Which is first, Chiral or Deconfinement?

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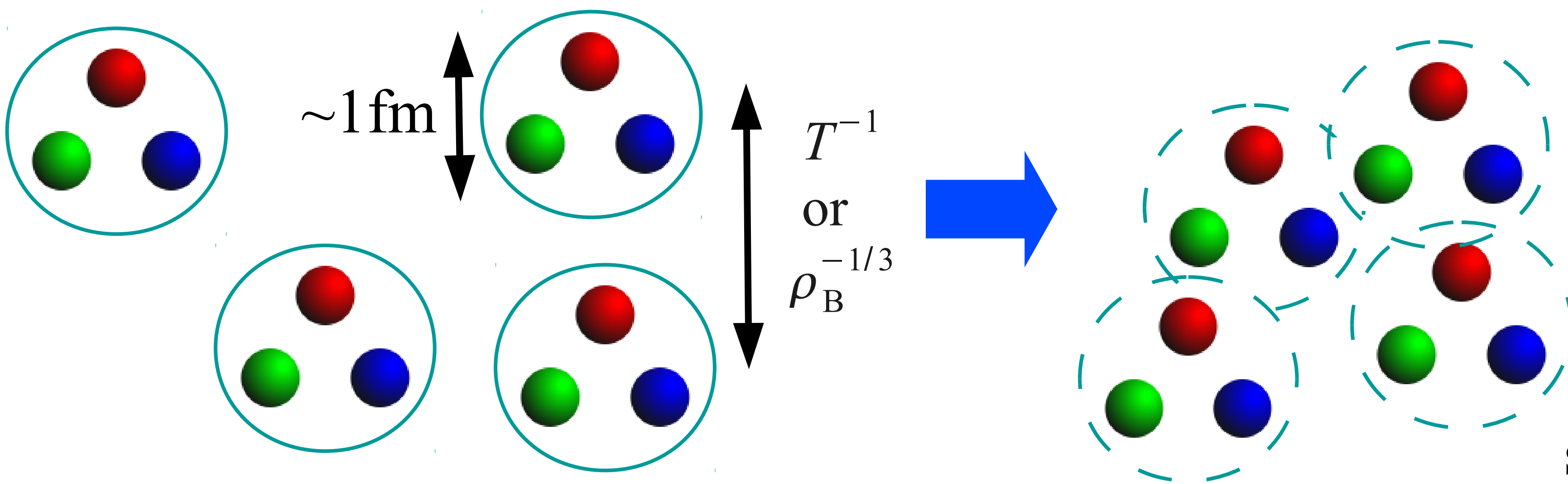
Two QCD Phase Transitions

