$SU(2)_c$ PNJL model and hadronization at finite temperature and density

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NJL model and hadronization

- Eguchi [PRD(1976)] showed that we can get the sigma model chiral Lagrangian by bosonization
- NJL model with Polyakov line generates a good thermodynamical property
- We want to derive meson-baryon dynamics and matter property from quarks (QCD)
- We derive here hadron physics from NJL Lagrangian with N_c=2

Bosonization and baryonization

NJL Lagrangian

Eguchi PRD(1976)

$$L = i\bar{\psi}\gamma_\mu\partial^\mu\psi + \frac{G_0}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

Auxiliary fields

$$Z = \int D\bar{\psi}D\psi D\sigma D\vec{\pi} \exp i \int d^4x[\bar{\psi}S^{-1}\psi - \frac{1}{2}\delta\mu_0^2(\sigma^2 + \vec{\pi}^2)]$$

$$S^{-1} = i\gamma_\mu\partial^\mu - g_0(\sigma + i\gamma_5\vec{\pi}\vec{\tau}) \qquad G_0 = \frac{g_0^2}{\delta\mu_0^2}$$

Quark integration

$$\int dx e^{-ax^2} = \sqrt{\frac{\pi}{a}}$$

$$Z = \int D\sigma D\vec{\pi} \exp(i \int d^4x[-\frac{1}{2}\delta\mu_0^2(\sigma^2 + \vec{\pi}^2) - iTr \ln S^{-1}])$$

Expand the Tr-log term (mean field)

$$U = -iTr \ln[1 - \frac{1}{i\gamma_\mu\partial^\mu - m} g_0(s + i\gamma_5\vec{\pi}\vec{\tau})] = \sum_{n=1}^{\infty} U^{(n)}$$

$$m = g_0 v_0 \quad \sigma = v_0 + s$$

NJL bosonization-2

$$U^{(n)} = \frac{i}{n} Tr \left[\frac{1}{i\gamma_\mu \partial^\mu - m} g_0 (s + i\gamma_5 \vec{\pi}\vec{\tau}) \right]^n$$

n=1

$$U_s^{(1)} = g_0 i Tr_{fcD} \int d^4x \langle x | \frac{1}{i\gamma_\mu \partial^\mu - m} s | x \rangle$$

⇒ Gap equation

$$U_s^{(1)} = 4 i g_0 Tr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \int d^4x \frac{m}{(p^2 - m^2)} s(x)$$
$$= 2 g_0 m I_2 \int d^4x s(x)$$

$$I_2 = 2iTr \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2}$$
$$I_0 = -2iTr \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^2}$$

n=2

$$U_s^{(2)} = g_0^2 \frac{i}{2} Tr \int d^4x \langle x | \frac{1}{i\gamma_\mu \partial^\mu - m} s \frac{1}{i\gamma_\mu \partial^\mu - m} s | x \rangle$$

$$s(y) = s(x) + (y-x)_\mu \partial^\mu s(x) + \frac{1}{2} (y-x)_\mu (y-x)_\nu \partial^\mu \partial^\nu s(x) + \dots$$

NJL bosonization-3

$$T^{(1)} = 2ig_0^2 Tr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \int d^4x \frac{p^2+m^2}{(p^2-m^2)(p^2-m^2)} s(x)s(x)$$
$$= (g_0^2 I_2 - 2m^2 g_0^2 I_0) \int d^4x s^2(x)$$

$$T^{(3)} = \frac{1}{2} g_0^2 \int d^4x \partial_\nu s(x) \partial^\nu s(x) I_0$$

⇨ Mass
Kinetic energy

n=3, 4

$$U_s^{(3)} = g_0^3 \frac{i}{3} Tr \int d^4x \langle x| \frac{1}{i\gamma_\mu \partial^\mu - m} s \frac{1}{i\gamma_\mu \partial^\mu - m} s \frac{1}{i\gamma_\mu \partial^\mu - m} s |x\rangle$$

$$U_s^{(3)} = g_0^3 \frac{4i}{3} Tr \int d^4x s^3(x) \int \frac{d^4p}{(2\pi)^4} \frac{3p^2m + m^3}{(p^2-m^2)^3}$$
$$\sim -2mg_0^3 I_0 \int d^4x s^3(x)$$

$$U_s^{(4)} = g_0^4 \frac{4i}{4} Tr \int d^4x s(x)^4 \int \frac{d^4p}{(2\pi)^4} \frac{p^4 + 6p^2m^2 + m^4}{(p^2-m^2)^4}$$
$$\sim -\frac{1}{2} g_0^4 I_0 \int d^4x s^4(x)$$

NJL bosonization-4

Pion part

$$U_p^{(2)} = g_0^2 \frac{i}{2} Tr \int d^4x \langle x| \frac{1}{i\gamma_\mu \partial^\mu - m} (i\gamma_5 \vec{\pi}\vec{\tau}) \frac{1}{i\gamma_\mu \partial^\mu - m} (i\gamma_5 \vec{\pi}\vec{\tau}) |x\rangle$$

$$\begin{aligned} U_p^{(2)} &= g_0^2 \frac{8i}{2} Tr_c \int d^4x \vec{\pi}^2(x) \int \frac{d^4p}{(2\pi)^4} \frac{p^2 - m^2}{(p^2 - m^2)^2} \\ &= g_0^2 I_2 \int d^4x \vec{\pi}^2(x) \end{aligned}$$

We can work out other pion terms in the similar way as sigma.

sigma+pion

$$\begin{aligned} L &= -\frac{1}{2}\delta\mu_0^2(\sigma^2 + \vec{\pi}^2) + 2g_0 m I_2 s(x) + \frac{1}{2} g_0^2 I_0 (\partial_\nu s(x) \partial^\nu s(x) + \partial_\nu \vec{\pi}(x) \partial^\nu \vec{\pi}(x)) \\ &+ (g_0^2 I_2 - 2m^2 g_0^2 I_0) s(x)^2 + g_0^2 I_2 \vec{\pi}^2(x) - 2m g_0^3 I_0 s^3(x) - 2 g_0^3 m I_0 \vec{\pi}^2(x) s(x) \\ &- \frac{1}{2} g_0^4 I_0 s^4(x) - g_0^4 I_0 \vec{\pi}^2(x) s^2(x) - \frac{1}{2} g_0^4 I_0 \vec{\pi}^4(x) \end{aligned}$$

NJL bosonization-5

Renormalization

$$I_0 g_0^2 = \frac{1}{Z_M}$$
$$2I_0 g_0^4 = \frac{\lambda_0}{Z_\lambda}$$
$$-\delta\mu_0^2 + 2I_2 g_0^2 = 0$$

$$\sigma_R = Z_M^{-1/2}\sigma$$
$$\vec{\pi}_R = Z_M^{-1/2}\vec{\pi}$$
$$v = Z_M^{-1/2} v_0$$
$$\lambda = Z_\lambda^{-1} Z_M^2 \lambda_0$$

$$L_s = \frac{1}{2}((\partial_\mu s_R)^2 + (\partial_\mu \vec{\pi}_R)^2) - \frac{1}{2}\lambda v^2 s_R^2 - \lambda v (s_R^2 + \vec{\pi}_R^2) s_R - \frac{1}{4}\lambda (s_R^2 + \vec{\pi}_R^2)^2$$

$$L_R = \frac{1}{2}((\partial_\mu \sigma_R)^2 + (\partial_\mu \vec{\pi}_R)^2) - \frac{1}{4}\lambda(\sigma_R^2 + \vec{\pi}_R^2 - v^2)^2$$

We can obtain sigma model Lagrangian by bosonization of the NJL model written in quarks.

