

Nuclear matter from strong coupling lattice QCD

Philippe de Forcrand
ETH Zürich and CERN

PhD thesis of Michael Fromm (ETH)

arXiv:0811.1931, 0907.1915 → PRL, 0912.2524
and in progress

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Scope of lattice QCD simulations

Physics of **color singlets**

- “One-body” physics: **confinement**
hadron masses
form factors, etc..



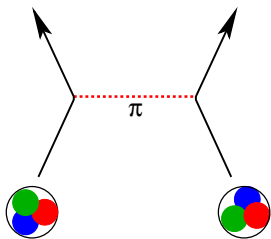
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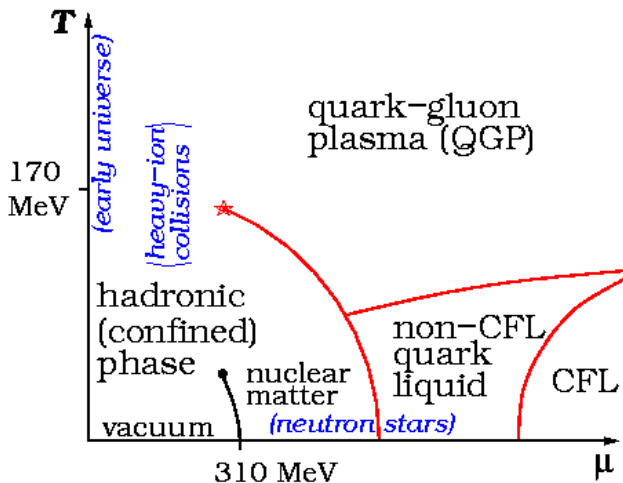


- “Two-body” physics: **nuclear interactions**
pioneers **Hatsuda et al, Savage et al**



hard-core
+
pion exchange?

QCD phase diagram according to Wikipedia




- Here:
- "many-body" physics: **hadron** \leftrightarrow **nuclear matter** transition
 - "two-body": $T = 0$ **nuclear interactions**

A different approach to the sign problem

$$Z = \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp \left(-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i (\not{D} + m_i + \mu_i \gamma_0) \psi_i \right)$$

$\det(\not{D} + m + \mu \gamma_0)$ complex \rightarrow try **integrating over the gauge field first!**

- Problem: $-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} \rightarrow \beta_{\text{gauge}} \text{Tr} U_{\text{Plaquette}}$, ie. **4-link interaction** 
 - Solution: set $\beta_{\text{gauge}} = \frac{2N_c}{g^2}$ to **zero**, ie. $g = \infty$, strong coupling limit
 - Then integral over gauge links **factorizes**: $\sim \int \prod dU \exp(\bar{\psi}_x U_{x,\hat{\mu}} \psi_{x+\hat{\mu}})$
 - analytic 1-link integral \rightarrow only **color singlets** survive
 - perform Grassmann integration last \rightarrow hopping of color singlets
- \rightarrow **hadron worldlines**
- sample gas of worldlines by Monte Carlo

Note: when $\beta_{\text{gauge}} = 0$, quarks are *always* confined $\forall(\mu, T)$, ie. **nuclear matter**

The price to pay: not continuum QCD

Strong coupling LQCD: why bother ?

Asymptotic freedom: $a(\beta_{\text{gauge}}) \propto \exp\left(-\frac{\beta_{\text{gauge}}}{4N_c b_0}\right)$

ie. $a \rightarrow 0$ when $\beta_{\text{gauge}} \equiv \frac{2N_c}{g^2} \rightarrow +\infty$. Here $\beta_{\text{gauge}} = 0$!!

