Thermodynamics of dense hadronic matter in chiral models

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Outline:

1. Introduction: baryons near chiral symmetry restoration
2. Thermodynamics of hadronic matter in a parity doublet model
3. Dynamical χSB and confinement
4. An exotic phase with four-quark condensate

Baryons near chiral symmetry restoration?

- dynamical origin of nucleon mass?
  - Gell-Mann Levy model: spontaneous χSB generates $m_N = g\langle\sigma\rangle$.

$$\mathcal{L}_{\rm GL} = i\bar{N}\partial\!\!\!/ N - g\bar{N}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})N + \mathcal{L}_{\rm meson}$$

  - chiral transf. for a nucleon *assumed* to be the same as for a quark
  - in Nature: excited states of nucleon, $N^*N\pi$ interaction?
- m_N at χ-symmetry restoration?
  - standard (naive): DχSB generates masses $m_N \stackrel{\sigma\to 0}{\to} 0$
  - parity doublet (mirror): DχSB generates mass difference
    $m_{N_+} \stackrel{\sigma\to 0}{\to} m_{N_-} = m_0 \neq 0$ [Detar-Kunihiro (89)]
  - in general linear combinations: $|\alpha\rangle = \sum_J c_J |J\rangle$
- emergence of a scale in QCD:
  - trace anomaly $\Theta^\mu_\mu = \frac{\beta}{2g}G^2 + m(1+\gamma)\bar{q}q$
  - $\langle\frac{\beta}{2g}G^2\rangle^{\rm lattice}_{T_\chi} \sim \frac{1}{2}\langle\frac{\beta}{2g}G^2\rangle_{\rm vac} \neq 0$ [Miller (07)]

- naive vs. mirror: not yet discriminated
  - axial couplings: $g_A^{++} = g_A^{--}$ (naive) $g_A^{++} = -g_A^{--}$ (mirror)
    as a group-theoretical consequence in a simple LσM for σ and π.

    However,
    cf. other chiral invariant operators allowed [Jaffe-Pirjol-Scardicchio (06)]
    cf. lattice QCD: $g_A^{--} = 0.2 \pm 0.3$ [Takahashi-Kunihiro (07)]
    AdS/QCD: $g_A^{++} = 0.73\,,\ g_A^{--} = 0.38$ [Hashimoto-Sakai-Sugimoto (08)]
    cf. explicit a_1 mesons [Gallas-Giacosa-Rischke (09)]
  - which state is the true chiral partner of $N(940)$?
    if $N(1535)$ then $m_0 = 270$ MeV (from $\Gamma^{(\text{exp})}(N^* \to N\pi) = 70$ MeV)
    $\Leftrightarrow$ cannot reproduce $\Gamma^{(\text{exp})}(N^* \to N\eta) \sim 80$ MeV
    a speculative candidate closer to N? and/or large OZI-violation?

Dense nuclear matter in chiral models

- **nuclear matter: known properties**
  - binding energy: $E/A(\rho_0) - m_N = -16$ MeV
  - saturation density: $\rho_0 = 0.16$ fm^{-3}
  - incompressibility: $K = 9\rho_0^2\partial^2(E/A)/\partial\rho^2|_{\rho=\rho_0} = 200\text{-}400$ MeV

- **in-medium chiral perturbation theory**
  $\Rightarrow$ talk by Wolfram Weise

- **mean-field models**
  - LSM: no stable ground state corr. to nuclear matter [Kerman-Miller (74)]
  - nucleonic NJL: possible if 4F vector and 8F scalar-vector int. incld.
    [Koch-Biro-Kunz-Mosel (87), Buballa (96), Mishustin-Satarov-Greiner (03)]
  - NJL with diquarks: baryon as a bound state of a quark and a diquak
    [Bentz-Thomas (01), Bentz-Horikawa-Ishii-Thomas (03)]
  - parity doublet model: large $m_0 \sim 800$ MeV needed?
    [Hatsuda-Prakash (89), Zschiesche-Tolos-Schaffner-Bielich-Pisarski (07)]

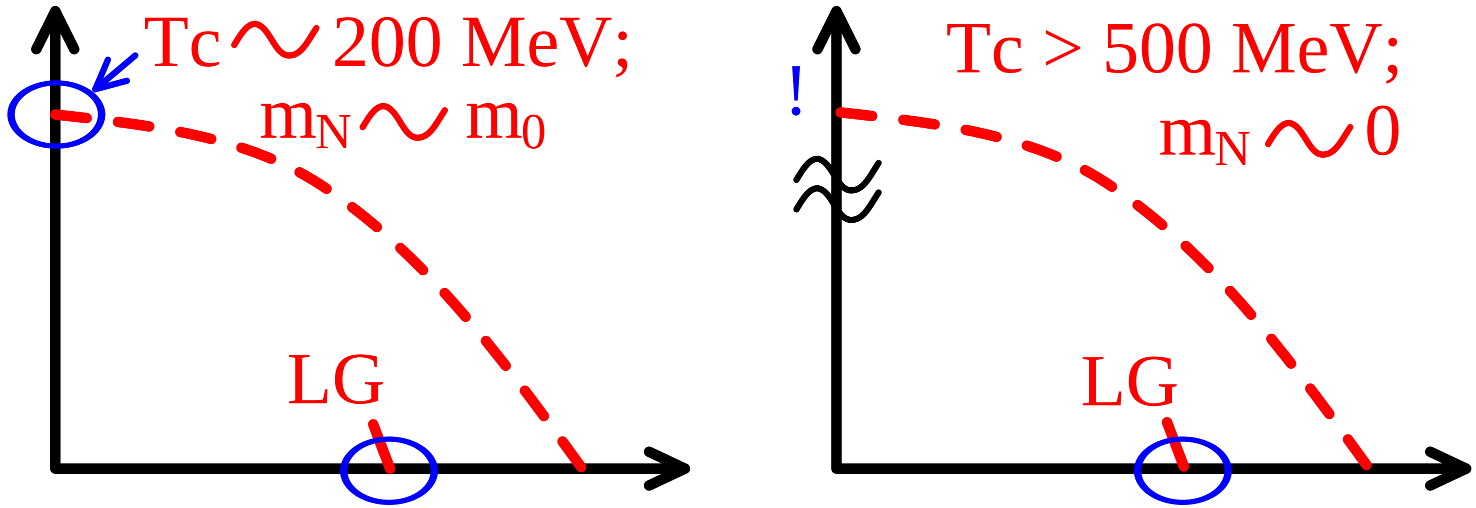
- thermodynamics on top of the nuclear matter ground state [CS-Mishustin (2010)]

  e.g. extrapolation of nucleonic NJL at $\mu_B = 0$??