NJL+diquark baryonization

Diquark Lagrangian

Ebert, Nagata, Hosaka

$$L_d = d^\dagger \Delta^{-1} d + \tilde{G}\bar{\psi} d^\dagger d \psi$$

$$\begin{aligned}
Z &= \int D\psi D\bar{\psi} D d D d^\dagger \exp(i\int d^4x [L_{NJL} + L_d]) = N \int D\psi D\bar{\psi} D\sigma D\vec{\pi} \\
&\times \exp[i\int d^4x (-\frac{1}{2G}(\sigma^2+\vec{\pi}^2) + \bar{\psi}(i\partial_\mu\gamma^\mu - m_0 - \sigma - i\gamma_5\vec{\tau}\vec{\pi})\psi)] \\
&\times N' \int D d D d^\dagger D B D\bar{B} \exp[i\int d^4x (-\frac{1}{\tilde{G}}\bar{B}B + \bar{\psi} d^\dagger B + \bar{B} d\psi + d^\dagger \Delta^{-1} d)]
\end{aligned}$$

Quark integral

$$\begin{aligned}
Z &= N\int D\sigma D\vec{\pi} D B D\bar{B} D d D d^\dagger \exp[i\int d^4x ((-\frac{1}{2G}(\sigma^2+\vec{\pi}^2) - iTr\ln S^{-1} - \frac{1}{\tilde{G}}\bar{B}B \\
&+ d^\dagger \Delta^{-1} d + d\bar{B}SBd^\dagger)]
\end{aligned}$$

Diquark integral

$$\begin{aligned}
Z &= N\int D\sigma D\vec{\pi} D B D\bar{B} \exp[i\int d^4x ((-\frac{1}{2G}(\sigma^2+\vec{\pi}^2) - iTr\ln S^{-1} - \frac{1}{\tilde{G}}\bar{B}B \\
&+ iTr\ln\Delta^{-1} + iTr\ln(1+\tilde{\Delta}\bar{B}SB))]
\end{aligned}$$

DNJL baryonization-2

$$iTr\ln(1+\tilde{\Delta}\bar{B}SB)=\sum_{n=1}^{\infty}U_B^{(n)} \qquad U_B^{(n)}=-(-1)^n\frac{i}{n}Tr[\tilde{\Delta}\bar{B}SB]^n$$

$$\frac{1}{i\gamma_\mu\partial^\mu-m-g_0(s+i\gamma_5\vec{\pi}\vec{\tau})}=\frac{1}{i\gamma_\mu\partial^\mu-m}(1-\frac{g_0(s+i\gamma_5\vec{\pi}\vec{\tau})}{i\gamma_\mu\partial^\mu-m})^{-1}$$
$$=\frac{1}{i\gamma_\mu\partial^\mu-m}+\frac{1}{i\gamma_\mu\partial^\mu-m}\frac{g_0(s+i\gamma_5\vec{\pi}\vec{\tau})}{i\gamma_\mu\partial^\mu-m}+...$$

$$U_B^{(1)}=iTr\int d^4x\langle x|\frac{1}{-\partial_\mu\partial^\mu-m_D^2}\bar{B}\frac{1}{i\gamma_\mu\partial^\mu-m}B|x\rangle$$

$$B(y)=B(x)+(y-x)_\mu\partial^\mu B(x)+\frac{1}{2}(y-x)_\mu(y-x)_\nu\partial^\mu\partial^\nu B(x)+...$$

$$\begin{aligned}T^{(1)} &= iTr\int\frac{d^4p}{(2\pi)^4}\int d^4x\frac{1}{p^2-m_D^2}\bar{B}(x)\frac{\gamma_\mu p^\mu+m}{p^2-m^2}B(x)\\ &= imTr\int d^4x\bar{B}(x)B(x)\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m_D^2}\frac{1}{p^2-m^2}\\ &= -\frac{1}{2}mI_0'\int d^4x\bar{B}(x)B(x)\end{aligned}$$

$$I_0'=-2iTr_{cf}\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m_D^2}\frac{1}{p^2-m^2}$$

DNJL baryonization-3

$$T^{(2)} = iTr\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\int d^4x\int d^4z e^{i(p-p')z}\frac{1}{p^2-m_D^2}z^\mu\bar{B}(x)\partial_\mu B(x)\frac{\gamma_\mu p'^\mu+m}{p'^2-m^2}$$

$$= \frac{1}{2}I_0'\int d^4x\bar{B}(x)i\partial_\mu\gamma^\mu B(x)$$

$$U^{(2)} = iTr\int\frac{d^4p}{(2\pi)^4}\int d^4x\frac{1}{p^2-m_D^2}\bar{B}(x)\frac{\gamma_\mu p^\mu+m}{p^2-m^2}g_0(s(x)+i\gamma_5\vec{\pi}(x)\vec{\tau})\frac{\gamma_\mu p^\mu+m}{p^2-m^2}B(x)$$

$$= iTr\int d^4x\bar{B}(x)g_0(s+i\gamma_5\vec{\pi}\vec{\tau})B(x)\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m_D^2}\frac{1}{p^2-m^2}$$

$$L_B = \frac{1}{2}I_0'\bar{B}(x)i\partial_\mu\gamma^\mu B(x)-\frac{1}{2}mI_0'\bar{B}(x)B(x)-\frac{1}{2}I_0'\bar{B}(x)g_0(s+i\gamma_5\vec{\pi}\vec{\tau})B(x)$$

$$-\frac{1}{\bar{G}}\bar{B}(x)B(x)$$

$$= \bar{B}_R(i\gamma^\mu\partial_\mu-M_B-g(s_R+i\gamma_5\vec{\pi}_R\vec{\tau}))B_R \qquad M_B=(\frac{m}{Z_B}+\frac{1}{\bar{G}})Z_B$$

$$\frac{1}{2}I_0' = \frac{1}{Z_B}$$

$$B_R = Z_B^{-1/2}B$$

$$g = 2g_0Z_M^{1/2}$$

By hadronization of quark model, we can derive sigma-model Lagrangian.

Polyakov line potential

4th gluon component

Fukushima, Weise, Sasaki, Arriola, ..

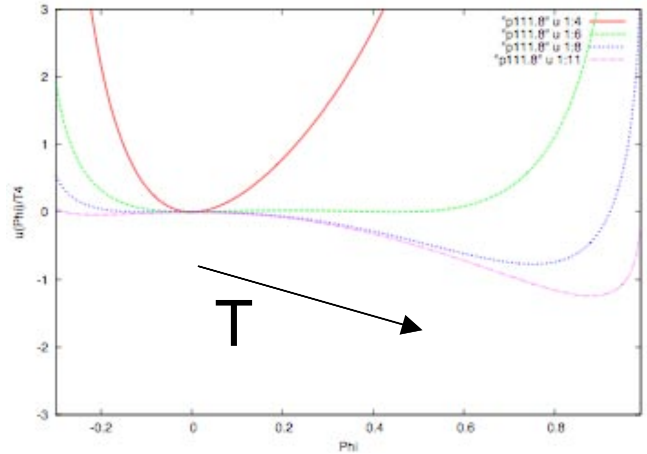
$$L_{PNJL} = \bar{\psi}(i\gamma_\mu D^\mu - m_0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - U(\Phi[A], \Phi^*[A], T)$$

$$D^\mu = \partial^\mu - iA^\mu$$

$$L(\vec{x}) = P_t \exp[-i\int_0^\beta dx_4 A_4(x_4, \vec{x})]$$

$$\Phi = (Tr_c L)/N_c$$

$$\Phi^* = (Tr_c L^\dagger)/N_c$$



$$\frac{U(\Phi, \Phi^*, T)}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln[1 - 6\Phi^*\Phi + 4(\Phi^{*3} + \Phi^3) - 3(\Phi^*\Phi)^2]$$

(Haar measure)

Thermo dynamical potential

$$\Omega_0 = U(\Phi, \Phi^*, T) - \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} Tr \ln[S^{-1}(i\omega_n, \vec{p})] + \frac{\sigma^2}{2G}$$

Polyakov-2

$$S^{-1}(i\omega_n,\vec{p}) = \begin{pmatrix} i\gamma_0\omega_n - \vec{\gamma}\cdot\vec{p} - m + \gamma_0(\mu - iA_4) & 0 \\ 0 & i\gamma_0\omega_n - \vec{\gamma}\cdot\vec{p} - m - \gamma_0(\mu - iA_4) \end{pmatrix}$$

Imaginary time formalism
Matsubara frequency

$$\begin{aligned}\Omega_0 &= U(\Phi,\Phi^*,T) + \frac{\sigma^2}{2G} - 2N_f N_c \int_\Lambda \frac{d^3p}{(2\pi)^3} E_p \\ &- 2N_f T \int_\Lambda \frac{d^3p}{(2\pi)^3} Tr_c(\ln[1 + Le^{-(E_p-\mu)/T}] + \ln[1 + L^\dagger e^{-(E_p+\mu)/T}])\end{aligned}$$

$$\begin{aligned}z_\Phi^+ &= Tr_c \ln[1 + Le^{-(E_p-\mu)/T}] \\ &= \ln[1 + 3\Phi e^{-(E_p-\mu)/T} + 3\Phi^* e^{-2(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}]\end{aligned}$$

Colored objects are removed by $\Phi = 0$ in confined region.

