

Fluctuations and the QCD phase diagram

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ECT*-EMMI International Workshop

on

Chiral Symmetry and Confinement in Cold Quark Matter

Trento, Italy

QCD Phase Transitions

QCD → two phase transitions:

1 restoration of chiral symmetry

$$SU_{L+R}(N_f) \rightarrow SU_L(N_f) \times SU_R(N_f)$$

order parameter:

$$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

2 de/confinement

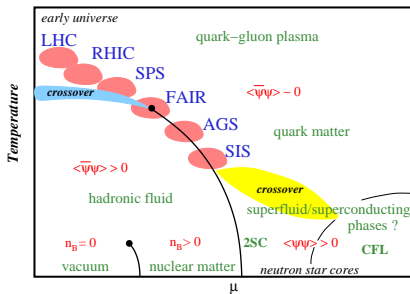
order parameter: Polyakov loop variable

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase, } T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase, } T > T_c \end{cases}$$

$$\Phi = \left\langle \text{tr}_c \mathcal{P} \exp \left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$

alternative: → dressed Polyakov loop (or dual condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator



At densities/temperatures of interest
only model calculations available

effective models:

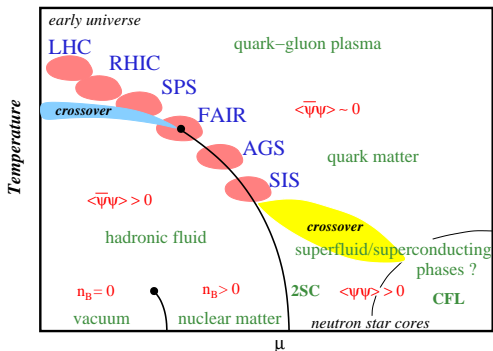
1 Quark-meson model

or other models e.g. NJL

2 Polyakov-quark-meson model

or PNJL models

The conjectured QCD Phase Diagram



At densities/temperatures of interest
only model calculations available

Open issues:

related to chiral & deconfinement transition

- ▷ existence of CEP?
- ▷ its location?
- ▷ additional CEPs?
How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/
deconfinement CEP?
- ▷ so far only MFA results
effect of fluctuations
(e.g. size of crit. reg.)?
- ▷ ...

Outline

- **Three-Flavor Chiral Quark-Meson Model**
- **...with Polyakov loop dynamics**
- **The important role of fluctuations**
- **Finite density extrapolations**

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling h :

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\not{\partial} - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

$$\text{fields: } M = \sum_{a=0}^8 \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$$

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1 (\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)] \\ & + \text{tr}[H(M + M^\dagger)] \end{aligned}$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

Phase diagram for $N_f = 2 + 1$ ($\mu \equiv \mu_q = \mu_s$)

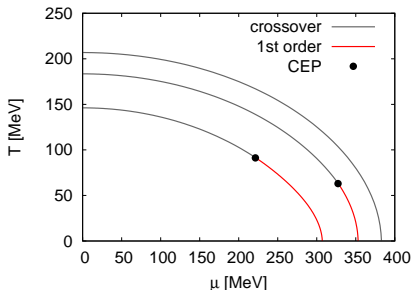
- Model parameter fitted to (pseudo)scalar meson spectrum:
- PDG: $f_0(600)$ mass=(400 . . . 1200) MeV \rightarrow broad resonance

\rightarrow existence of CEP depends on m_σ !

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)

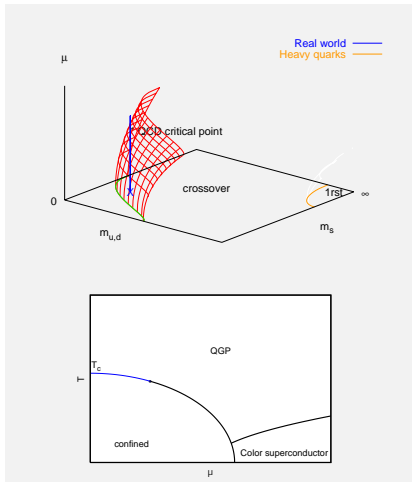
with $U(1)_A$

[BJS, M. Wagner '09]

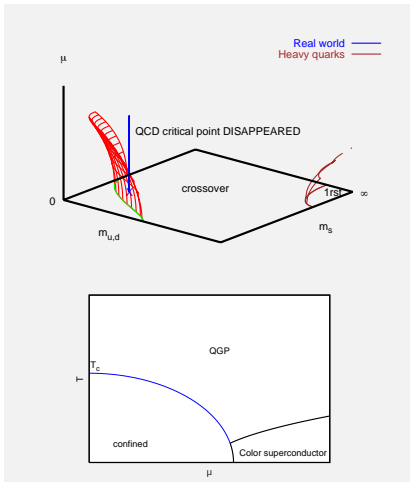


Mass sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

Standard scenario: $m_c(\mu)$ increasing



Nonstandard scenario: $m_c(\mu)$ decreasing

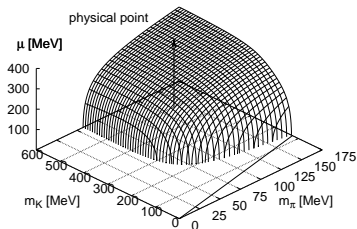


[de Forcrand, Philipsen: hep-lat/0611027]