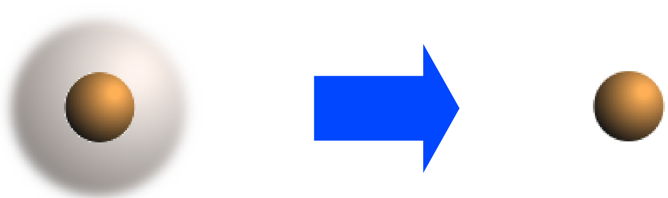
Quark Deconfinement Transition



Satz

$T \sim 10^{12}$ K

Chiral Phase Transition

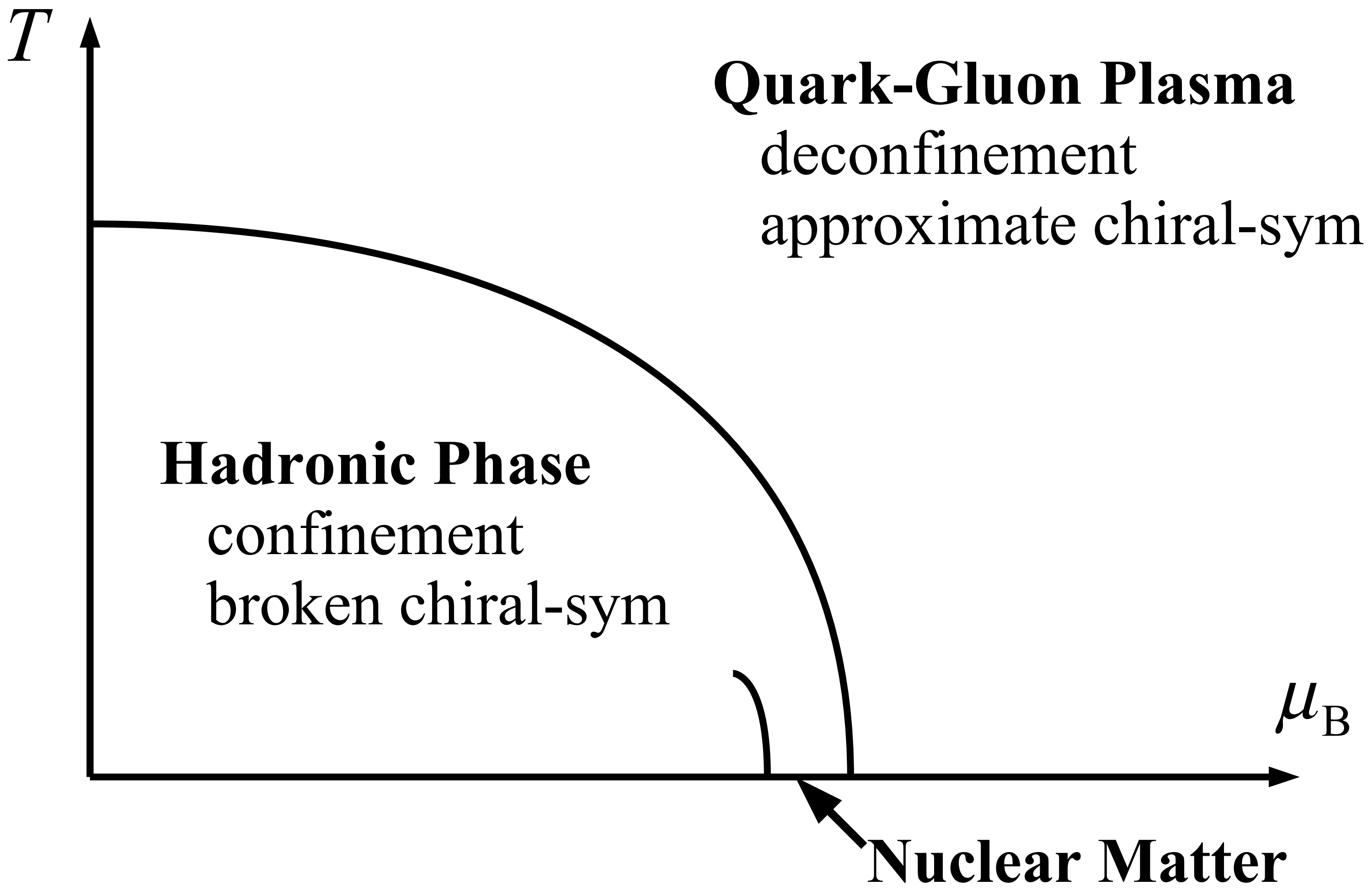


"Bare" Quarks
$m_q \sim$ 3-6 MeV

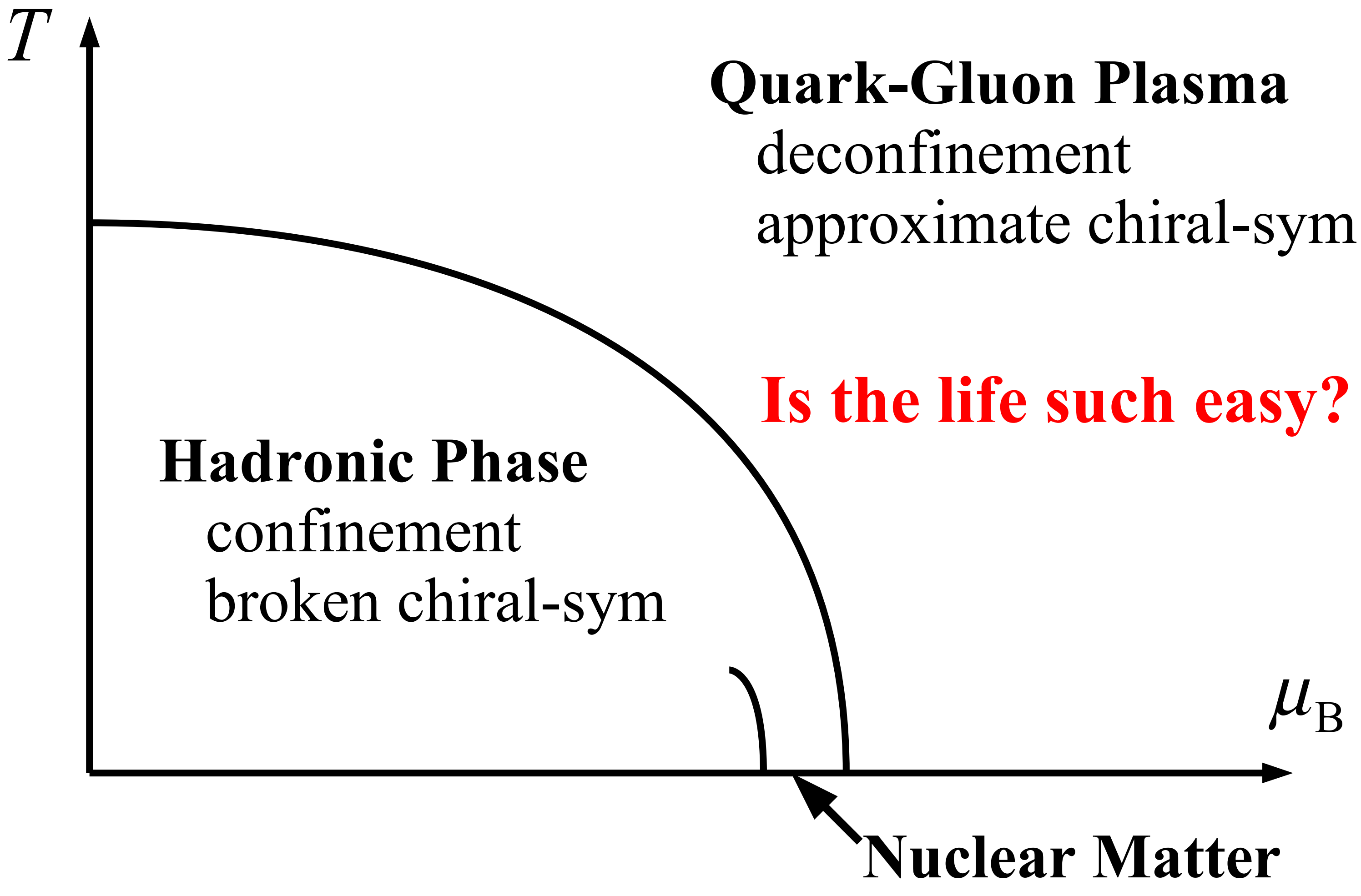
"Constituent" Quarks
$M_q \sim$ 350 MeV

Nambu-Jona-Lasinio

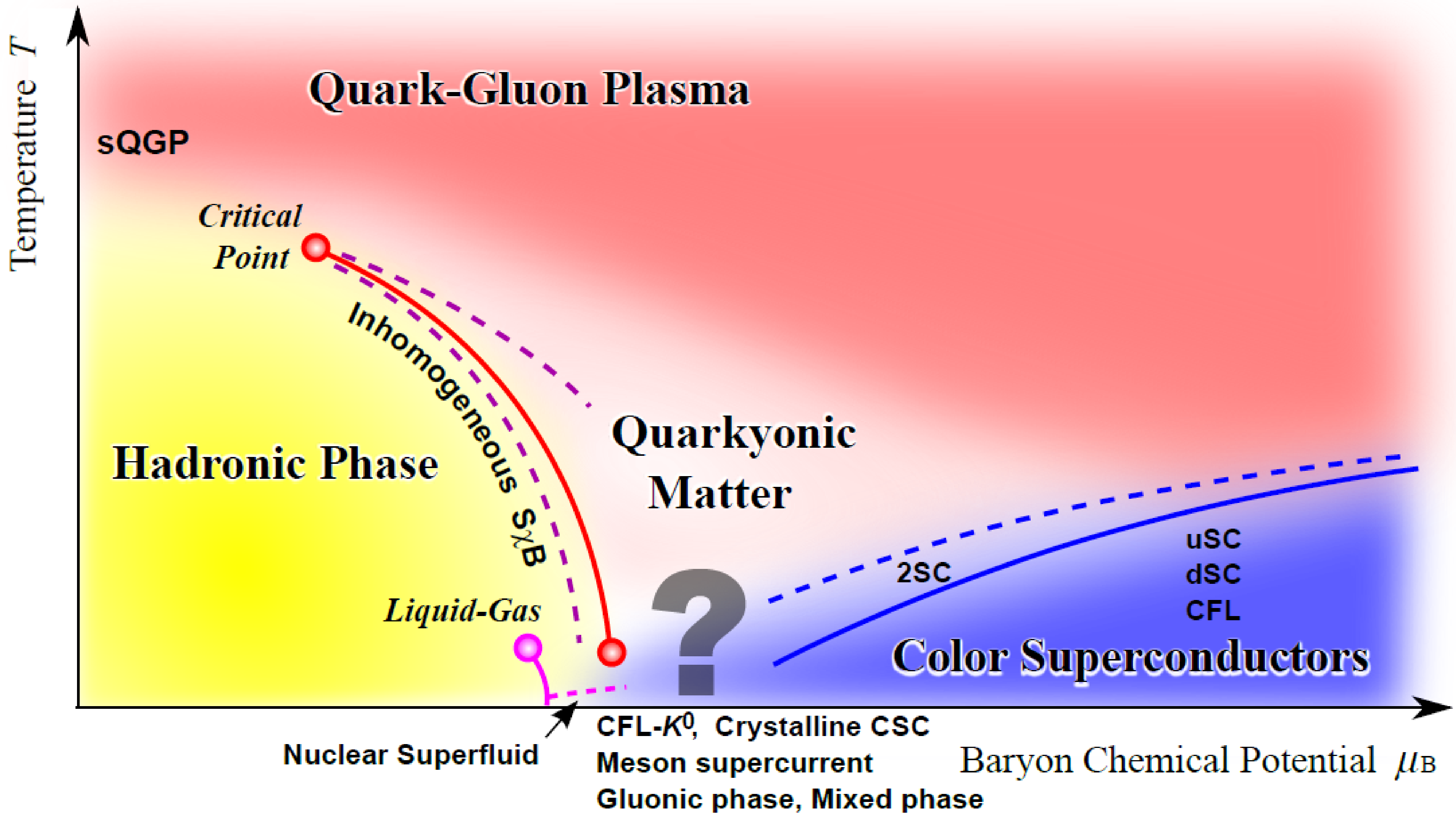
QCD Phase Diagram – Folklore



QCD Phase Diagram – Folklore



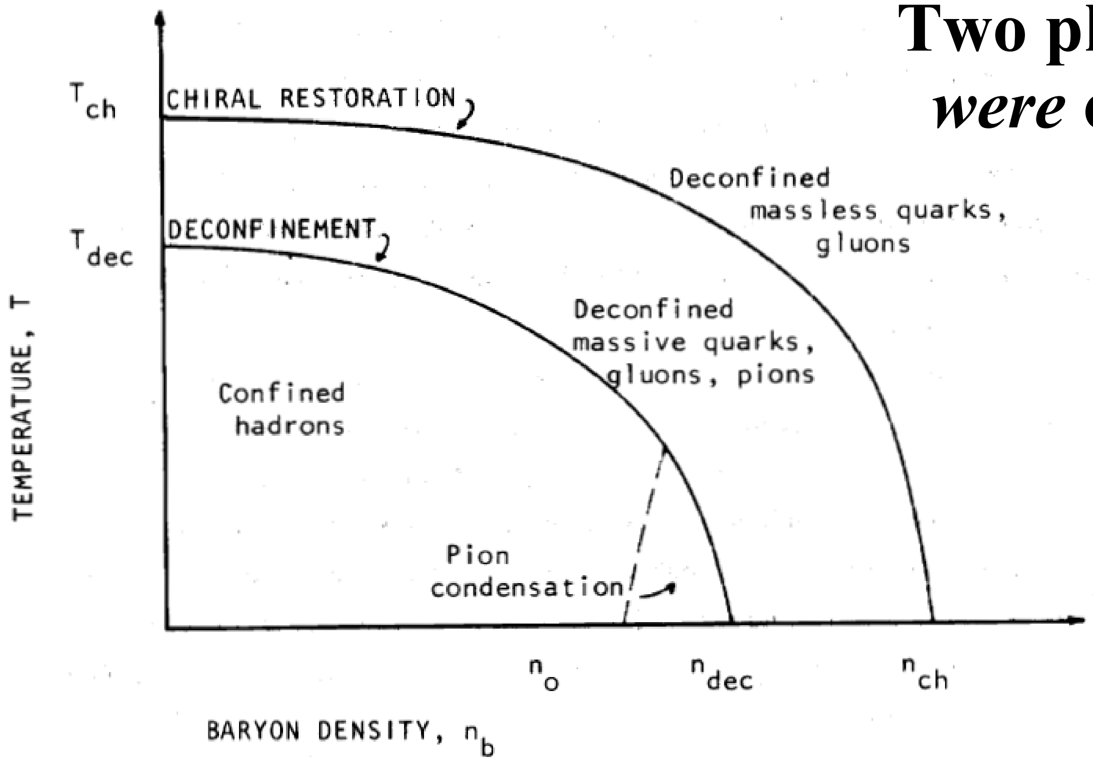
Modern QCD Phase Diagram



KF-Hatsuda (2010)

Conjectured Phase Diagram in 1980's

Gordon Baym (1982)