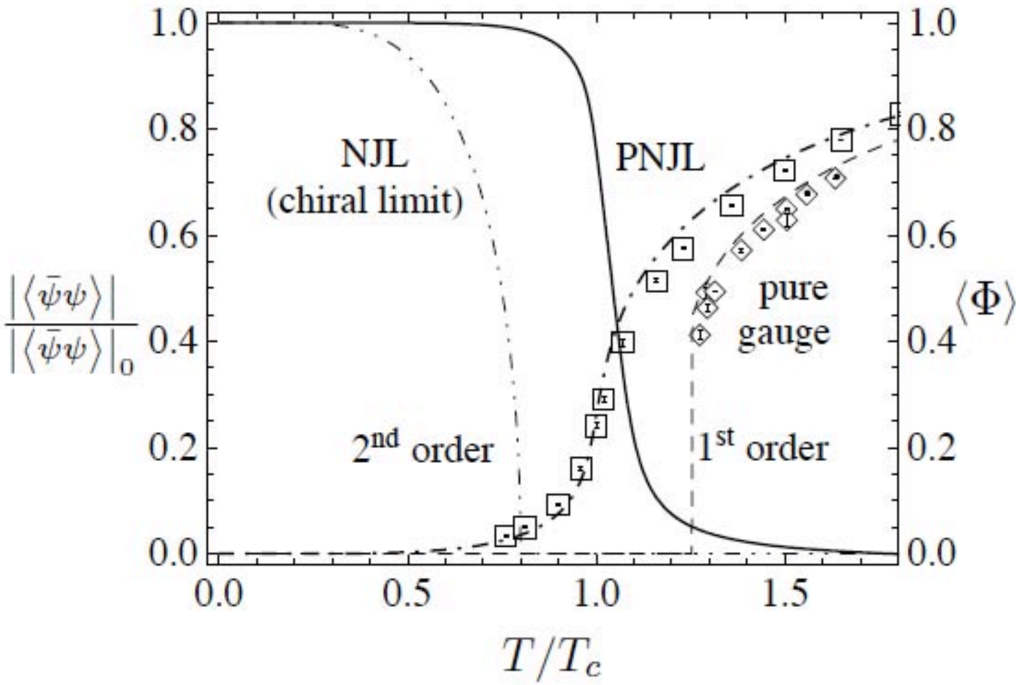
Meson part contributes to thermodynamics.

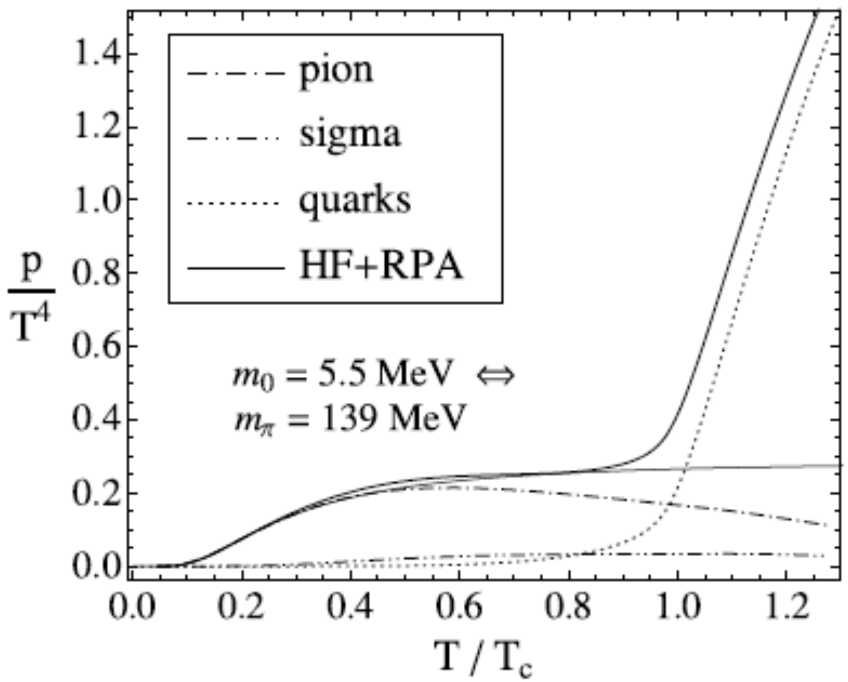
$$\Omega_\pi = \int \frac{d^3p}{(2\pi)^3} 3(\frac{1}{2}E_p^\pi + T\ln(1 - e^{-E_p^\pi/T}))$$

$$\Omega_\sigma = \int \frac{d^3p}{(2\pi)^3} (\frac{1}{2}E_p^s + T\ln(1 - e^{-E_p^s/T}))$$

PNJL results

Kuti, Rosner, Weise





1. T_c and T_p coincide at one temperature.
2. Meson contributes to EOS of quark-hadron matter.

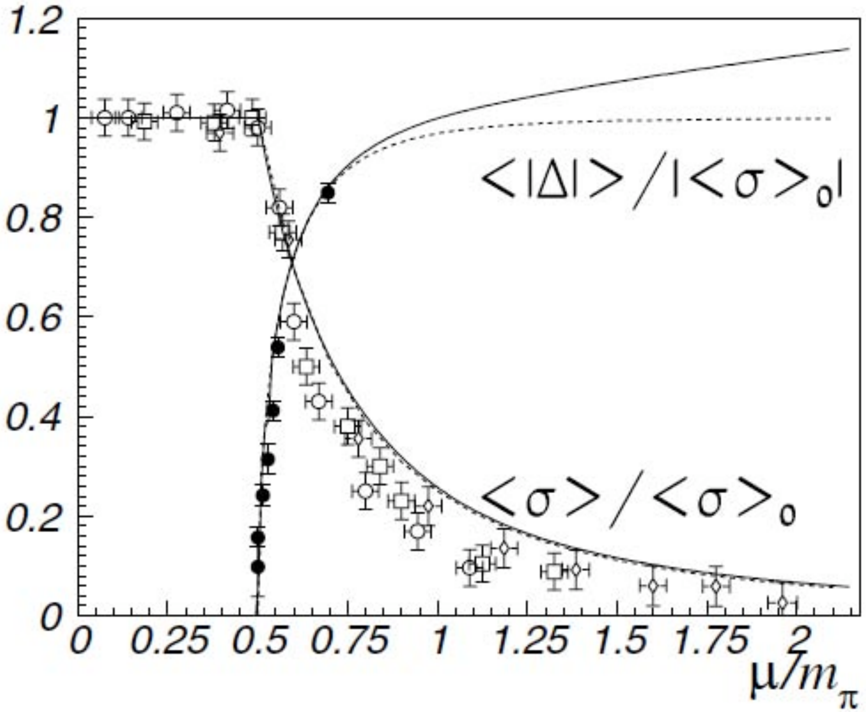
PNJL with baryons

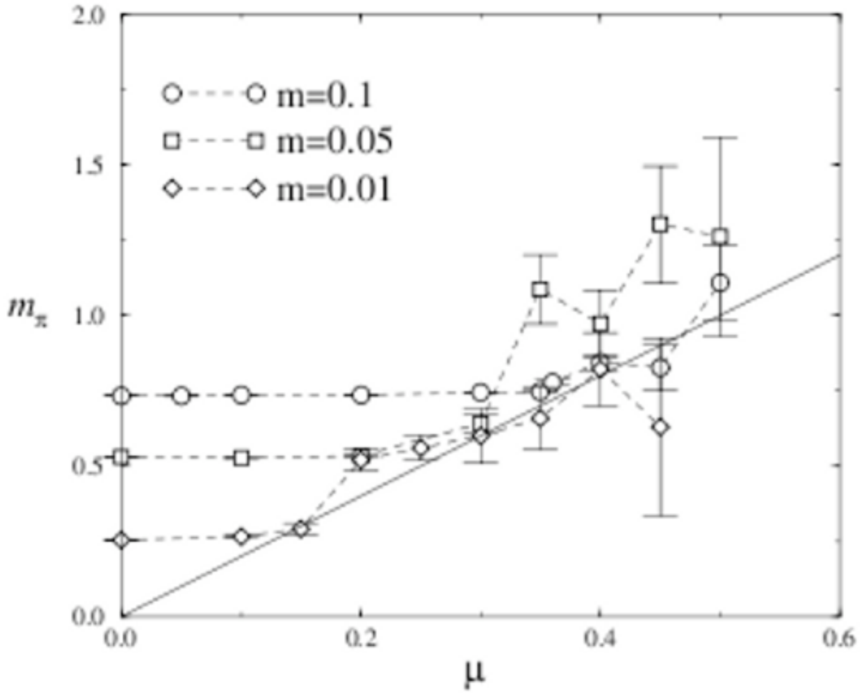
- Diquark quark model was used.
- There are several difficulties (diquark is boson) to go to finite temperature.
- Diquark auxiliary field method was developed for strong coupling QCD.

  (Kawamoto et al., Azcoiti et al.)
- We move to $SU(2)_c$ PNJL

SU2-lattice data

Ratti, Weise, Kogut





Lattice calculation by S. Hands et al, Eur.Phys.J.C22(2001)451

SU2 PNJL is interesting, since there exist lattice data particularly at finite density in addition to finite temperature.