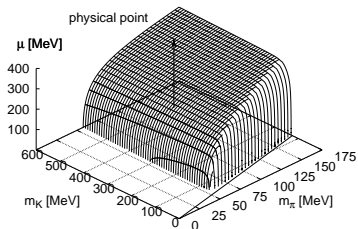
Chiral critical surface ($m_\sigma = 800$ MeV)

→ standard scenario for $m_\sigma = 800$ MeV (as expected)

with $U(1)_A$



without $U(1)_A$



[BJS, M. Wagner, '09]

Note: 't Hooft coupling μ -independent

PNJL with (unrealistic) large vector int. → bending of surface

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- Three-Flavor Chiral Quark-Meson Model
- **...with Polyakov loop dynamics**
- The important role of fluctuations
- Finite density extrapolations

Polyakov-quark-meson (PQM) model

■ Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

■ polynomial Polyakov loop potential:

Polyakov 1978, Meisinger 1996, Pisarski 2000

$$\frac{\mathcal{U}(\phi, \bar{\phi})}{T^4} = -\frac{b_2(T, T_0)}{2} \phi \bar{\phi} - \frac{b_3}{6} (\phi^3 + \bar{\phi}^3) + \frac{b_4}{16} (\phi \bar{\phi})^2$$

with

$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

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■ logarithmic potential:

Rössner et al. 2007

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2} a(T) \bar{\phi} \phi + b(T) \ln \left[1 - 6 \bar{\phi} \phi + 4 (\phi^3 + \bar{\phi}^3) - 3 (\bar{\phi} \phi)^2 \right]$$

with

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

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■ Fukushima

Fukushima 2008

$$\mathcal{U}_{\text{Fuku}} = -bT \left\{ 54e^{-a/T} \phi \bar{\phi} + \ln \left[1 - 6\bar{\phi}\phi + 4(\phi^3 + \bar{\phi}^3) - 3(\bar{\phi}\phi)^2 \right] \right\}$$

with

a controls deconfinement b strength of mixing chiral & deconfinement

Polyakov-quark-meson (PQM) model

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in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$

BJS, Pawłowski, Wambach, 2007

N_f	0	1	2	2 + 1	3
T_0 [MeV]	270	240	208	187	178

$\mu \neq 0$: $\bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

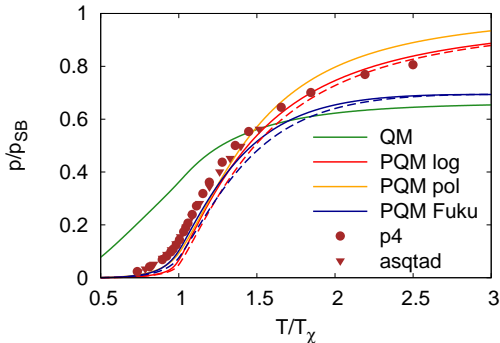
in medium with more quarks \rightarrow antiquarks are more easily screened.

QCD Thermodynamics $N_f = 2 + 1$

[BJS, M. Wagner, J. Wambach '10]

$$\text{SB limit: } \frac{p_{\text{SB}}}{T^4} = 2(N_c^2 - 1) \frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



- ▷ solid lines:
PQM with lattice masses
(HotQCD)
 $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines:
(P)QM with realistic masses

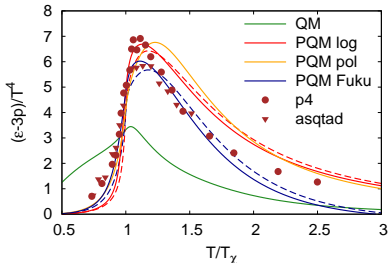
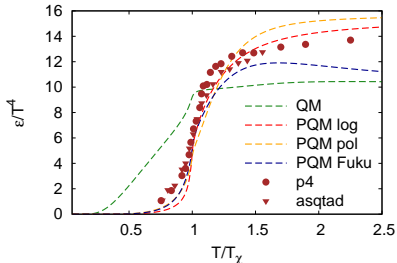
lattice data: [Bazavov et al. '09]

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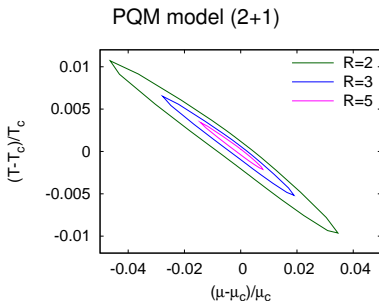
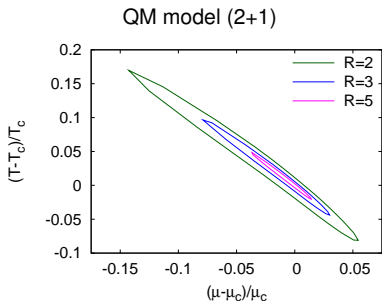
solid lines: $m_\pi \sim 220, m_K \sim 503$ MeV (HotQCD)
[Bazavov et al. '09]

