Density fluctuations around the QCD critical point: role of the vector interaction

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ECT*--EMMI International Workshop on
Chiral Symmetry and Confinement in Cold, Dense Quark Matter
scheduled for July 19 - 23, 2010, ECT*

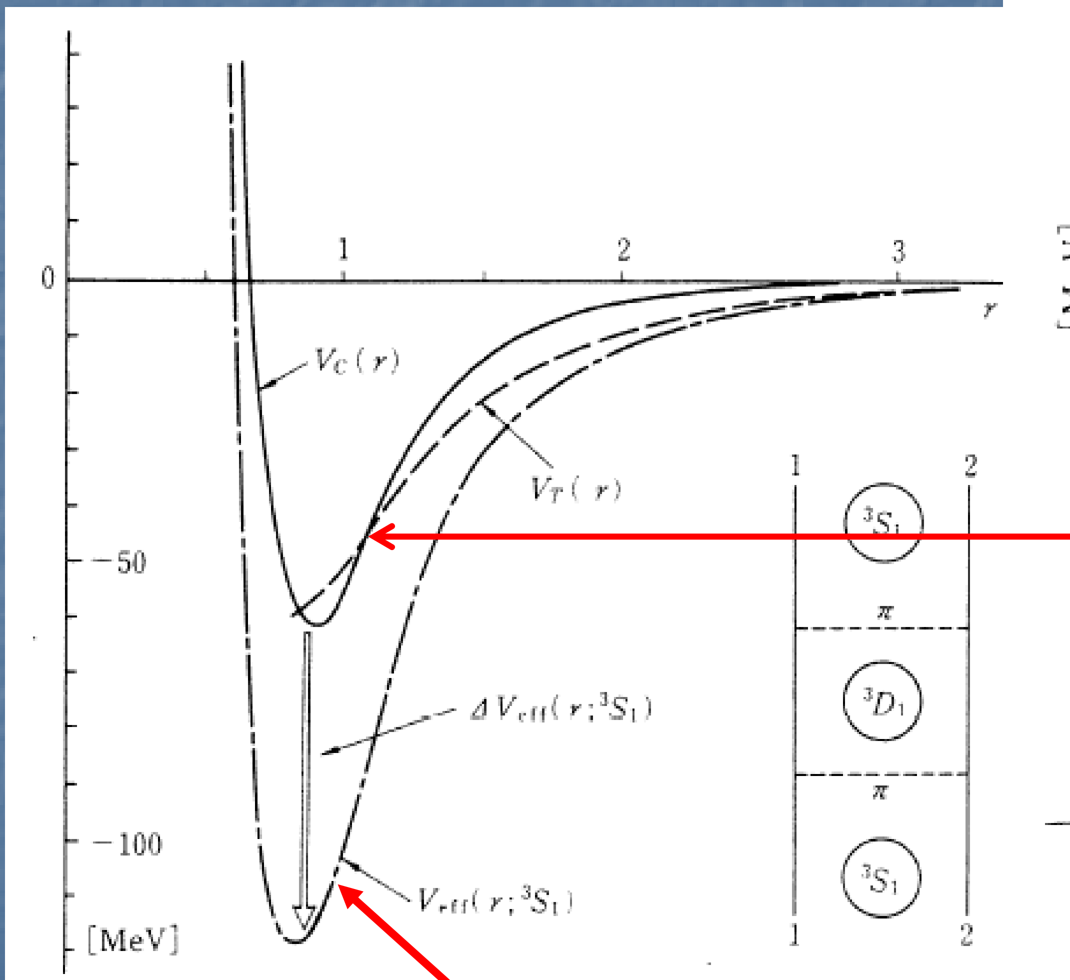
Contents

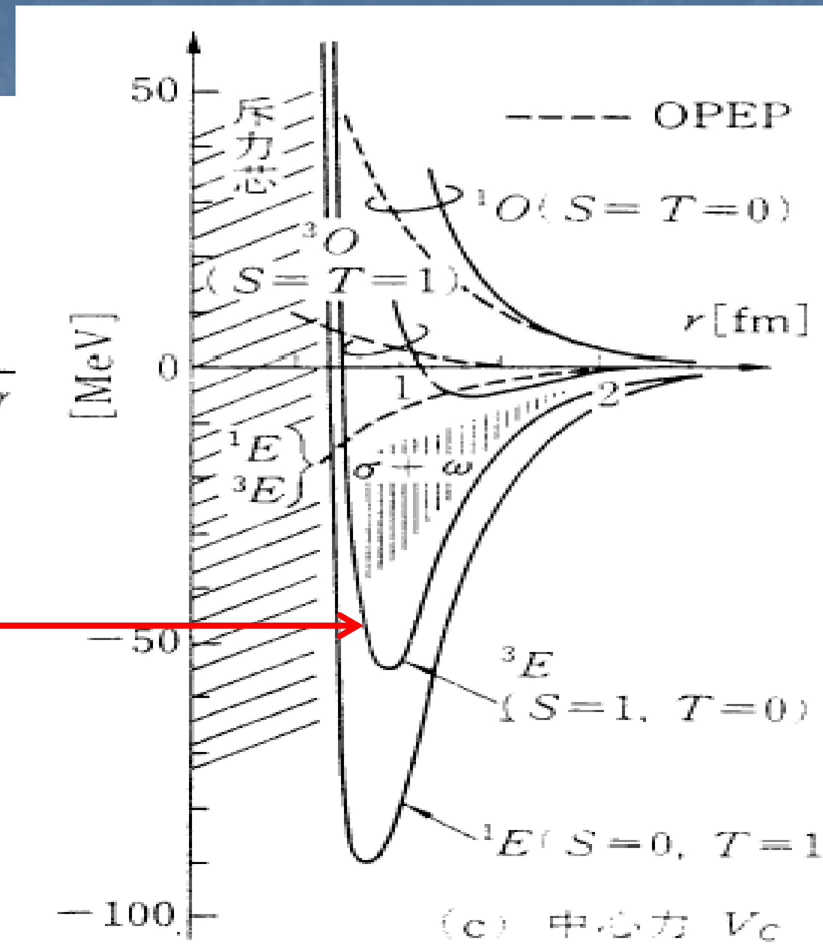
The binding mechanism of deuteron

---- the basis of nuclear physics -----

Bare central forces

Deuteron (T=0, ^3E) $^3S_1 - ^3D_1$



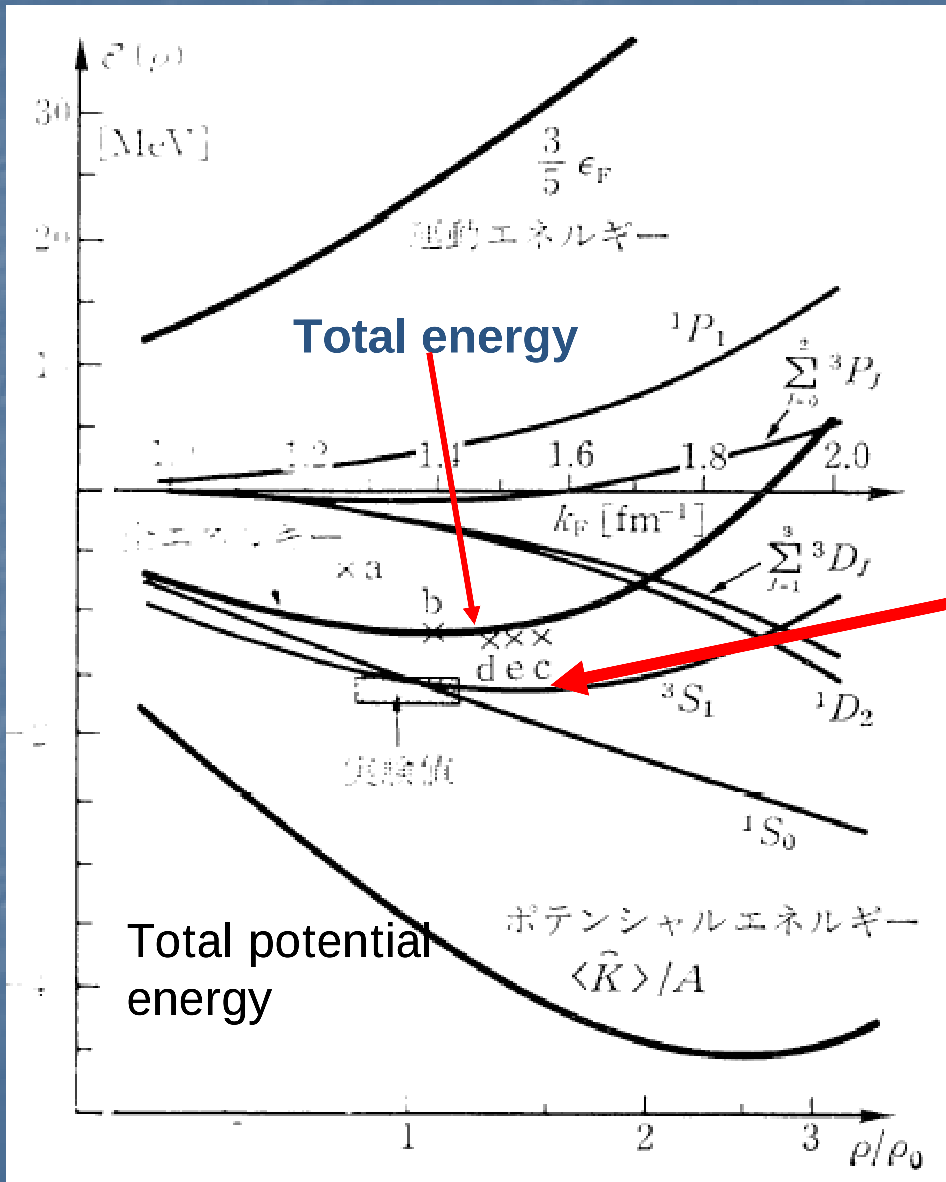


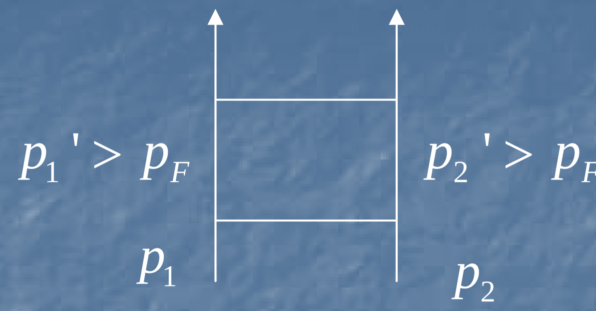
No di-neutron bound state (T=1)
V_C(1E)>V_C(3E)

Effective Central force

Text book by R. Tamagaki(1996)

Saturation mechanism of N=Z nuclear matter





The second-order quantum effect due to Tensor force gives rise to a peculiar density dependence of the potential energy.

The primay cause of the saturation of density

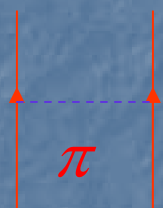
Pauli principle and Tensor force

OPEP(One-Pion-Exchange Potential)

$$V_{\rm OPEP}(r) = f^2 m_\pi \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{3}\left[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 Y(m_\pi r) + S_{12} Z(m_\pi r)\right],$$

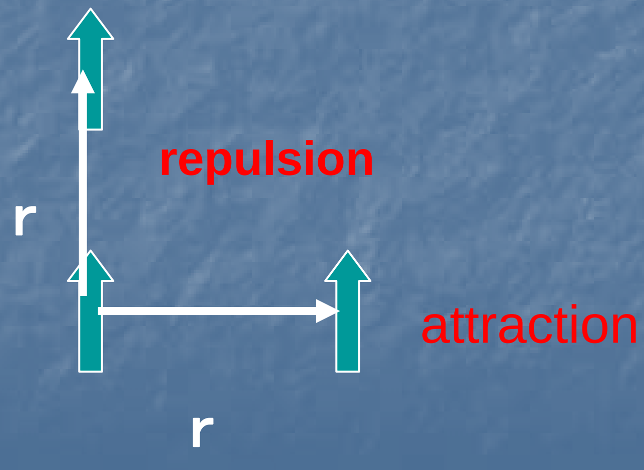
Central Tensor

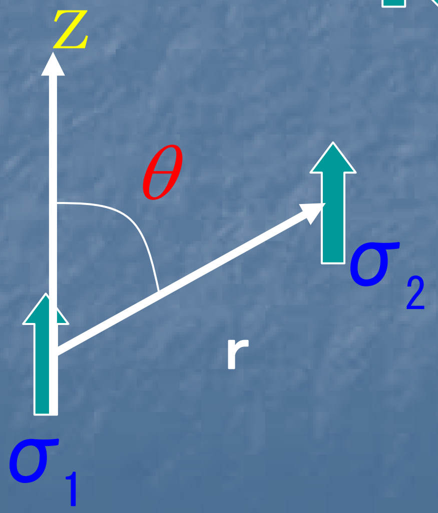
$$Y(x) = \exp(-x)/x,\ Z(x) = (1 + 3/x + 3/x^2)Y(x),$$



$$S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_2 \cdot \hat{\boldsymbol{r}}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \Rightarrow 3\cos^2\theta - 1$$

Tensor operator $(\hat{\boldsymbol{r}} = \boldsymbol{r}/r). \Rightarrow -3\cos^2\theta + 1$





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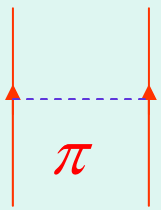

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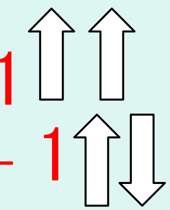
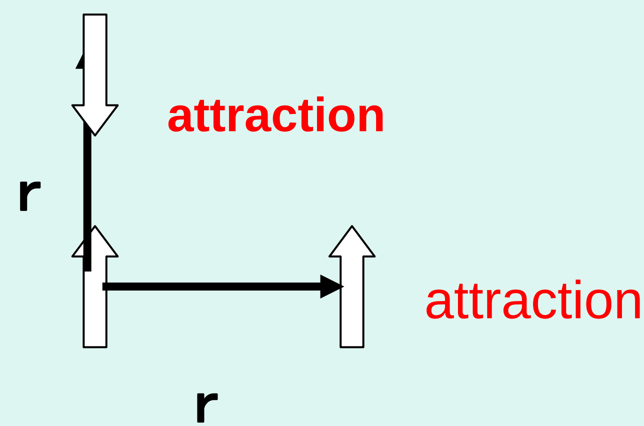
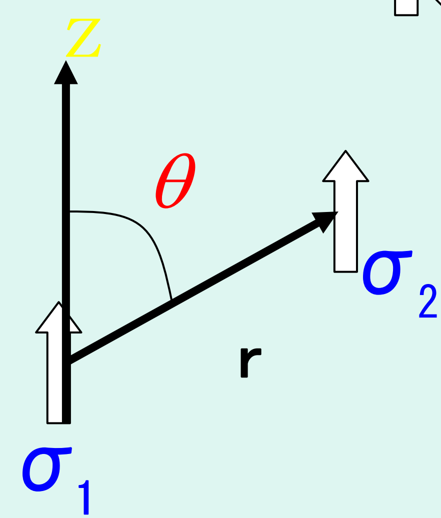
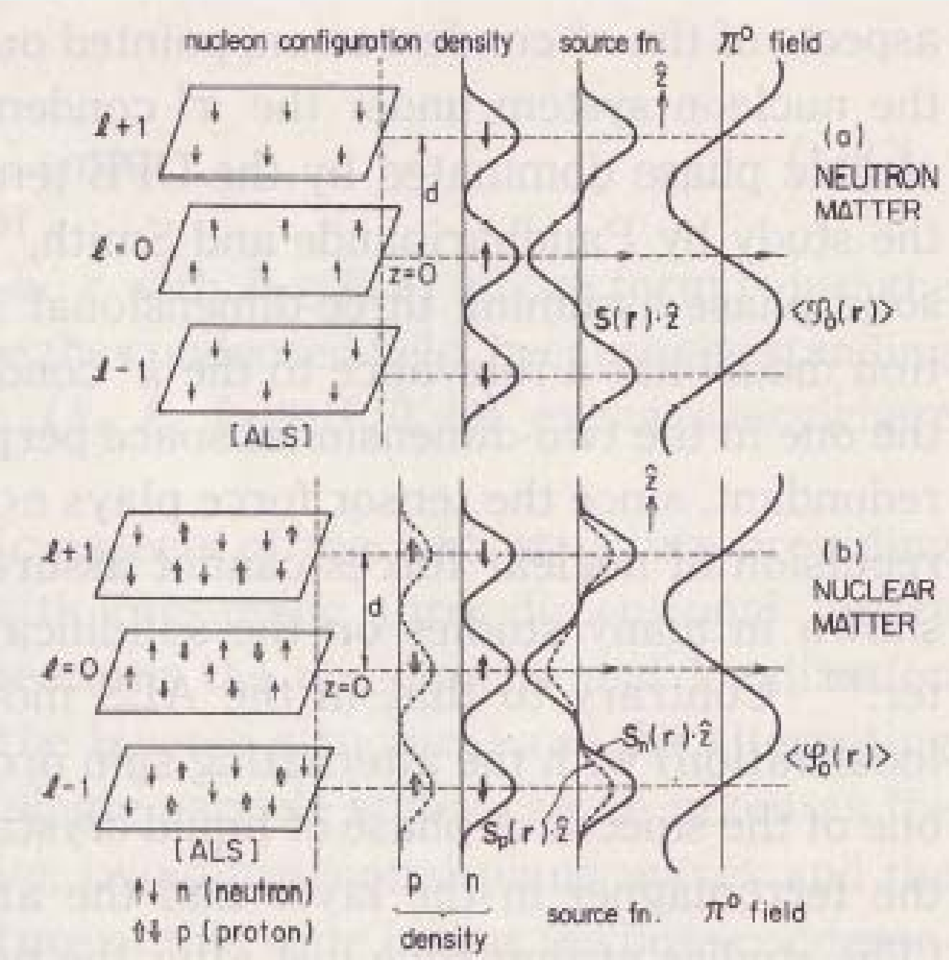