SU(2)$_c$ PNJL

NJL Lagrangian with diquark correlation

$$L = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi + \frac{G_0}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] + \frac{H_0}{2}[(\bar{\psi}i\gamma_5\lambda_2\tau_2 C\bar{\psi}^T)(\psi^T C i\gamma_5\lambda_2\tau_2\psi)]$$

$G_0 = H_0$ for $SU(2)_c$ and $H_0 = \frac{3}{4}G_0$ for $SU(3)_c$

$(\bar{\psi}\lambda\gamma_\mu\psi)^2$

Pauli-Guersey sym.

Path integral with auxiliary fields

$$Z = \int D\bar{\psi} D\psi D\sigma D\vec{\pi} D\Delta^* D\Delta \exp(i \int d^4x L_{aug})$$

$$\begin{aligned} L_{aux} &= \bar{\psi}(i\gamma_\mu\partial^\mu - m_0 + \gamma_0\mu) - g_0(\sigma + \vec{\pi} i\gamma_5\tau))\psi - \frac{1}{2}\delta\mu_0^2(\sigma^2 + \vec{\pi}^2) \\ &+ \frac{1}{2}g_d[(\bar{\psi}\gamma_5\lambda_2\tau_2 C\bar{\psi}^T)\Delta - \Delta^*(\psi^T C\gamma_5\lambda_2\tau_2\psi)] - \frac{1}{2}M_d^2\Delta^*\Delta \end{aligned}$$

$\frac{g_0^2}{\delta\mu_0^2} = G_0$ and $\frac{g_d^2}{M_d^2} = H_0$

SU(2)$_c$ PNJL-3

Pfaffian form (Nambu-Gorkov)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \bar{\psi} & \psi^T C \end{pmatrix} \begin{pmatrix} S^{-1}(\mu) & g_d \gamma_5 \lambda_2 \tau_2 \Delta \\ -g_d \Delta^* \gamma_5 \lambda_2 \tau_2 & S^{-1}(-\mu) \end{pmatrix} \begin{pmatrix} \psi \\ C\bar{\psi}^T \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\begin{aligned} S^{-1}(\mu) &= i\gamma_\mu \partial^\mu - m_0 + \gamma_0 \mu - g_0(\sigma + \vec{\pi} i \gamma_5 \tau) \\ S^{-1}(-\mu) &= i\gamma_\mu \partial^\mu - m_0 - \gamma_0 \mu - g_0(\sigma - \vec{\pi} i \gamma_5 \tau) \end{aligned}$$

$$\int D\bar{\psi} D\psi e^{\bar{X} G^{-1} X} = \sqrt{Det[G^{-1}]}$$

$$-i\frac{1}{2} Tr[\ln S^{-1}(\mu) + \ln S^{-1}(-\mu) + \ln(1 + S(-\mu) g_d \Delta^* \gamma_5 \lambda_2 \tau_2 S(\mu) g_d \gamma_5 \lambda_2 \tau_2 \Delta)]$$

$$Det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = Det A \cdot Det(D - CA^{-1}B)$$

SU(2)$_c$ PNJL-4

Meson part

$$Z = \int D\sigma D\vec{\pi} \exp(i \int d^4x[-\frac{1}{2}\delta\mu_0^2(\sigma^2 + \vec{\pi}^2) - iTr \ln S^{-1}])$$

$$-iTr \ln S^{-1} = -iTr \ln(S_0^{-1} - g_0(s + i\gamma_5\vec{\pi}\vec{\tau}))$$

$$S_0^{-1} = i\gamma_\mu\partial^\mu - m + \gamma_0\mu$$

Expansion of log term

$$U = -iTr \ln[1 - S_0 g_0(s + i\gamma_5\vec{\pi}\vec{\tau})] = \sum_{n=1}^{\infty} U^{(n)}$$

$$U^{(n)} = \frac{i}{n} Tr \left[\frac{1}{i\gamma_\mu\partial^\mu - m + \gamma_0\mu} g_0(s + i\gamma_5\vec{\pi}\vec{\tau})\right]^n$$

$$U_s^{(2)} = g_0^2 \frac{i}{2} Tr \int d^4x \langle x| \frac{1}{i\gamma_\mu\partial^\mu - m + \gamma_0\mu}(s + i\gamma_5\vec{\pi}\vec{\tau}) \frac{1}{i\gamma_\mu\partial^\mu - m + \gamma_0\mu}(s + i\gamma_5\vec{\pi}\vec{\tau})|x\rangle$$

SU(2)$_c$ PNJL-5

Mass and kinetic energy part

$$U_s^{(2)} = g_0^2 \int d^4x \int d^4y [\Gamma_s(y-x)s(y)s(x) + \Gamma_p(y-x)\vec{\pi}(y)\vec{\pi}(x)]$$

$$\Gamma_s(y-x) = 2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p'}{(2\pi)^4} e^{i(p-p')(y-x)} \frac{(p_0+\mu)(p'_0+\mu) - \vec{p}\vec{p'} + m^2}{((p_0+\mu)^2 - E_p^2)((p'_0+\mu)^2 - E_p^2)}$$

$$\Gamma_p(y-x) = 2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4p'}{(2\pi)^4} e^{i(p-p')(y-x)} \frac{(p_0+\mu)(p'_0+\mu) - \vec{p}\vec{p'} - m^2}{((p_0+\mu)^2 - E_p^2)((p'_0+\mu)^2 - E_p^2)}$$

$$\Gamma_s(q) = \int d^4z e^{-iqz} \Gamma_s(z) = 2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \frac{(p_0+\mu+q_0)(p_0+\mu) - (\vec{p}+\vec{q})\vec{p} + m^2}{((p_0+\mu+q_0)^2 - E_{p+q}^2)((p_0+\mu)^2 - E_p^2)}$$

$$\Gamma_p(q) = \int d^4z e^{-iqz} \Gamma_p(z) = 2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \frac{(p_0+\mu+q_0)(p_0+\mu) - (\vec{p}+\vec{q})\vec{p} - m^2}{((p_0+\mu+q_0)^2 - E_{p+q}^2)((p_0+\mu)^2 - E_p^2)}$$

Low momentum expansion

$$\Gamma(q) = \Gamma(0) + q^\mu \partial_\mu(q)\Gamma(q)|_{q=0} + \frac{1}{2} q^\mu q^\nu \partial_\mu \partial_\nu \Gamma(q)|_{q=0}$$

$$\Gamma(z) = \int \frac{d^4q}{(2\pi)^4} \Gamma(q) e^{iqz} = \Gamma(0)\delta(z) - i\partial^\mu \delta(z) \partial_\mu \Gamma(q)|_{q=0} - \frac{1}{2}\partial^\mu \partial^\nu \delta(z) \partial_\mu \partial_\nu \Gamma(q)|_{q=0} + ..$$

$$U_s^{(2)} = g_0^2 \int d^4x [\Gamma_s(0)s(x)s(x) + i\partial_\mu \Gamma_s(0)(\partial^\mu s(x))s(x) - \frac{1}{2}\partial_\mu \partial_\nu \Gamma_s(0)(\partial^\mu \partial^\nu s(x))s(x) +$$

SU(2)$_c$ PNJL-6

KG-equation

$$U_s^{(2)} = g_0^2 \int d^4x[(I_2 - 2m^2 I_0)s(x)s(x) - \frac{1}{2} I_0(\partial_\mu \partial^\mu s(x))s(x) + I_2 \vec{\pi}(x)\vec{\pi}(x) - \frac{1}{2} I_0(\partial_\mu \partial^\mu \vec{\pi}(x))\vec{\pi}(x)]$$

$$L_{MR} = \frac{1}{2}(\partial_\mu s_R)^2 - \frac{1}{2} M_{sR}^2 s_R^2 + \frac{1}{2}(\partial_\mu \vec{\pi}_R)^2 - \frac{1}{2} M_{pR}^2 \vec{\pi}_R^2$$

$$M_{sR}^2 = (\tfrac{1}{G_0} - 2I_2 + 4m^2 I_0)/I_0$$

$$M_{pR}^2 = (\tfrac{1}{G_0} - 2I_2)/I_0$$

$$I_2(\omega) = \Gamma_p(\omega) = 2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \frac{(p_0 + \mu + \omega)(p_0 + \mu) - E_p^2}{((p_0 + \mu + \omega)^2 - E_p^2)((p_0 + \mu)^2 - E_p^2)}$$

$$I_0(\omega) = -2iTr_{(cf)} \int \frac{d^4p}{(2\pi)^4} \frac{1}{((p_0 + \mu + \omega)^2 - E_p^2)((p_0 + \mu)^2 - E_p^2)}$$

We can take higher order terms for the non-linear couplings.