Critical region

contour plot of **size of the critical region** around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

→ compressed with Polyakov loop



[BJS, M. Wagner; in preparation]

Outline

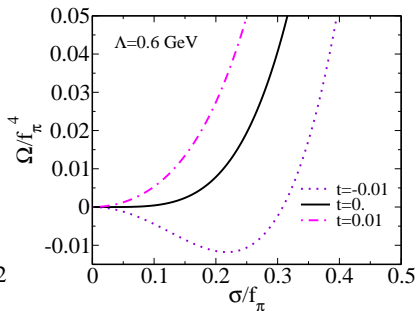
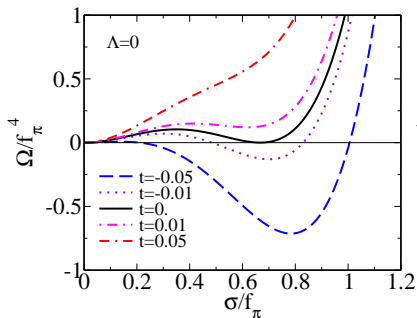
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- **The important role of fluctuations**
- Finite density extrapolations

Importance of Dirac term

[V. Skokov, B. Friman, K.Redlich, BJS; arXiv:1005.3166]

Thermodynamic potential (numerical results for $\mu = 0$)

$$\begin{aligned}\Omega &= U_{\text{Pol}} + U_{\text{meson}} + \Omega_{q\bar{q}} && \text{with} \\ \Omega_{q\bar{q}} &= -2N_f \int \frac{d^3p}{(2\pi)^3} \left\{ N_c E_q \theta(p^2 - \Lambda^2) + T \ln N_q + T \ln N_{\bar{q}} \right\} \\ N_q &= 1 + 3\Phi e^{-\beta(E_q - \mu)} + 3\bar{\Phi} e^{-2\beta(E_q - \mu)} + e^{-3\beta(E_q - \mu)}\end{aligned}$$

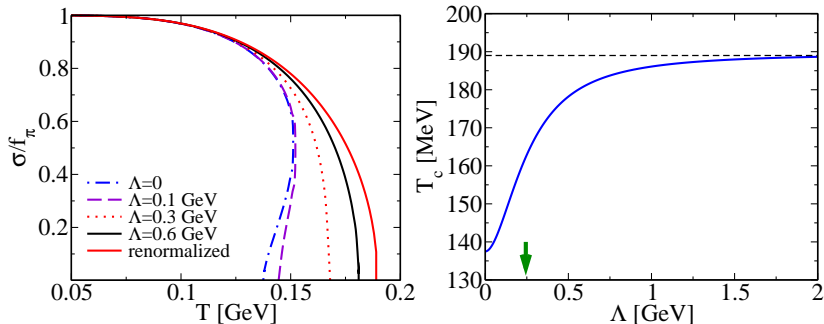


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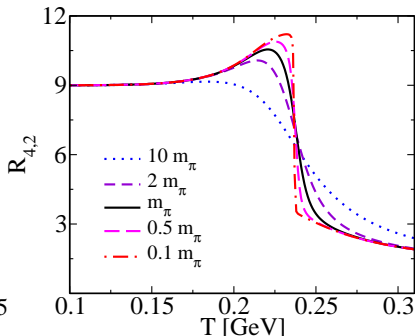
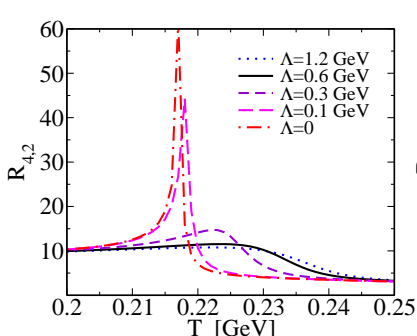


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Isentropes $s/n = \text{const}$ and Focussing

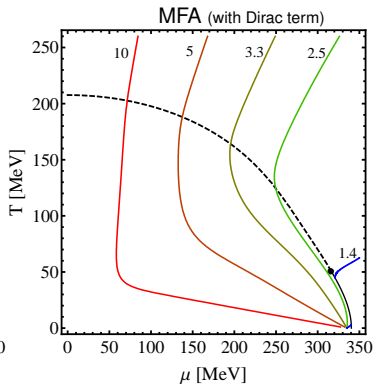
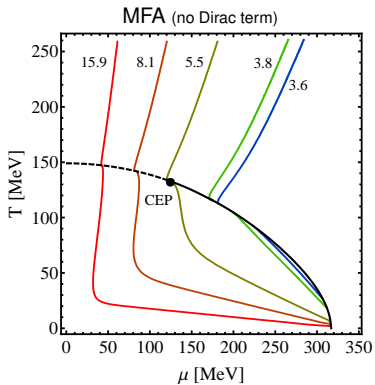
[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich '10]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

