Non-uniform chiral condensate at the finite density

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Outline

1. **Introduction**
   - The problem
   - Spontaneously broken symmetries
2. **Non-uniform chiral condensate**
   - Chiral waves (in effective models)
   - NJL model
   - Mean-field solution
3. **Results**
   - Phase diagram

Chiral condensate: uniform or not uniform?

You probably have got your own opinion after the previous talks. . .

Fabri - Picasso theorem

Wigner-Weyl realization

- $Q|0\rangle = 0$,
- $Q|0\rangle$ is an eigenstate of Q with eigenvalue 0.

Nambu-Goldstone realization

- $Q|0\rangle$ does not exists in $\mathcal{H}$ (its norm is infinite),
- as a consequence sime field possesses non-vanishing vacuum expectation value.

Homogeneous phases.

Condensed matter physics

- BCS superconductivity.
- Superfluidity.

Particle physics

- Chiral condensate.
- Gluon condensate.
- Color superconductivity (2SC, CFL).

Inhomogeneous phases.

Condensed matter physics

- Crystal structures at low T (and/or high P).
- Cold atoms (^6Li).
- FFLO superconductivity (non-zero B field).
- Spin density waves.
- Uniaxial magnetization.

Particle physics

- LOFF phase of color superconductivity.
- Skyrme crystals.
- Chiral density waves.
- Chiral spirals (rather helis).

Inhomogeneous phases II

1+1-dimensional

- Peierls instability.
- Gross - Neveu model.

2+1-dimensional

- Overhauser effect in QCD.
- Kosterlitz-Thouless

3+1-dimensional

- NJL - type models (LOFF, CDW).
- Gell-Mann - Levy σ-model (CDW).
- Large N_c QCD.

One can risk. . .

Corollary

The assumption that the phases of QCD at finite density are homogeneous is not well supported. Rather the opposite may be closer to reality.

References

Time-ordered

F. Dautry, E. M. Nyman, Nucl. Phys. **A319** (1979) 323; T. Tatsumi, Prog. Theor. Phys. **63** (1980) 1252; M. Kutschera, W. Broniowski and A. Kotlorz, Nucl. Phys. **A516** (1990) 566; W. Broniowski, MS, Phys. Lett. **B488** (2000) 63, MS Phys. Lett. **B553** (2003) 45, Phys. Lett. **B642** (2006) 238; E. Nakano and T. Tatsumi, Phys. Rev. **D71** (2005) 114006; B. Bringoltz, JHEP 0703:016,2007; D. Nickel, Phys. Rev. **D80** (2009) 074025, T. Partyka, MS J. Phys. **G36** (2009) 025004; S. Maedan, Prog. Theor. Phys. **123** (2010)285, T. Partyka, arXiv:1005.2667, arXiv:1005.5688.

Hamiltonian

Scalar and diquark interactions

$$\begin{aligned} H &= \int_x \Big\{ \bar{\psi}(i\gamma^\nu \partial_\nu + \mu\gamma_0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2\right] \\ &+ G' \sum_{A=2,5,7} (\bar{\psi}_c i\gamma_5 \tau_2 \lambda^A \psi)(\bar{\psi} i\gamma_5 \tau_2 \lambda^A \psi_c) \Big\}, \end{aligned}$$

where

$$\int_x = \int_0^\beta d\tau \int d^3x, \quad \partial_\nu = (i\partial_\tau, \vec{\nabla}).$$

Hamiltonian II

After Hubbard-Stratonovich transformation

Partial bosonization

$$\begin{aligned}
H = \frac{1}{2}\int_x \{ & \bar{\psi}(i\gamma^\nu \partial_\nu + \mu\gamma_0 + \sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau})\psi \\
& + \bar{\psi}_c(i\gamma^\mu \partial_\mu - \mu\gamma_0 + \sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}^T)\psi_c \\
& + G'(\Delta_A \bar{\psi}_c i\gamma_5 \tau_2 \lambda^A \psi + \Delta_A^* \bar{\psi} i\gamma_5 \tau_2 \lambda^A \psi_c) \\
& - \frac{\sigma^2 + \vec{\pi}^2}{2G} - \frac{\Delta_A \Delta_A^*}{2G'} \Big\} \quad .
\end{aligned}$$