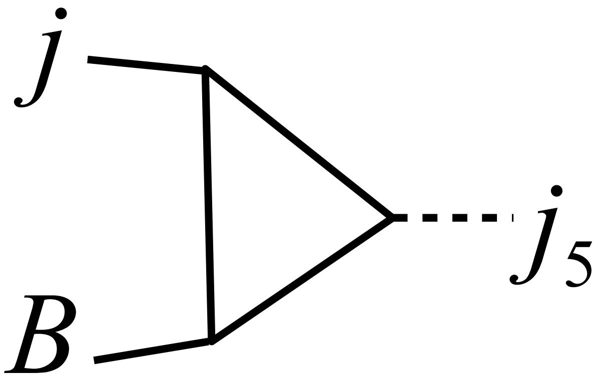


Two phase transitions *were* considered separately

This possibility is coming back now (30 years later)!

Why $T_{ch} > T_{dec}$?

Anomaly Matching Condition **('t Hooft)**



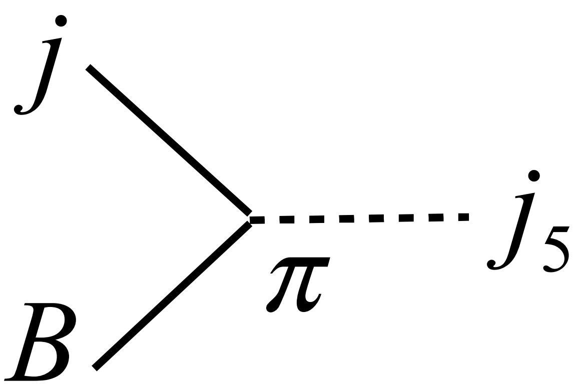
$$K^{AB}_{\mu\nu}(q) = i\int d^{4x}\, e^{iq\cdot x}\langle T\, j^{A}_{\mu 5}(x)\, j^{B}_{\nu}(0)\rangle_B$$

$$\sim -N_c \frac{e_A B}{2\pi^2}\frac{q_\mu \tilde{q}_\nu}{q^2}\frac{1}{2}\delta^{AB}$$

c.f. Chiral Magnetic Effect $j_\mu = \dfrac{eB}{2\pi^2}\mu_5$

Fukushima-
-Kharzeev-
-Warringa