$$m_N = m_N^0 + \gamma_N G_S \int \frac{d^3p}{(2\pi)^3} \frac{m_N}{E} \left[1 - 2n_f(m_N; T)\right]$$

parameters fixed to reproduce nuclear matter properties [Mishustin et al. (03)]

$\Lambda = 0.4\,\mathrm{MeV}$, $G_S = 1.7\,\mathrm{GeV\ fm}^3$, $m_N^0 = 41\,\mathrm{MeV}$ $\Rightarrow$ $T_c \sim 500\,\mathrm{MeV}$



$T_c^{\mathrm{lattice}}(\mu \sim 0)$ & nuclear matter ground state

$\Rightarrow$ a minimal set of constraint on modeling

- $N_f = 2$ **parity doublet model** [Zschiesche et al. (07)]
  - 2 nucleon fields

$$\psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2)$$
$$\psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0)$$

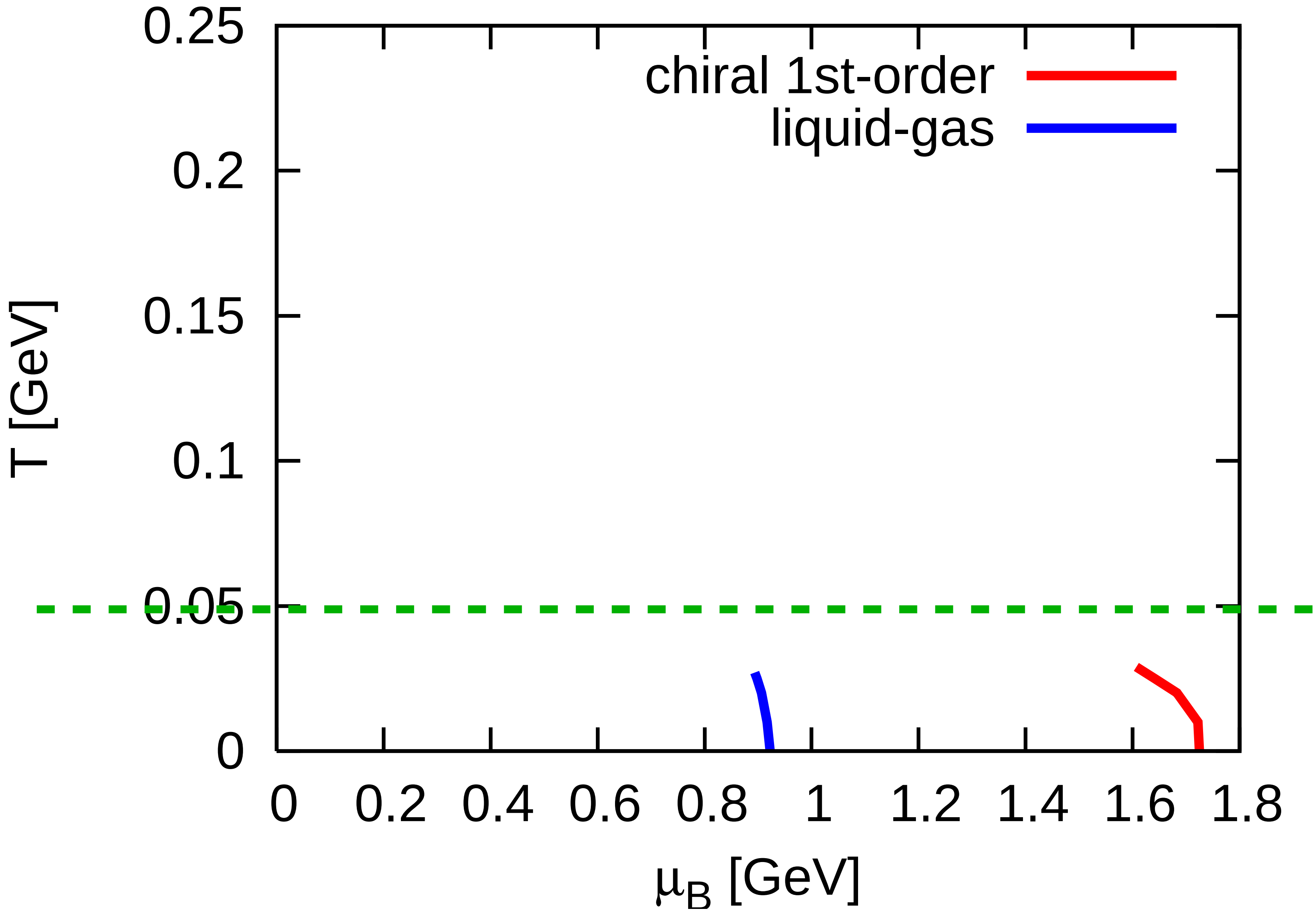
  - Lagrangian

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_1 i\partial\!\!\!/ \psi_1 + \bar{\psi}_2 i\partial\!\!\!/ \psi_2 + m_0 \left(\bar{\psi}_2\gamma_5\psi_1 - \bar{\psi}_1\gamma_5\psi_2\right) \\ &\quad + a\bar{\psi}_1(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_1 + b\bar{\psi}_2(\sigma - i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi_2 - g_\omega\bar{\psi}_1\omega\!\!\!/\psi_1 - g_\omega\bar{\psi}_2\omega\!\!\!/\psi_2 + \mathcal{L}_M\,, \\ \mathcal{L}_M &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\bar{\mu}^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 + \epsilon\sigma \\ &\quad - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + (g_4)^4(\omega_\mu\omega^\mu)^2 \end{aligned}$$

