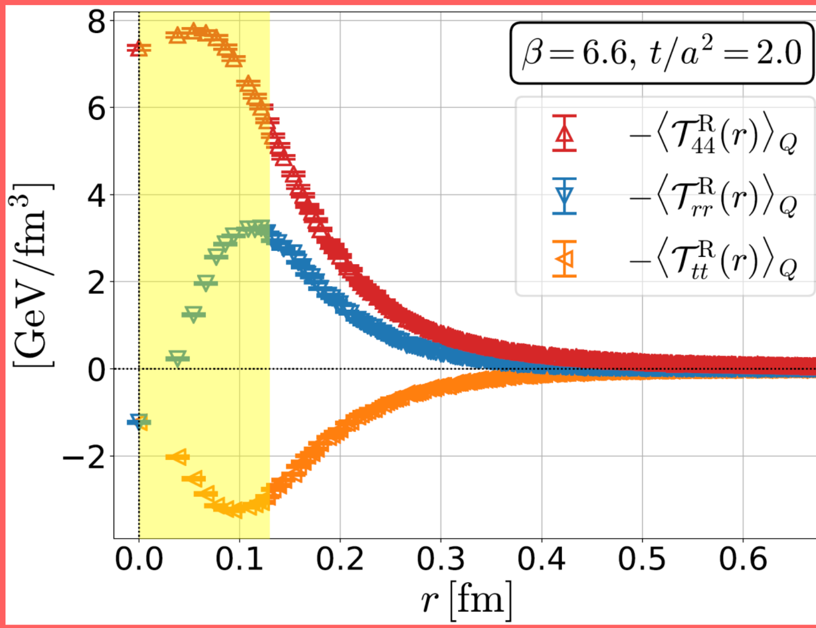
Th.





Shanahan *et al.*, PRL **122** (2019) no7, 072003.
Kumano *et al.*, PRD **97** (2018) 014020.

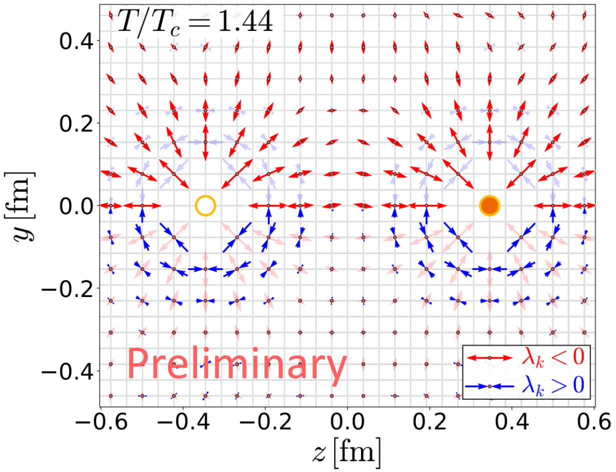
Our study

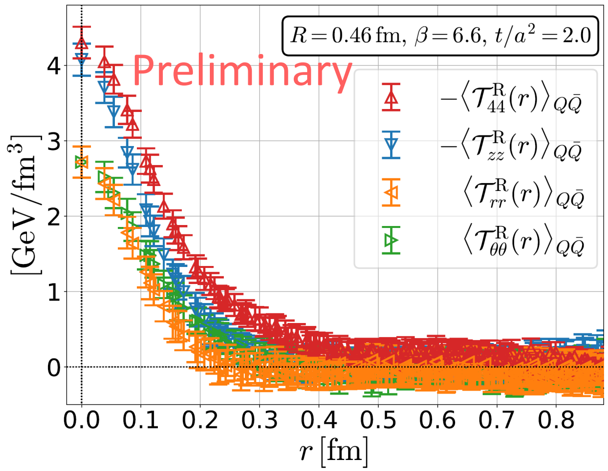


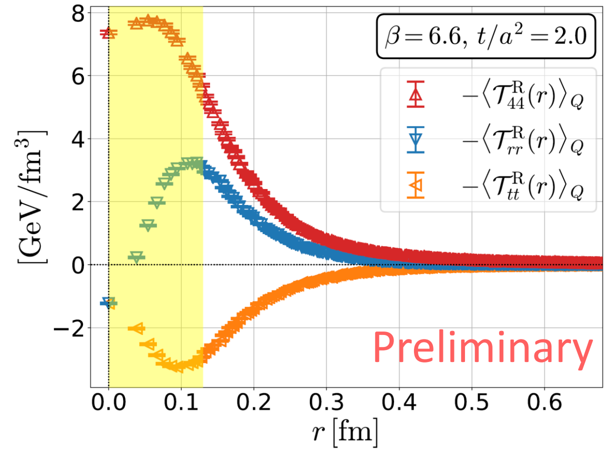
Summary and Outlook

Summary

✓ We first measure stress distribution around $Q\bar{Q}/Q$ at zero/nonzero temperature on the lattice