SU(2)$_c$ PNJL-7

Finite temperature

$$I_2(\omega) \rightarrow -2Tr_{(cf)}T\sum_n \int \frac{d^3p}{(2\pi)^3} \frac{(i\omega_n+\mu+\omega)(i\omega_n+\mu)-E_p^2}{((i\omega_n+\mu+\omega)^2-E_p^2)((i\omega_n+\mu)^2-E_p^2)}$$

$$I_2(\omega) = 2Tr_{(cf)} \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}[1-f(-\mu-\omega+E_p)-f(\mu+E_p))\frac{1}{-\omega+2E_p}$$
$$+ (1-f(\mu+\omega+E_p)-f(-\mu+E_p))\frac{1}{\omega+2E_p}]$$

$$I_0(\omega) = 2Tr_{cf} \int \frac{d^3p}{(2\pi)^3}\frac{1}{4E_p^2}$$
$$[(f(-\mu+E_p)-f(-\mu-\omega+E_p))\frac{1}{\omega}$$
$$+(1-f(-\mu+E_p)-f(\mu+\omega+E_p))\frac{1}{\omega+2E_p}$$
$$+(1-f(\mu+E_p)-f(-\mu-\omega+E_p))\frac{1}{-\omega+2E_p}$$
$$+(-f(\mu+E_p)+f(\mu+\omega+E_p))\frac{1}{\omega}]$$

$$f(x) = \frac{1}{1+e^{x/T}}$$

SU(2)$_c$ PNJL-8

Diquark boson

$$L_d = -i\frac{1}{2}TrS(-\mu)g_d\Delta^*\gamma_5\lambda_2\tau_2S(\mu)g_d\gamma_5\lambda_2\tau_2\Delta$$

$$\begin{aligned} L_d &= i2Tr_{cf}g_d^2\int d^4x\int d^4y\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}e^{i(p-p')(y-x)} \\ &\quad \frac{(p_0-\mu)(p_0'+\mu)-E_pE_p'}{((p_0-\mu)^2-E_p^2)((p_0'+\mu)^2-E_p'^2)}\Delta^*(y)\Delta(x) \\ &= g_d^2\int d^4x\int d^4y\Gamma_\Delta(y-x)\Delta^*(y)\Delta(x) \end{aligned}$$

$$\begin{aligned} \Gamma_\Delta(q) &= \int d^4ze^{-iqz}\Gamma_\Delta(z) = 2iTr_{cf}\int\frac{d^4p}{(2\pi)^4} \\ &\quad \frac{(p_0-\mu+\omega)(p_0+\mu)-(\vec{p}+\vec{q})\vec{p}-m^2}{((p_0-\mu+\omega)^2-E_{p+q}^2)((p_0+\mu)^2-E_p^2)} \end{aligned}$$

SU(2)$_c$ PNJL-9

Diquark boson has fermi energy

$$L_D = -\frac{1}{2}[(M_d^2 - 2g_d^2 I_2 - (2\mu)^2 g_d^2 I_0)\Delta^*\Delta + ig_d^2 4\mu I_0(\partial^0\Delta^*)\Delta + g_d^2 I_0(\partial_\mu^2\Delta^*)\Delta]$$
$$= -\frac{1}{2}[(M_d^2 - 2g_d^2 I_2)\Delta^*\Delta + g_d^2 I_0((\partial_\mu + i2\mu\delta_{\mu 0})^2\Delta^*)\Delta]$$

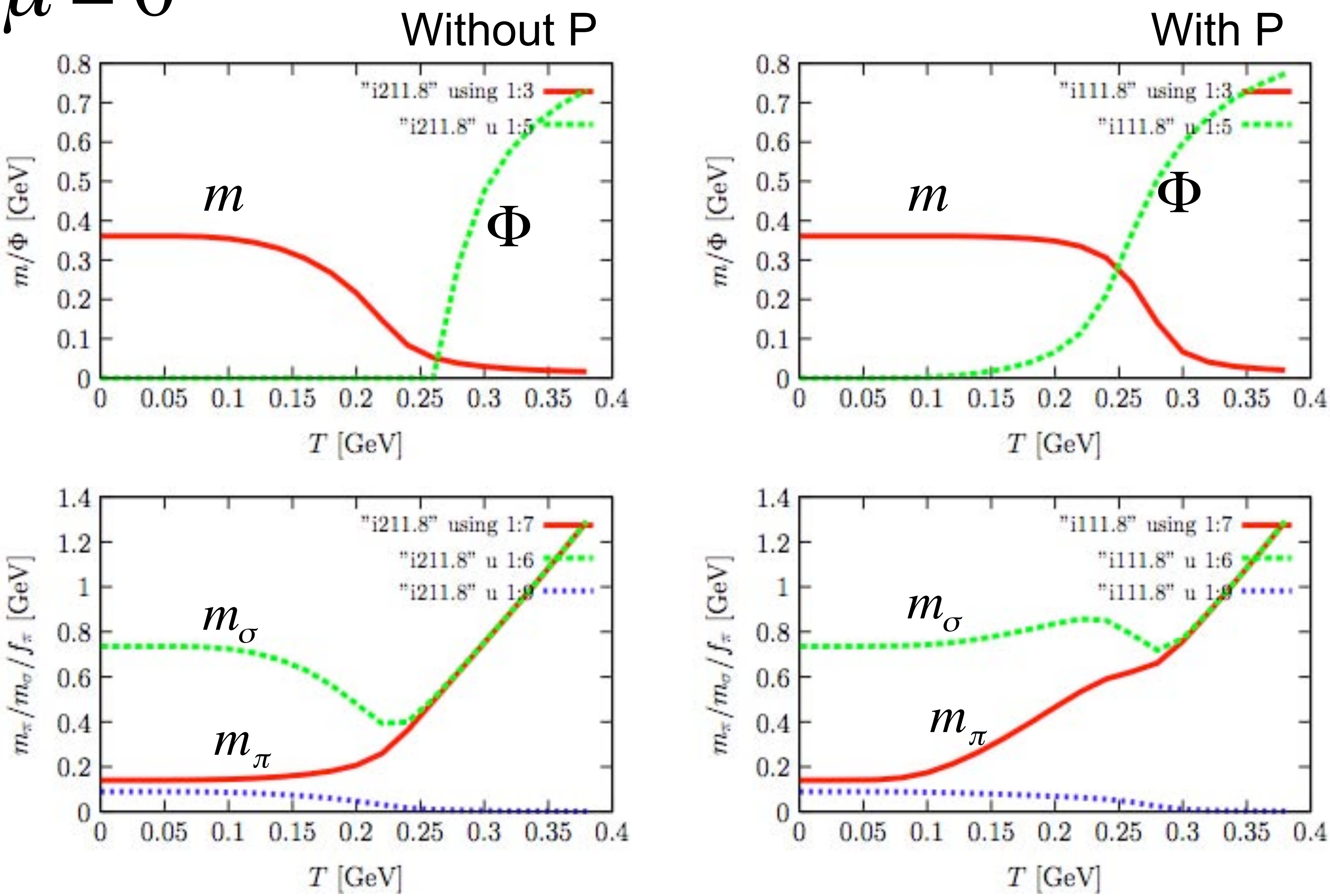
$$L_D = -\frac{1}{2}[M_{dR}^2\Delta_R^*\Delta_R + ((\partial_\mu + i2\mu\delta_{\mu 0})^2\Delta_R^*)\Delta_R]$$
$$= \frac{1}{2}|(\partial_\mu - i2\mu\delta_{\mu 0})\Delta_R|^2 - \frac{1}{2}M_{dR}^2\Delta_R^*\Delta_R$$