Pion condensation and Tensor force



Pion condensed phase
=Alternating-Layer Spin (ALS) structure of the nucleon System
(R.Tamagaki et al (1976~))

cf. PTP suppl.112(1993)

$$(\nabla^2 - m_\pi{}^2)\langle\varphi_0(\boldsymbol{r})\rangle = \tilde{f}\,\nabla\cdot\boldsymbol{S}(\boldsymbol{r})$$

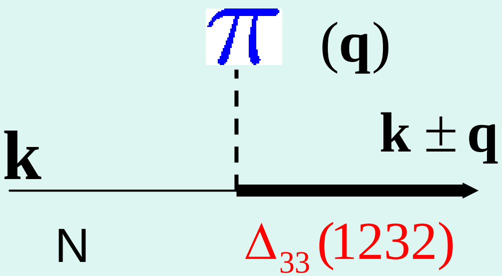
$$\boldsymbol{S} = \langle\Phi_N|\psi^\dagger(\xi,t)\tau_3\boldsymbol{\sigma}\psi(\xi,t)|\Phi_N\rangle$$

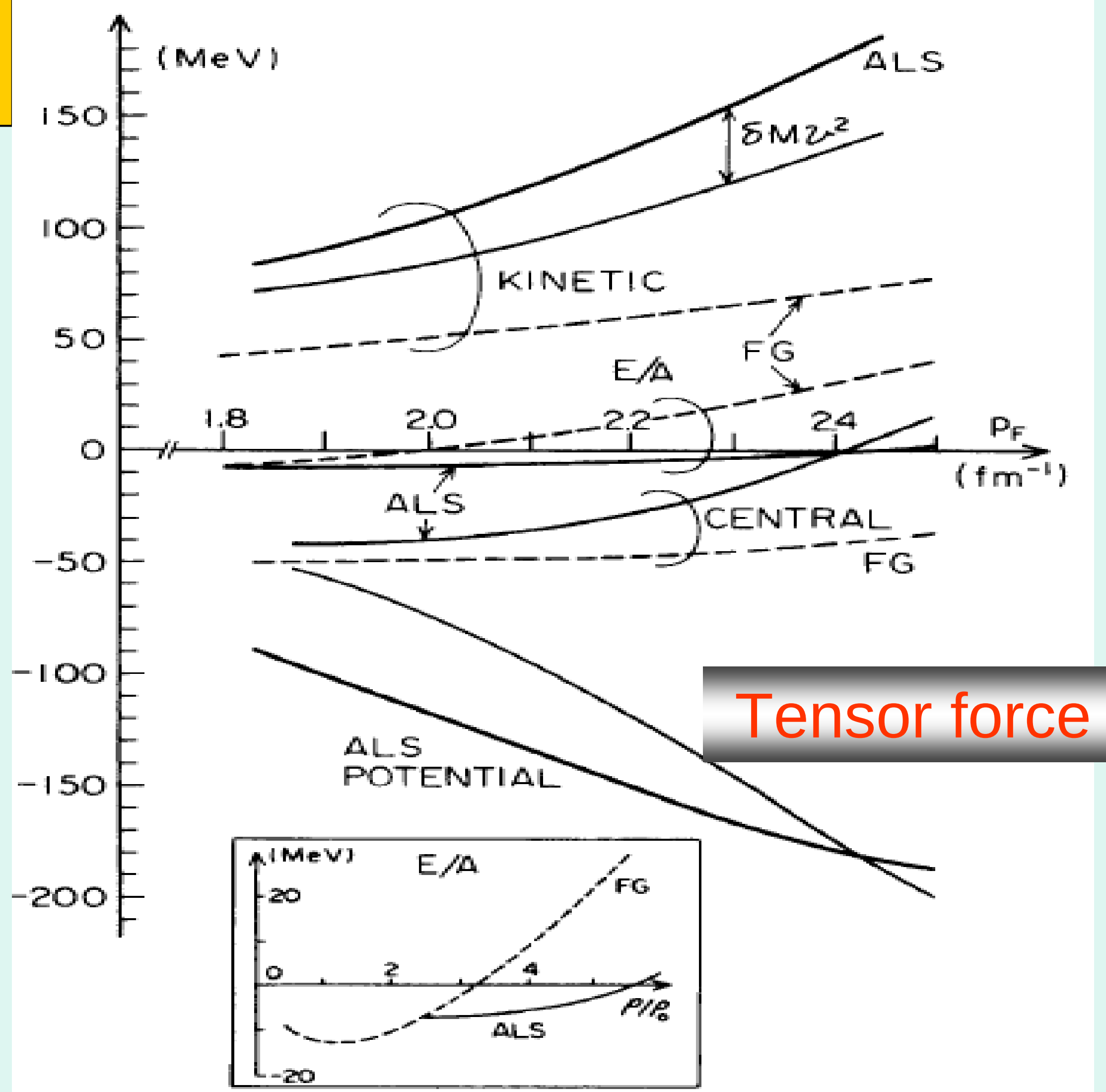
$$\langle\varphi_0(\boldsymbol{r})\rangle = \frac{f}{4\pi}\int d\xi'\langle\psi^\dagger(\xi')\tau_3'\boldsymbol{\sigma}'\psi(\xi')\rangle\cdot\nabla' Y(m_\pi|\boldsymbol{r}-\boldsymbol{r}'|)$$

p-wave Pion-condensed phase*=Tensor-force dominating phase

T.K.,T.Takatsuka, R.Tamagaki and T.Tatsumi,
PTP Suppl. 112('93), 123

EOS for pion-condensed N=Z Matter





and higher harmonics.

⇨ Wannier type rather than Bloch one

*D.W.L.Sprung and P.K. Banerjee,NPA168('71);
D.W.L. Sprung,NPA182('72),97.

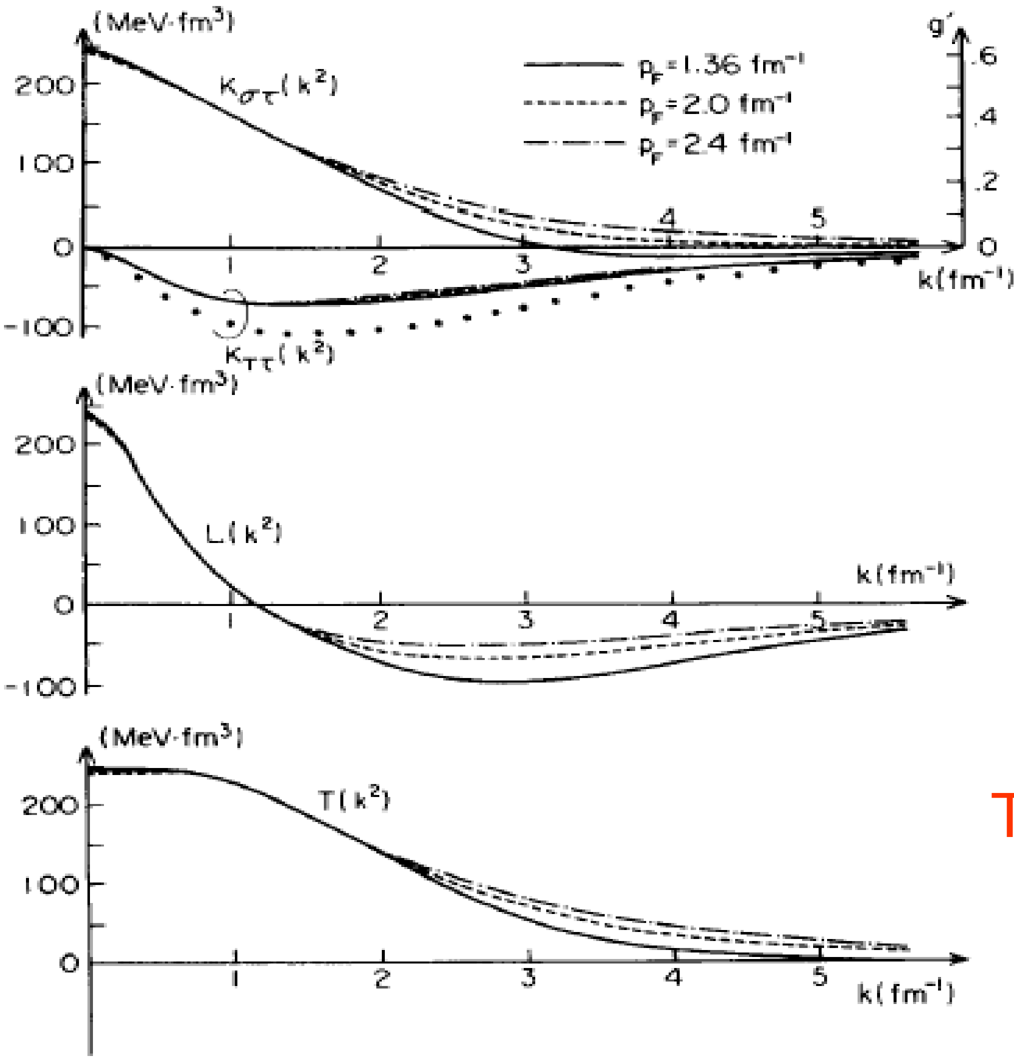
Effective Interaction(G-0 force)* in the $\sigma\tau$ channel

$$V_{ph}(1,2) = \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[\,V_{\sigma\tau}(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_{T\tau}S_{12}(\hat{\boldsymbol{r}})]\,,$$

$$= \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\int\frac{d\boldsymbol{k}}{(2\pi)^3}[(\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{k}})(\boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{k}})L(k^2)+(\boldsymbol{\sigma}_1\times\hat{\boldsymbol{k}})\cdot(\boldsymbol{\sigma}_2\times\hat{\boldsymbol{k}})T(k^2)]e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$L(k^2) = 2K_{T\tau}(k^2)+K_{\sigma\tau}(k^2),\quad T(k^2) = -K_{T\tau}(k^2)+K_{\sigma\tau}(k^2)$$

*D.W.L.Sprung and P.K. Banerjee,NPA168('71); D.W.L. Sprung,NPA182('72),97.



Central and Tensor

Longitudinal (pion) channel

Transverse(ρ meson) channel

Role of the vector mesons

ρ and ω mesons

Tensor coupling v.s. vector coupling

- cf. Chiral bag model (G.E.Brown)

The E.M. form factor of the nucleon based on
the Vector Meson Dominance

$$\bar{\psi}\sigma_{\mu\nu}\tau^k\psi(\partial^\mu\rho_k{}^\nu - \partial^\nu\rho_k{}^\mu) \Rightarrow (\sigma_1\times\mathbf{q})\cdot(\sigma_2\times\mathbf{q}) = -S_{12}(\mathbf{q}) + \frac{2}{3}\sigma_{\mathbf{1}}\cdot\sigma_2 q^2$$

Tensor coupling,
couplibng to the transverse
Spin density

the opposite sign to OPEP

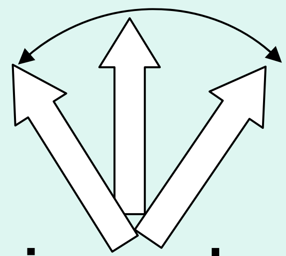
the same sign as OPEP

Determining the range of
Tensor force

Rho meson condensation

T.K., PTP 60 (1978), 1229; master thesis .

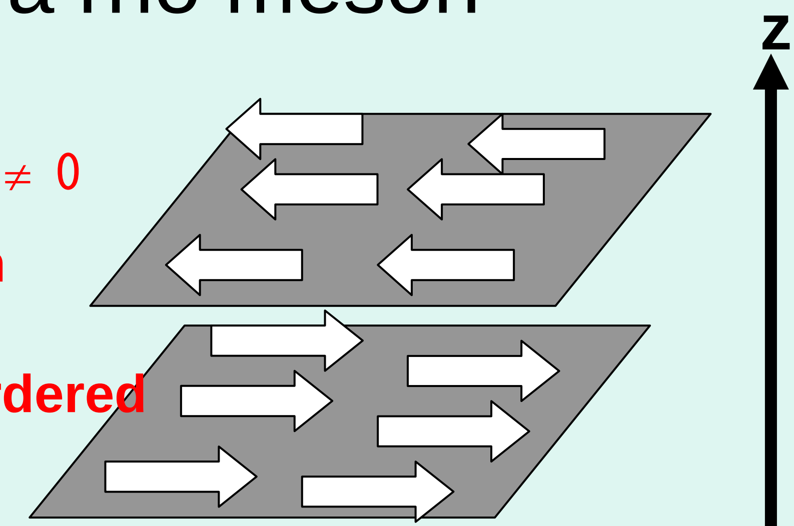
- The nuclear spin direction oscillates at higher densities,



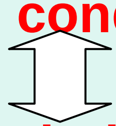
- And then

bend down, which is a rho meson condensed state.

$$\langle \rho_z^0 \rangle \neq 0$$



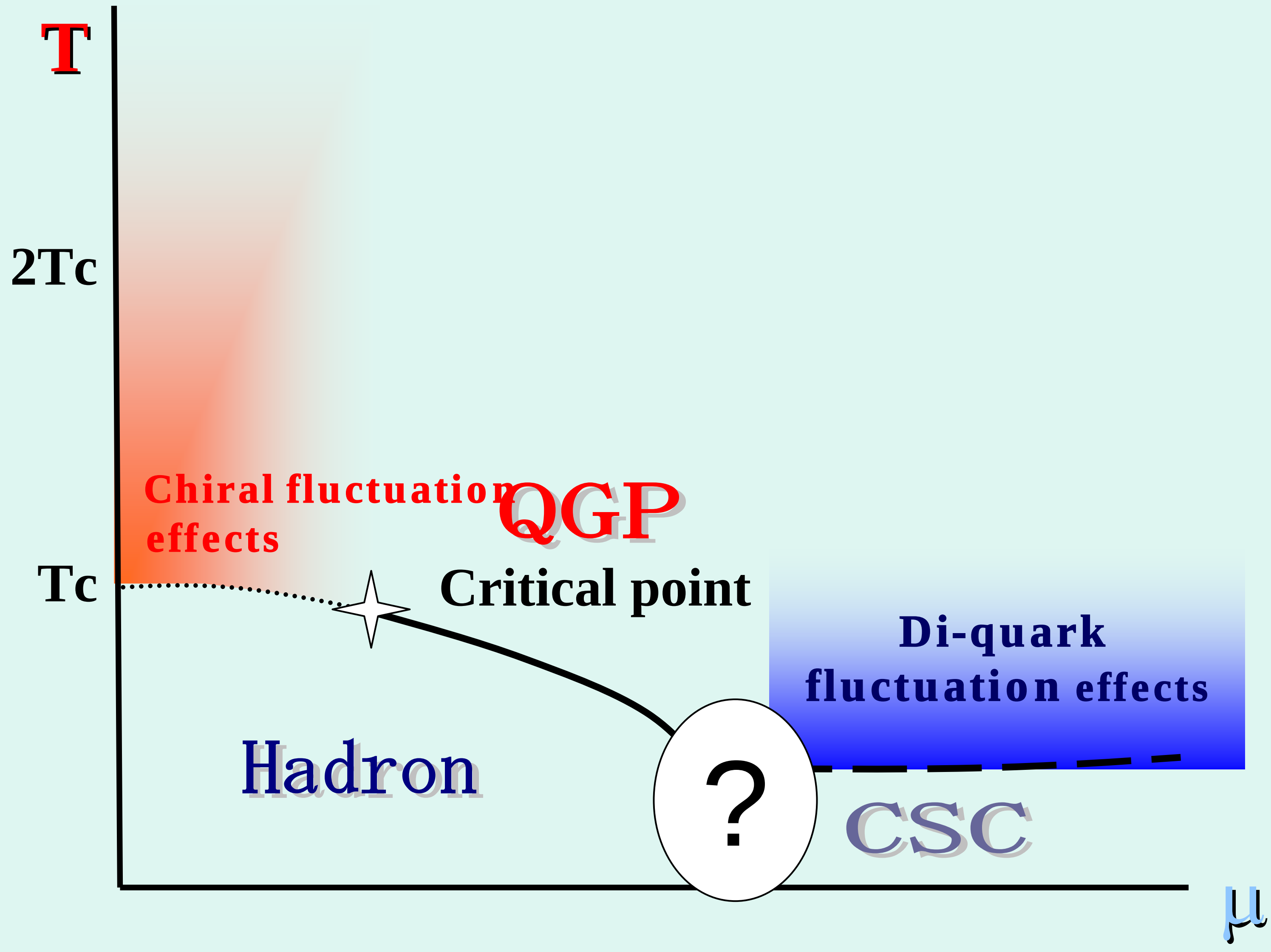
Rho meson condensation



Transverse spin-isospin ordered baryonic matter

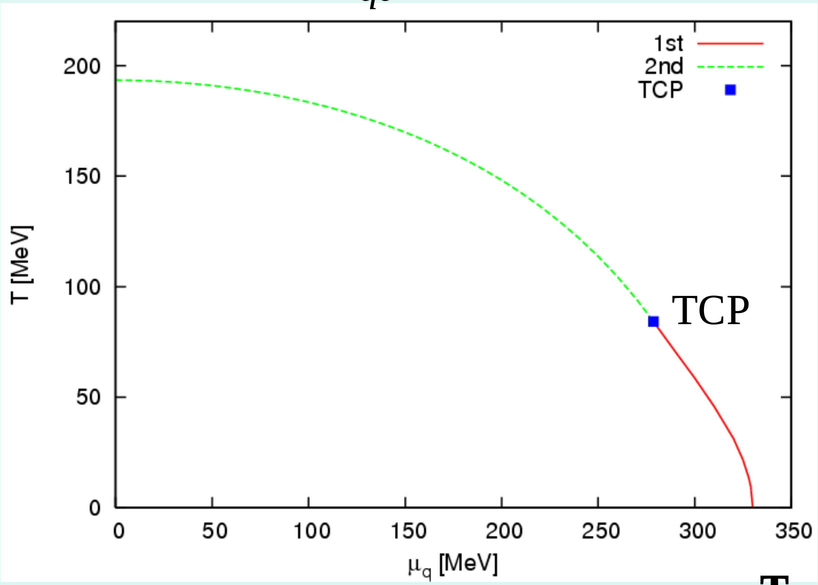
c.f. H. Toki and J.R. Comfort, PRL 47, (1981), 1716.