- Lattice “infinitely coarse”
- Physics not universal

Nevertheless:

- Properties similar to QCD: **confinement** and χSB
- Include (perhaps) next term in strong coupling expansion, ie. $\beta_{\text{gauge}} > 0$

When $\beta_{\text{gauge}} = 0$, sign problem is **manageable** \rightarrow **complete solution**

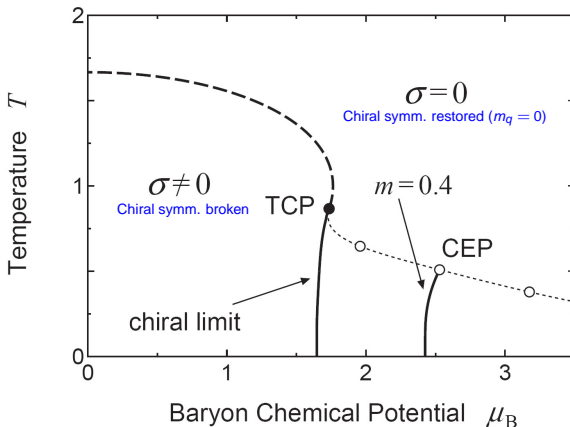
Valuable insight?

Further motivation

- 25⁺ years of analytic predictions:
 - 80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto
 $\mu_c(T=0) = 0.66, T_c(\mu=0) = 5/3$
 - 90's: Petersson et al., $1/g^2$ corrections
 - 00's: detailed (μ, T) phase diagram: Nishida, Kawamoto,...
 - now: Ohnishi et al. $o(\beta)$ & $o(\beta^2)$, Münster & Philipsen,...
 - How accurate is mean-field ($1/d$) approximation?
- Almost no Monte Carlo crosschecks:
 - 89: Karsch-Mütter \rightarrow MDP formalism $\rightarrow \mu_c(T=0) \sim 0.63$
 - 92: Karsch et al. $T_c(\mu=0) \approx 1.40$
 - 99: Azcoiti et al., MDP ergodicity ??
 - 06: PdF-Kim, HMC \rightarrow hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram from Nishida (2004, mean field, cf. Fukushima)



- Very similar to conjectured phase diagram of $N_f = 2$ QCD
- But no deconfinement here: high density phase is **nuclear matter**
- Baryon mass = $M_{\text{proton}} \Rightarrow$ lattice spacing $a \sim 0.6$ fm not universal

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D} U \mathcal{D} \bar{\psi} \mathcal{D} \psi \exp(-\bar{\psi}(\not{D}(U) + m_q)\psi)$, no plaquette term ($\beta_{\text{gauge}} = 0$)

- **One** complex colored fermion field per site (**no Dirac indices**, **spinless**)
- $\not{D}(U) = \frac{1}{2} \sum_{x,v} \eta_v(x) (U_v(x) - U_v^\dagger(x - \hat{v}))$, $\eta_v(x) = (-)^{x_1 + \dots + x_{v-1}}$

$U(1)_V \times U(1)_A$ **symmetry** when $m = 0$:

$$\left. \begin{array}{l} \psi(x) \rightarrow e^{i\theta} \psi(x) \\ \bar{\psi}(x) \rightarrow e^{-i\theta} \bar{\psi}(x) \end{array} \right\} \text{unbroken} \Rightarrow \text{quark number} \Rightarrow \text{chem. pot.}$$

$$\left. \begin{array}{l} \psi(x) \rightarrow e^{i\gamma_5(x)\theta} \psi(x) \\ \bar{\psi}(x) \rightarrow e^{i\gamma_5(x)\theta} \bar{\psi}(x) \\ \gamma_5(x) = (-)^{x_1+x_2+x_3+x_4} \end{array} \right\} \text{spont. broken } (m = 0) \Rightarrow \text{quark condensate}$$

($N_f = 1 \rightarrow U(1)$ chiral symmetry)

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- Chemical potential $\mu \rightarrow \exp(\pm a\mu) U_{\pm 4}$
- $\mathcal{D} U = \prod dU$ factorizes \rightarrow integrate over **links** Rossi & Wolff

\rightarrow **Color singlet** degrees of freedom:

- **Meson** $\bar{\psi}\psi$: *monomer*, $M(x) \in \{0, 1, 2, 3\}$
- **Meson hopping**: *dimer*, non-oriented $n_v(x) \in \{0, 1, 2, 3\}$
- **Baryon hopping**: oriented $\bar{B}B_v(x) \in \{0, 1\} \rightarrow$ *self-avoiding loops* C