$$\tilde{p}^\mu = (p_0 + \mu, \vec{p})$$
$$\bar{p}^\mu = (p_0 - \mu, \vec{p})$$

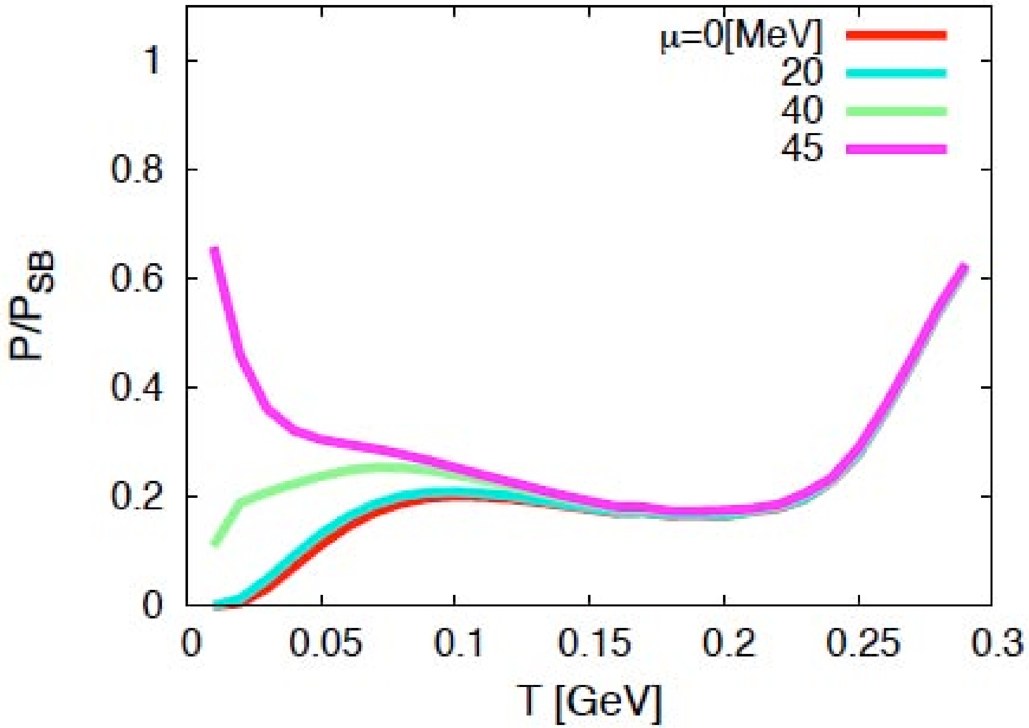
$$\Gamma_\Delta(q)|_{q=0} = 2iTr_{cf}\int \frac{d^4p}{(2\pi)^4}\frac{\bar{p}^\mu\tilde{p}_\mu - m^2}{(\bar{p}_\mu^2 - m^2)(\tilde{p}_\mu^2 - m^2)}$$
$$\sim 2iTr_{cf}\int \frac{d^4p}{(2\pi)^4}\frac{p_\mu^2 - \mu^2 - m^2}{(p_\mu^2 - m^2)(1 + \frac{-2p_0\mu+\mu^2}{p_\mu^2 - m^2})(p_\mu^2 - m^2)(1 + \frac{2p_0\mu+\mu^2}{p_\mu^2 - m^2})}$$
$$\sim 2iTr_{cf}\int \frac{d^4p}{(2\pi)^4}[\frac{1}{p_\mu^2 - m^2} - \frac{2\mu^2}{(p_\mu^2 - m^2)^2}]$$
$$= I_2 + 2\mu^2 I_0 = I_2 + \frac{1}{2}(2\mu)^2 I_0$$

Figures($SU2_c$)-1

$\mu = 0$

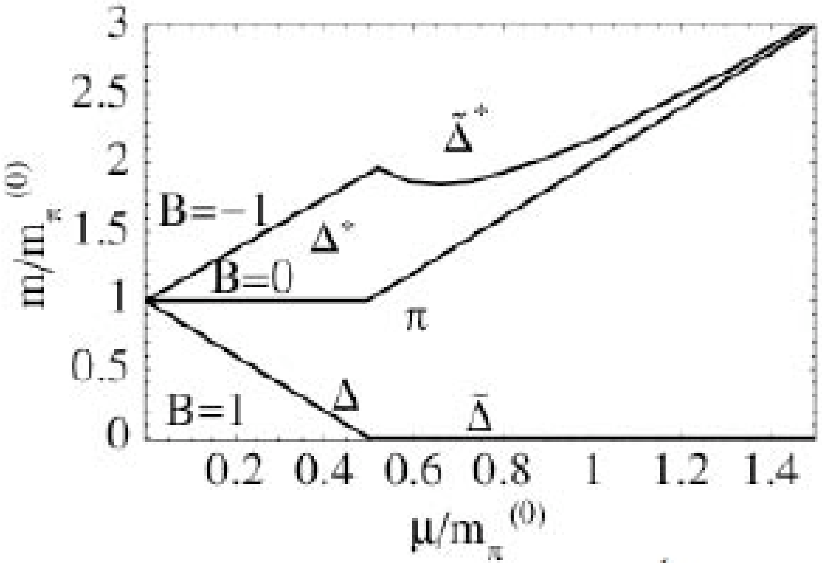


Figures($SU2_c$)-2



Pressure diverges at T=0 for some finite density

⇨ Diquark condensation



SU(2)$_c$ PNJL-10

Mean field approximation (BCS)

$$\sigma = \sigma_0 + s \qquad \Delta = \Delta_0 + d \qquad \Delta^* = \Delta_0^* + d^*$$

$$\begin{aligned}\mathcal{L} &= -\frac{i}{2}\mathrm{tr}\{\ln \hat{S}^{-1} + \ln(1+\hat{S}\hat{K})\} - \frac{1}{2}\delta\mu_0^2\sigma_0^2 - \frac{1}{2}\delta\mu_0^2(s^2+\vec{\pi}^2) - \delta\mu_0^2\sigma_0 s \\ &\quad -\frac{1}{2}M_d^2\Delta_0^*\Delta_0 - \frac{1}{2}M_d^2(\Delta_0^* d + d^*\Delta_0) - \frac{1}{2}M_d^2 d^* d,\end{aligned}$$

$$\hat{S}^{-1} = \begin{pmatrix} S_0^{-1}(\mu) & \Delta^- \\ \Delta^+ & S_0^{-1}(-\mu)\end{pmatrix} \quad \hat{K} = \begin{pmatrix} -g_0(s+i\gamma_5\vec{\pi}\cdot\vec{\tau}) & g_d\gamma_5\lambda_2\tau_2 d \\ -g_d d^*\gamma_5\lambda_2\tau_2 & -g_0(s-i\gamma_5\vec{\pi}\cdot\vec{\tau})\end{pmatrix}$$

$$S_0^{-1} = i\gamma_\mu\partial^\mu - m + \gamma_0\mu \quad \Delta^- = g_d\gamma_5\lambda_2\tau_2\Delta_0 \quad \Delta^+ = -g_d\Delta_0^*\gamma_5\lambda_2\tau_2$$

$$\hat{S} = \begin{pmatrix} G^+ & H^- \\ H^+ & G^-\end{pmatrix} \qquad G^\pm = \frac{p_0 \pm E_p^-}{p_0^2 - E_\Delta^{-2}}\Lambda_\pm\gamma_0 + \frac{p_0 \mp E_p^+}{p_0^2 - E_\Delta^{+2}}\Lambda_\mp\gamma_0$$

$$\Lambda_\pm = \frac{1}{2}\left(1 \pm \frac{\gamma_0(\vec{\gamma}\cdot\vec{p}+m)}{E_p}\right) \qquad H^\pm = \frac{\Delta^\pm}{p_0^2 - E_\Delta^{\pm 2}}\tilde{\Lambda}_+ + \frac{\Delta^\pm}{p_0^2 - E_\Delta^{\mp 2}}\tilde{\Lambda}_-$$

$$\tilde{\Lambda}_\pm = \frac{1}{2}\left(1 \pm \frac{\gamma_0(\vec{\gamma}\cdot\vec{p}-m)}{E_p}\right)$$

$$E_\Delta^{\pm 2} = E_p^{\pm 2} + g_d\Delta_0^2$$

SU(2)$_c$ PNJL-11

$$-\frac{i}{2}\mathrm{tr}\ln\hat{S}^{-1} = -\frac{i}{2}\ln\det\hat{S}^{-1}$$

$$Det\hat{S}^{-1} = \sqrt{(p_0^2 - E_\Delta^{-2})(p_0^2 - E_\Delta^{+2})}$$

$$\Omega = -2Tr\int\frac{d^3p}{(2\pi)^3}\frac{1}{2}(E_\Delta^+ + E_\Delta^-) - 2Tr\int\frac{d^3p}{(2\pi)^3}T[\ln(1+e^{-\beta E_\Delta^+}) + \ln(1+e^{-\beta E_\Delta^-})] + \Delta^2/(2H) + (m-m_0)^2/(2G)$$

$$-\frac{i}{2}\mathrm{tr}\ln(1+\hat{S}\hat{K}) = -\frac{i}{2}\mathrm{tr}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}(\hat{S}\hat{K})^k \equiv \sum_{k=1}^{\infty}U^{(k)}$$