→ no focussing if fluctuations taken into account

a) influence of Dirac term

b) smallest of critical region



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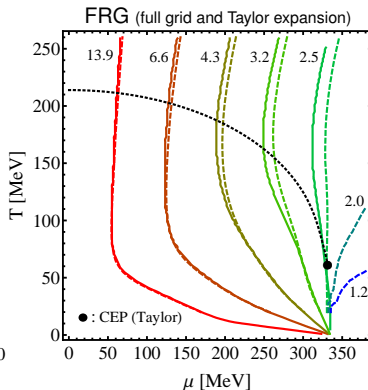
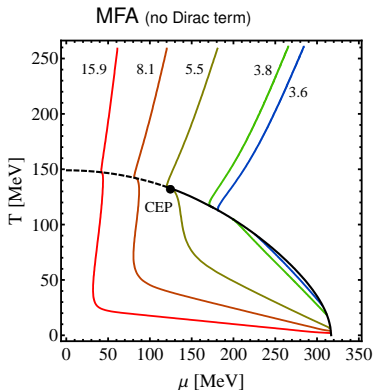
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a) influence of Dirac term b) smallest of crit region

kink structure at boundary in mean field approximation

⇒ remnant of first-order transition in chiral limit

if Dirac term neglected

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in presence of dynamical quarks: $T_0 = T_0(N_f, \mu)$

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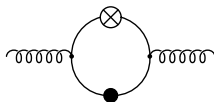
in medium with more quarks \rightarrow antiquarks are more easily screened.

$T_0(N_f, \mu)$ modification

full QCD FRG flow: gluon, ghosts, quark and meson (via hadronization) fluctuations
 cf talk by J.M.Pawlowski

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Polyakov loop} - \text{ghost loop} - \text{quark loop} + \frac{1}{2} \text{meson loop} \right)$$

in presence of dynamical quarks
 gluonic contribution modified:



pure YM flow

(\rightarrow Polyakov loop potential):

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Polyakov loop} - \text{ghost loop} \right)$$

$$T_0 \leftrightarrow \Lambda_{QCD} \quad :$$

$$T_0 \rightarrow T_0(N_f, \mu)$$

[BJS, Pawlowski, Wambach, 2007]

[Herbst, Pawlowski, BJS; in preparation]

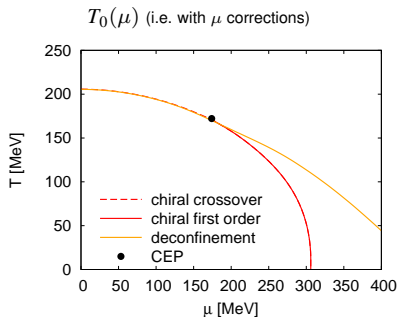
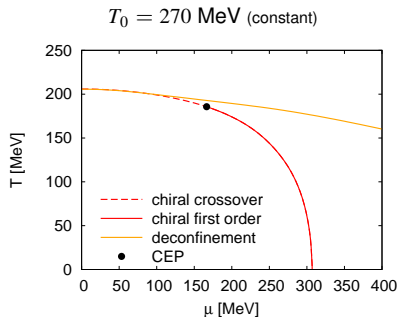
Phase diagram $N_f = 2 + 1$

[BJS, M. Wagner; in preparation]

influence of Polyakov loop

Logarithmic Polyakov loop potential

Mean-field approximation



shrinking of possible quarkyonic phase

Functional Renormalization Group

similar conclusion if **fluctuations** are included

cf. talk by Jan Pawłowski (yesterday)

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right)$$

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

PQM truncation $N_f = 2$

T.K.Herbst, J.M. Pawłowski, BJS, in preparation

$$\Gamma_k = \int d^4x \left\{ \bar{\psi} (\not{D} + \mu \gamma_0 + ih(\sigma + i\gamma_5 \vec{\tau} \vec{\pi})) \psi + \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} (\partial_\mu \vec{\pi})^2 + \Omega_k[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] \right\}$$

Initial action at UV scale Λ :

$$\Omega_\Lambda[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] = U(\sigma, \vec{\pi}) + \mathcal{U}(\Phi, \bar{\Phi}) + \Omega_\Lambda^\infty[\sigma, \vec{\pi}, \Phi, \bar{\Phi}]$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

Functional Renormalization Group

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi}$$

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Flow equation for PQM $N_f = 2$