Conjectured QCD phase diagram

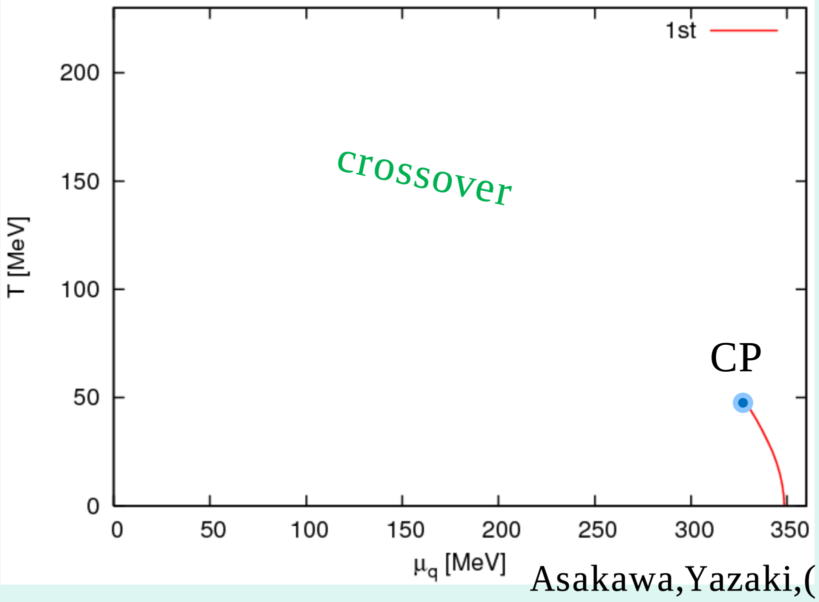


Phase diagram in NJL model

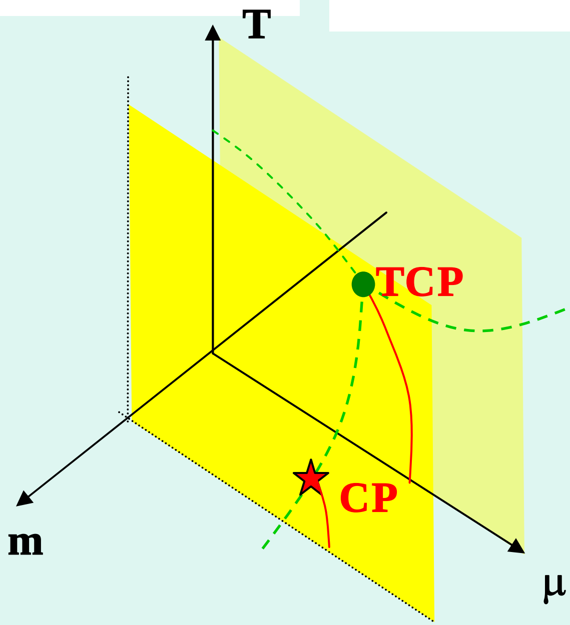
$m_{q0} = 0$



$m_{q0} = 5.5$ MeV

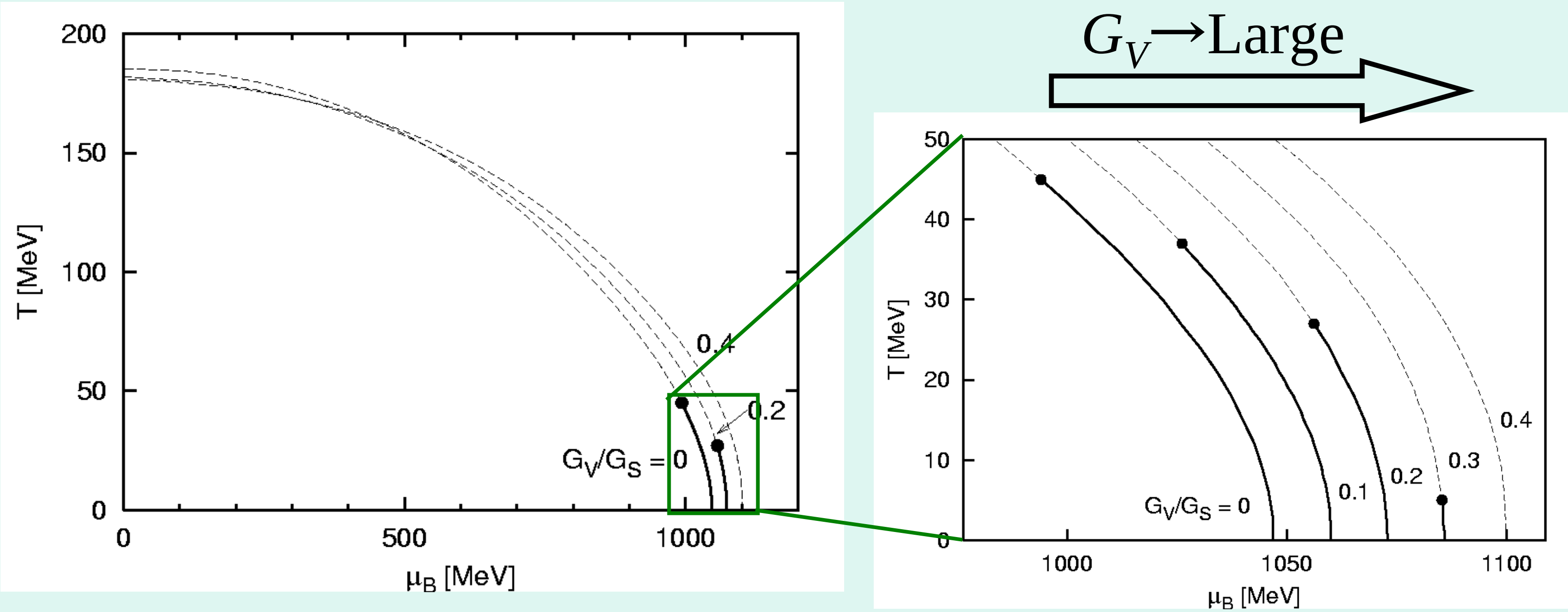


Asakawa,Yazaki,(1989)



Caution!

★ Effects of G_V on Chiral Restoration



First Order
Cross Over

As G_V is increased,

- Chiral restoration is shifted to higher densities.
- The phase transition is weakened.

Asakawa,Yazaki ’89 /Klimt,Luts,&Weise ’90 / Buballa,Oertel ’96

What would happen when the CSC joins the game?

★ Importance of Vector-type Interaction

- Vector interaction naturally appears in the effective theories.
  - Instanton-anti-instanton molecule model Shaefer,Shuryak ('98)

$$L = G\left\{\frac{2}{N_C^2}\left[(\bar\psi\tau^a\psi)^2 + (\bar\psi\tau^a i\gamma_5\psi)^2\right] - \frac{1}{2N_C^2}\left[(\bar\psi\tau^a\gamma^\mu\psi)^2 + (\bar\psi\tau^a\gamma^\mu\gamma_5\psi)^2\right]\right\} + L_8$$

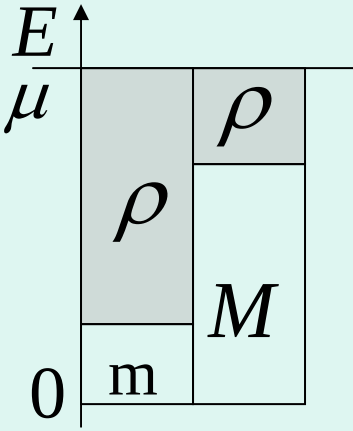
$\Rightarrow$ $G_V / G_S = 1/4$

  - Renormalization-group analysis N.Evans et al. ('99)

$$L_{LL}^0 = G_{ll}\left\{(\bar\psi_L\gamma^0\psi_L)^2 - (\bar\psi_L\gamma^\mu\psi_L)^2\right\}$$

$-G_V\left(\bar\psi\gamma^\mu\psi\right)^2$ $\Leftrightarrow$ density-density correlation

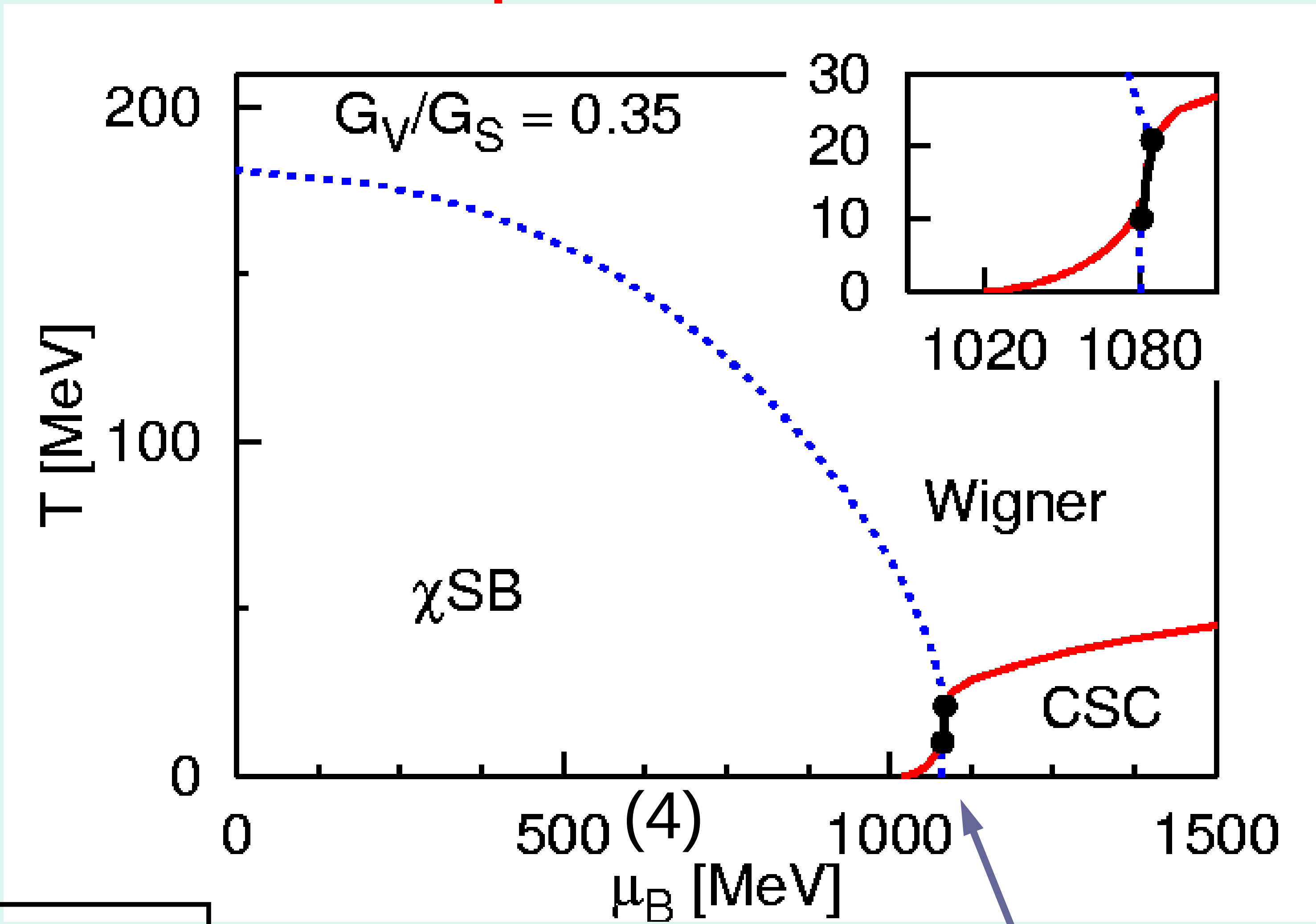
- $-G_V\left(\bar\psi\gamma^0\psi\right)^2 \to -G_V\left\langle\bar\psi\gamma^0\psi\right\rangle^2 = -G_V\rho^2 \quad \rho = \left\langle\bar\psi\gamma^0\psi\right\rangle$



Chiral restoration is punished by the vector interaction!

With color superconductivity transition incorporated: Two critical end point!

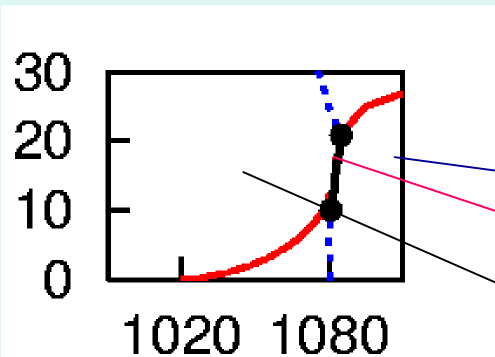
M. Kitazawa, T. Koide, Y. Nemoto and T.K. ('02)

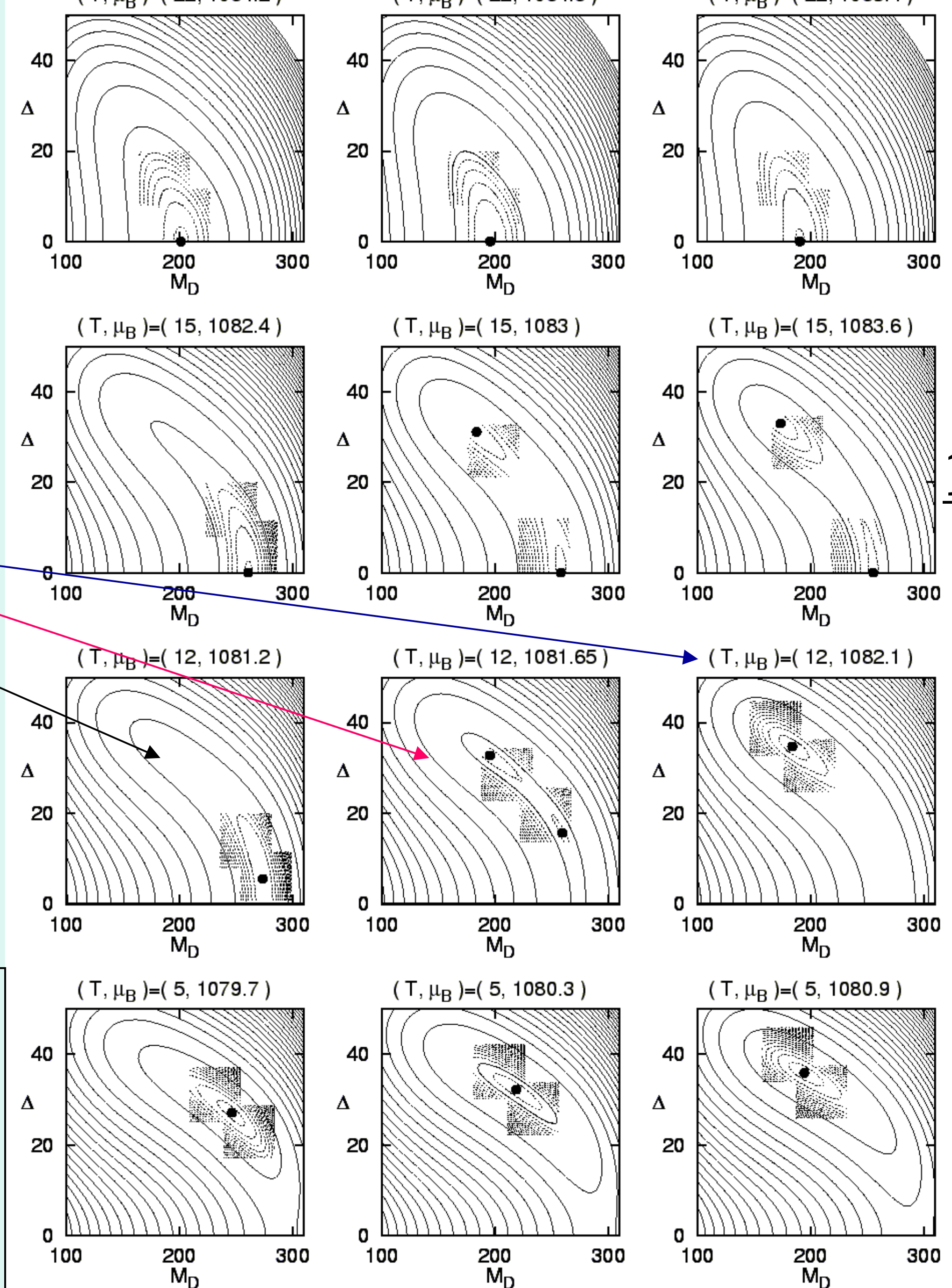


$G_V / G_S = 0.35$

Another end point appears from lower temperature, and hence **there can exist two end points** in some range of G_V!

Contour of ω with G_V/G_S=0.35





T=

22MeV

15MeV

12MeV

5MeV

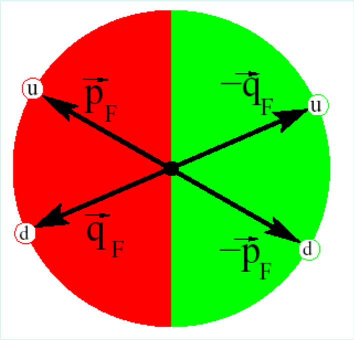
μ

Large fluctuation owing to the **interplay between χSB and CSC** enhanced by G_V.