k=1

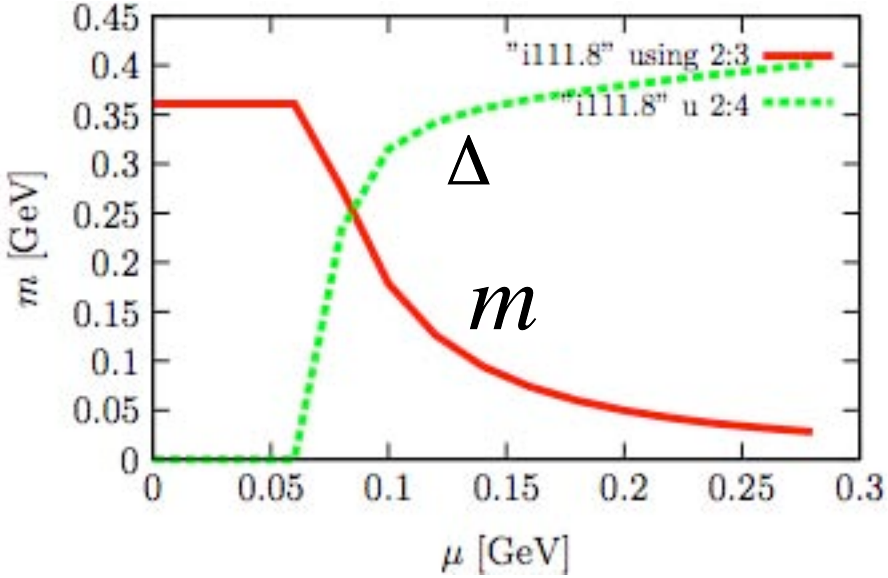
$$-\delta\mu_0^2\sigma_0 + 2g_0 m I_2 = 0, \qquad I_2 = iTr_{fc}\int\frac{d^4p}{(2\pi)^4}\frac{1}{E_p}\left(\frac{E_p^+}{p_0^2 - E_\Delta^{+2}} + \frac{E_p^-}{p_0^2 - E_\Delta^{-2}}\right)$$

$$-\frac{1}{2}M_d^2 + ig_d^2 I_2' = 0. \qquad I_2' = iTr_{fc}\int\frac{d^4p}{(2\pi)^4}\left(\frac{1}{p_0^2 - E_\Delta^{+2}} + \frac{1}{p_0^2 - E_\Delta^{-2}}\right)$$

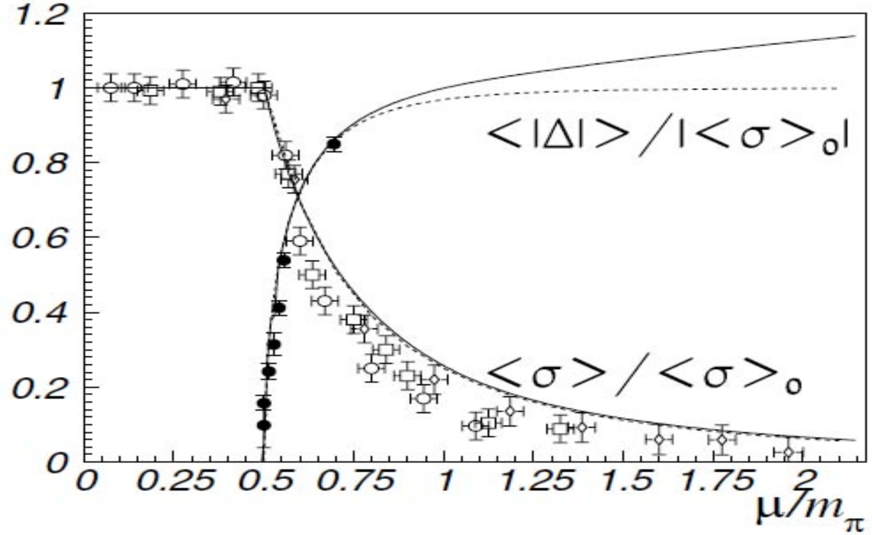
k=2

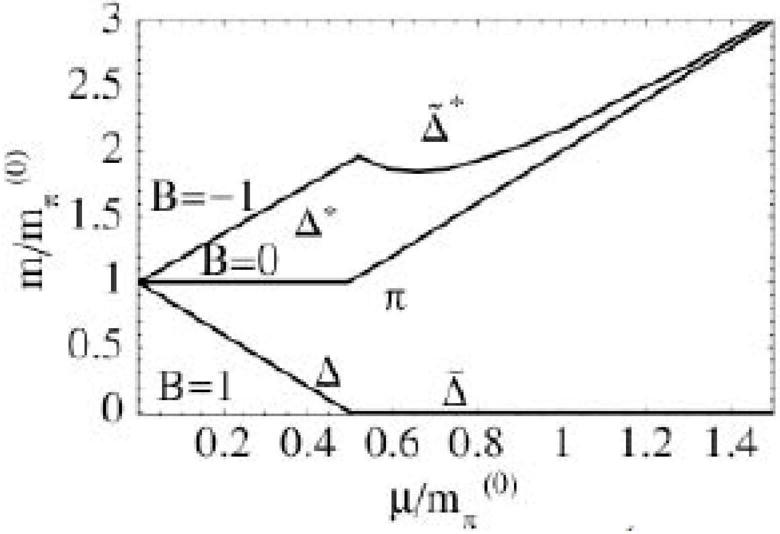
Mass and kinetic energy terms for pi, sigma and delta.

T=0



Ratti Weise

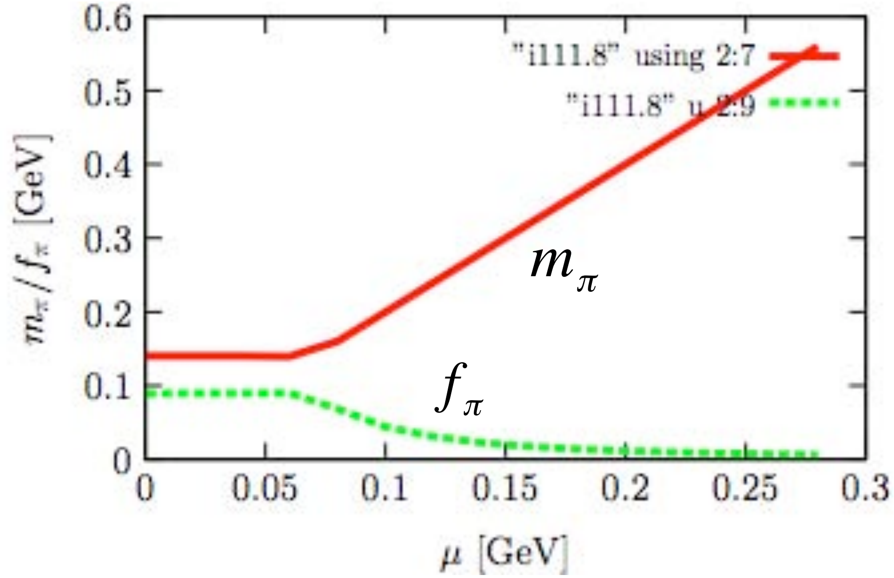




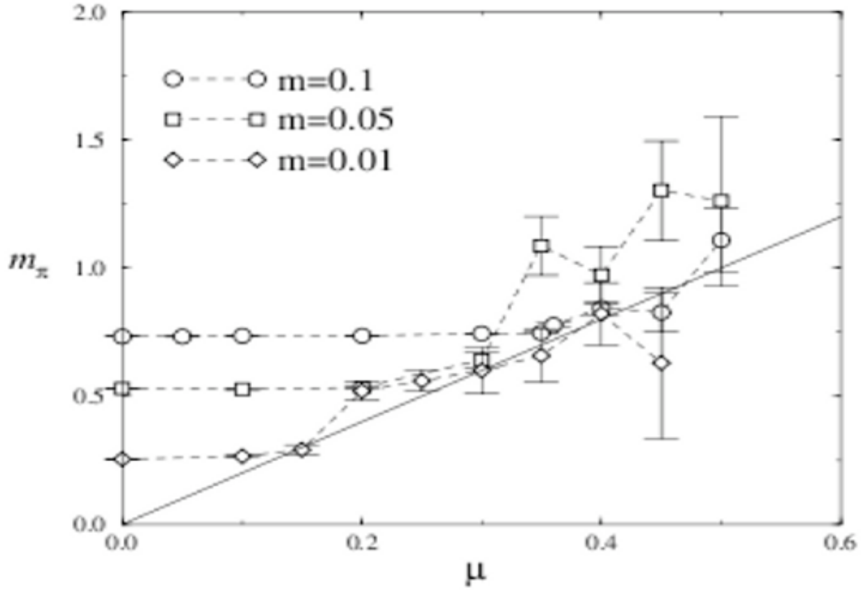
Diquark condensation appears at $\mu = m_\pi / 2$
This means that the color vector current coupling NJL is good.