T.K.Herbst, J.M. Pawłowski, BJS, in preparation

$$\begin{aligned} \partial_t \Omega_k = & \frac{k^5}{12\pi^2} \left[-\frac{2N_f N_c}{E_q} \left\{ 1 - N_q(T, \mu; \Phi, \bar{\Phi}) + N_{\bar{q}}(T, \mu; \Phi, \bar{\Phi}) \right\} \right. \\ & \left. + \frac{1}{E_\sigma} \coth\left(\frac{E_\sigma}{2T}\right) + \frac{3}{E_\pi} \coth\left(\frac{E_\pi}{2T}\right) \right] \end{aligned}$$

with
and

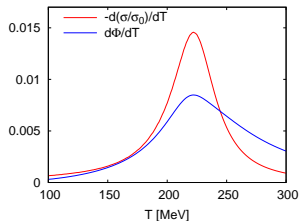
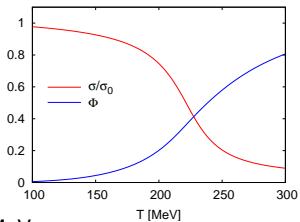
$$E_{\sigma, \pi, q} = \sqrt{k^2 + m_{\sigma, \pi, q}^2}, \quad m_\sigma^2 = 2\Omega_k' + 4\sigma^2 \Omega_k'', \quad m_\pi^2 = 2\Omega_k', \quad m_q^2 = g^2 \sigma^2$$

$$N_q(T, \mu; \Phi, \bar{\Phi}) = \frac{1 + 2\bar{\Phi} e^{\beta(E_q - \mu)} + \Phi e^{2\beta(E_q - \mu)}}{1 + 3\bar{\Phi} e^{\beta(E_q - \mu)} + 3\Phi e^{2\beta(E_q - \mu)} + e^{3\beta(E_q - \mu)}}$$

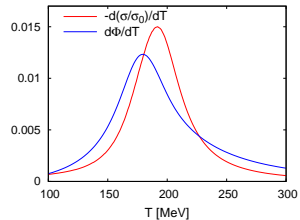
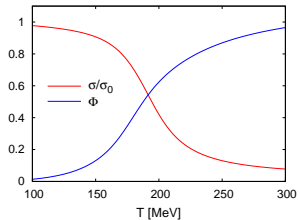
$$N_{\bar{q}}(T, \mu; \Phi, \bar{\Phi}) = N_q(T, -\mu; \Phi, \bar{\Phi})|_{\mu \rightarrow -\mu}$$

$\mu = 0$: order parameters and T -derivatives

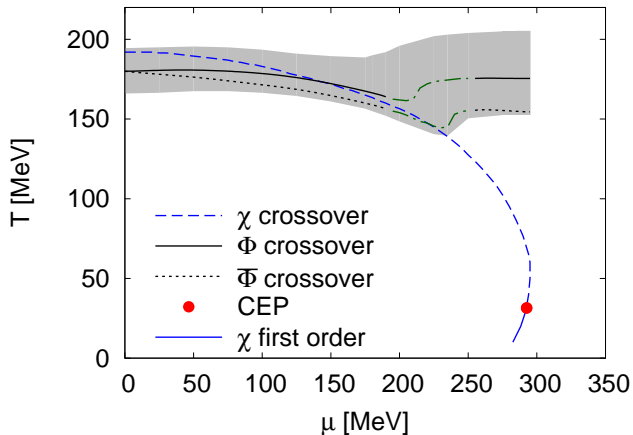
$T_0 = 270$ MeV



$T_0 = 208$ MeV

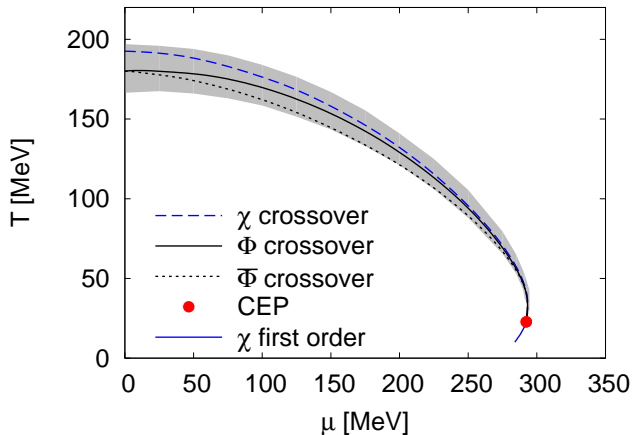


Phase diagram $T_0 = 208$ MeV



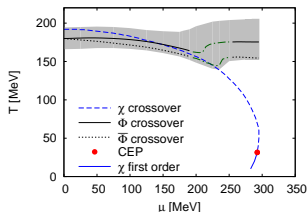
[Herbst, Pawłowski, BJS; in preparation]

Phase diagram $T_0(\mu), T_0(0) = 208 \text{ MeV}$

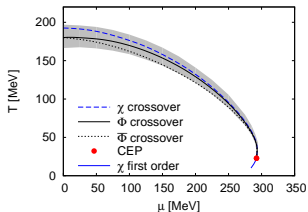


[Herbst, Pawłowski,BJS; in preparation]