Ansatz

Non-uniform

$$\begin{aligned} \sigma &= -M\cos\vec{q}\cdot\vec{x}, \quad \pi_a = -M\delta_{a3}\sin\vec{q}\cdot\vec{x}, \\ \Delta_A &= \Delta\delta_{A2}, \end{aligned}$$

- uniform chiral phase Ch: $M \neq 0$, $\vec{q} = 0$, $\Delta = 0$;
- non-uniform chiral phase NCh: $M \neq 0$, $\vec{q} \neq 0$, $\Delta = 0$;
- 2SC phase $M = 0$, $\vec{q} = 0$, $\Delta \neq 0$,
- mixed phase $M \neq 0$, $\vec{q} \neq 0$, $\Delta \neq 0$.

Partition function and field rotation

Partition function

$$Z = \int D\bar{\psi} D\psi D\bar{\psi}_c D\psi_c D\sigma D\vec{\pi} D\Delta_A D\Delta_A^* \exp H .$$

Field rotation

$$\psi' = \sqrt{U}\psi, \quad U = \exp(i\gamma_5 \tau_3 \vec{q} \cdot \vec{x})$$

Mean field partition function

$$Z_{MF} = \exp\left[- \int_x \left(\frac{M^2}{4G} + \frac{|\Delta|^2}{4G'} \right) + \frac{1}{2} \ln\det S^{-1} \right],$$

Determinant

Inverse Nambu-Gorkov propagator

$$S^{-1}(x,y) = \begin{bmatrix} i\gamma^{\nu}(\partial_{\nu} - \frac{1}{2} i\gamma_5\tau_3 q_{\nu}) + \mu\gamma_0 - M & i\gamma_5\tau_2\lambda_2\Delta \\ i\gamma_5\tau_2\lambda_2\Delta^{*} & i\gamma^{\nu}(\partial_{\nu} - \frac{1}{2} i\gamma_5\tau_3 q_{\nu}) - \mu\gamma_0 - M \end{bmatrix}.$$

For $Nc = 3$, $N_f = 2$ this is 48×48 matrix but it decays into 2 sub-diagonal matrices 16×16 and 36×36.

Potential

Mean field potential

$$\frac{\Omega}{V} = V_0 - 4T\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\sum_{i,k=\pm}\left(\ln\frac{1}{2}\left(1+e^{\frac{-\epsilon_{ik}}{T}}\right)+\frac{1}{2}\ln\frac{1}{2}\left(1+e^{\frac{-\epsilon^0_{ik}}{T}}\right)\right),$$

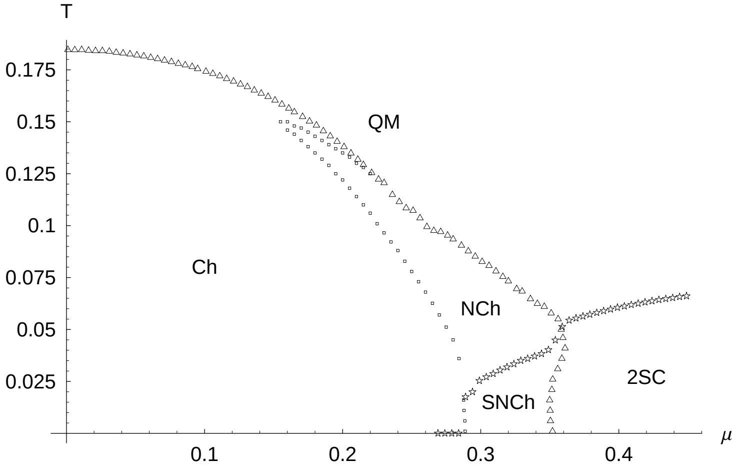
$$\begin{aligned} V_0 \;&=\; \frac{M^2}{4G}+\frac{|\Delta|^2}{4G'}+\frac{M^2F_\pi^2\vec{q}^{\,2}}{2M_0^2}-12\int^{\Lambda}\frac{d^3k}{(2\pi)^3}E_0 \\ &-\; 2\sum_{i=\pm}\left[\int^{\Lambda}\frac{d^3k}{(2\pi)^3}\Big(\sum_{k=\pm}\epsilon_{ik}-2E_i\Big)-\int_{E_i\leq\mu}\frac{d^3k}{(2\pi)^3}(E_i-\mu)\right], \end{aligned}$$