Effect of electric chemical potential with neutral CSC

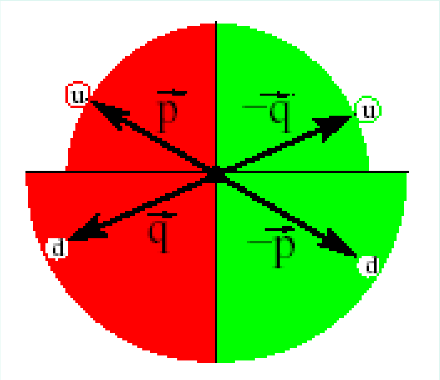
Asymmetric homogenous CSC with charge neutrality

$n_d > n_u > n_s$ ⟶ Mismatch cooper paring



Standard BSC paring , rare cace

$$\delta\mu, \delta m \rightarrow \delta p_F$$ ⇨



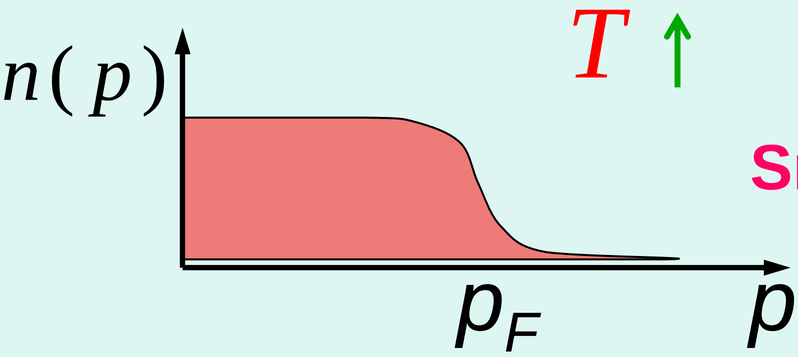
Mismatch paring or pair breaking, real case

For two flavor asymmetric homogenous CSC

$$\delta\mu = \frac{\mu_e}{2}$$ ⇨

- Abnormal thermal behavior of diquark gap
- Chromomagnetic instibility, imaginary meissner mass

Abnormal thermal behavior of diquark energy gap

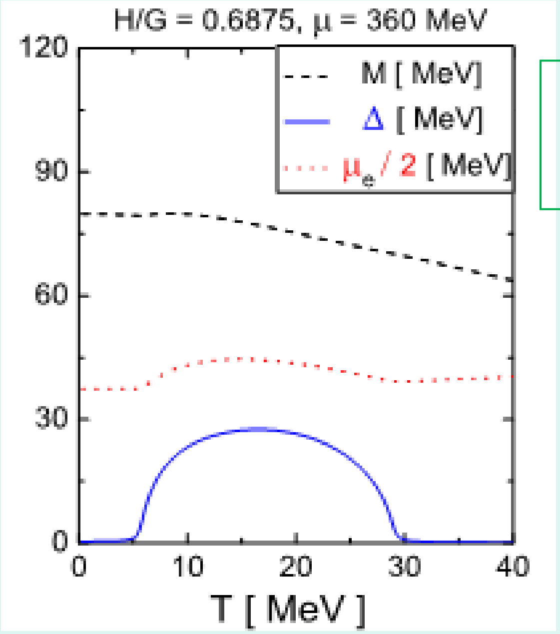


Smearing by T induces the pairing!

Double effects of T :

- Melting the condensate
- More and more components take part in cooper pairing

Competition between these two effects gives rise to abormal thermal behavior of diquark condensate



⇨ **Enhancing the competition between chiral condensate and diquark condensate for somewhat larger T,** leading to a nontrivial impact on chiral phase transition

Shovkovy and Huang, PLB 564, (2003) 205

Vector Interaction with Charge Neutrality
---- Multiple critical points ---

Z. Zhang and T. K., **Phys.Rev.D80:014015,2009**.;

Cf T.Hatsuda et al (2006) ; in chiral limit.

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - \hat{m}\,)\psi + G_S\left[(\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma_5\vec{\tau}q(x))^2\right]$$ chiral

$$+\; G_D\sum_A\left[\bar{q}(x)\gamma_5\tau_2\lambda_A q_C(x)\right]\left[\bar{q}_C(x)\gamma_5\tau_2\lambda_A q(x)\right]$$ di-quark

$$-\,G_V\sum_{i=0}^{3}\left[(\bar{q}(x)\gamma^\mu\tau_i q(x))^2 + (\bar{q}(x)i\gamma^\mu\gamma_5\tau_i q(x))^2\right]$$ vector

$$-\,K\left\{\det_f\left[\bar{\psi}\,(1+\gamma_5)\,\psi\right] + \det_f\left[\bar{\psi}\,(1-\gamma_5)\,\psi\right]\right\}$$ anomaly

for 2+1 flavors

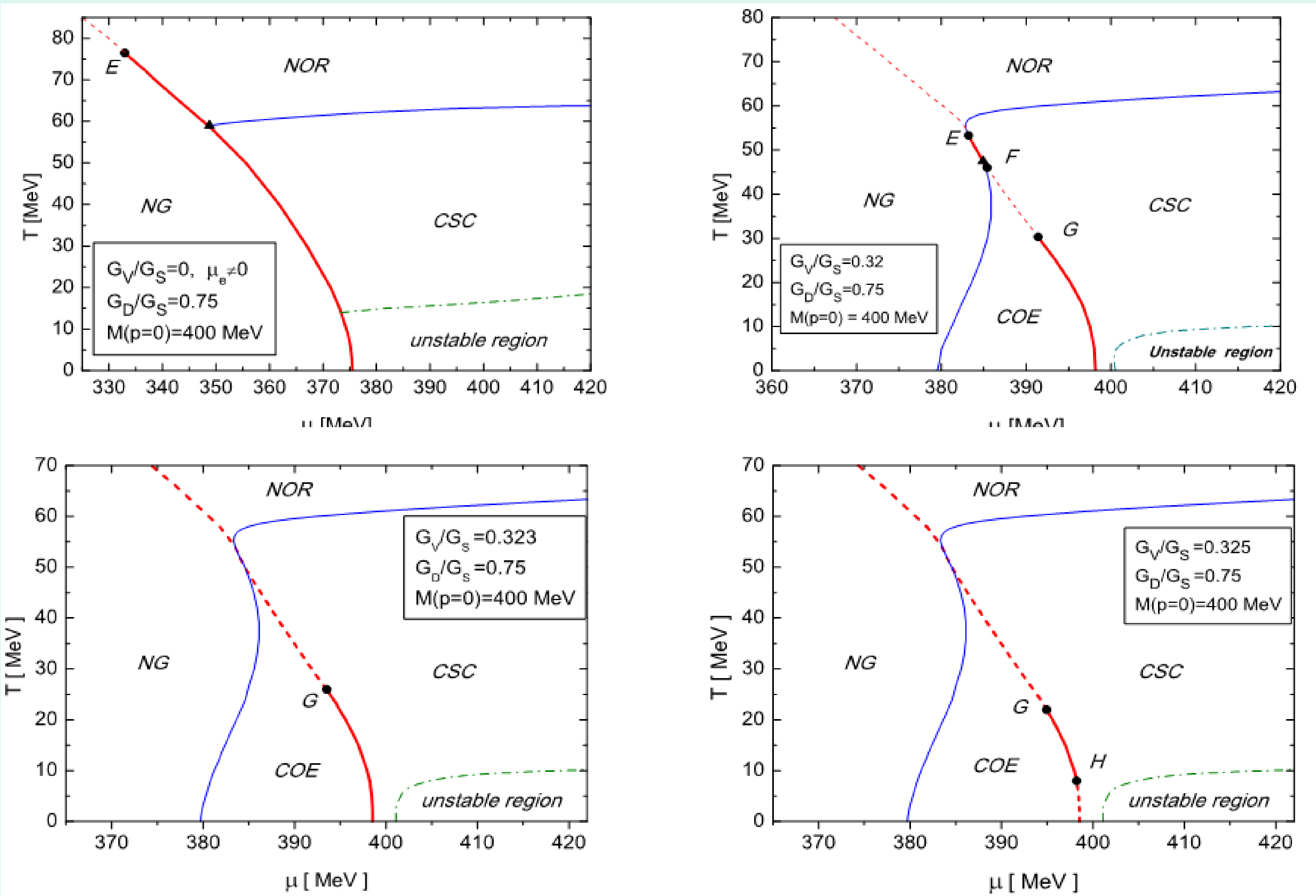
T.Hatsuda and T.K. PLB206 (1988), 385; T.K. PLB219 (1989), 363 $T \neq 0$

Z. Zhang and T. K., Phys.Rev.D80:014015,2009.;

Cf T.Hatsuda et al (2006) ; in chiral limit.

see also for the similar role of charge neutrality,

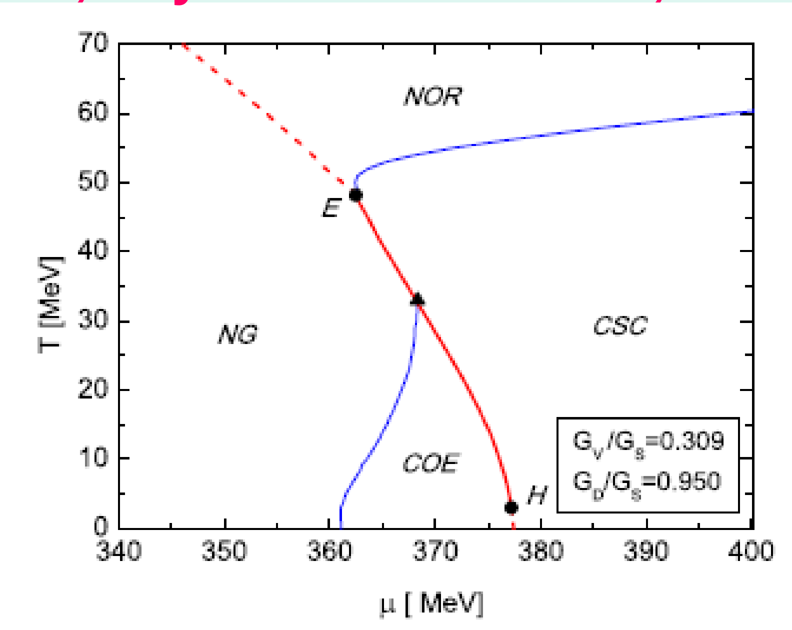
Z.Zhang, K.Fukushima and T.K. Phys. Rev. D **79**, 014004 (2009).

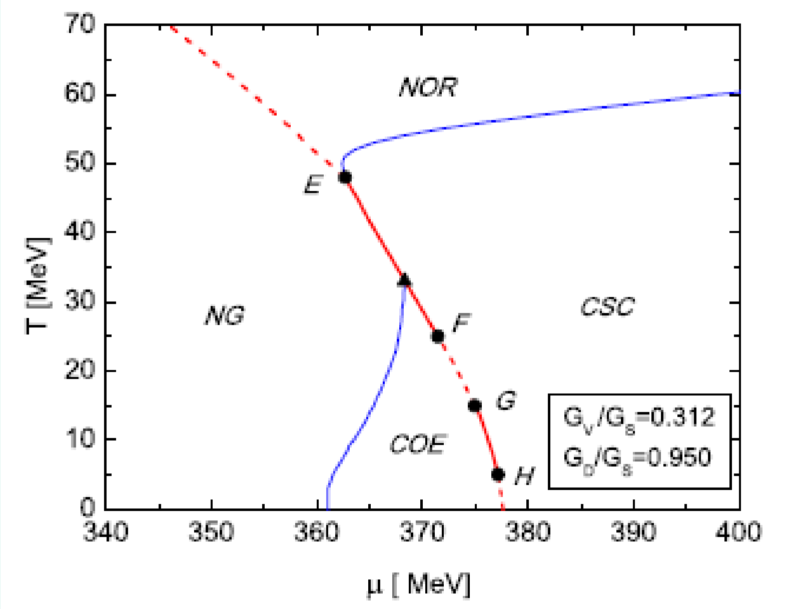


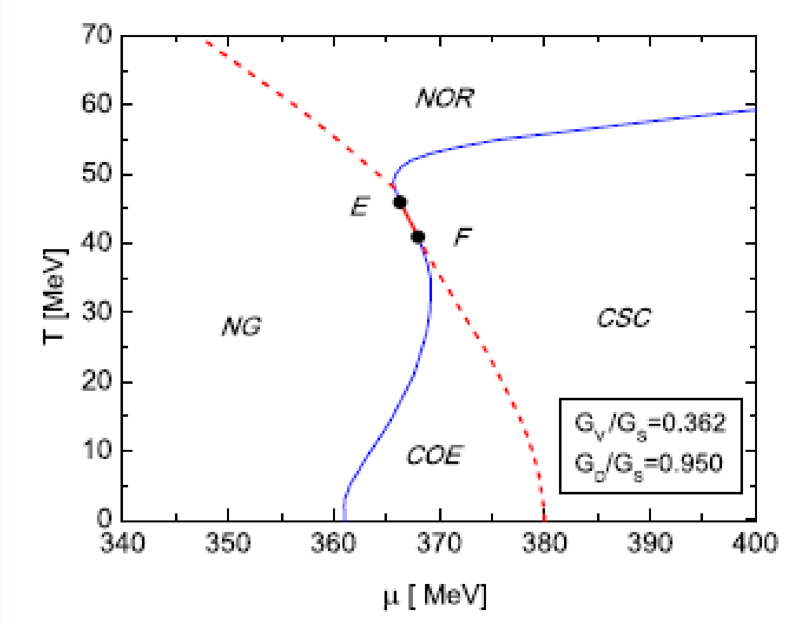
Z. Zhang and T. K., Phys.Rev.D80:014015,2009.;

2+1 flavor case

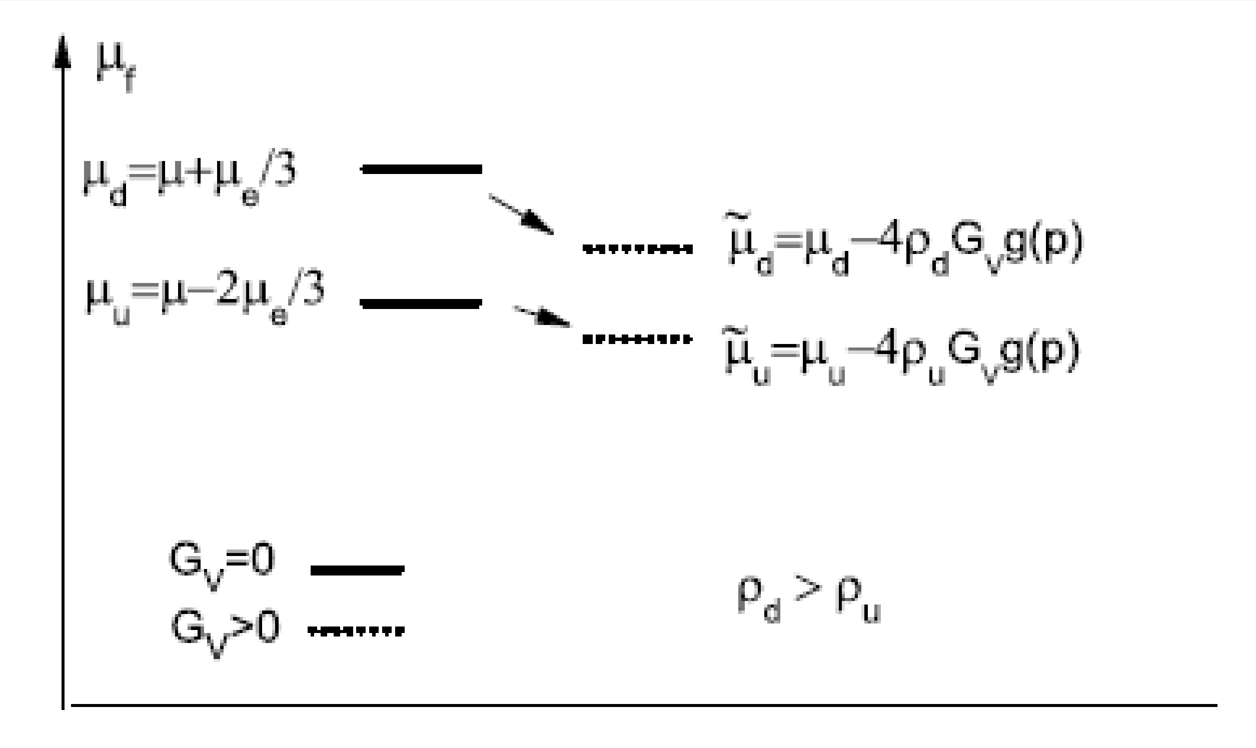
$$m_{u,d} = 5.5\mathrm{MeV} \quad m_s = 140\mathrm{MeV}$$







Effects of the vector interaction on the effective chemical potentials

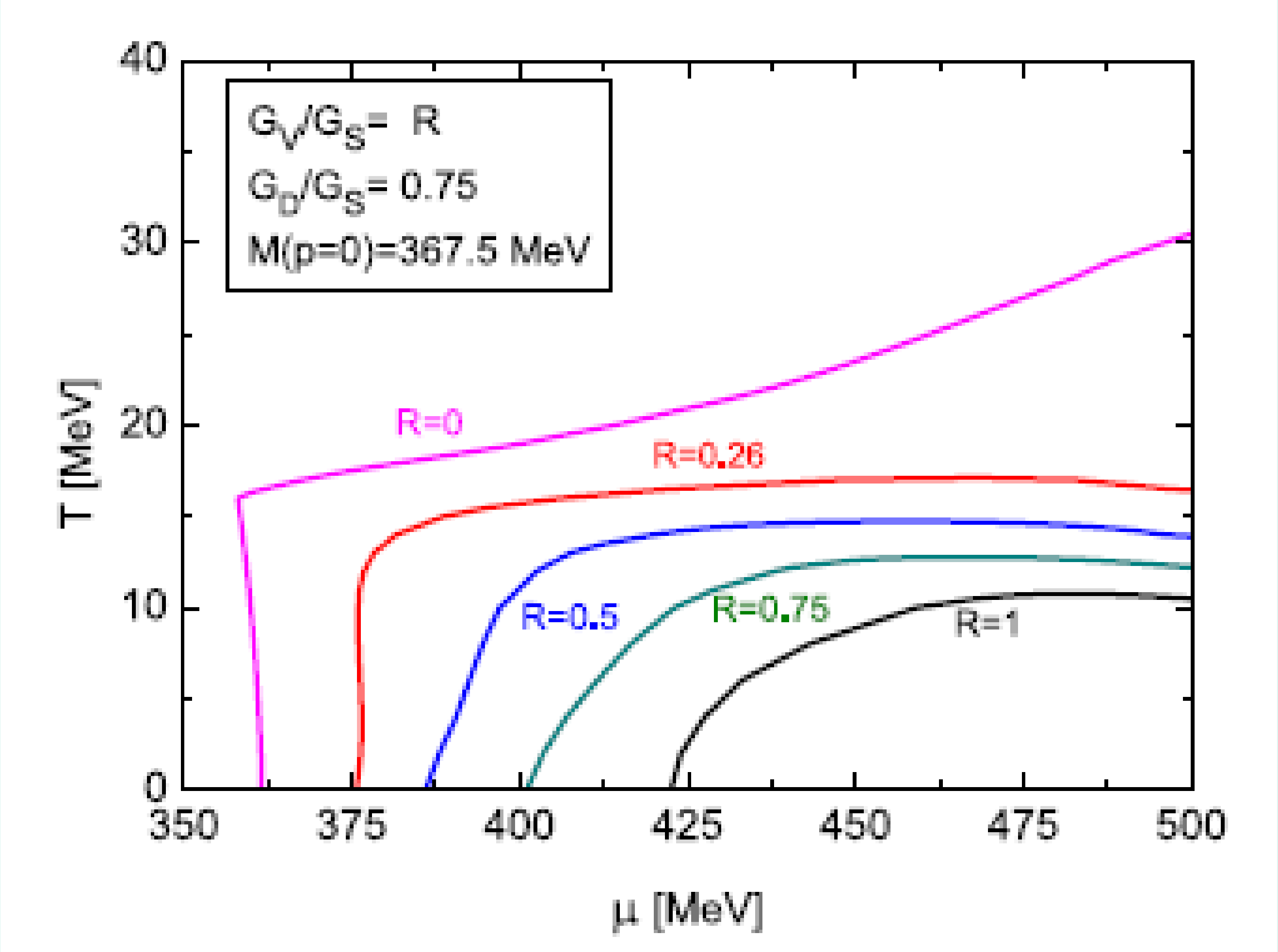


The vector interaction tends to suppress the mismatch of the Fermi spheres of the Cooper pairs.