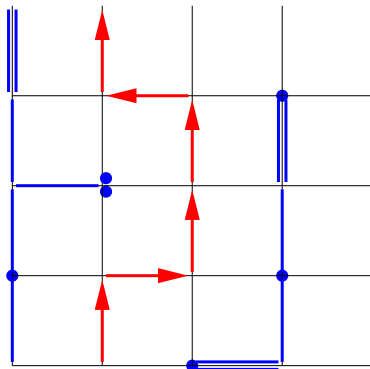
Point-like, hard-core baryons in pion bath

No πNN vertex

MDP Monte Carlo

$$Z(m_q, \mu) = \sum_{\{M, n_v, C\}} \prod_x \frac{m_q^{M(x)}}{M(x)!} \prod_{x,v} \frac{(3 - n_v(x))!}{n_v(x)!} \prod_{\text{loops } C} \rho(C)$$

with **constraint** $(M + \sum_{\pm v} n_v)(x) = 3 \forall x \notin \{C\}$

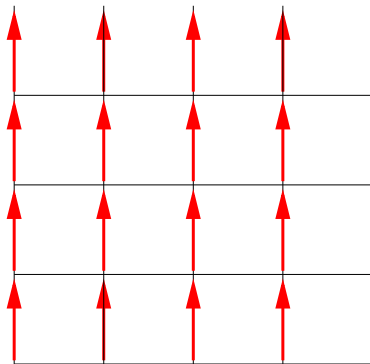


Constraint: 3 **blue** symbols or a **baryon** loop at every site

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The **dense** (crystalline) phase: 1 baryon per site

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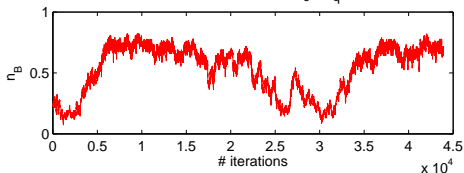
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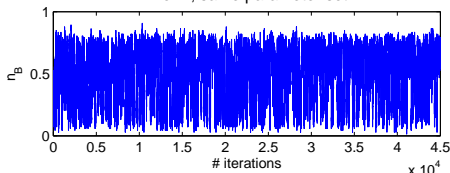
Remaining difficulties:

- Baryons are fermions: mild sign problem from $\rho(C)$ Karsch & Mütter
 \rightarrow volumes up to $16^3 \times 4 \forall \mu$
- tight-packing **constraint** \rightarrow local update inefficient, esp. as $m \rightarrow 0$
 Solved with **worm algorithm** (Prokof'ev & Svistunov 1998)
 Efficient even when $m_q = 0$

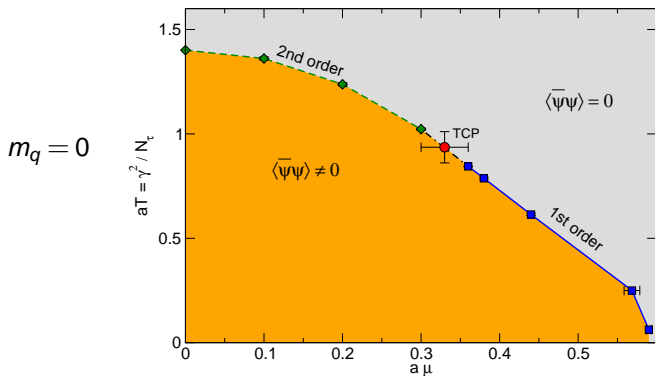
Local Metropolis, $4^3 \times 2$ at μ_c , $m_q = 0.025$