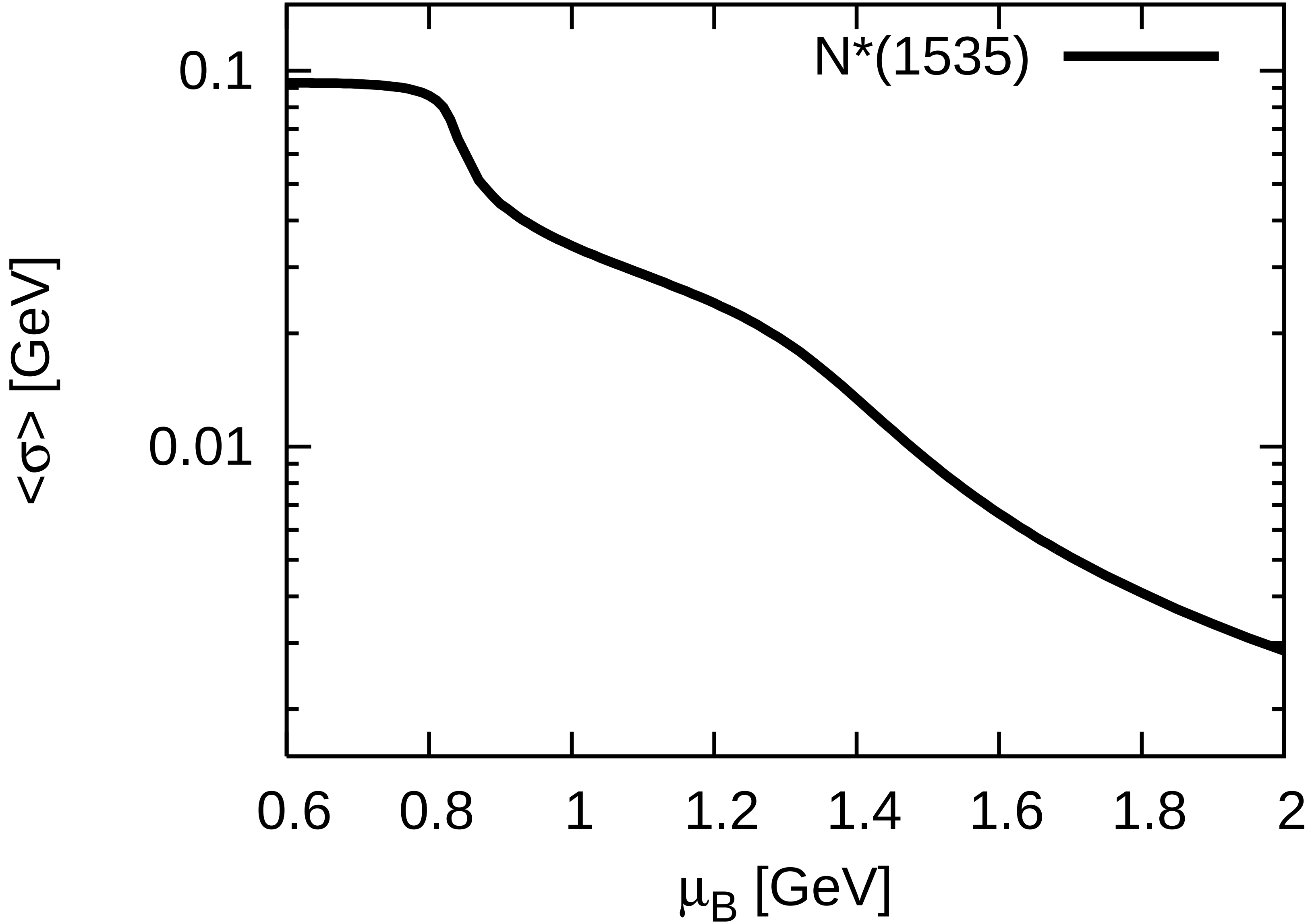
  - masses: $m_\pm = \frac{1}{2}\left[\sqrt{(a+b)^2\sigma^2 + 4m_0^2} \mp (a-b)\sigma\right] \stackrel{\sigma\to 0}{\longrightarrow} m_0$
  - "transition" from meson-rich to baryon-rich matter in hadronic phase

$$\rho_{\rm meson}/\rho_{\rm baryon} \sim 1\,, \quad \rho_i = \gamma_i \int \frac{d^3p}{(2\pi)^3} n_i(T, \mu; m_i)$$

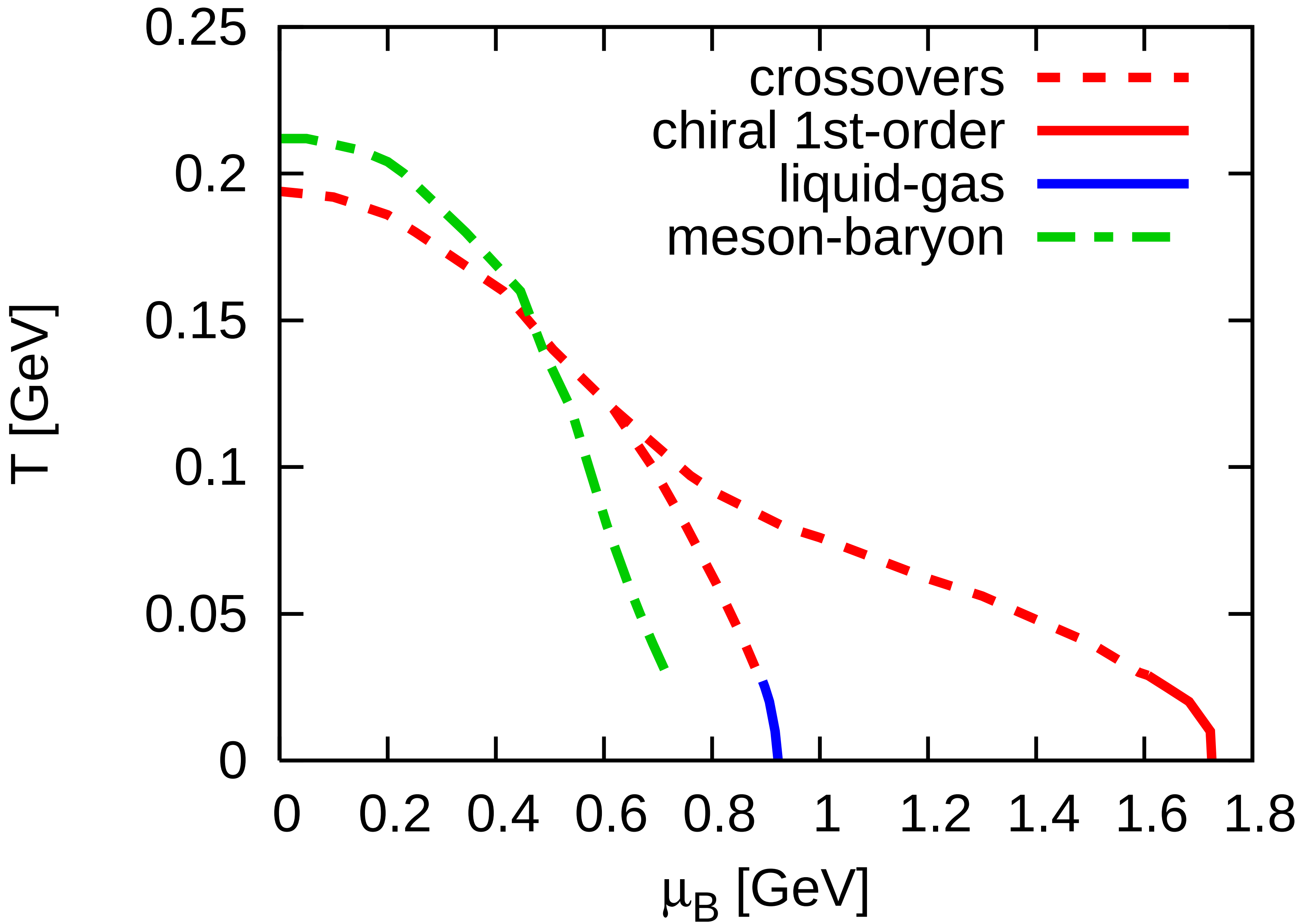
• **phase diagram in PDM**: $m_{N_-} = 1.5$ GeV