Suppression of the Chromomagnetic instability!

Suppression of the Chromomagnetic instability due to the vector interaction!

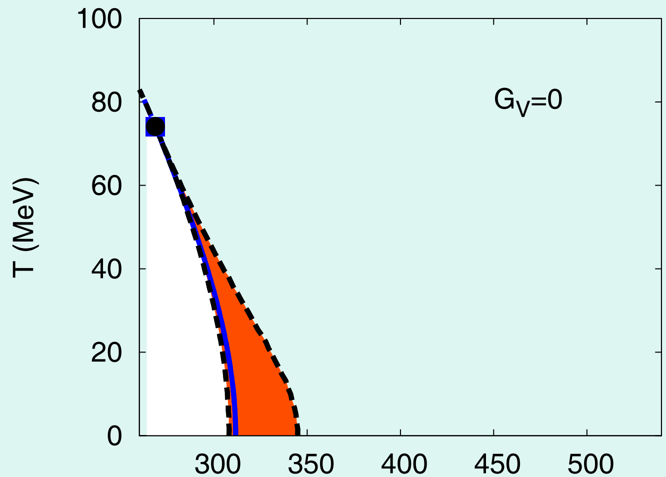
Z. Zhang and T. K., **Phys.Rev.D80:014015,2009**.;

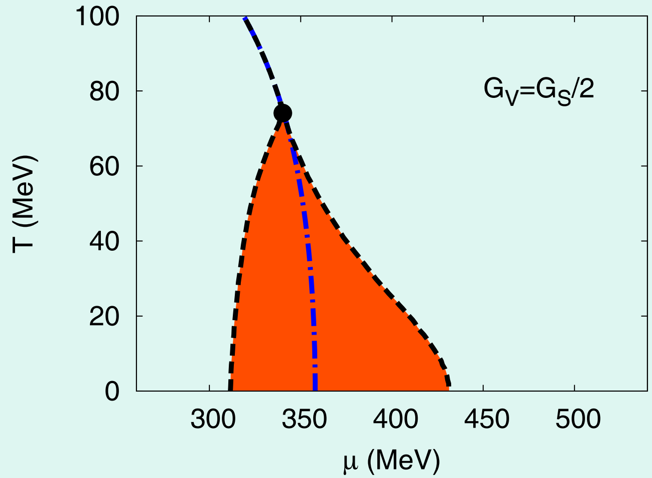


(Partial) resolution of the chromomagnetic instability problem!

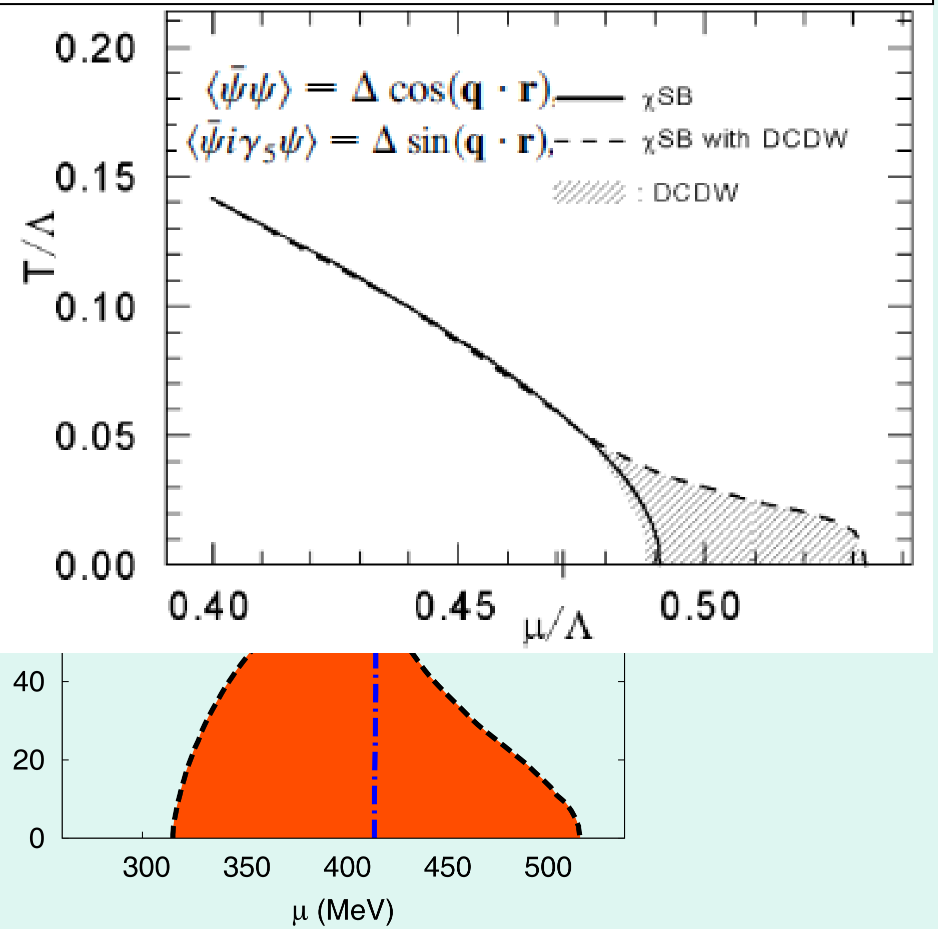
S.Carignano, D. Nickel and M. Buballa, arXiv:1007.1397

$$M(\mathbf{x}) = \mathbf{m} - \mathbf{2G}_s\left(S(\mathbf{x}) + iP(\mathbf{x})\right)$$





E. Nakano and T.Tatsumi, PRD71 (2005)



Interplay between G_V and Polyakov loop is not incorporated;
see also P. Buescher, Mater thesis submitted by Darmstadt University,where Ginzburg-Levanyuk analysys shows also an existence of Lifschitz point at finite G_V.

⇒ Spatial dependence of Polyakov loop should be considered explicitly.

S.Carignano, D. Nickel and M. Buballa, arXiv:1007.1397

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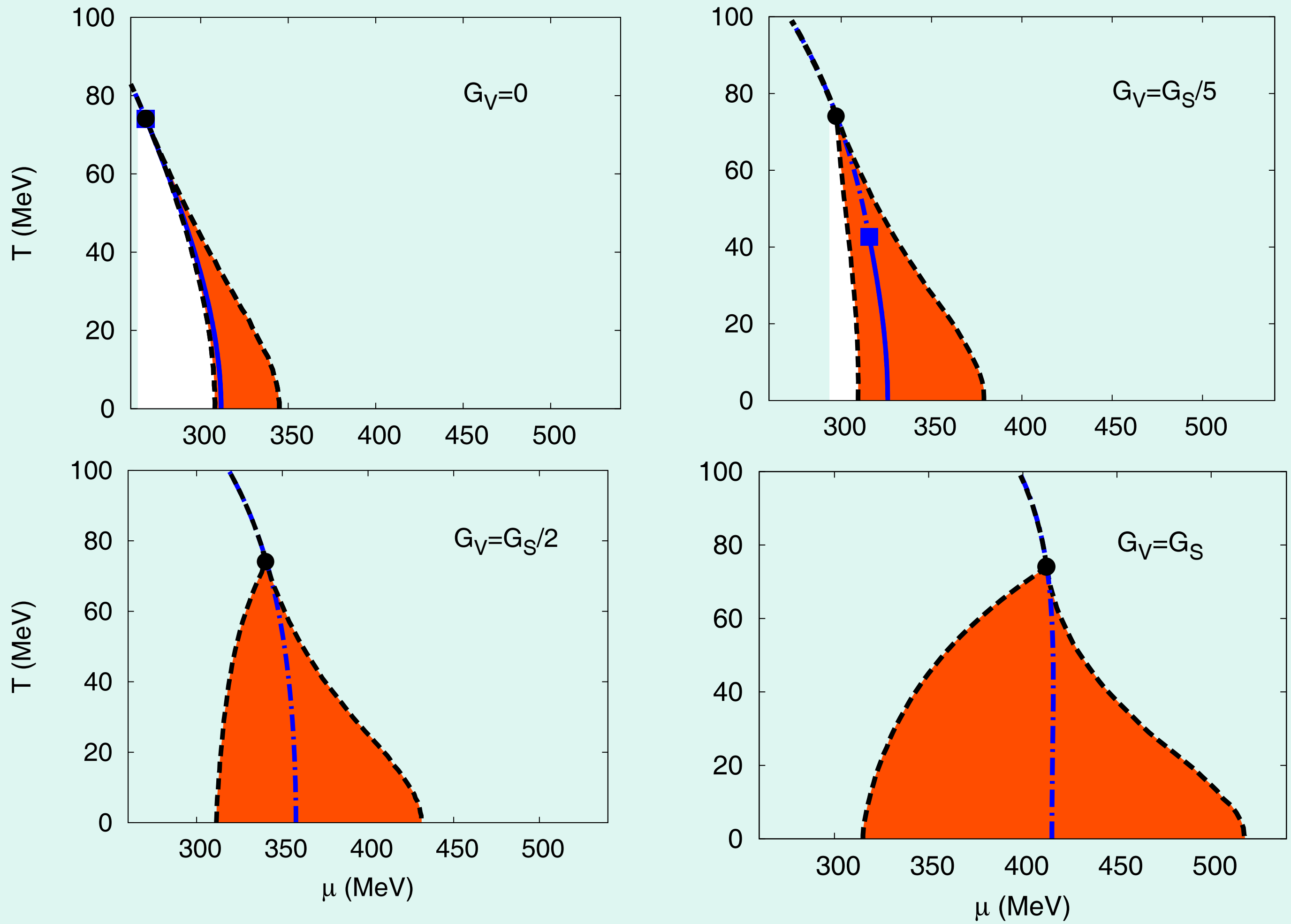

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⇨ Spatial dependence of Polyakov loop should be considered explicitly.

Possible (T, μ) dependence of g_V

T.K. Phys. Lett. B271 (1991), 395:

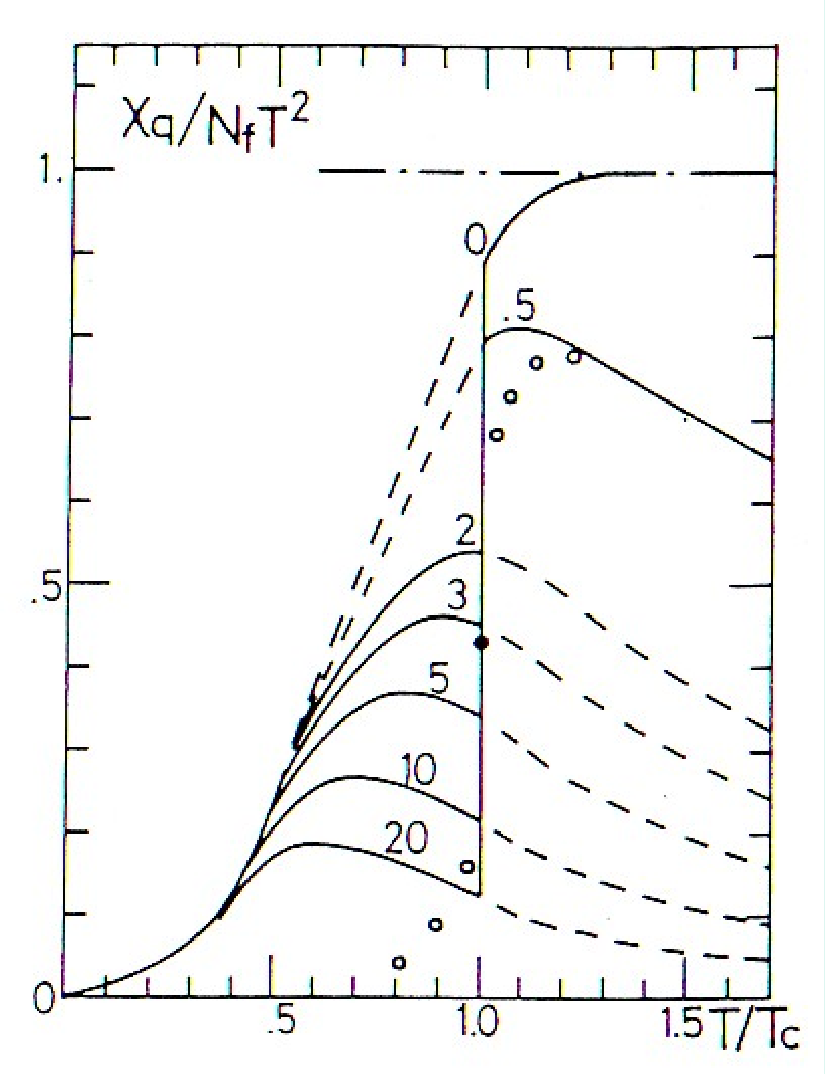
$$\kappa_T = \frac{\chi_q}{\rho^2}$$

$$\kappa_T \equiv -N_q^{-1}(\partial V/\partial \mu_q)_{T,N_q}$$

isothermal compressibility

$$\chi_q = \frac{\chi_q^{(0)}(T)}{1 + 2g_V\, \chi_q^{(0)}(T)}$$

g_V sould decrease as T is raised.



$\mu \neq 0$

The vector-scalar mixing
Arises:

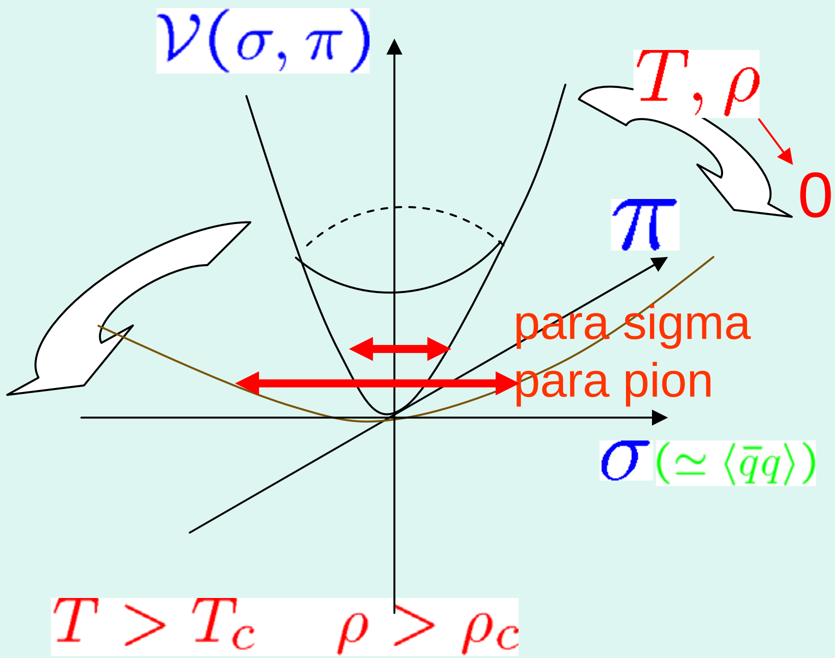
$$\chi_q = \frac{D_s^{-1}(0)\chi_q^{(0)} + 2g_s(\chi_{VS}^{(0)})^2}{D_s^{-1}(0)D_v^{-1}(0) + 4g_v g_s \chi_{VS}^{(0)}\chi_q^{(0)}}$$

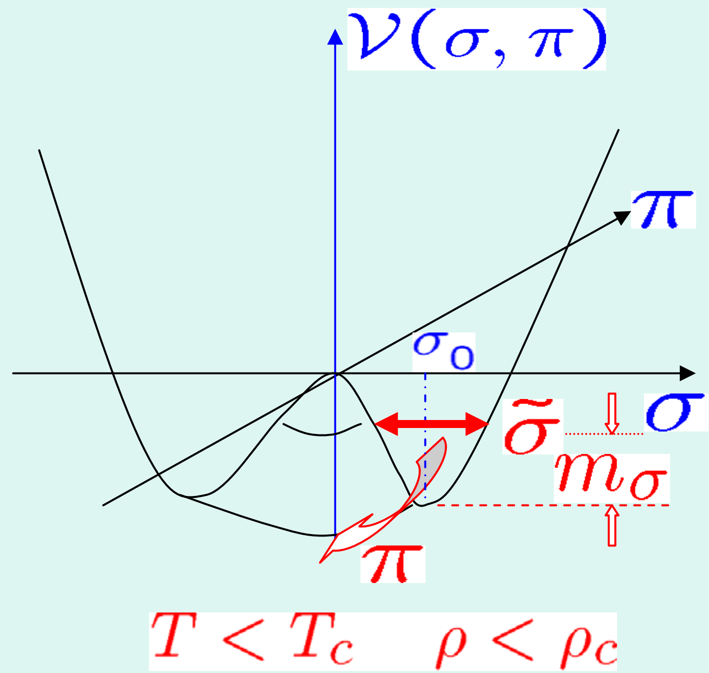
The diverging behavior of the density fluctuation around the CP ;
T.K., *CONFINEMENT 2000* (World Scientific, 2001), p. 287;
hep-ph/0007173.