Worm, same parameter set

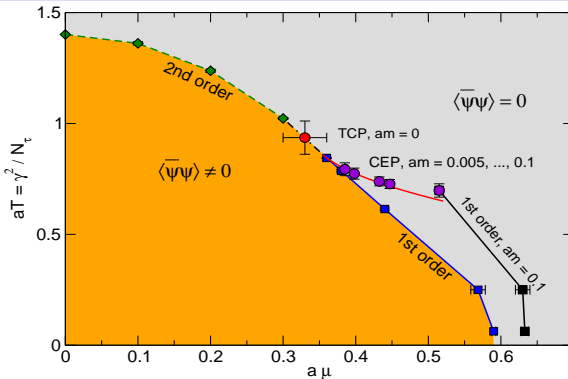


(μ, T) phase diagram in the chiral limit $m_q = 0$, and for $m_q \neq 0$



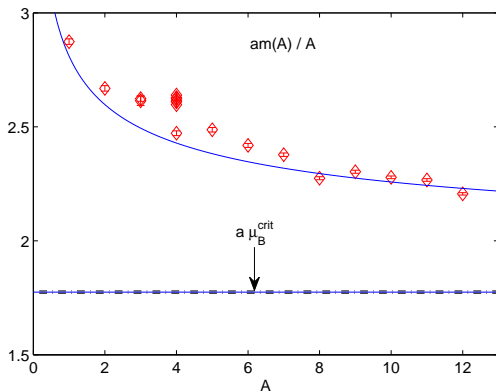
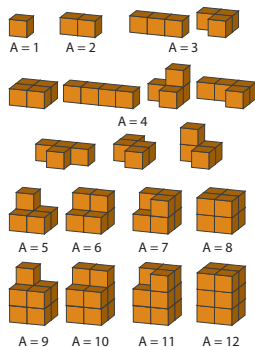
- Phase boundary for breaking/restoration of $U(1)$ chiral symmetry
 - 2nd order at $\mu = 0$: 3d O(2) universality class
 - 1st order at $T = 0$: ρ_B jumps from 0 to 1 baryon per site \implies **tricrit. pt. TCP**
- Finite-size scaling: $(\mu, T)_{TCP} = (0.33(3), 0.94(7))$ vs $(0.577, 0.866)$ (mean-field)
- Beware of quantitative mean-field predictions for phase diagram**

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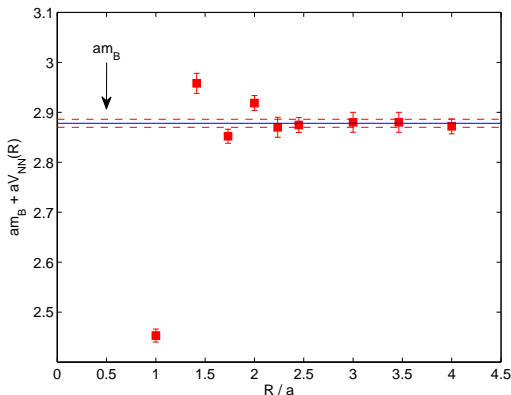
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- $m_q \neq 0$: liquid-gas transition $T_{CEP} \sim 200\text{MeV}$ – traj. of CEP obeys tricrit. scaling

Nuclear matter: spectroscopy



- Can compare masses of differently shaped “isotopes”
- $E(B=2) - 2E(B=1) \sim -0.4$, ie. “deuteron” binding energy ca. 120 MeV
- $am(A) \sim a\mu_B^{\text{crit}} A + (36\pi)^{1/3} \sigma a^2 A^{2/3}$, ie. (bulk + surface tension)
Bethe-Weizsäcker parameter-free (μ_B^{crit} and σ measured separately)
- “Magic numbers” with increased stability: $A = 4, 8, 12$ (reduced area)

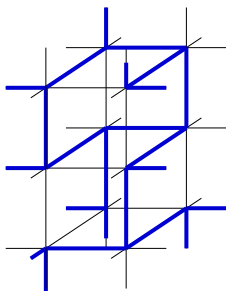
Nuclear potential: more than hard core



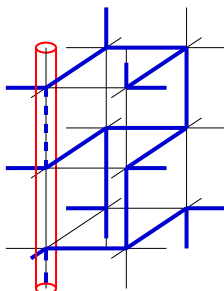
- Nucleons are point-like \rightarrow no ambiguity with definition of static potential
- Nearest-neighbour attraction ~ 120 MeV at distance ~ 0.5 fm: cf. real world
- Baryon worldlines self-avoiding \rightarrow no direct meson exchange (just **hard core**)
Attraction due to bath of neutral pions: cf. **Casimir effect** (see next)

How the nucleon got its mass

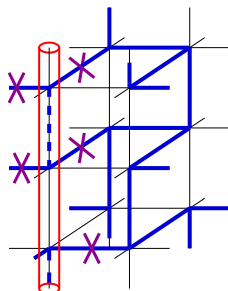
- Point-like nucleon **distorts pion bath** cf. Casimir



vacuum



static baryon



effect on pions

- Energy = nb. time-like pion lines

Constraint: 3 pion lines per site ($m_q = 0$) \rightarrow energy density = 3/4 in vacuum

