Holographic Chiral Currents in a Magnetic Field

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Chiral Symmetry and Confinement in Cold, Dense Quark Matter
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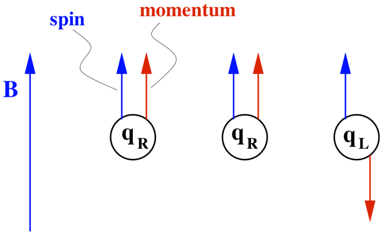
A. Schmitt, A. Rebhan, SS, JHEP, 1001, 026 (2010)

Outline

The Chiral Magnetic Effect (CME)

D.E. Kharzeev, L.D. McLerran, H.J. Warringa, NPA 803, 227 (2008)



- topological charge +QCD anomaly → **nonzero chirality** $N_5 = N_R - N_L$
- **magnetic field B**

→induces **electric vector current** parallel to B

$$\mathcal{J} = \frac{e^2 N_c}{2\pi^2}\mu_5 B$$

This is the **Chiral Magnetic Effect**

- analogously for **axial current:** $\mathcal{J}_5 = \frac{e^2 N_c}{2\pi^2}\mu B$
  M.A. Metlitski, A.R. Zhitnitsky, PRD 72, 045011 (2005)

- **CME possibly responsible for charge separation observed at RHIC**
- **compute currents via gauge gravity duality**
  - weakly coupled string theory $\Longleftrightarrow$ strongly coupled gauge theory
  - II B string theory on $AdS_5 \times S_5 \Longleftrightarrow \mathcal{N} = 4$ SYM on $\mathbb{R}^{1,3}$
  - radial coordinate in AdS $\Leftrightarrow$ energy scale in CFT
  - $(L/\ell_s)^4 = \lambda$, $g_s = \lambda/(4\pi N_c)$
- **we use Sakai Sugimoto model**

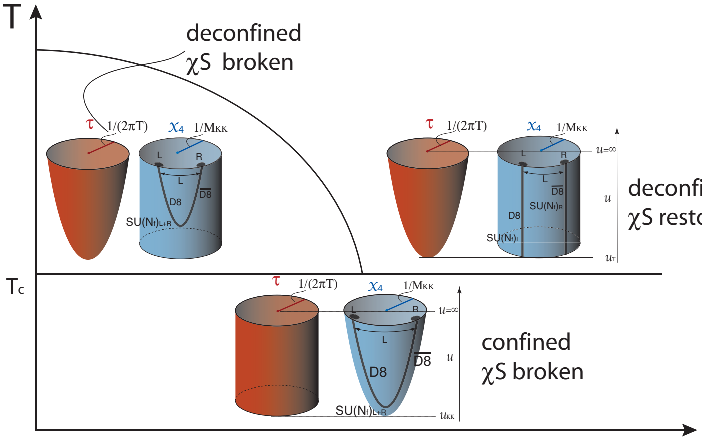
The Sakai-Sugimoto Model

Witten 98, T.Sakai, S.Sugimoto, Prog.Theor.Phys. 113, 843 (2005)

- **close to QCD**
  - chiral symmetry breaking
  - confinement
  - SUSY is completely broken
- **N$_c$ D4 branes in 10 dim. SUGRA with one compact dimension**
  - 4-4 strings: adjoint scalars & fermions, gauge fields
  - **compact dimension**: $x_4 = x_4 + 2\pi/M_{KK}$ → **breaks SUSY** by giving mass to scalars & fermions
    → **SU(N$_c$) gauge theory**
- **add flavor branes separated in coordinate x_4**
  - fundamental degrees of freedom
  - D4-D8/$\overline{\text{D8}}$ strings : fundamental chiral massless fermions
  - D8-$\overline{\text{D8}}$ strings: bound states, i.e. mesonic d.o.f.

T-μ phase diagram

N. Horigome, Y. Tanii, JHEP 0701, 072 (2007)



similar for large N_c-QCD: *L. McLerran, R. D. Pisarski, Nucl. Phys. A796, 83 (2007)*

Our Setup

A. Rebhan, A. Schmitt, SS, JHEP 1001.026 (2010)

- **maximal separated branes**
  - D8 branes follow geodesics, trivial embedding
- **one flavor**
- $S = S_{\mathrm{YM}} + S_{\mathrm{CS}}$

$$S_{\mathrm{YM}} = \kappa M_{\mathrm{KK}}^2 \int d^4x \int_{-\infty}^{\infty} dz \left[k(z) F_{z\mu} F^{z\mu} + \frac{h(z)}{2M_{\mathrm{KK}}^2} F_{\mu\nu} F^{\mu\nu} \right], \qquad k(z) = 1+z^2,$$

$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2} \int d^4x \int_{-\infty}^{\infty} dz\, A_\mu F_{z\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}, \qquad \kappa = \frac{\lambda N_c}{216\pi^2}, \qquad h(z) = k(z)^{-\frac{1}{3}}$$

- **boundary conditions**
  - $A_0(x, \pm\infty) = \mu_{L/R}$
  - $A_2(x_2, \pm\infty) = -x_2 B$
- gauge choize: $A_z(x, z) = 0$

Anomalies

- **add Bardeen counterterm to action**

$$\Delta S = c \int d^4x (A^L_\mu A^R_\nu F^L_{\rho\sigma} + A^L_\mu A^R_\nu F^R_{\rho\sigma}) \epsilon^{\mu\nu\rho\sigma}$$

W.A. Bardeen, Phys. Rev. 184, 1848 (1969)

  - correct QED covariant anomaly: correct decay rate $\pi^0 \to 2\gamma$
  - vector current vanishes
  - can be written as *holographic counterterm*