3.Dynamical Critical Phenomena

Y. Minami and T.K.,
Prog. Theor. Phys.122,
(2009),881

Chiral Transition and the collective modes





The low mass sigma in vacuum is now established:

pi-pi scattering; Colangero, Gasser, Leutwyler('06) and many others

Full lattice QCD ; SCALAR collaboration ('03)

pi-pi,q-qbar, tetra quark, glue balls, or their mixed st's?

c.f. The sigma as the Higgs particle in QCD

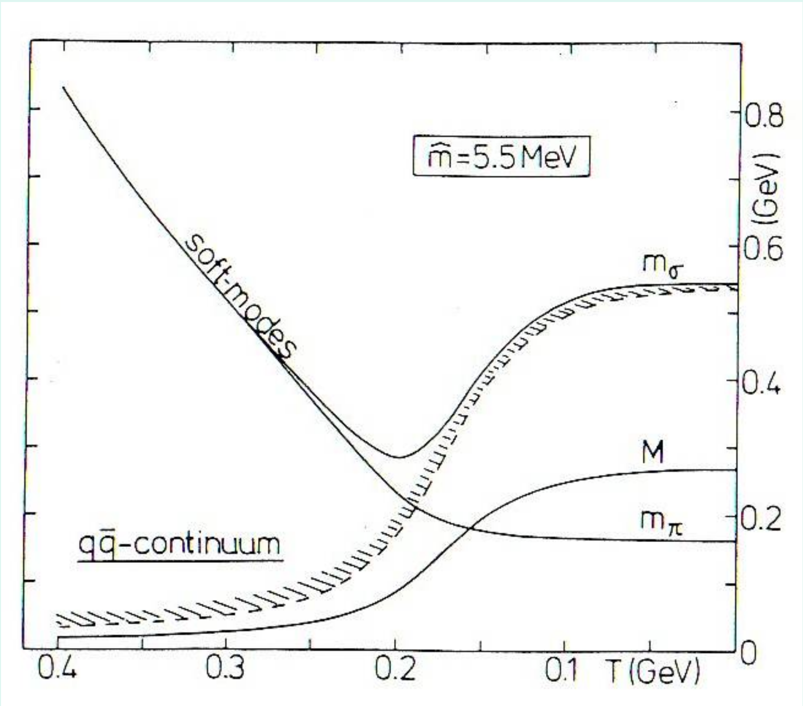
$$\sigma = \sigma_0 + \tilde{\sigma}$$

ϕ ; Higgs field $\Rightarrow$ $\phi = \langle\phi\rangle + \tilde{\phi}$

Higgs particle

Restoration of symmetry and change of hadron properties in the (classic) NJL model

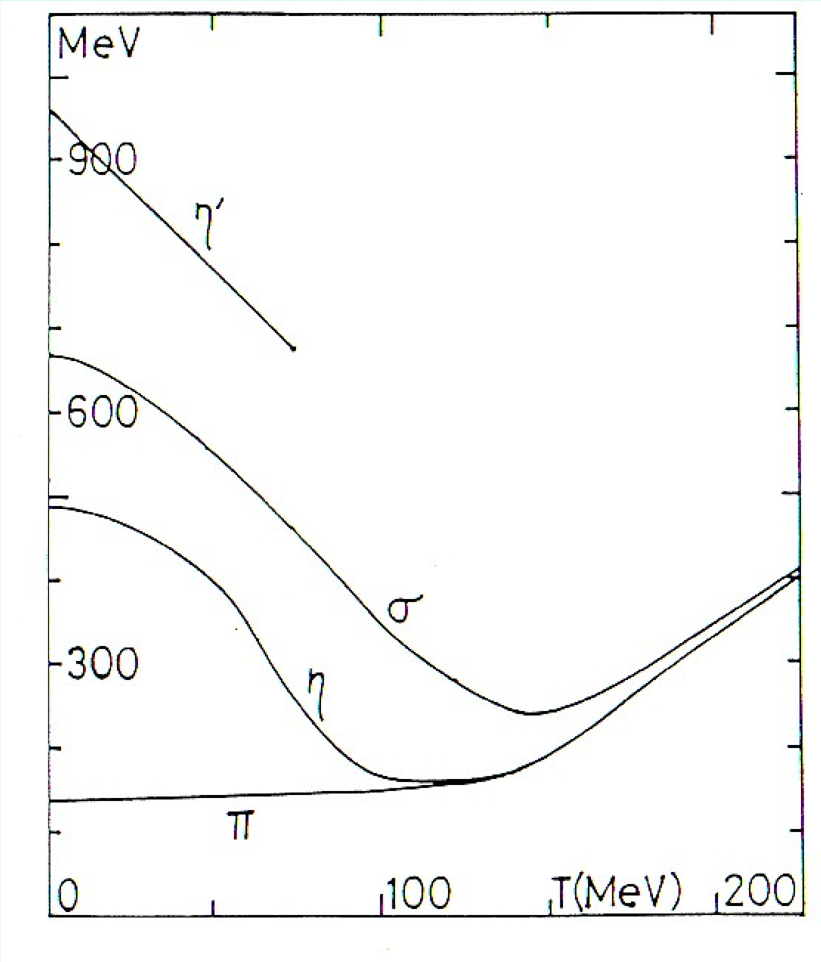
Softening (decrease) of the sigma meson



T. Hatsuda and T.K., PRL 55(1985),158

Effective restoration of U_A(1) symm.

(R. Pisarski and F. Wilczek (1985))

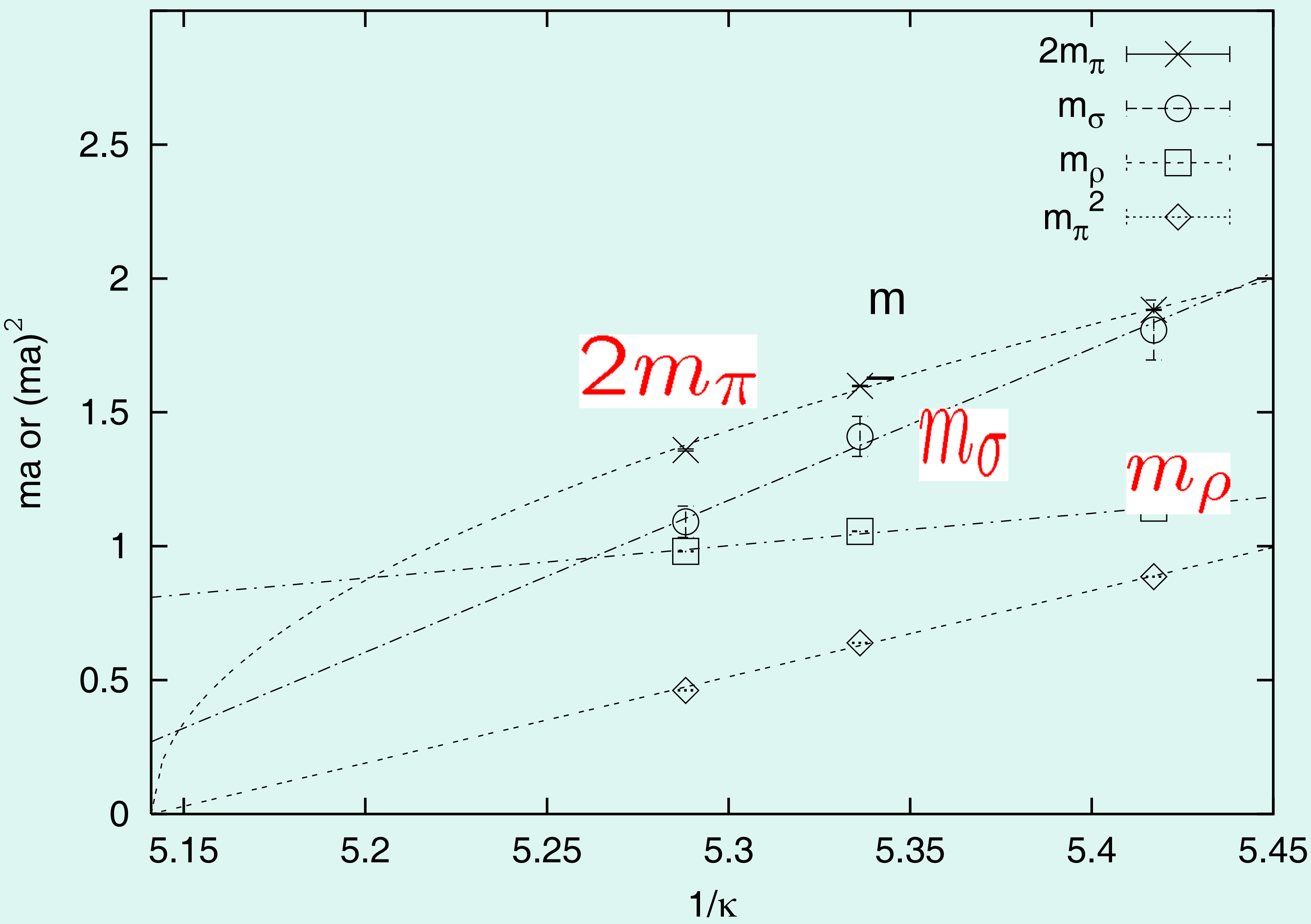


NJL with KMT term; T.K. PLB219 (1989),363

A full QCD calculation

The Scalar Collaboration:
S. Muroya, A. Nakamura, C. Nonaka, M. Sekiguchi, H. Wada, T. K.

Phys. Rev. D70, 034504(2004)



What's next ?

MANY!

- Better Simulation
  - Larger lattice; continuum limit → Glue ball is strongly dep. on a, lattice spacing. (UKQCD)
  - Error reduction, eg. smearing
    - Large errors come from

$$\left\langle (\sigma(x) - \langle \sigma \rangle)(\sigma(y) - \langle \sigma \rangle) \right\rangle$$

  --variational method with multiple interpolating op's.
  an explicit inclusion of tetraquark operator.
  - volume dependence; resonance or scattering
  - chiral fermions, Nc-dependence, etc

- Observables which are sensitive to the inner structure? Possible role of axial anomaly?....
- the sigma at finite T?

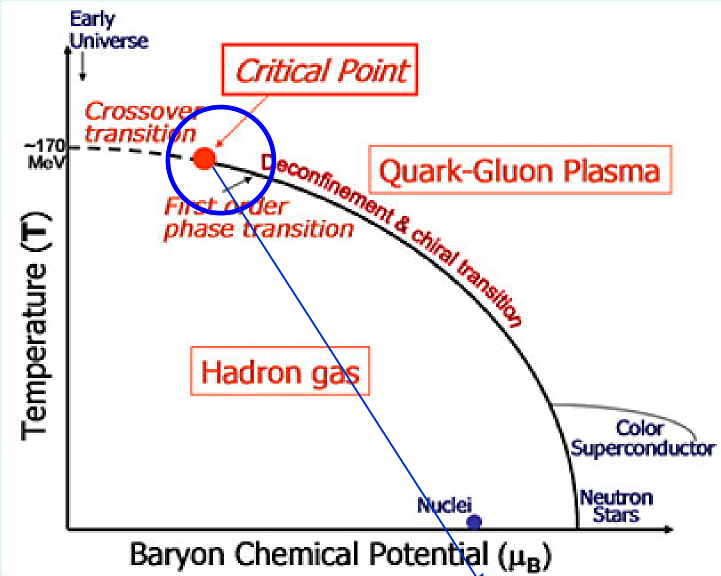
The κ

- full QCD simulation with multiple interpolating op's including a tetraquark
- operator with variational method, and analysis of the volume dependence.

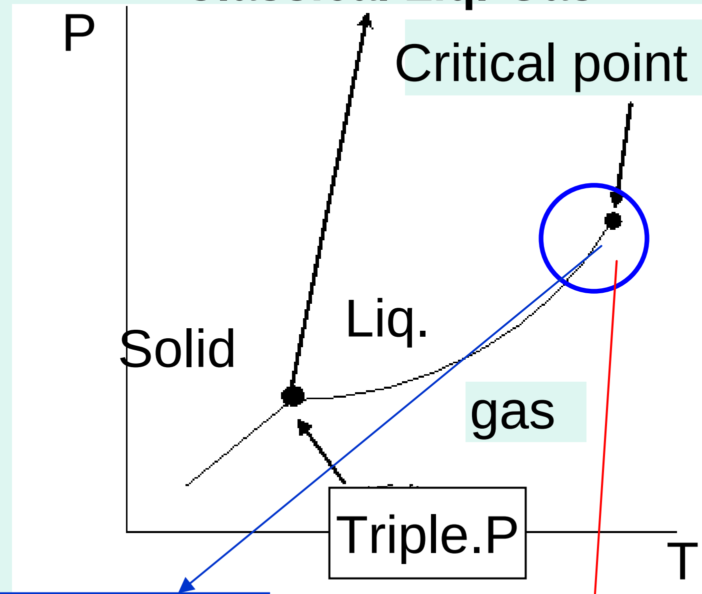
Actually, there have been a quite remarkable development in the studies of the low-lying scalar mesons using (un)quenched lattice QCD. However, no full-QCD simulation with disconnected diagrams included and these conditions satisfied so far, despite of the vigorous activities on this problem.

See the following nice review articles for more detailed accounts:
Craig McNeil, arXiv:0710.0985, arXiv:0710.24708hep-lat]
Sasa Prelovsek, arXiv:0804.2549[hep-lat]

Plausible QCD phase diagram:



Classical Liq.-Gas



The same universality class; Z2

H. Fujii, PRD 67 (03) 094018;H. Fujii and M.Ohtani,Phys.Rev.D70(2004)
Dam. T. Son and M. A. Stephanov, PRD70 ('04) 056001

Fluctuations of conserved quantities such as the number and energy are the soft mode of QCD critical point!
The sigma mode is a slaving mode of the density.

What is the soft mode at CP?

Sigma meson has still a non-zero mass at CP.
This is because the chiral symmetry is explicitly broken.

What is the soft mode at CP?

At finite density, scalar-vector mixing is present.

Phonon mode in the space-like region softens at CP.

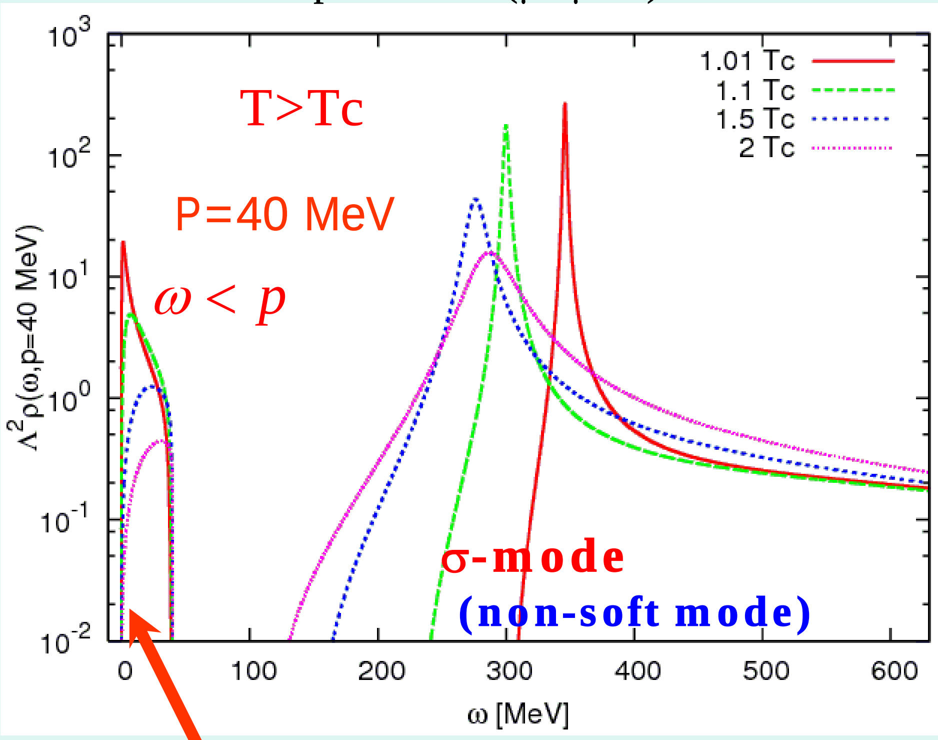
H. Fujii (2003)
H. Fujii and M.Ohtani(2004)

See also, D. T. Son and M. Stephanov (2004)

does not affect particle creation in the time-like region.

It couples to hydrodynamical modes,
leading to interesting dynamical critical phenomena.