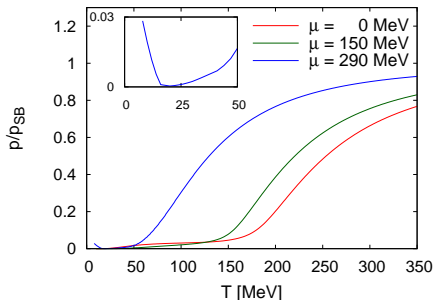
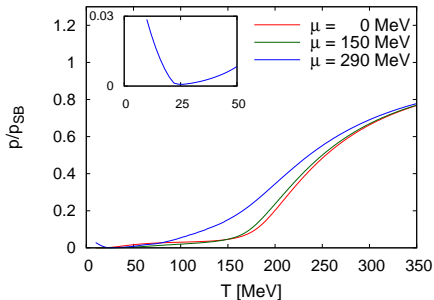
Thermodynamics



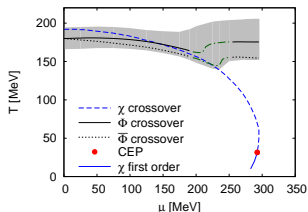
$T_0 = 208$ MeV



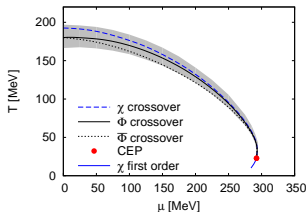
$T_0(\mu)$ MeV



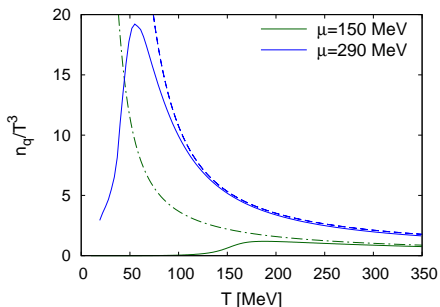
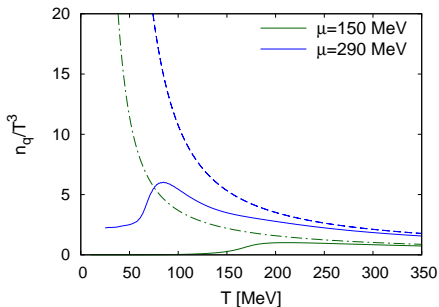
Thermodynamics



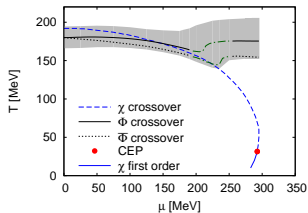
$T_0 = 208$ MeV



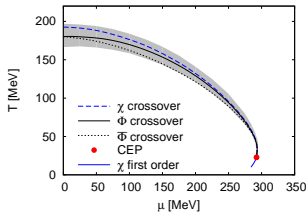
$T_0(\mu)$ MeV



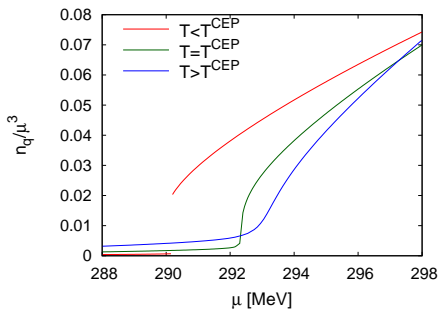
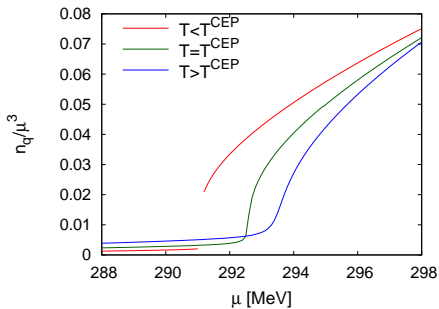
Thermodynamics



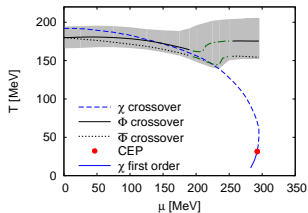
$T_0 = 208$ MeV



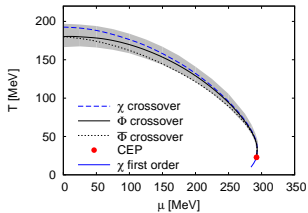
$T_0(\mu)$ MeV



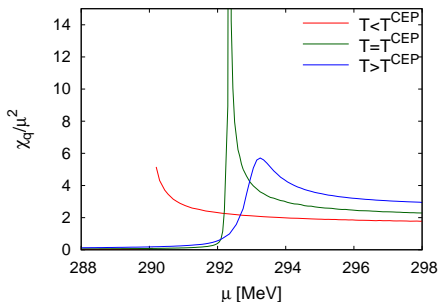
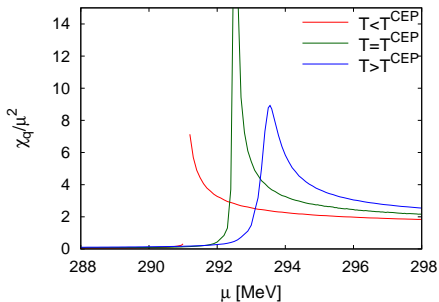
Thermodynamics