• **pion decay constant at an intermediate temperature**

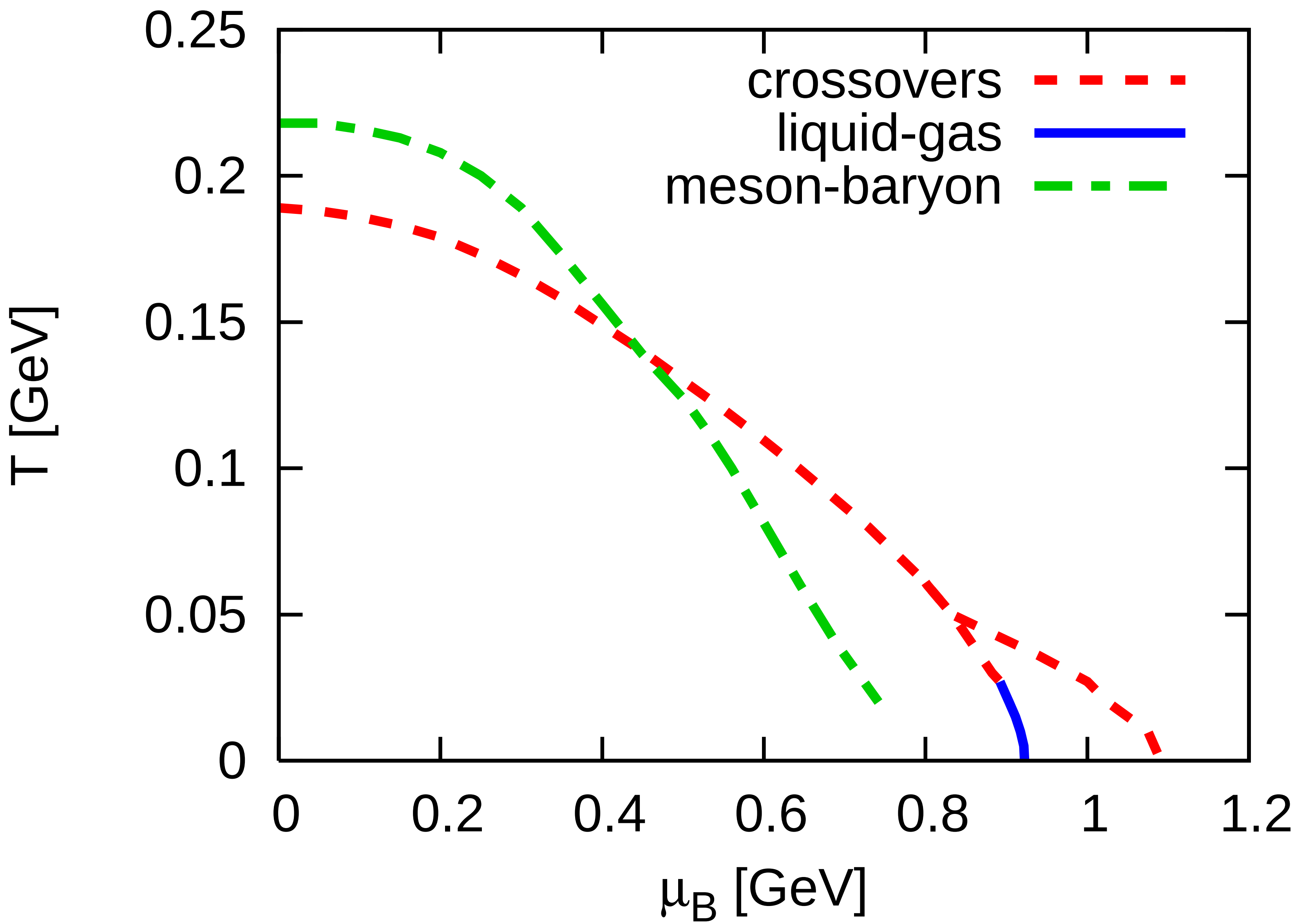


- **phase diagram in PDM**: $m_{N_-} = 1.5$ GeV



broken phase (meson-rich & baryon-rich) and restored phase

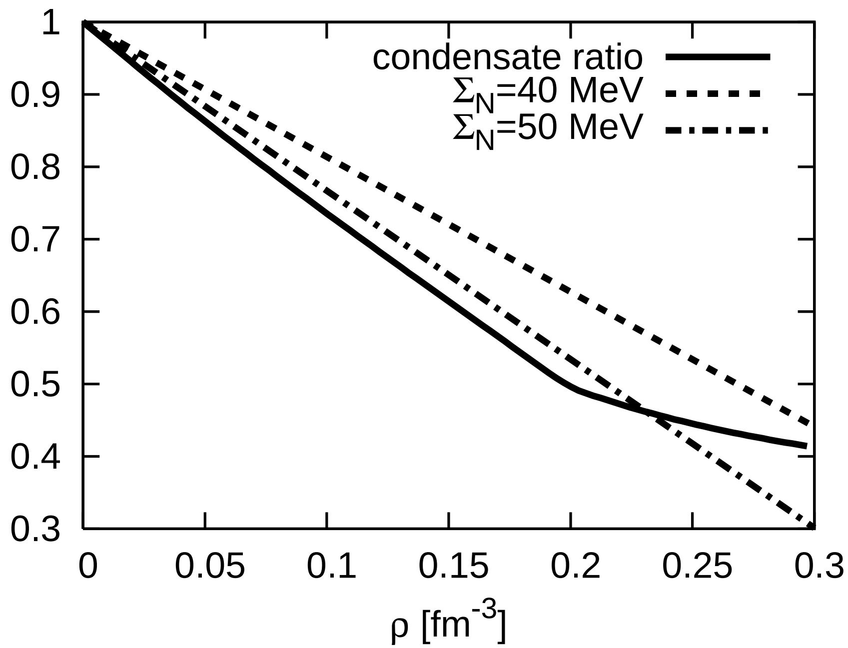
• **phase diagram in PDM**: $m_{N_-} = 1.2$ GeV