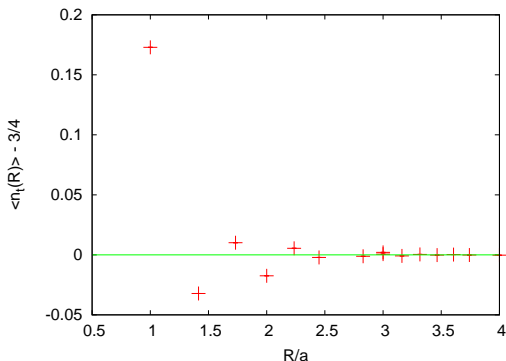
No spatial pion lines connecting to site occupied by nucleon \rightarrow **energy increase**

Steric effect

- $am_B \approx 2.88 = (3 - 0.75) + \Delta E_\pi$, ie. "valence"(78%) + "pion cloud"(22%)

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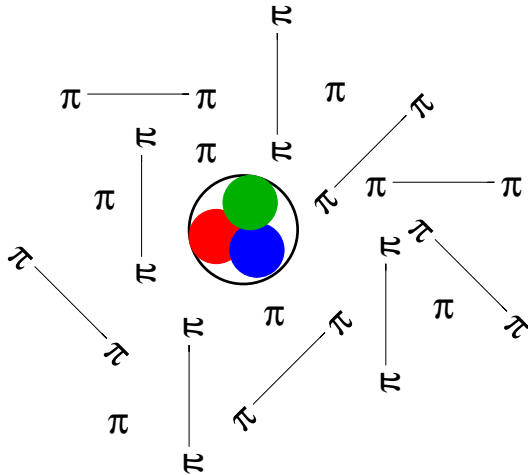
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So, in fact, nucleon is *not* point-like

Point-like “bag” of 3 valence quarks \rightarrow **macroscopic** disturbance in pion vacuum

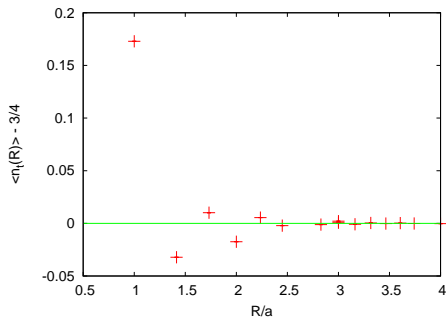


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Static baryon prevents monomers = static (t -invariant) monomer “source”

Linear response \propto Green's fct. of lightest t -invariant meson, ie. **rho/omega**
(pion has factor $(-1)^t$)

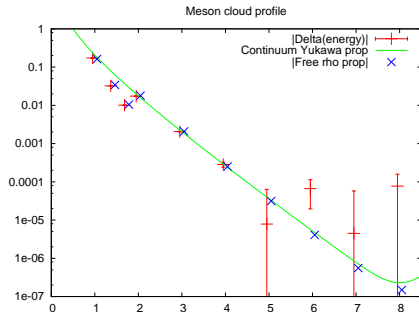
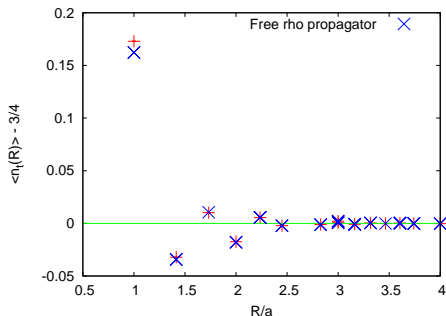