If quarks are confined, how this IR singularity appears?



In the confined world there must be massless NG bosons (chiral broken).

For $N_f = 2$, massless proton and neutron can saturate the IR singularity...

In real world $N_f > 2$ but *s* quarks are massive !?

Why $T_{ch} > T_{dec}$?

Potential Model Argument (Casher)

Assume that massless quarks are confined in a bag by *s*-wave potential (which does not change the spin).

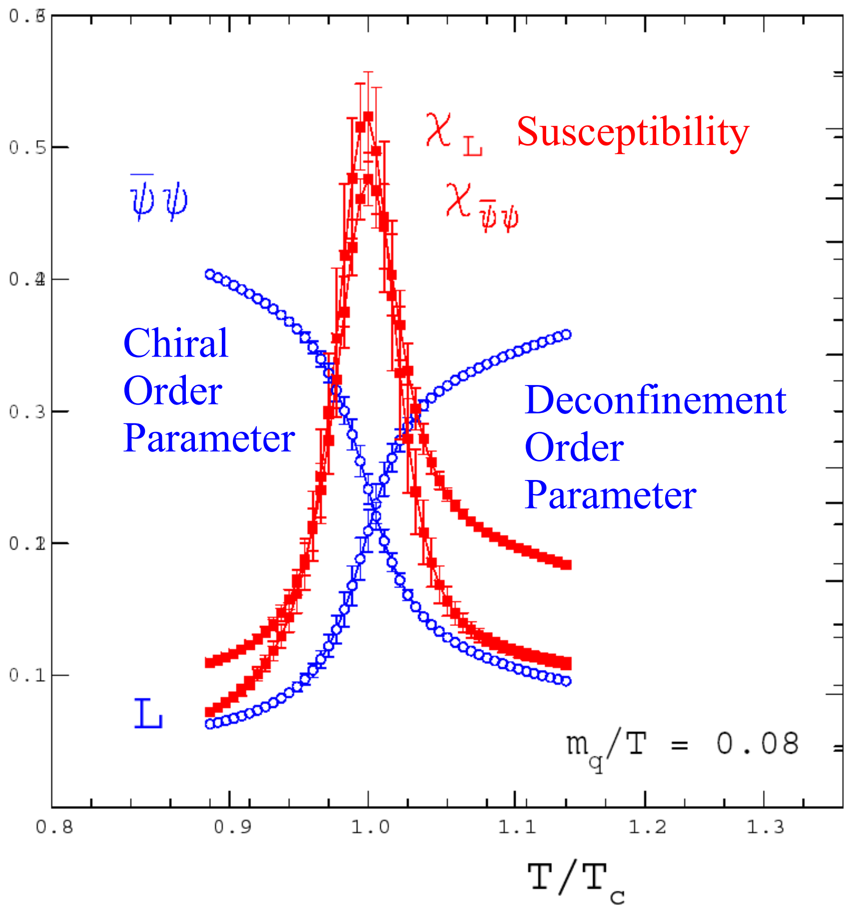
When quarks turn back, they change the chirality, which should be compensated by chiral condensate.