- **covariant anomaly**

$$\begin{aligned} \partial_\mu \bar{\mathcal{J}}^\mu &= 0\,, \\ \partial_\mu \bar{\mathcal{J}}^\mu_5 &= \frac{N_c}{8\pi^2} F^V_{\mu\nu} \widetilde{F}^{\mu\nu}_V + \frac{N_c}{24\pi^2} F^A_{\mu\nu} \widetilde{F}^{\mu\nu}_A\,. \end{aligned}$$

Chiral Currents

- **definition of the currents**

$$\mathcal{J}^{\mu}_{L/R} \equiv -\frac{\delta S}{\delta A_{\mu}(x, z = \pm\infty)} = \mathcal{J}^{\mu}_{L/R,YM} + \mathcal{J}^{\mu}_{L/R,CS} + \Delta\mathcal{J}^{\mu}_{L/R}$$

- **axial current**

$$\mathcal{J}^{5} = \left(1 - \frac{1}{3} + \frac{1}{3}\right)\frac{N_c}{2\pi^2}B\mu = \frac{N_c}{2\pi^2}B\mu$$

  - expected result
- **vector current**

$$\mathcal{J} = \left(1 - \frac{1}{3} - \frac{2}{3}\right)\frac{N_c}{2\pi^2}B\mu_5 = 0$$

  - **CME vanishes**
- but in the asympotics of the gauge field:

$$A_{\mu}(x, z) = A_{\mu}(x, z = \pm\infty) \pm \frac{\mathcal{J}^{L/R}_{\mu,\mathrm{YM}}}{2\kappa M^2_{\mathrm{KK}}}\frac{1}{z} + \mathcal{O}\left(\frac{1}{z^2}\right)$$

Collection of results

		$\mathcal{J}_{\rm YM}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}$	$\mathcal{J}_{\rm YM}+\mathcal{J}_{\rm CS}+\Delta\mathcal{J}$
	anomaly	"semi-covariant":	consistent:	covariant:
	$\partial_\mu \mathcal{J}_5^\mu \,/\, \frac{N_c}{24\pi^2}$	$\underline{3F_V\widetilde{F}_V}+3F_A\widetilde{F}_A$	$F_V\widetilde{F}_V + F_A\widetilde{F}_A$	$\underline{3F_V\widetilde{F}_V + F_A\widetilde{F}_A}$
	$\partial_\mu \mathcal{J}^\mu \,/\, \frac{N_c}{24\pi^2}$	$6F_V\widetilde{F}_A$	$2F_V\widetilde{F}_A$	$\underline{0}$
$T > T_c$	$\mathcal{J}_5 \,/\, \frac{\mu B N_c}{2\pi^2}$	1	$\frac{2}{3}$	1
	$\mathcal{J} \,/\, \frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0
$T < T_c$	$\mathcal{J}_5 \,/\, \frac{\mu B N_c}{2\pi^2}$	$\frac{\beta \coth \beta\pi}{2\rho(\beta)}$	$\frac{\beta \coth \beta\pi}{2\rho(\beta)} - \frac{1}{3}$	$\frac{\beta \coth \beta\pi}{2\rho(\beta)}$
	$\mathcal{J} \,/\, \frac{\mu_5 B N_c}{2\pi^2}$	1	$\frac{2}{3}$	0

- in the **chirally broken phase** the result seems quite reasonable

  W. j. Fu, Y. x. Liu, Y. l. Wu, arXiv:1003.4169 [hep-ph]

Arising questions

- **only vector current gets strong coupling corrections?**
  K. Fukushima, M. Ruggieri, arXiv:1004.2769 [hep-ph]
- **need to consider conserved N_5?**
  V. A. Rubakov, arXiv:1005.1888 [hep-ph],
  A.Y. Alekseev, V.V. Cheianov, J. Fröhlich, PRL 81, 3503 (1998)
- **careful distinction between chemical potential and sources for currents**
  A. Gynther, K. Landsteiner, F. Pena-Benitez, A. Rebhan, arXiv:1005.2587 [hep-th]
- **need the canonical ensemble ?**
  H.U. Yee, Talk at BNL Workshop, April 26 - 30, 2010,
  A. Rebhan,A. Schmitt, SS, work in progress
- **or is $\mathcal{J}_{YM}$ the more appropriate quantity?**
  - often done in the literature without justification

can we justify to keep YM part only

A. Rebhan, A. Schmitt, SS, work in progress

- **anomaly for YM:**
  $\partial_\mu \mathcal{J}^\mu = \frac{N_c}{4\pi^2} F^V_{\mu\nu} \widetilde{F}^{\mu\nu}_A \,, \qquad \partial_\mu \mathcal{J}^\mu_5 = \frac{N_c}{8\pi^2} \left(F^V_{\mu\nu} \widetilde{F}^{\mu\nu}_V + F^A_{\mu\nu} \widetilde{F}^{\mu\nu}_A \right)$
  - OK if $F_A = 0$
- **definition of chemical potential**

  $\mu = \int_0^\infty dz \, \partial_z A_0 \qquad$ instead of $A_0(\infty) = \mu$
- **interprete $A_0(\infty)$ as source only**
  - meaning that we set $A_0(\infty) = 0$ at the physical point
    $\rightarrow$ **CS contribution** to current **vanishes** trivially
    but induces singularity at horizon
- **amounts to (holographic) Legendre transformation**
- **modified action for thermodynamic consistency**
  O. Bergman, G. Lifschytz and M. Lippert, PRD 79, 105024 (2009)