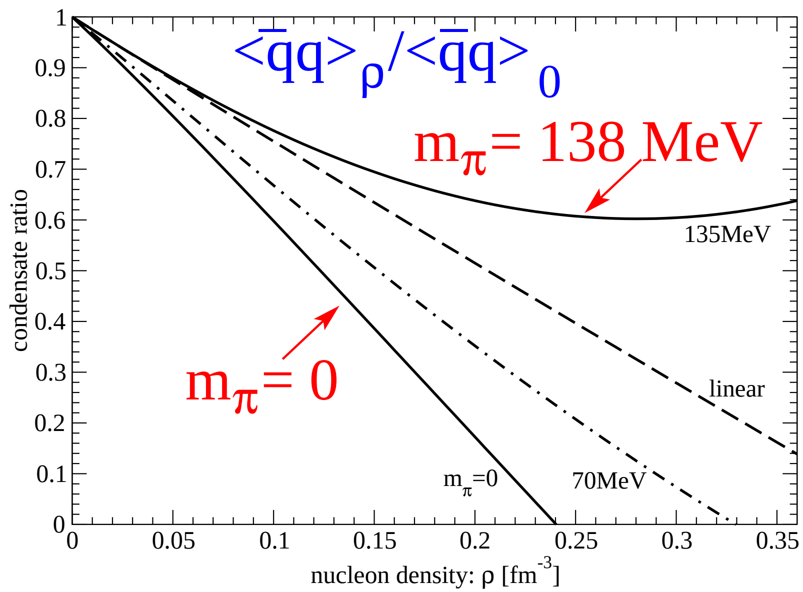
$\mu_{\rm chiral} - \mu_{\rm LG}$ dep. on m_{N-} $\Rightarrow$ baryon-rich domain shrinks.

- **in-medium quark condensate and the low-energy theorem**

present MF model



ChPT [Kaiser-de Homont-Weise (08)]



up to the leading order in ρ:

$$R(\rho) = \frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\rm vac}} = 1 - \frac{\Sigma_N}{m_\pi^2 f_\pi^2}\rho\,, \quad \Sigma_N = 45 \pm 8\,\text{MeV}$$

beyond linear-density-approximation:

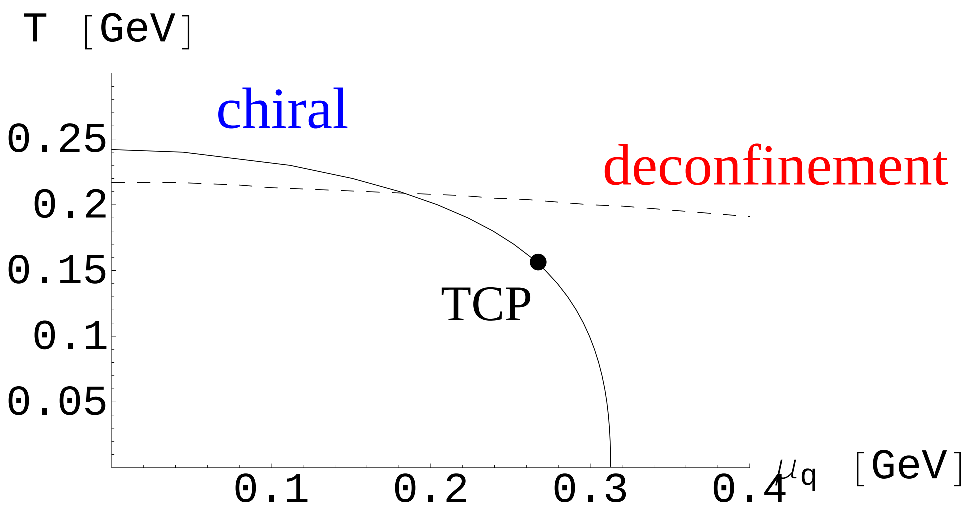
$$R^{(\rm PDM)}(\rho \sim 0.2\,\text{fm}^{-3}) \sim 0.5\,, \quad R^{(\rm ChPT)}(\rho \sim 0.2\,\text{fm}^{-3}) \sim 0.7$$

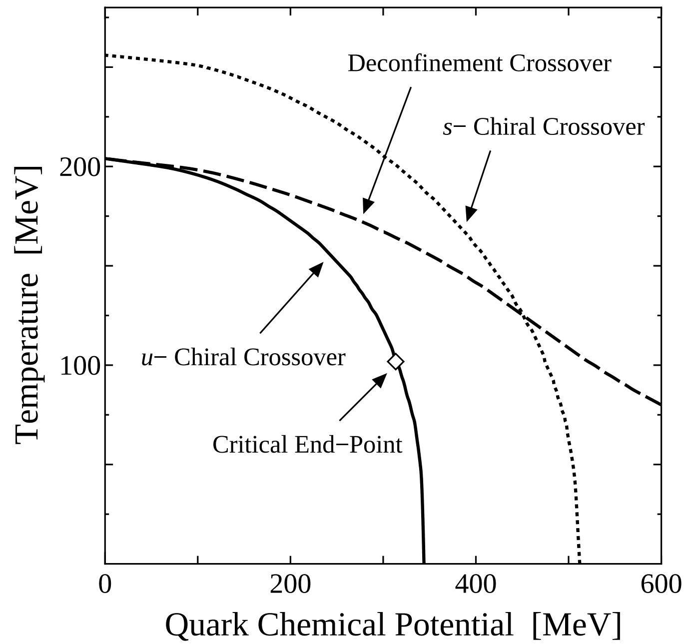
$\Rightarrow$ importance of two-pion exchange correlations with $\Delta(1232)$

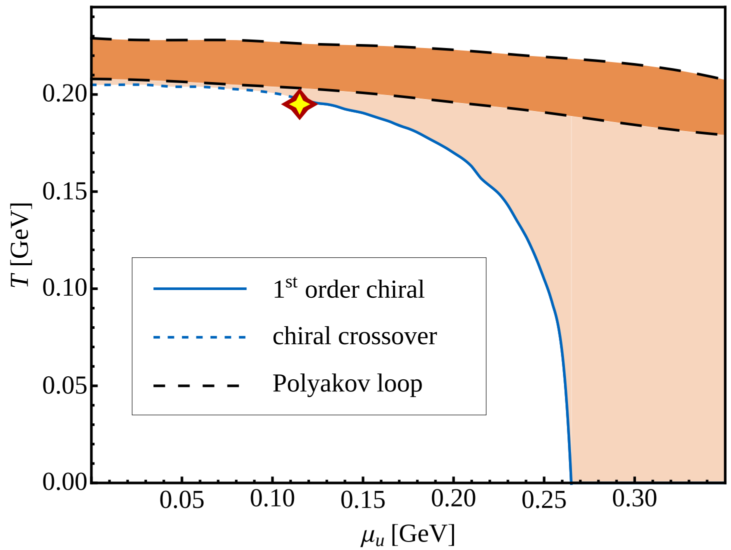
$\ast$ explicit breaking term in baryonic sector too