Confinement requires chiral symmetry breaking

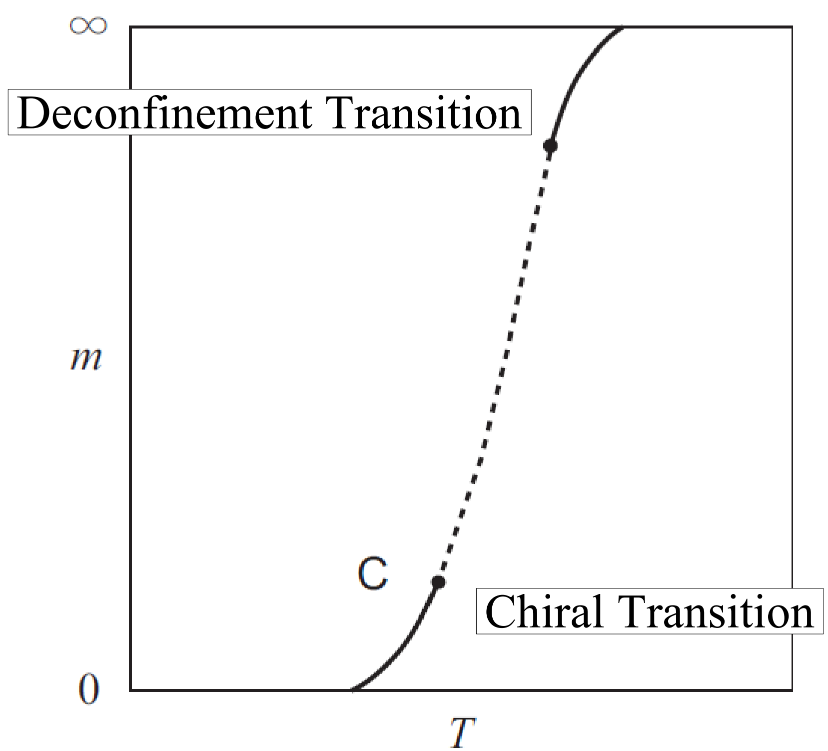
Can this be an exact argument from QCD?

Coincidence at T≠0 and μ_B=0

Old Lattice-QCD Data
Hands (physics/0105022)



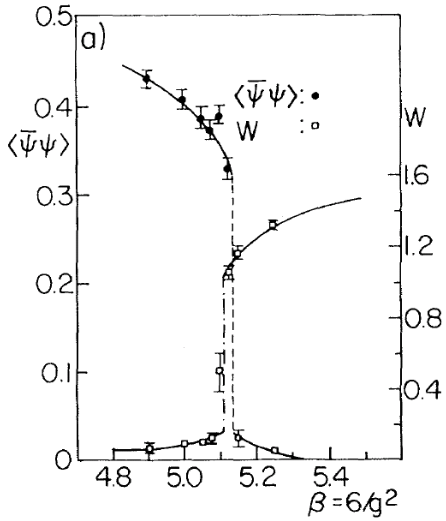
Two-crossovers connected
(Fukushima-Hatta 2003)



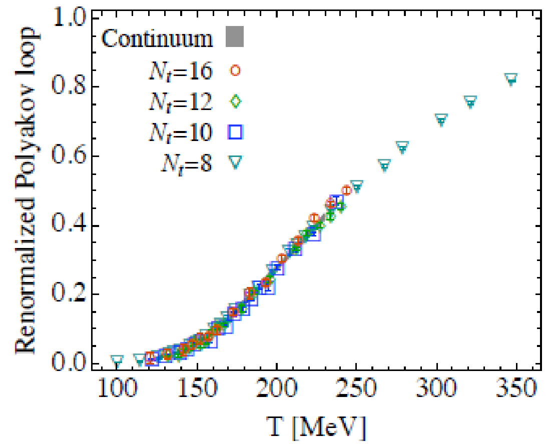
Deconfinement and Chiral Transitions (Nearly) Coincide