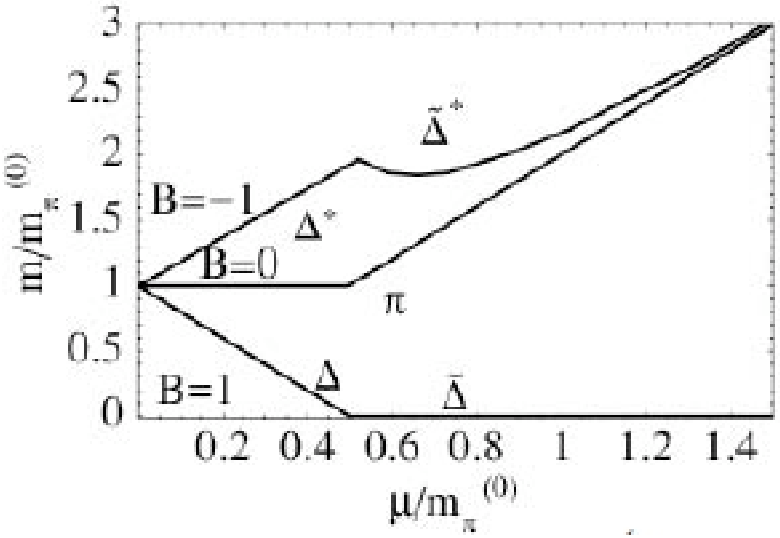
Figures(SU2_c)-4

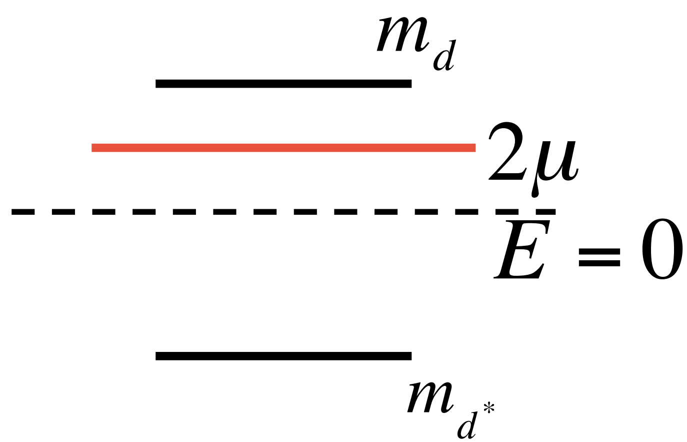
T=0



Lattice calculation

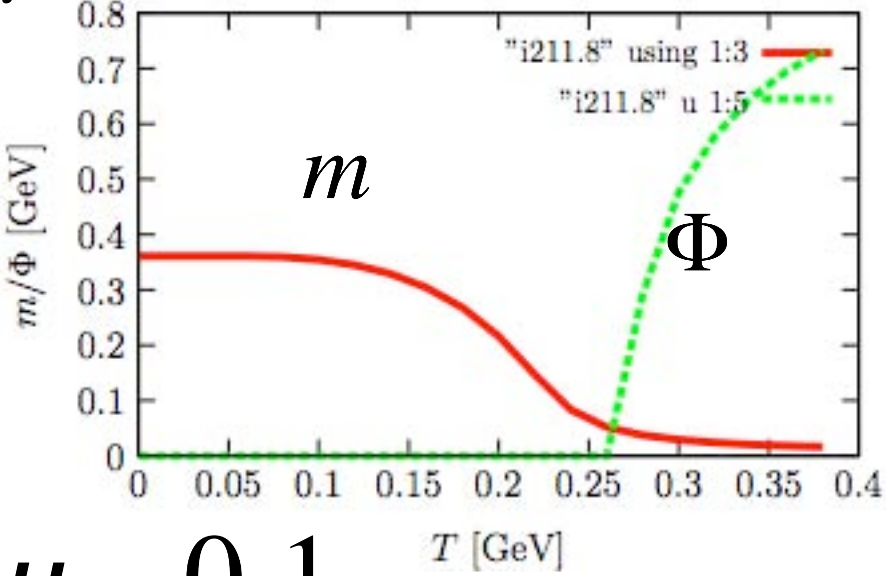


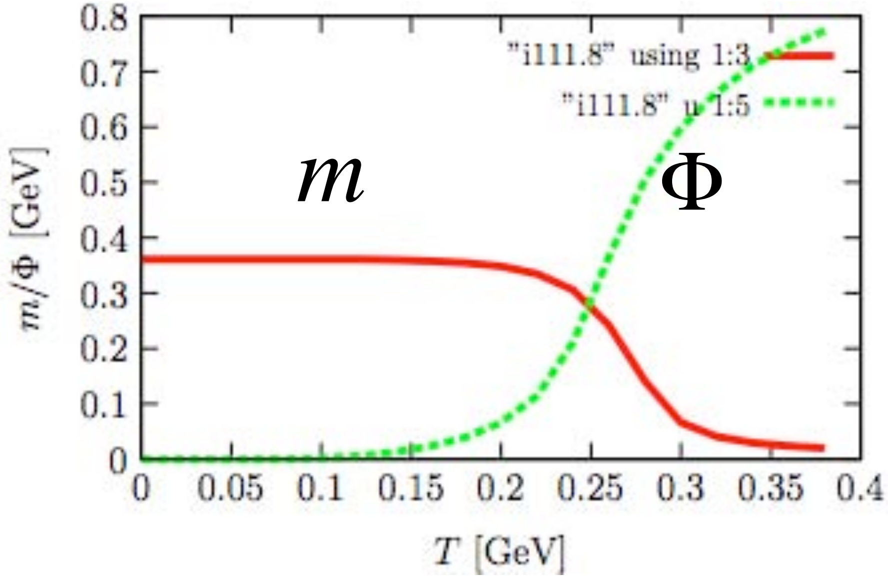




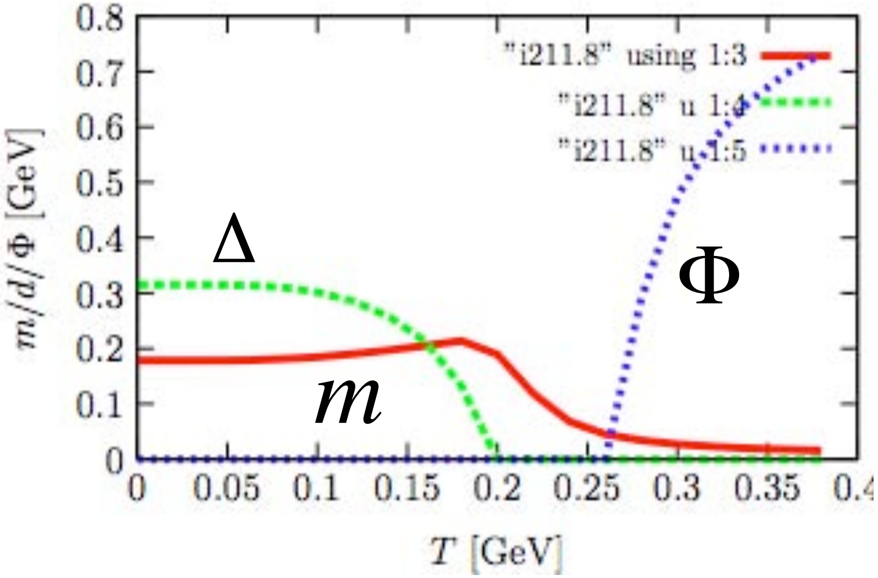
Figures(SU2$_c$)-5

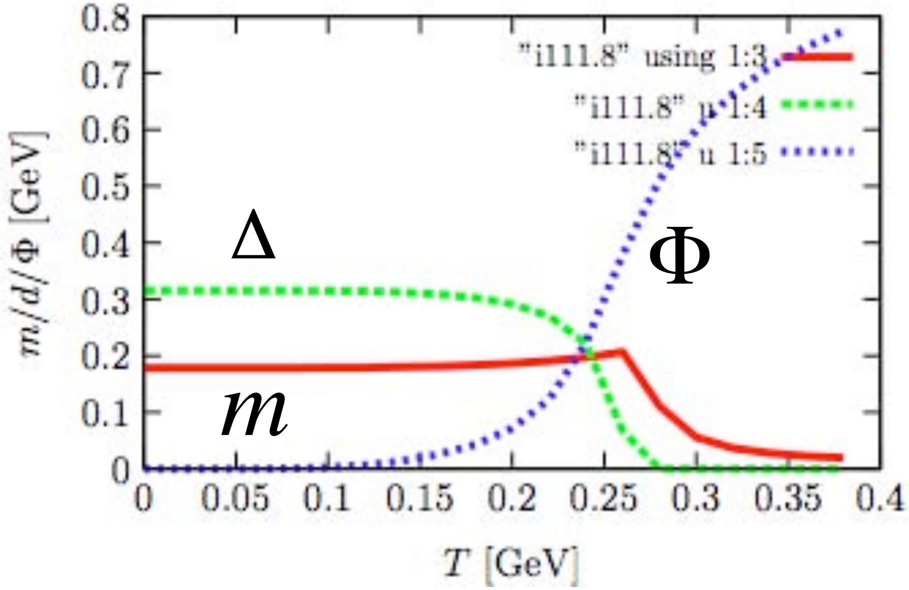
$\mu = 0$





$\mu = 0.1$





Conclusion

- By path integration method, we get sigma-model hadron Lagrangian.
- We work out $SU2_c$ Lagrangian for mesons and diquark bosons.
- We work out the thermodynamics with the Polyakov line.
- We see the property of diquark condensation at finite density.
- We want to work out baryon for SU3 case.