where

$$\epsilon_{+,\pm} = \sqrt{(\mu+E_\pm)^2+|\Delta|^2}, \quad \epsilon_{-,\pm} = \sqrt{(\mu-E_\pm)^2+|\Delta|^2},$$

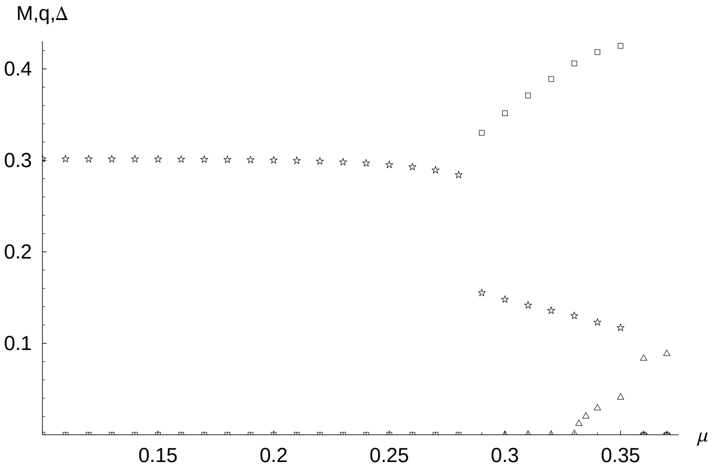
$$E_\pm = \sqrt{\vec{k}^2+M^2+\frac{\vec{q}^{\,2}}{4}\pm\sqrt{(\vec{q}\cdot\vec{k})^2+M^2\vec{q}^{\,2}}}.$$

Phase diagram

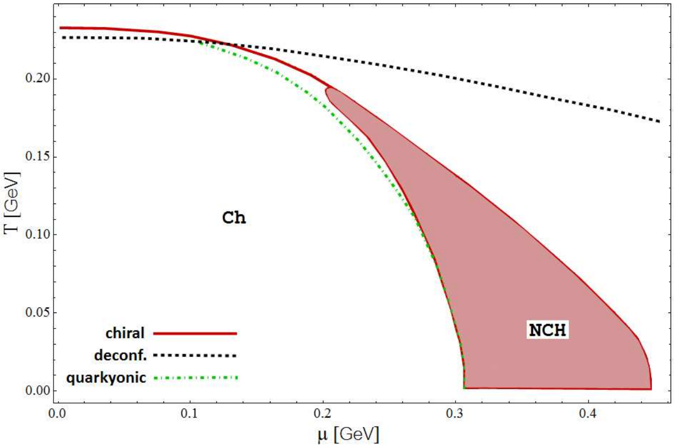


MS, Phys. Lett. **B642** (2006) 238

Order parameters



Phase diagram - PNJL model



T. Partyka, MS in preparation.

Summary and outlook

- Non-uniform chiral phase is a non-trivial solution.
- Sequence of the phase transitions at finite density: uniform chiral phase $\rightarrow$ inhomogeneous chiral phase $\rightarrow$ chiral spirals $\rightarrow$ superconducting phases.
- Signatures of the new phases (DCC).
- Caution (regularization, mass).
- Study of the PNJL model including both non-uniform chiral phase and chiral spirals.

Peierls instability