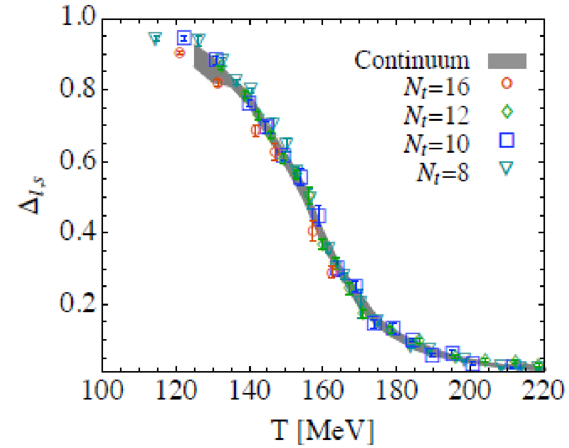
Some Lattice-QCD Highlight

Kogut et al (1983)

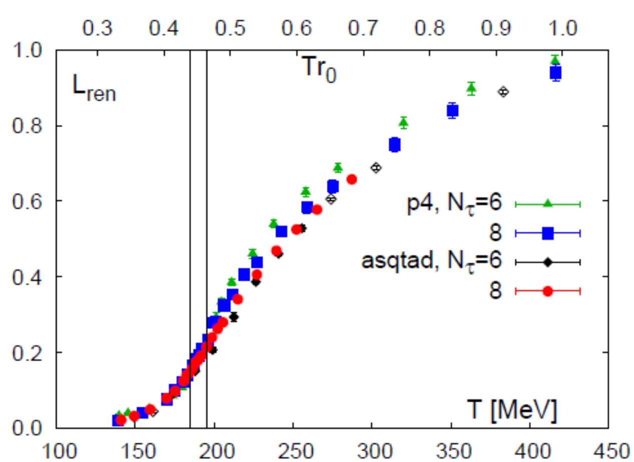


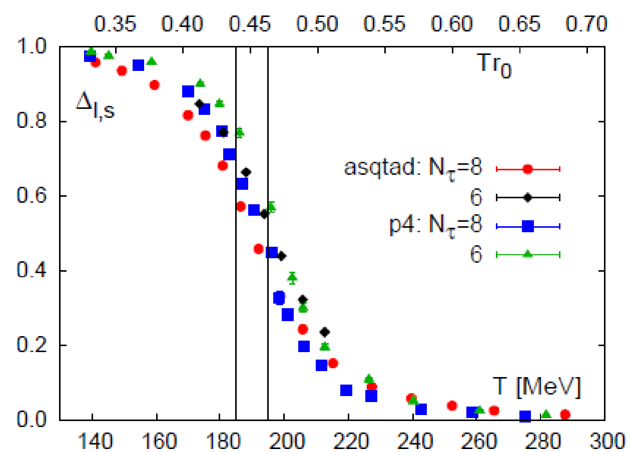
Wuppertal-Budapest (2010)





Bazavov et al (2009)





Why $T_{ch} = T_{dec}$?

Maybe there is no “deep” reason for this locking.

Most likely true that $T_{ch} > T_{dec}$

If it happens that the deconfinement scale T_{pure}=270MeV is larger than the chiral scale $f_\pi \sim$ 130MeV, only possible compromise is $T_{ch} = T_{dec}$

This is the case in the PNJL model

Any chance to realize $T_{ch} > T_{dec}$?

Probably "Yes"

QCD Phase Transitions in a Strong Magnetic Field

- ***Deconfinement transition***

  There is no direct coupling between gluons and a magnetic field, so the deconfinement is affected less.

  What happens in the infinite magnetic field limit?

- ***Chiral transition***

  Magnetic catalysis induces a larger chiral condensate. Chiral restoration is delayed to a higher temperature.

  A strong magnetic field causes the dimensional reduction from 3+1 to 1+1 (chiral magnetic spiral at finite density!)

Basar-Dunne-Kharzeev

However…

Recent Lattice-QCD Simulation (M.D'Elia et al)

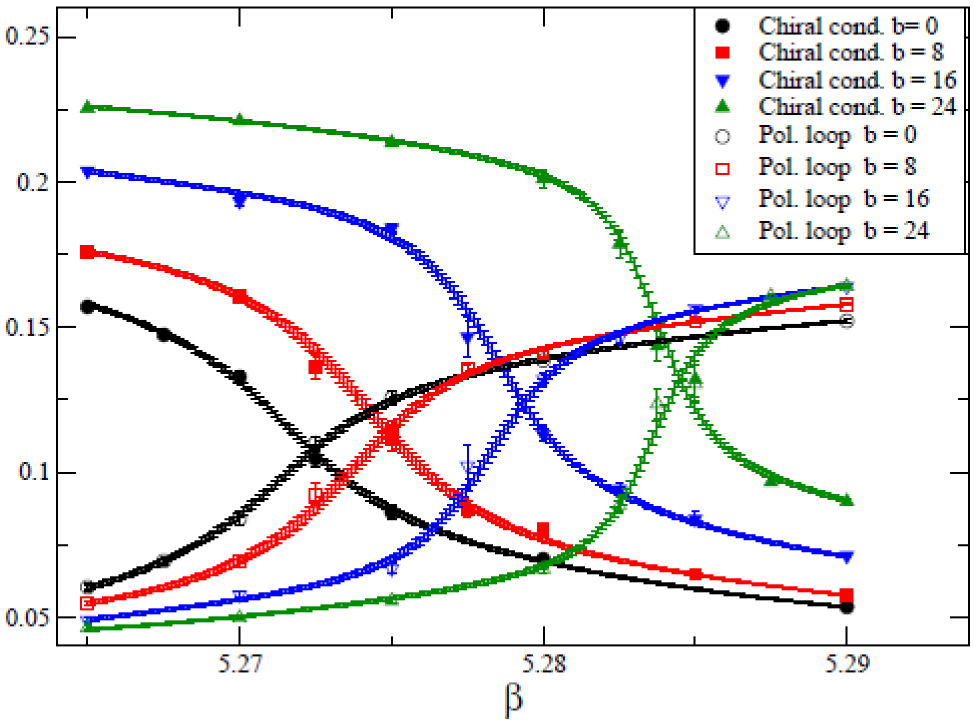


FIG. 3: Same as in Fig. 1 for $am = 0.01335$

Change in T_c is 10~20%

Magnetic field $eB \sim 0.75\text{GeV}^2$ should be large enough to affect QCD physics.