Quark—Anti-Quark

Single quark

Outlook

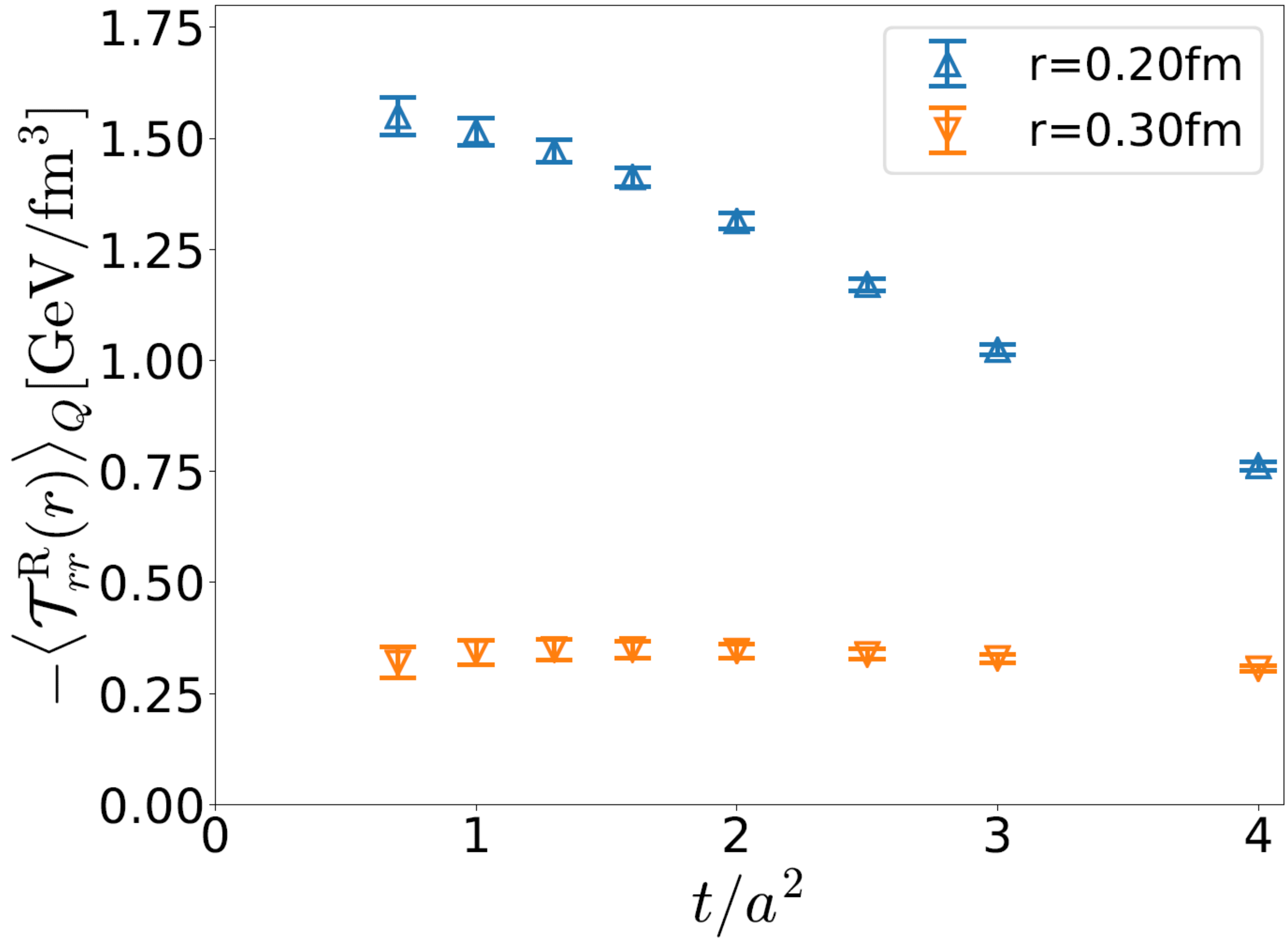
✓ $a, t \to 0$ (double limit)

✓ Temperature dependence

✓ Application : QQ、QQQ、excited state、hadron (full QCD)…

Back up

Flow time dependence (single quark system)



Interference b/w singlet and octet state