Dynamical chiral symmetry breaking vs. confinement

- **phase diagram from PNJL models: 3 regions**







[upper-left] CS-Friman-Redlich (06); [right] Fukushima (08)

[lower] Hell-Roessner-Cristoforetti-Weise (09)

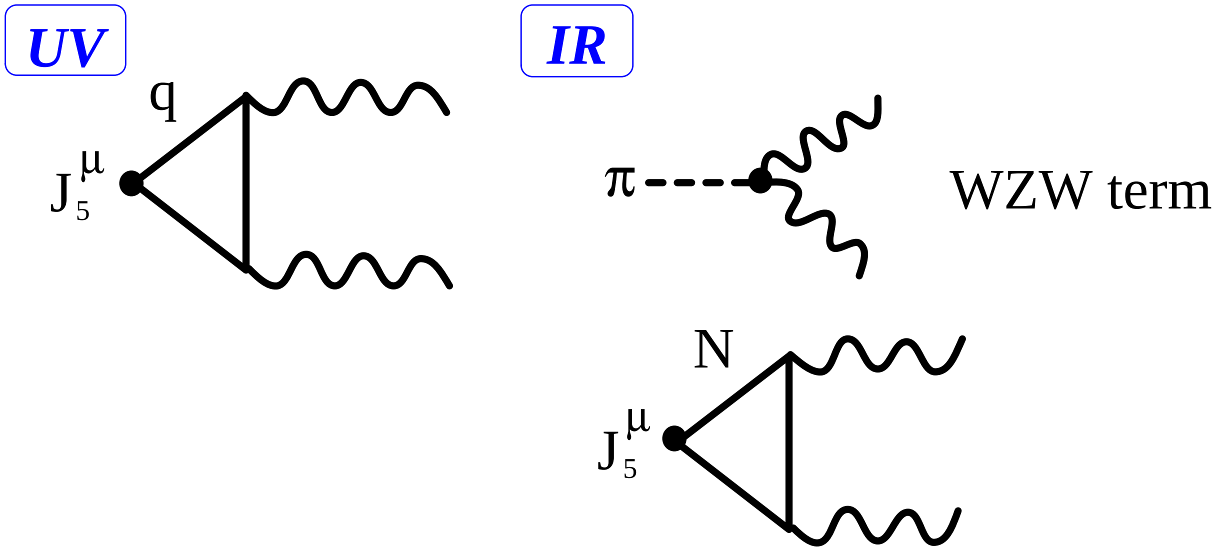
confinement in Wigner-Weyl phase?

– back reaction from quarks to $\mathcal{U}(\Phi)$

[Schaefer et al. (07), Fukushima (10)]

– anomaly matching: $N_f = 2$ or 3

- **anomaly matching between UV and IR theories**



– chirally restored phase: no NG boson thus no WZW term

– triangle diagrams with baryon: matched for $N_f = 2$ but not for $N_f = 3$

[Shifman (89)]

$$\text{triangle graph} \propto \text{tr}[T^a\{Q, Q\}]$$

$$N_f = 2 : (\text{quark})\ 3N_c/9 = 1 \quad (\text{hadron})\ = 1$$

$$N_f = 3 : (\text{quark})\ 3N_c/9 = 1 \quad (\text{hadron})\ = 0$$

– parity doubled nucleons: anomalies are cancelled because of $g_A^+ = -g_A^-$

- **how are anomalies saturated in matter?**
  - chirally-restored phase with confinement with “known” hadrons
    - allowed for $N_f = 2$
    - not allowed for $N_f = 3$
    - not allowed in mirror baryon scenario for $N_f = 2, 3$
  - gapless modes might appear on the Fermi surface? (either “boson” or “fermion”) **what are they? dynamical origin?**
  - unless lack of Lorentz covariance spoils them totally,
    - if relevant low-energy excitations known,
      then anomaly matching constrains phases.
    - if relevant low-energy excitations unknown,
      then they should be constrained by anomaly matching.

Exotic phase? role of tetra-quark states

[Harada-CS-Takemoto (09)]

- Goldstone theorem: $\langle 0| [iQ_5, \mathcal{O}(x)] |0\rangle \neq 0$
  $\Rightarrow$ possibility of $\langle \bar{q}q \rangle = 0$ but χ-SB due to higher dim. operators?
  [SDE: e.g. Holdom-Triantaphyllou, Maris-Wang]
- symmetry breaking: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V \times Z_{N_f} \to SU(N_f)_V$
  e.g.

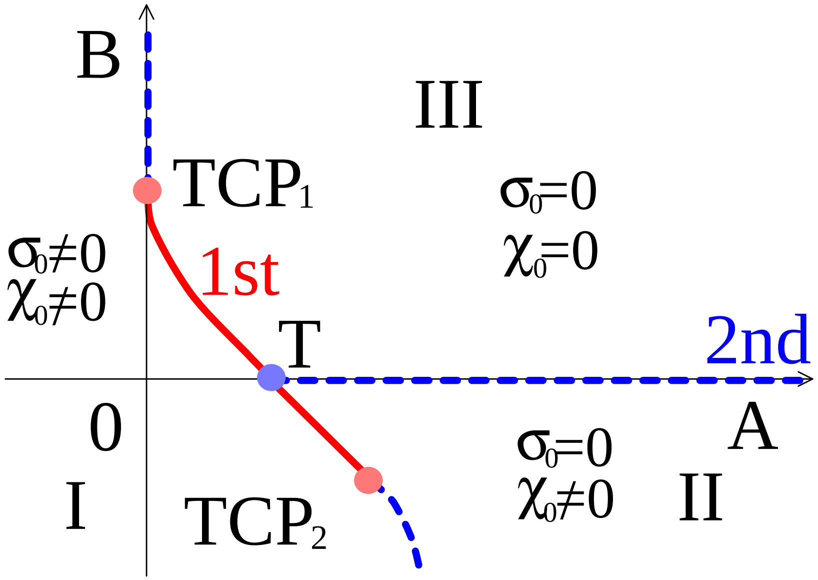
$$
\begin{aligned}
\mathcal{O}_1 &= \bar{q}T_a\gamma_\mu(1-\gamma_5)q \cdot \bar{q}T_a\gamma^\mu(1+\gamma_5)q \sim V_\mu V^\mu - A_\mu A^\mu \,, \\
\mathcal{O}_2 &= \bar{q}T_a(1-\gamma_5)q \cdot \bar{q}T_a(1+\gamma_5)q \sim \sigma^2 - \pi^2
\end{aligned}
$$