Why locked together??

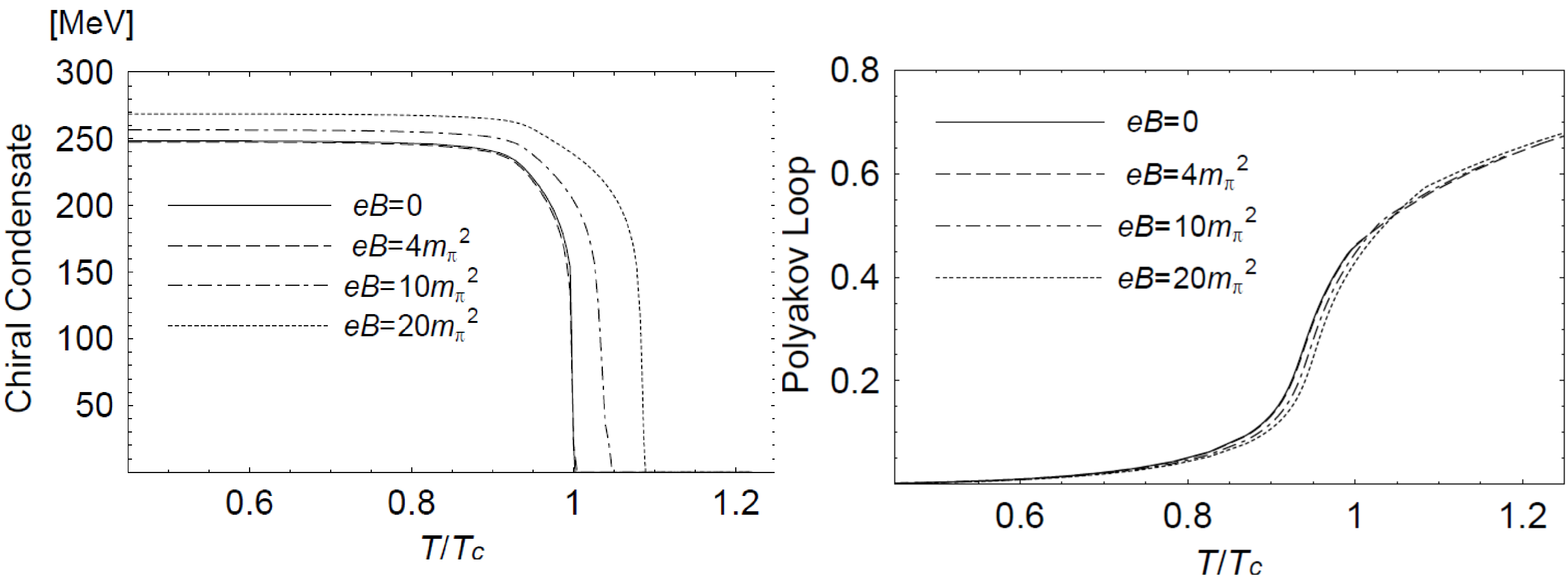
As long as the chiral condensate is large, heavy quarks suppress center symmetry breaking.

Maybe necessary to realize

$$T_{ch} > T_{dec} \sim 270\text{MeV}$$

Inconsistent with Model Calculations

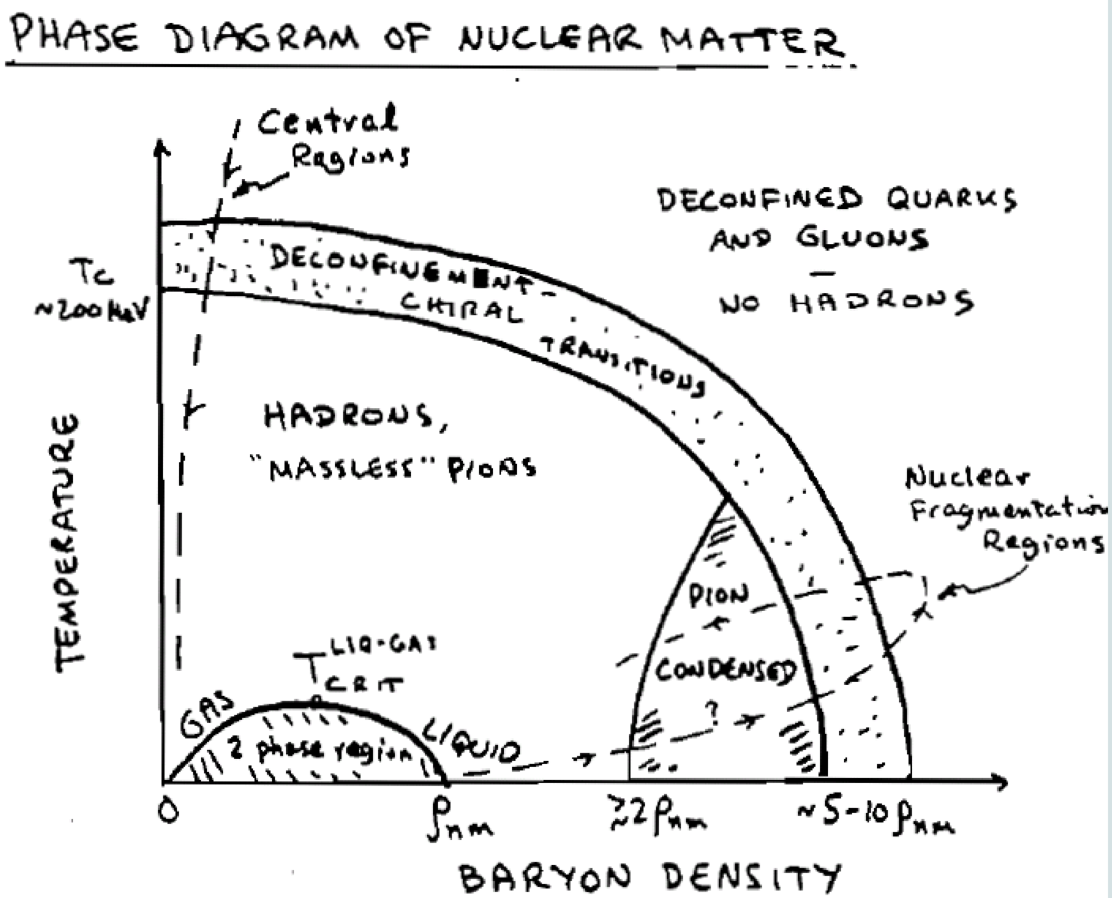
Chiral and Deconfinement (Fukushima-Ruggieri-Gatto)