Spectral function of the chiral condensate
T-dependence ($\mu=\mu_{CP}$)



Space-like region $\omega < p$

(the soft modes)

Spectral function of conserved quantities

The density/energy fluctuation depends on the transport as well as thermodynamic quantities which show an anomalous behavior around the critical point.

For non-relativistic case with use of Navier-Stokes eq.

L.D. Landau and G.Placzek(1934),
L. P. Kadanoff and P.C. Martin(1963),
R. D. Mountain, Rev. Mod. Phys. 38 (1966), 38
H.E. Stanley, `Intro. To Phase transitions and critical phenomena' (Clarendon, 1971)

We apply for the first time relativistic hydrodynamic equations to analyze the spectral properties of density and energy fluctuations, and examine possible critical phenomena.

Notice: The 1st-order can be valid for describing the hydrodynamic modes with a long wave-length without encountering the causality problem.

Critical behavior of the density-fluctuation spectral functions

Y. Minami and T.K., Prog. Theor. Phys.122, (2009),881

$$S_{nn}(\vec{k},\omega) = < (n(\vec{k},t=0))^2 > [\ (1-\frac{1}{\gamma})\frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4} + \frac{1}{\gamma}\{\frac{\Gamma_B k^2}{(\omega - c_s k)^2+\Gamma_B^2 k^4} + \frac{\Gamma_B k^2}{(\omega + c_s k)^2+\Gamma_B^2 k^4}\}\ ]$$

$$\gamma = c_p / c_n = t^{-\tilde{\gamma}+\tilde{\alpha}} \to \infty$$

$$< (n(\vec{k},t=0))^2 > \frac{2\Gamma_R k^2}{\omega^2+\Gamma_R^2 k^4}$$

$$< (n(\vec{k},t=0))^2 > \sim (|T-T_c| + k^2)^{-1}$$

Critical opalescence

In the vicinity of CP, only the Rayleigh peak stays out, while the sound modes (Brillouin peaks) die out.

c.f. The dynamical critical exponent.

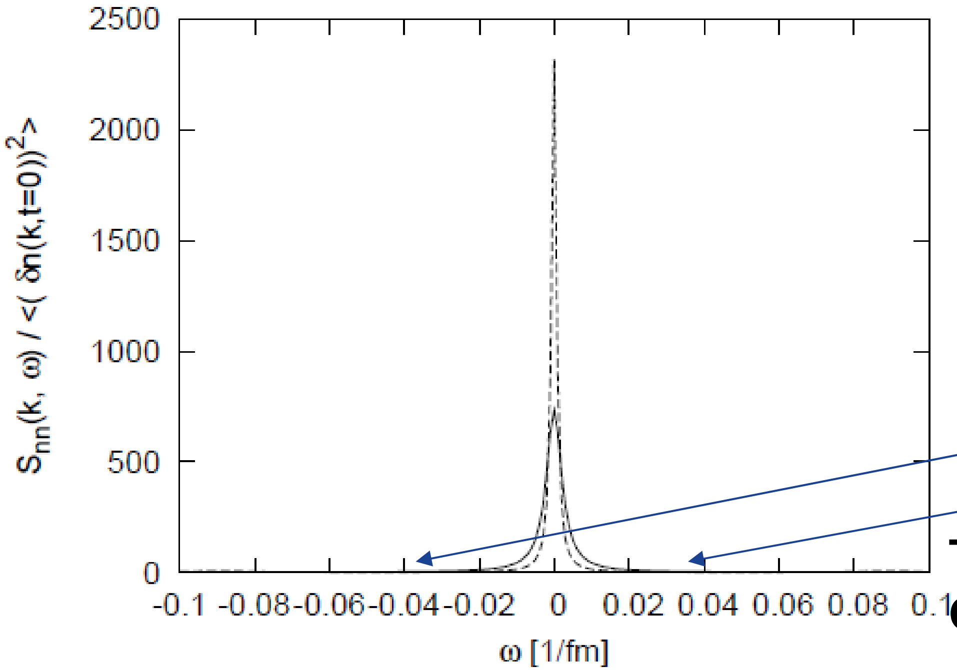
$$z = 2 + (\gamma - a)/\nu \simeq 3$$

So, the divergence of Γ_B and the viscosities therein can not be observed, unfortunately.

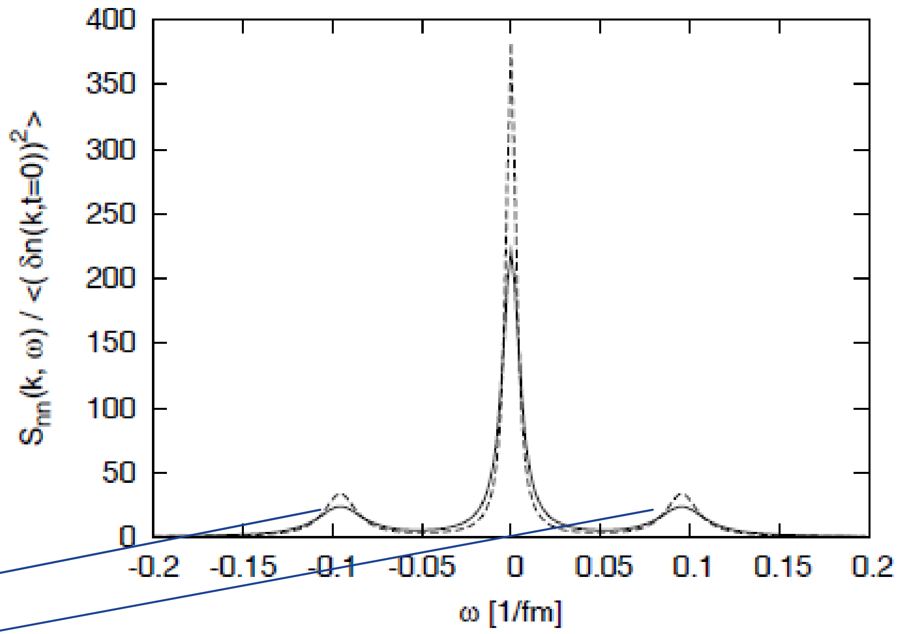
Spectral function of density fluctuation at CP

Y. Minami and T.K., (2009)

$t = 0.1$



$t \equiv (T - T_c) / T_c = 0.4$



Spectral function at CP

The sound mode (Brillouin) disappears
Only an enhanced thermal mode remains.
Furthermore, the Rayleigh peak is enhanced, meaning the large energy dissipation.

The soft mode around QCD CP is thermally induced density fluctuations, but not the usual sound mode.

➡ Suggesting interesting critical phenomena related to sound mode.

Why at all do sound modes die out at the Critical Point ?

The correlation length

$$\xi = \xi_0 t^{-\nu}$$

The wave length of sound mode

$$\lambda_s$$

The hydrodynamic regime!

$$\xi \ll \lambda_s$$

However, around the critical point $\xi \longrightarrow \infty$ as $t \longrightarrow 0$

$$\lambda_s \ll \xi$$

The system behaves as an aggregate of clumps of matter with a diameter ξ .

So the hydrodynamic sound modes can not be developed around CP!

$$C_P = n^2 \int d^3r \langle \delta s(\boldsymbol{r}, t=0)\delta s(0,0)\rangle$$

Ornstein-Zernike;

$$\langle \delta s(\boldsymbol{r}, 0)\delta(0,0)\rangle \sim \frac{e^{-r/\xi}}{r}$$

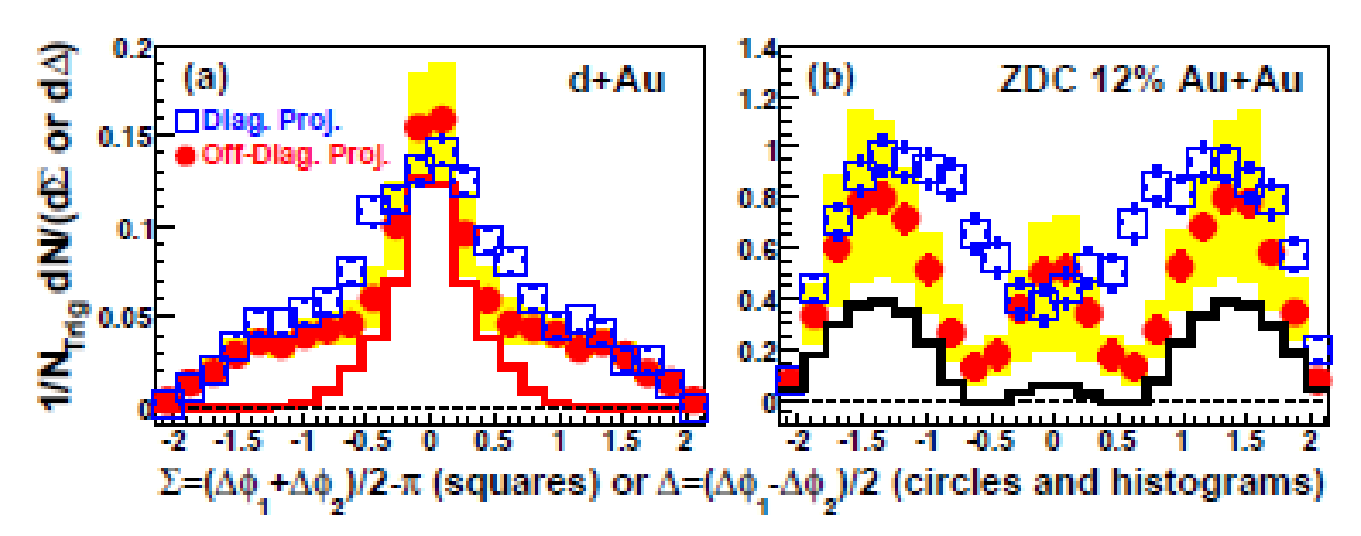
$\Longrightarrow$ $C_P \propto \xi^2$ $\Longrightarrow$ $\xi^{2-\eta}$

$$\eta \sim 0.03$$

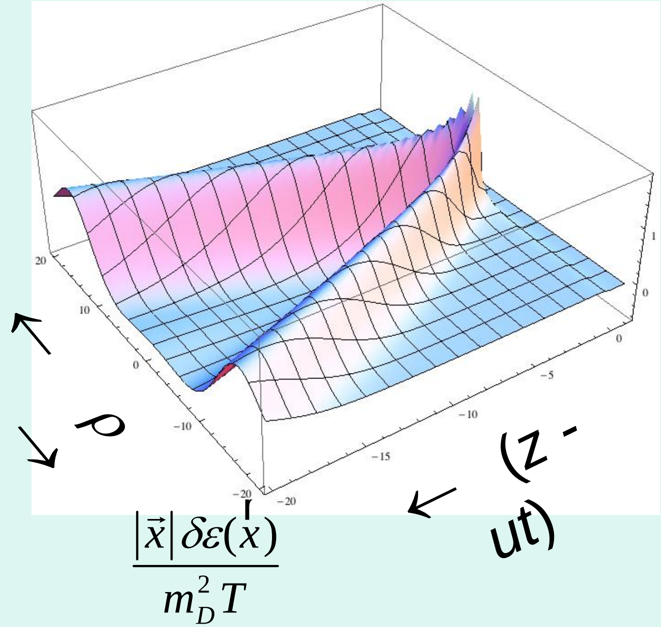
Cf. **STAR, arXiv:0805/0622.**

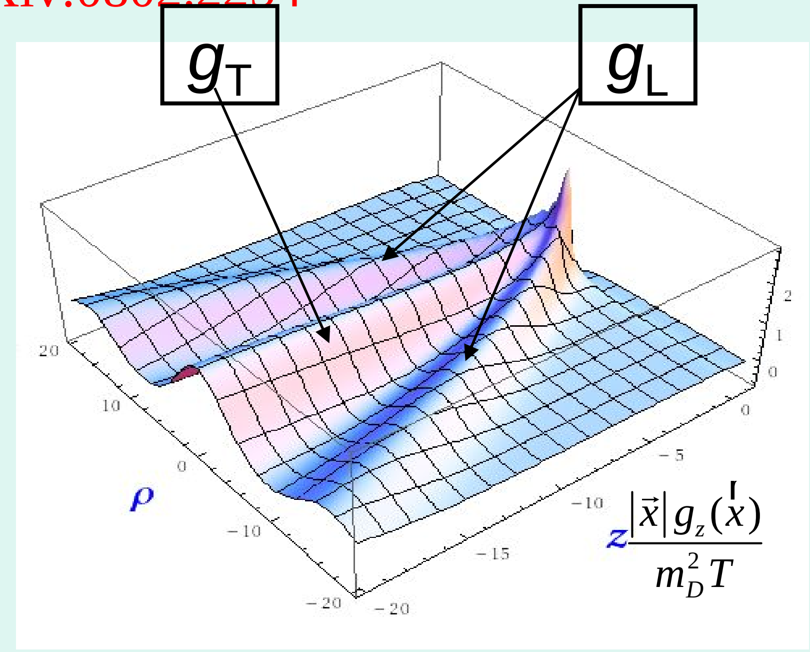
3-body correlations

Cf. the idea of Mach cone: E. Stoecker , E. Shuryak and many Others.

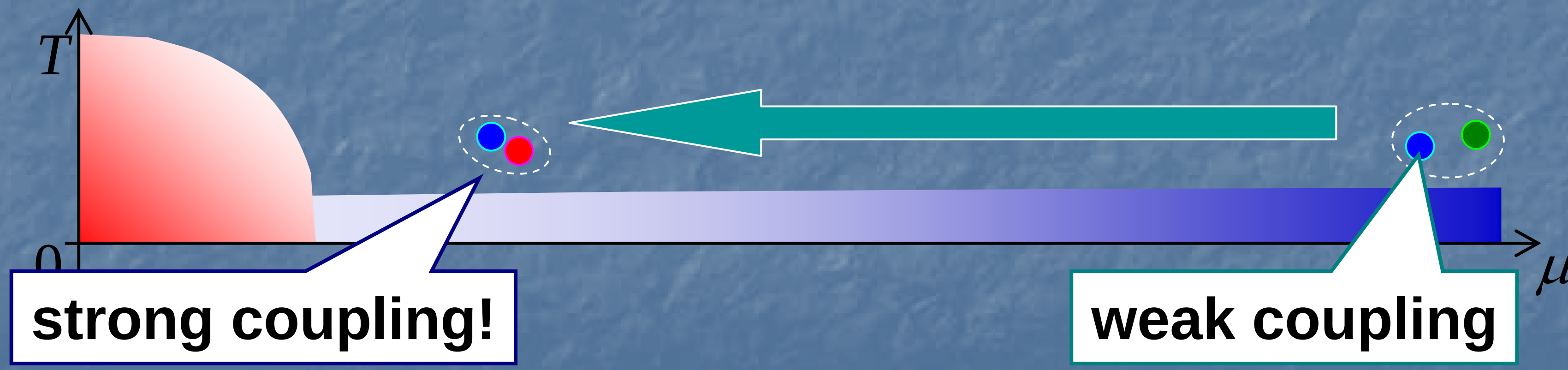


R. B. Neufeld, B. Muller, and J. Ruppert,arXiv:0802.2254





4. Diquark fluctuations and pseudogap in quark spectral function in hot and dense quark matter

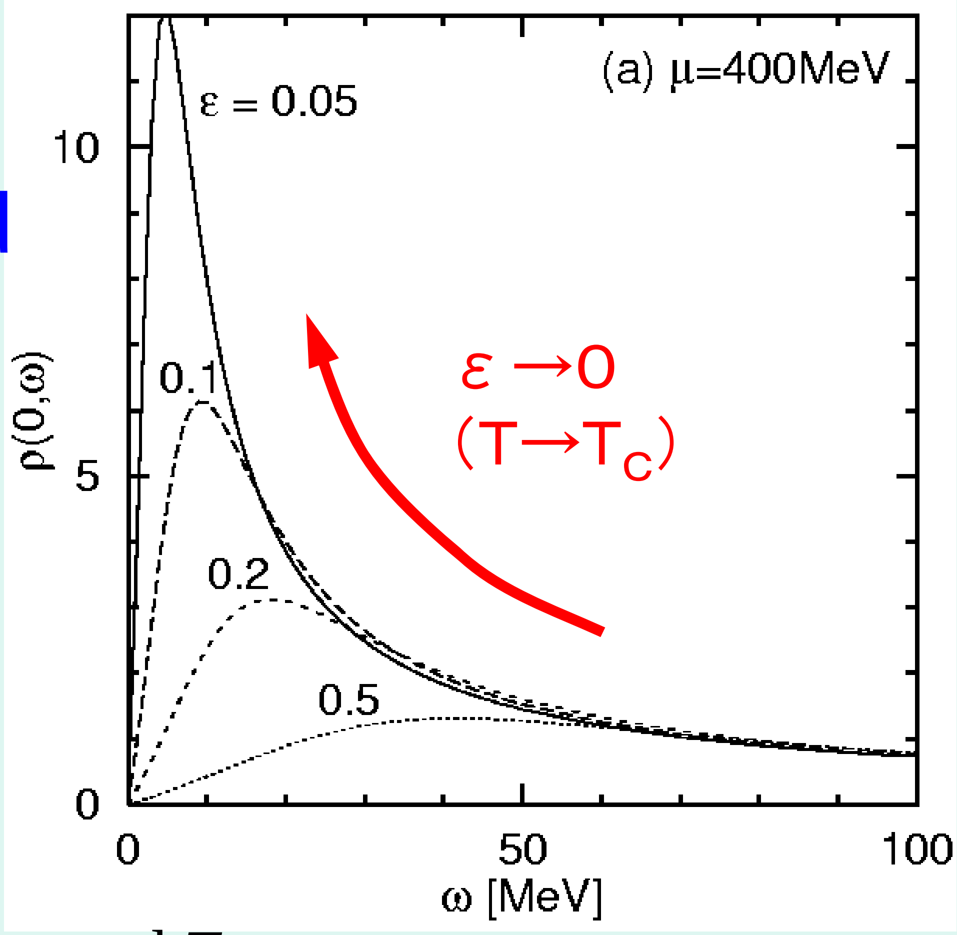


Precursory Mode in CSC

(Kitazawa, Koide, Nemoto and T.K., PRD 65, 091504(2002))