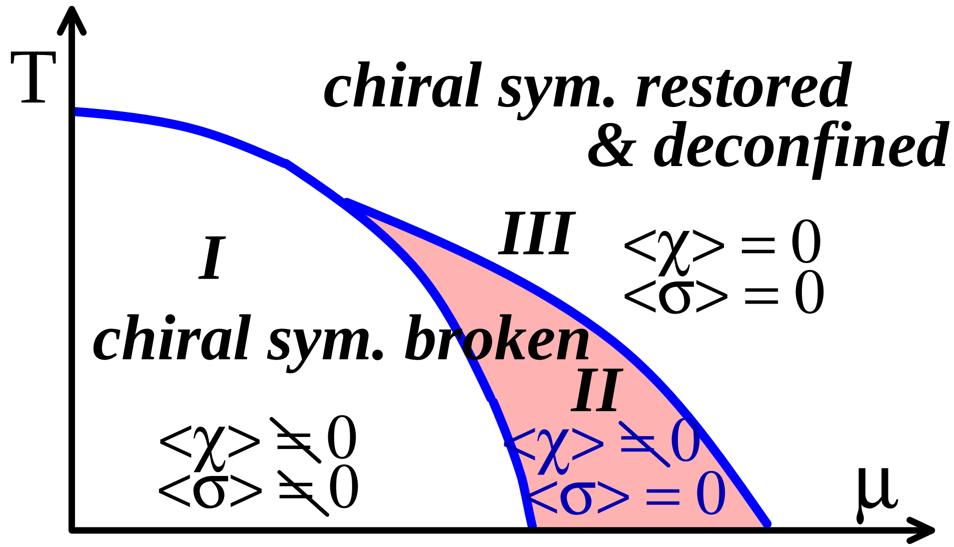
  impossible in vacuum but not excluded at $\mu \neq 0$ [Kogan-Kovner-Shifman (99)]
- **some "exotic phase" from dynamical model calculations**
  - O(2) model (scalar $\phi = \phi_1 + i\phi_2$) in CJT: [Watanabe-Fukushima-Hatsuda (03)]
    meta-stable phase where $\langle \phi^2 \rangle \neq 0$ while $\langle \phi \rangle = 0$
  - Skyrme model **w/o unbroken center** [Park et al. (02), Lee et al. (03)]
    an intermediate phase where $\langle \sigma \rangle = 0$ but $F_\pi \neq 0$

Exotic phase? role of tetra-quark states

[Harada-CS-Takemoto (09)]

- **2 phases with broken symmetry: distinguished by** n_B





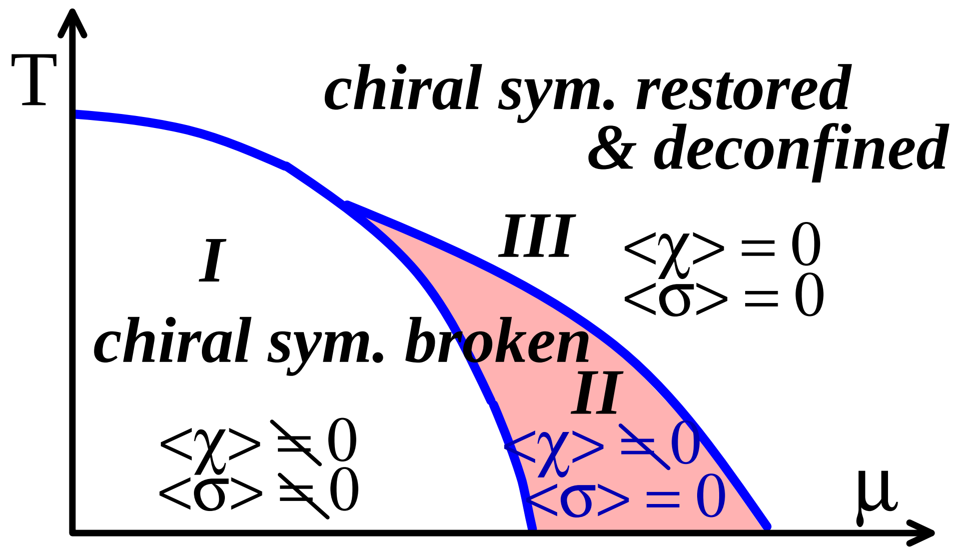
– order parameters: 2-quark state $\sigma \sim \bar{q}q$ and 4-quark state $\chi \sim (\bar{q}q)^2 + \bar{q}\bar{q}\text{-}qq$

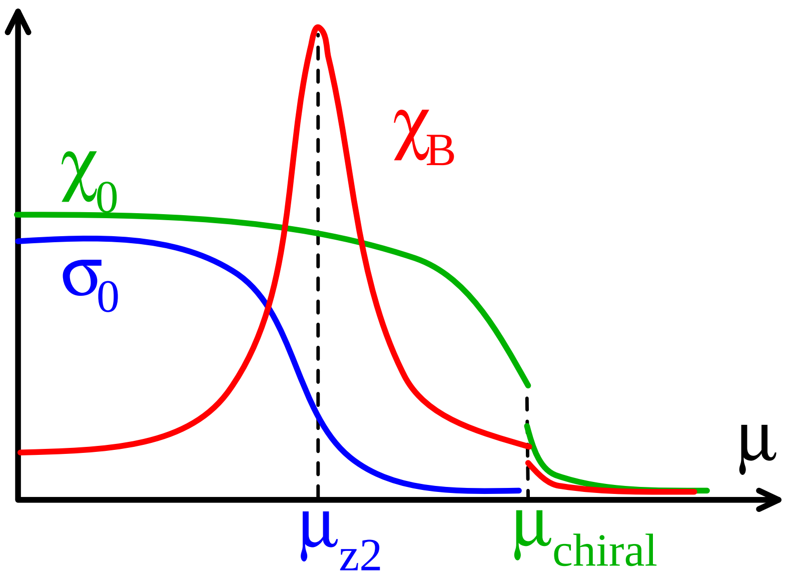
– 3 phases from a Ginzburg-Landau potential $(V = A\sigma^2 + B\chi^2 + \cdots)$

Exotic phase? role of tetra-quark states

[Harada-CS-Takemoto (09)]