$T_0 = 208$ MeV



$T_0(\mu)$ MeV



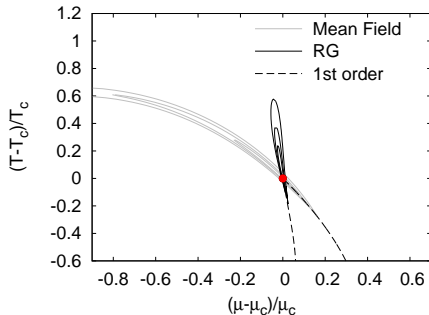
Critical region

similar conclusion if **fluctuations** are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ QM model

Mean Field \leftrightarrow RG analysis



[BJS, J. Wambach '06]

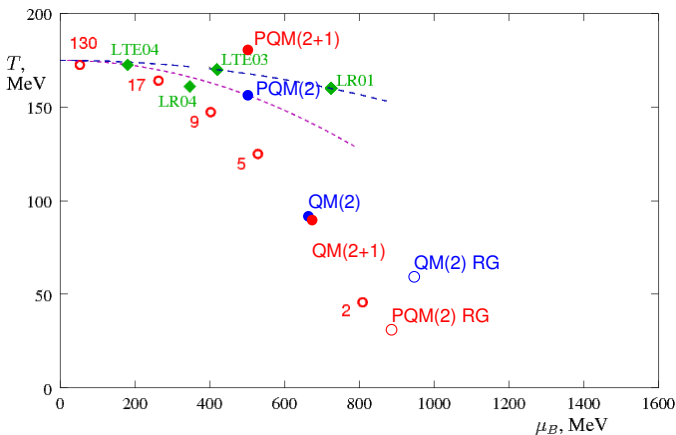
Critical Endpoints

model studies vs. lattice simulations

Blue points: models

Lines & green points: lattice

Red circles: Freezeout points for HIC



lattice methods:

- reweighting
- imaginary μ_B
- Taylor expansion around $\mu_B = 0$

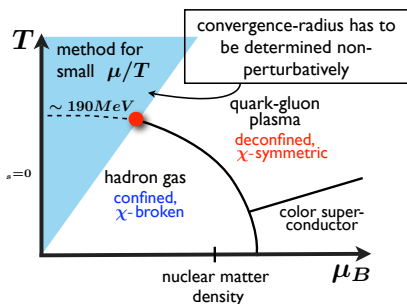
Outline

- Three-Flavor Chiral Quark-Meson Model
- ...with Polyakov loop dynamics
- The important role of fluctuations
- **Finite density extrapolations**

Finite density extrapolations $N_f = 2 + 1$

Taylor expansion:

$$\frac{p(T, \mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \left. \frac{\partial^n (p(T, \mu)/T^4)}{\partial (\mu/T)^n} \right|_{\mu=0}$$



convergence radii:

limited by first-order line?

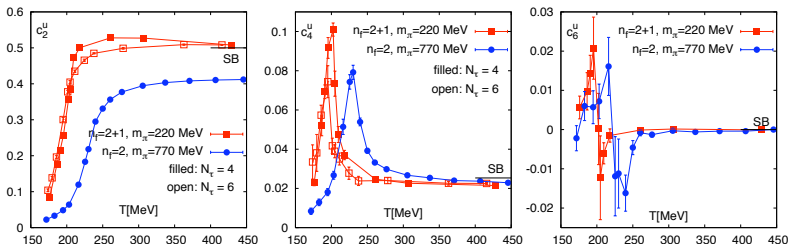
$$\rho_{2n} = \left| \frac{c_2}{c_{2n}} \right|^{1/(2n-2)}$$

[C. Schmidt '09]

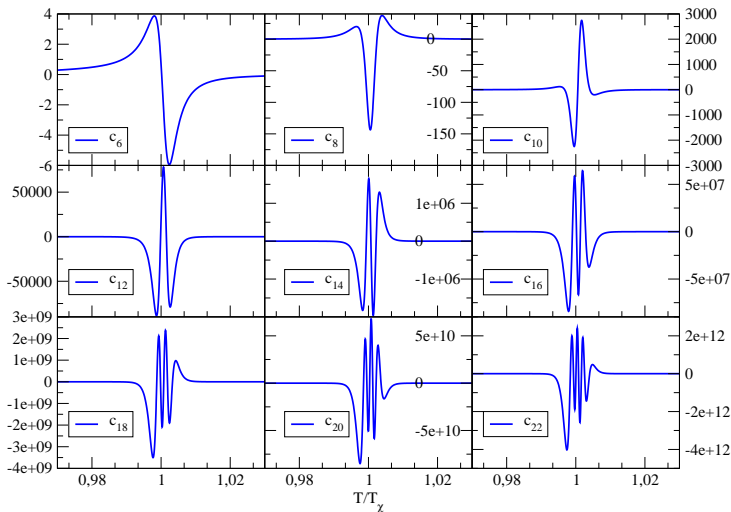
Finite density extrapolations $N_f = 2 + 1$