$$\rho(\mathbf{k}=0,\omega)$$

Spectral function of the pair field at T>0



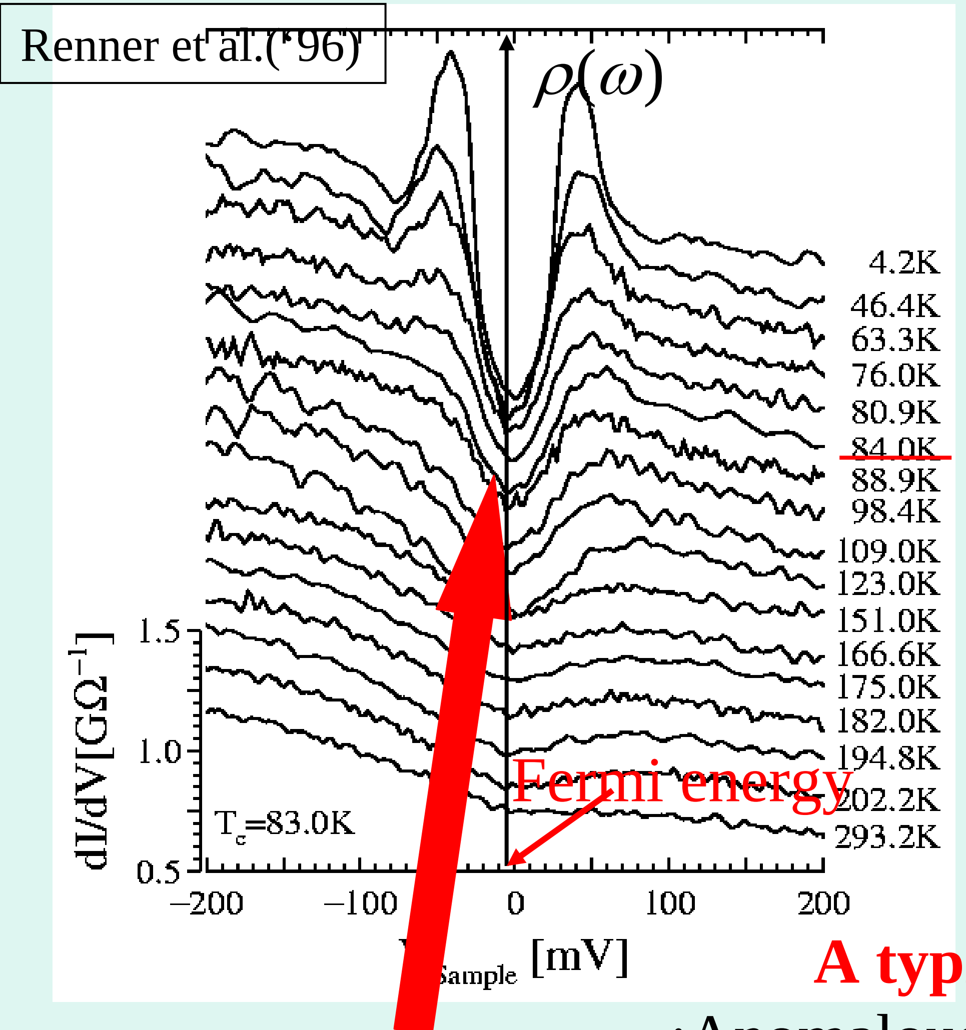
ε →0
(T→T$_C$)

at k=0

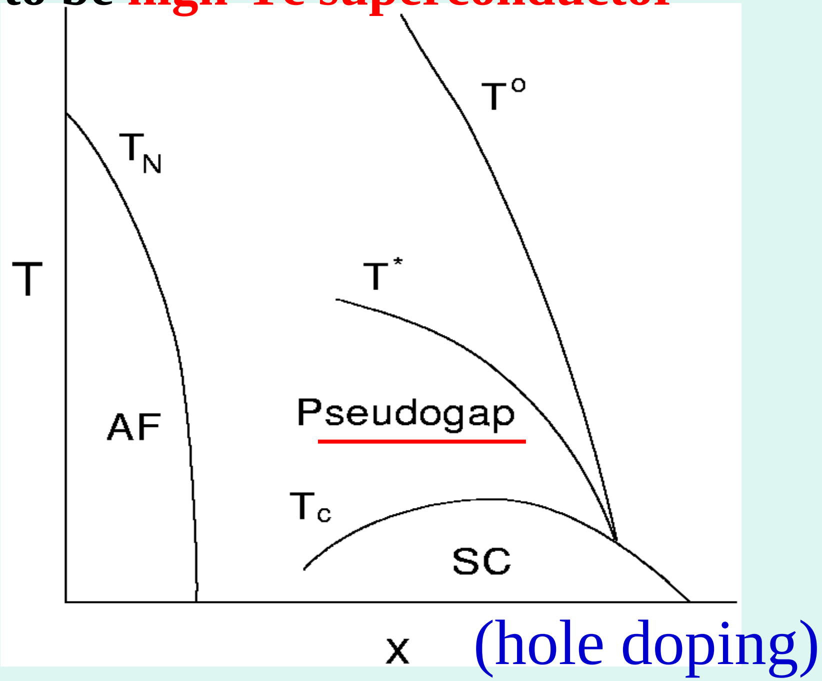
$$\varepsilon = \frac{T - T_C}{T_C}$$

- As T is lowered toward T_C,
  The peak of ρ becomes sharp. (Soft mode) ⇨ Pole behavior
- The peak survives up to $\varepsilon \sim 0.2$ ⟺ electric SC: $\varepsilon \sim 0.005$

Lesson from condensed matter physics on strong correlations



Phase diagram of cuprates
to be high-Tc superconductor



A typical non-Fermi liq. behavior!

Pseudogap

:Anomalous depression of the **density of states** near the Fermi surface in **the normal phase**.

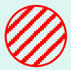
The mechanism of the pseudogap in High-TcSC is **still controversial**, but see,

Y. Yanase et al, Phys. Rep. 387 (2003),1, where the essential role of pair fluc. is shown.

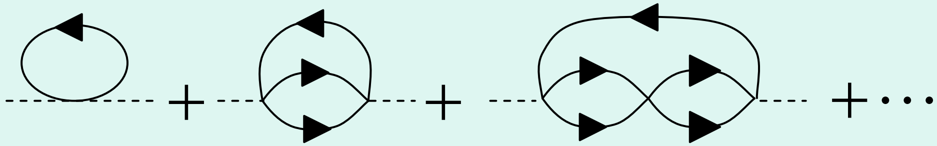
T-matrix Approximation for Quark Propagator

$$G(\mathbf{k},\omega_n) = \frac{1}{G^0(\mathbf{k},i\omega_n) - \Sigma(\mathbf{k},i\omega_n)} \qquad G^0(\mathbf{k},i\omega_n) = \left[(i\omega_n+\mu)\gamma^0 - \mathbf{k}\cdot\vec{\gamma}\right]^{-1} = $$

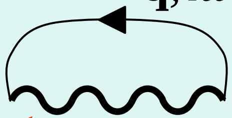
$$\Sigma(\mathbf{k},\omega_n) = $$



$$=$$



$$\equiv$$



$$= T\sum_m \int \frac{d^3q}{(2\pi)^3} \Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m) G^0(\mathbf{q},\omega_m)$$

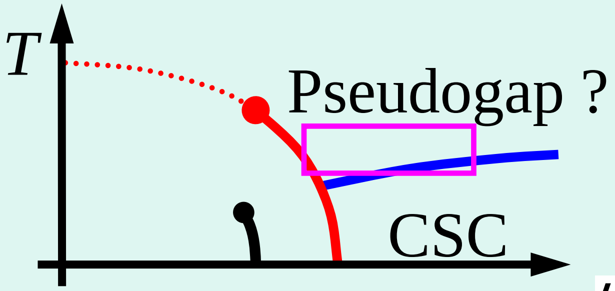
Di-quark soft mode

Density of States $N(\omega)$:

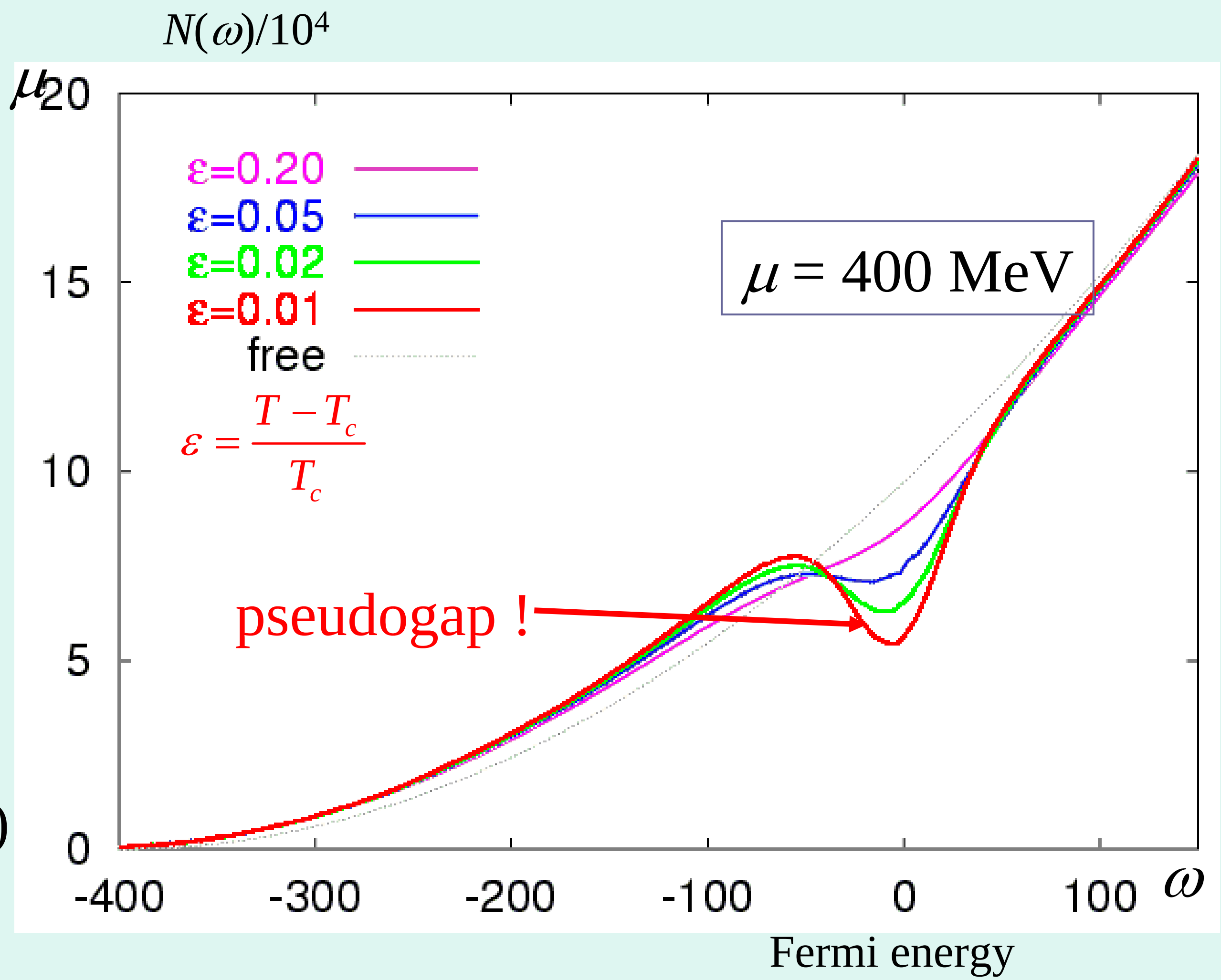
$$N = \int d^3x \left\langle \bar{\psi}\gamma^0\psi \right\rangle$$

$$N(\omega) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \rho^0(\mathbf{k},\omega) \Leftarrow \rho^0(\mathbf{k},\omega) = \frac{1}{4}\mathrm{Tr}\left[\gamma^0 \,\mathrm{Im}\, G^R(\mathbf{k},\omega)\right]$$

Possible pseudogap formation in heated quark matter



M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),



Pseudogap is formed above Tc of CSC in heated quark matter!

How?

$$N(\omega) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \rho(\mathbf{k}, \omega)$$

4. Summary and concluding remarks

★ QCD phase diagram with vector interaction under charge neutrality

There are still a room of other structure of the QCD phase diagram with **multiple critical points** when the **color superconductivity** and the **vector interaction** are incorporated.

It seems that the QCD matter is very soft along the critical line when the color superconductivity is incorporated; there can be a good chance to see large fluctuations of various observables like **diquark-density** mixed fluctuations,

$$a\overline{q_c}q + bq^\dagger q.$$

Inhomogenious phases should be considered as an alternative of the Low-temperature phase.

E. Nakano and T. Tatsumi,
D. Nickel and others.

Spatial dependence of Polyakov loop should be clarified; PL dynamics.

Summary and conl. remarks(continued)

- The dynamical density fluctuations have been analyzed using rel. hydro,

  in which the entropy fluctuations are automatically incorporated.
- The **sound modes** due to density fluctuations are **attenuated**, and the Rayleigh peak due to the entropy-energy fluctuation in turn gets enhanced around the QCD critical point.

- Need further explicit calculation for confirmation.

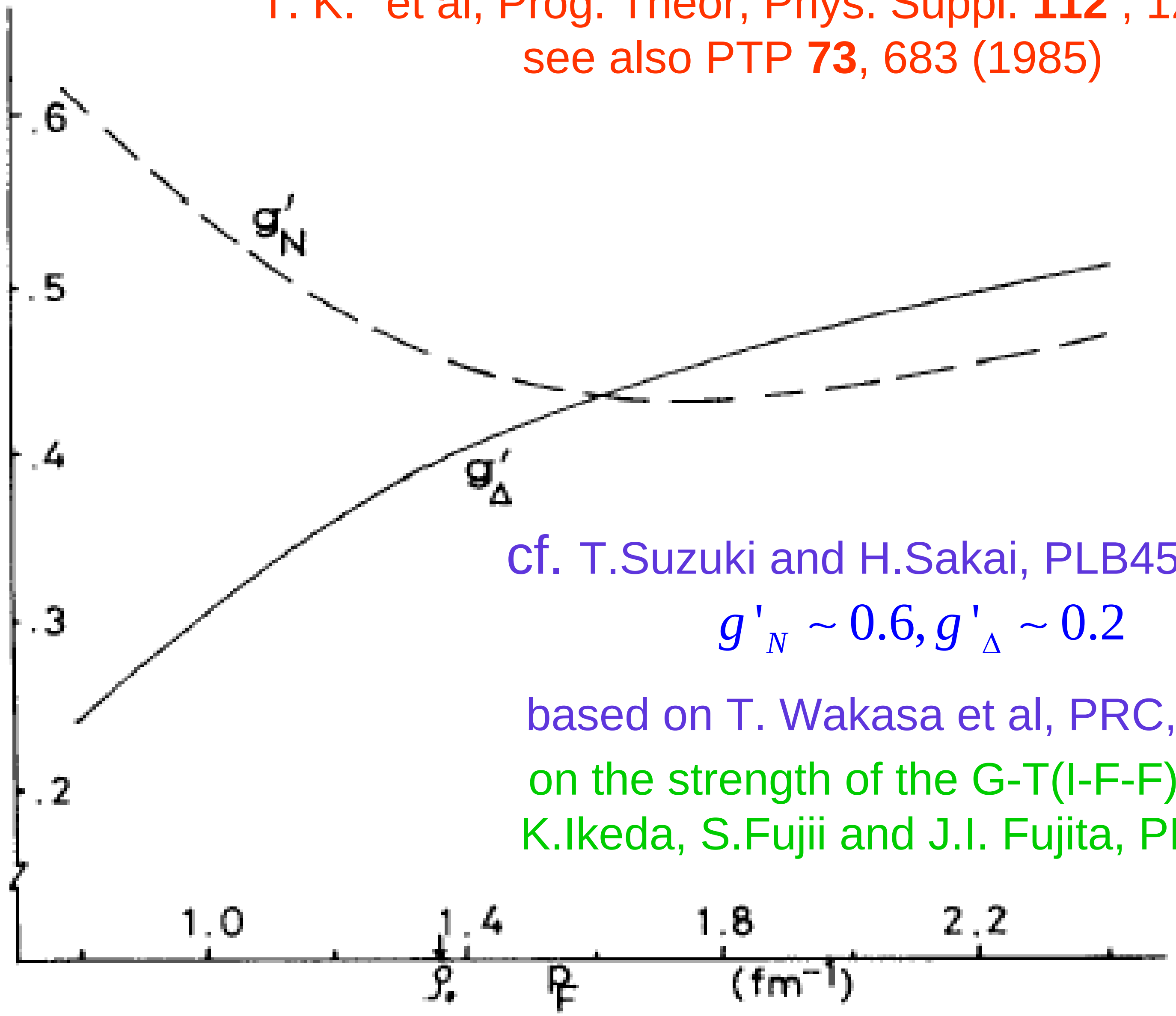
Future problems

- Application of the dynamical RG or mode-mode coupling theory a la Kawasaki, Onuki and others; in progress .
- Density fluctuations around the expanding back ground.
- Hydro. $\longrightarrow$ Dual Magneto-hydro.?

Back Up

The density dependence of the Landau-Migdal parameteters in the N-N and Δ -N channels

T. K. et al, Prog. Theor, Phys. Suppl. **112** , 123(1993);
see also PTP **73**, 683 (1985)



cf. T.Suzuki and H.Sakai, PLB455('99),25

$$g'_N \sim 0.6, g'_\Delta \sim 0.2$$

based on T. Wakasa et al, PRC, ('97), 2909

on the strength of the G-T(I-F-F) resonance,
K.Ikeda, S.Fujii and J.I. Fujita, PL .3('63),271