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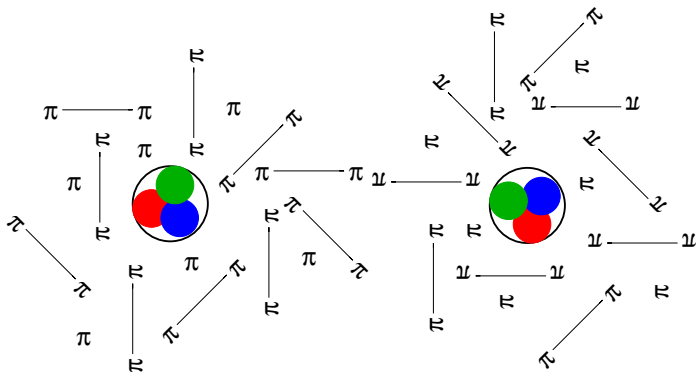
$$\text{Pion cloud profile} \propto \frac{\exp(-m_{\rho/\omega} r)}{r} \times (-1)^{x+y+z}$$

Nuclear interaction via pion clouds (thanks W. Weise)

- Here, baryons make self-avoiding loops \rightarrow **no direct meson exchange**
- Interaction comes because of **pion clouds**

The two pion clouds can interpenetrate at \approx constant energy (2nd order effect)
 But each set of *valence* quarks disturbs pion cloud of other baryon

$$\Rightarrow V_{NN}(R) \approx -2 \times \Delta E_{\pi}(R) \propto \frac{\exp(-m_{\pi}/\omega R)}{R} \times (-1)^{x+y+z}$$



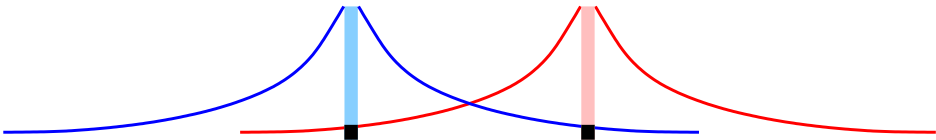
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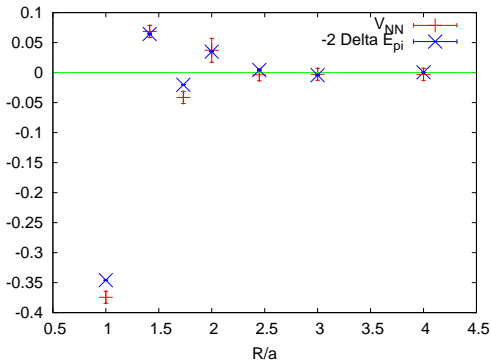
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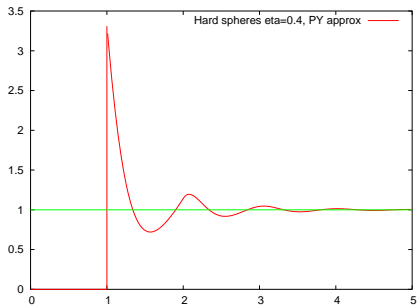
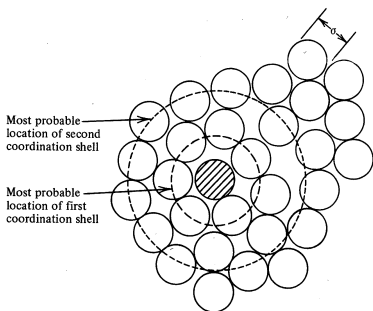
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Meson exchange potential without meson exchange!

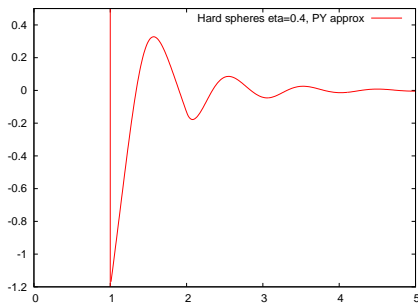
Is pion bath essential? Classical hard spheres



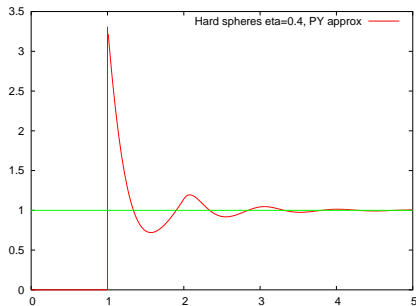
Density-density correlation
(Percus-Yevick approximation)

$g(r) \equiv \langle \rho(0)\rho(r) \rangle$ relaxes to $\langle \rho \rangle^2$ with **damped oscillations** \rightarrow **liquid**