Taylor expansion:

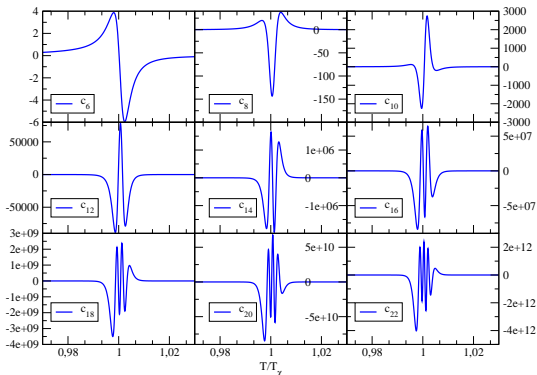
$$\frac{p(T, \mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \left. \frac{\partial^n (p(T, \mu)/T^4)}{\partial (\mu/T)^n} \right|_{\mu=0}$$



[Miao et al. '08]

Taylor coefficients c_n numerically known to high order, e.g. $n = 22$ 

Finite density extrapolations $N_f = 2 + 1$



- ▷ this technique applied to PQM model
- ▷ investigation of convergence properties of Taylor series
- ▷ properties of c_n
 - oscillating
 - increasing amplitude
 - no numerical noise
 - small outside transition region
 - number of roots increasing
 - 26th order

[F. Karsch, BJS, M. Wagner, J. Wambach; in preparation]

Can we locate the QCD critical endpoint with the Taylor expansion ?

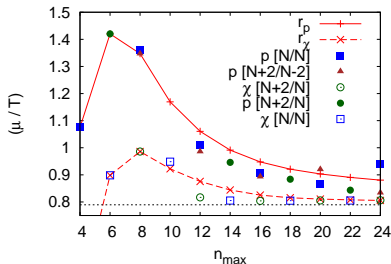
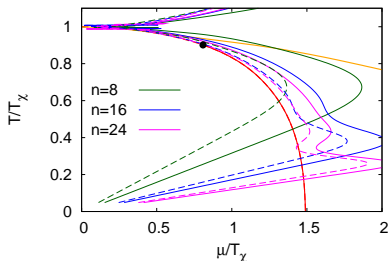
Susceptibility $N_f = 2 + 1$ PQM model

Findings:

- simply Taylor expansion: slow convergence
high orders needed
disadvantage for lattice simulations
- Taylor applicable within convergence radius
also for $\mu/T > 1$

$$r_{2n} = \left| \frac{c_{2n}}{c_{2n+2}} \right|^{1/2}, \quad r_{2n}^{\chi} = \left| \frac{(2n+2)(2n+1)}{(2n+3)(2n+4)} \right|^{1/2} r_{2n+2}$$

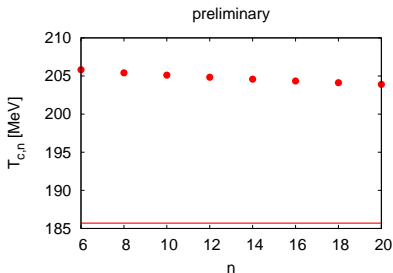
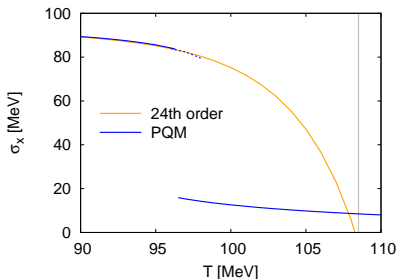
Padé [N/N]



Susceptibility $N_f = 2 + 1$ PQM model

Findings:

- simply Taylor expansion: slow convergence
high orders needed
disadvantage for lattice simulations
- Taylor applicable within convergence radius
also for $\mu/T > 1$
- but 1st order transition not resolvable
expansion around $\mu = 0$



Summary

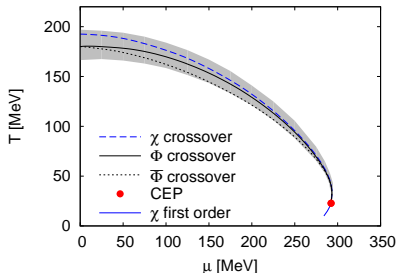
- $N_f = 2$ and $N_f = 2 + 1$ chiral (Polyakov)-quark-meson model study
 - Mean-field approximation and FRG
 - finite density extrapolation via novel AD technique

Findings:

- ▷ matter **back-reaction to YM sector**:
 $T_0 \Rightarrow T_0(N_f, \mu)$
- ▷ **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$ -corrections
- ▷ same conclusion for $N_f = 2 + 1$?
- ▷ **role of quantum fluctuations**
effects of Dirac term in a mean-field approximation
- ▷ **convergence properties** of Taylor expansion technique
 $c_n(T) \rightarrow$ **high order available**

Outlook:

- include glue dynamics with FRG \rightarrow full QCD





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