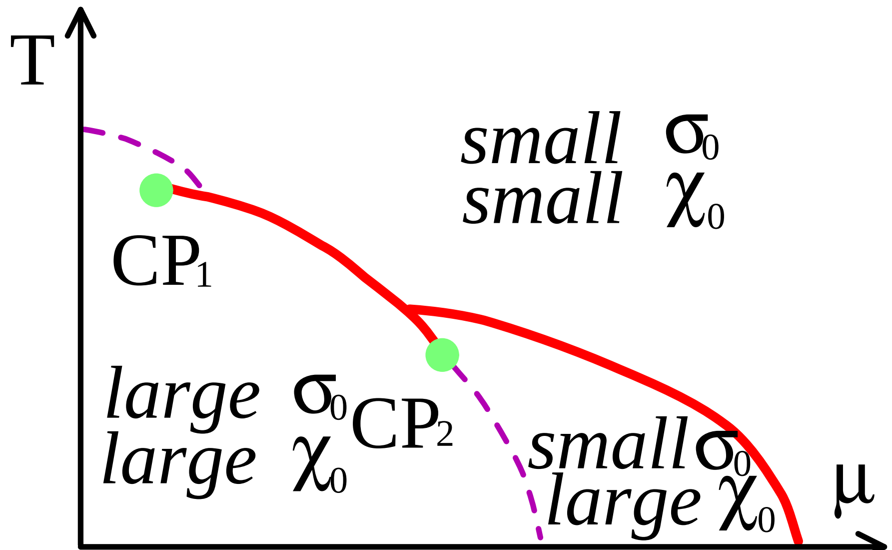
- **2 phases with broken symmetry: distinguished by n_B**

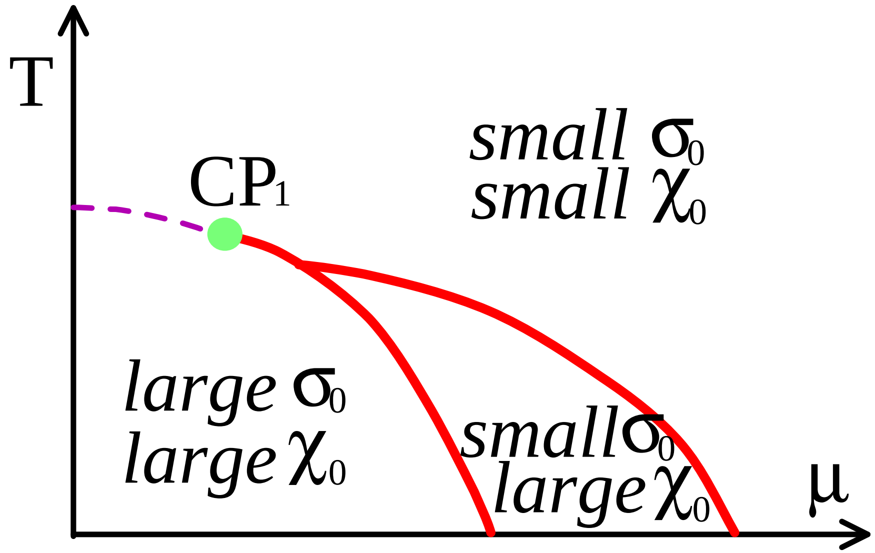




- 3 phases from a Ginzburg-Landau potential
  I-II: $\chi_B \sim \frac{\partial\sigma}{\partial\mu}$ max. $(\sigma \to 0)$
  II-III: χ_B no much change (no Yukawa term $\bar{N}N\chi$ in phase II)
- χ_B max. along Z_2 restoration

• **hypothetical phase diagram in T-μ plane (w/ explicit breaking)**





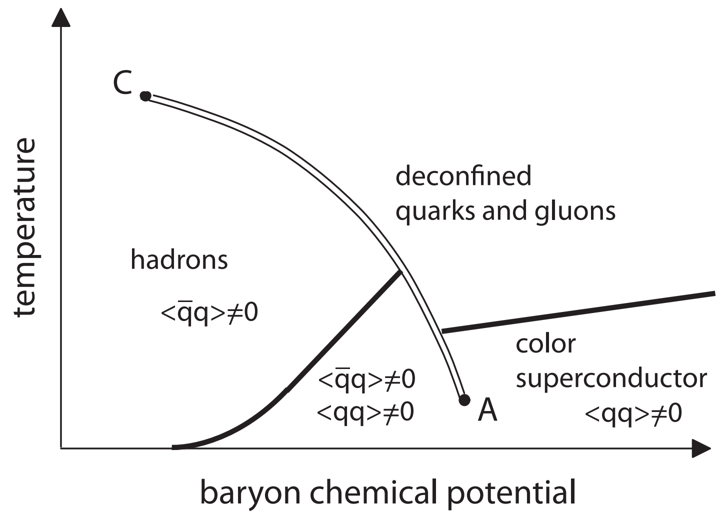
– 2 order parameters: σ (2-quark) and χ (4-quark)

$\Rightarrow$ 2 phase transitions: restoration of Z_2 center and chiral symmetries

– multiple CPs: CP1 and CP2 belong to the same universality class

$\Leftrightarrow$ different universality from anomaly induced CP [Hatsuda et al. (06-07)]

$\because$ $U(1)_B$ is broken in CFL phase.



- **hadron mass spectra**

phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$	phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$
$SU(2)_V$	$SU(2)_V \times (Z_2)_A$
$m_S \neq 0\,, m_P = 0$ $m_V \neq m_A$	$m_S \neq m_P \neq 0$ $m_V \neq m_A$
$m_{N^+} \neq 0$	(i) naive: $\begin{cases} m_{N^+} = 0 \text{ (ground state)} \\ m_{N'^+} = m_{N'^-} \neq 0 \\ \text{(excited states)} \end{cases}$ (ii) mirror: $\begin{cases} m_{N^+} = m_{N^-} \neq 0 \\ \text{(all states)} \end{cases}$

phase I: $\sigma_0 \neq 0\,, \chi_0 \neq 0$	phase II: $\sigma_0 = 0\,, \chi_0 \neq 0$
$SU(3)_V$	$SU(3)_V \times (Z_3)_A$
$m_S \neq 0\,, m_P = 0$ $m_V \neq m_A$	$m_S = m_P \neq 0$ $m_V \neq m_A$
$m_{N^+} \neq 0$	(i) naive: $m_{N^+} \neq 0$ (ii) mirror: $m_{N^+} = m_{N^-} \neq 0$

Summary and remarks

- **dense hadronic matter in chiral approaches**
  - parity doublet model (LG and chiral transitions)
    - saturation properties & pseudo critical temperature from LQCD
    - meson-baryon "transition": a trace of LG transition
  - unbroken center Z_{N_f} in dense matter (fluctuations generate DχSB)
    - a model for 2- and 4-quark states
    - enhancement of χ_B associated with Z_{N_f} symmetry restoration
      baryons are more activated in this $broken$ phase.
- **low-enery excitations constrained by anomaly matching**
  what are they in chirally-restored and confined phase?
- **origin of m_0?**