Is pion bath essential? Classical hard spheres



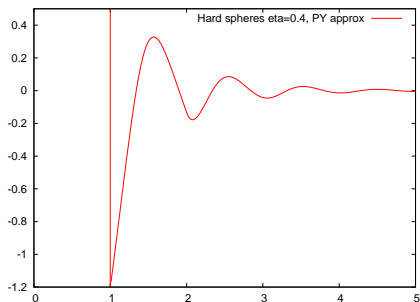
"Potential of mean force"



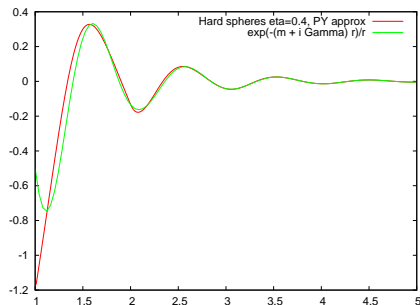
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"Potential of mean force" $V_{\text{eff}}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

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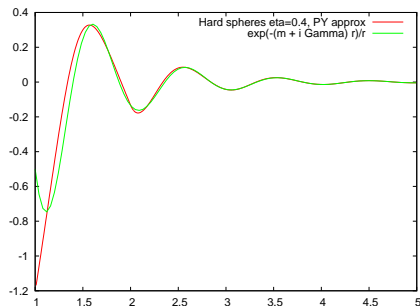
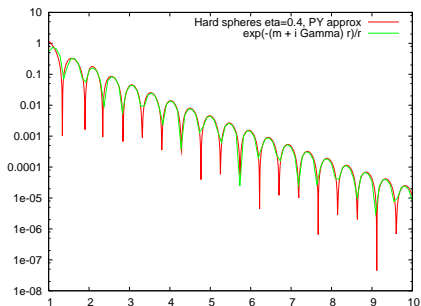


Compare with Yukawa potential

"Potential of mean force" $V_{\text{eff}}(r) \equiv -\log(g(r))$ is hard-core + damped oscillatory

Consistent with Yukawa form $\frac{\exp(-mr)}{r} \times \cos(\Gamma r)$

Is pion bath essential? Classical hard spheres



$\log(|V_{\text{eff}}(r)|) + \text{Yukawa fit}$

Perfect fit at large distance

Hard-sphere “potential of mean force” is of **Yukawa** form

$$V_{\text{eff}}(r) = \text{Re} \left[\frac{e^{-(m+i\Gamma)r}}{r} \right]$$

Recap & speculation

- Baryons are not point-like: pion cloud $\sim \exp(-m_{\rho/\omega}r)$
- Nuclear potential:
 - Hard-core from Pauli principle
 - Yukawa potential (times $(-1)^r$) from the two pion clouds
- Exactly like a **classical hard-sphere fluid**:
 - “Pion cloud” from ripples around tagged sphere
 - Density-density correlation $\leftrightarrow V_{\text{eff}}(r) \sim \exp(-(m + i\Gamma)r)/r$
- **Note**: at high density (packing fraction $\eta \in [0.4945, 0.6802]$),
hard-sphere system in solid phase cf. **Kepler, microsphere exp. @ ISS**
 $\rho_0 = 0.16/\text{fm}^3$ and $r \sim 0.5 \text{ fm} \rightarrow \eta \sim 0.08 \sim \frac{1}{6}\eta_{\text{crit}}$
- **Speculation**: IF baryons similar to hard [enough] billiard balls,
 THEN expect solid phase at high enough density ($\sim 6\rho_0$)

Solid phase due to (close packing + hard core) \Rightarrow **robust** w.r.t. details of potential

Conclusions

Summary

- Phase diagram: take mean-field results with a grain of salt
- [Crude, crystalline] **nuclear matter** from QCD:
tabletop simulations of first-principles nuclear physics
- Nucleon: point-like “bag” (\rightarrow hard core) + large **pion cloud** (\rightarrow Yukawa)
- **Hard core \implies solid phase at high density**

Outlook

- Include second quark species \rightarrow **isospin**
- Include $\mathcal{O}(\beta)$ effects ?