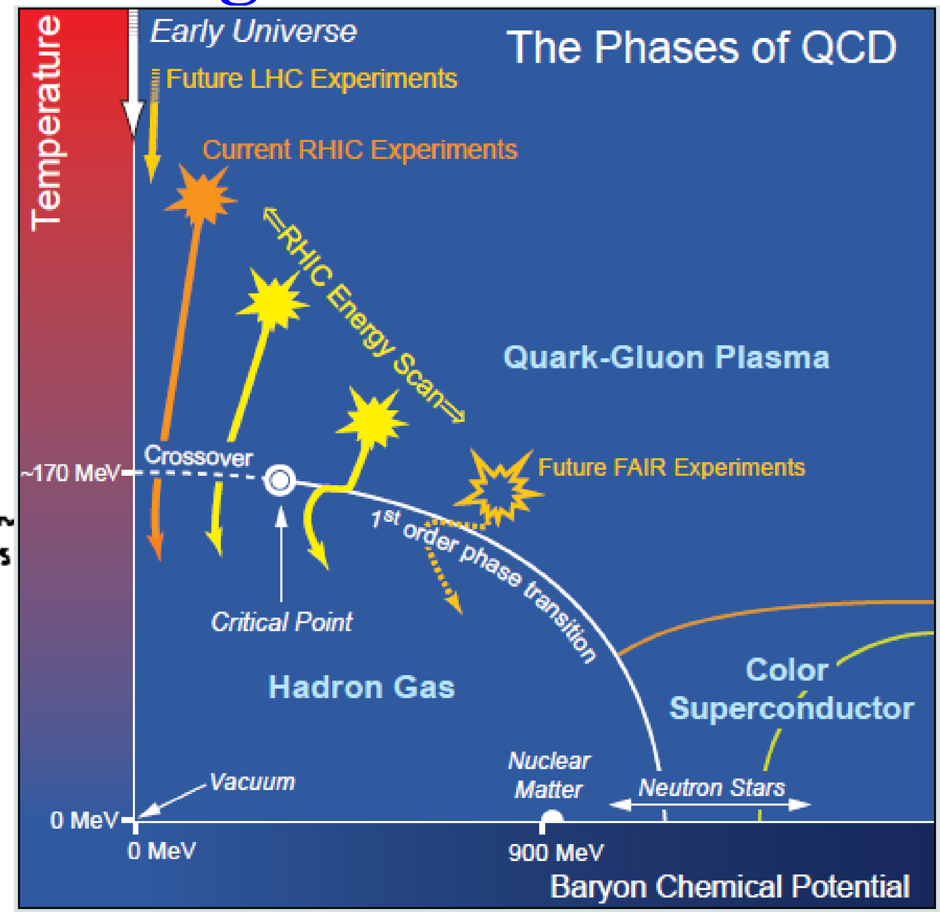


PNJL does not capture the Polyakov loop shift
Maybe some missing coupling between gluons and photons?
(Running coupling constant → opposite effect !?)

Separate boundaries are unlikely

Diagram in 1983 and Diagram in 2007





at least zero density (baryon chemical potential)

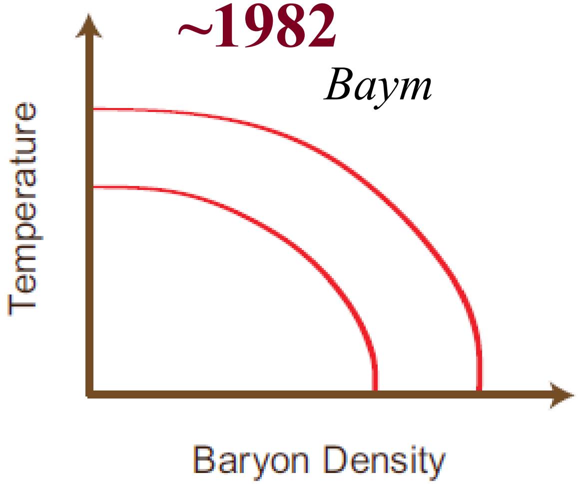
Open Question

Which is first? Chiral or Deconfinement??

KF (2008)

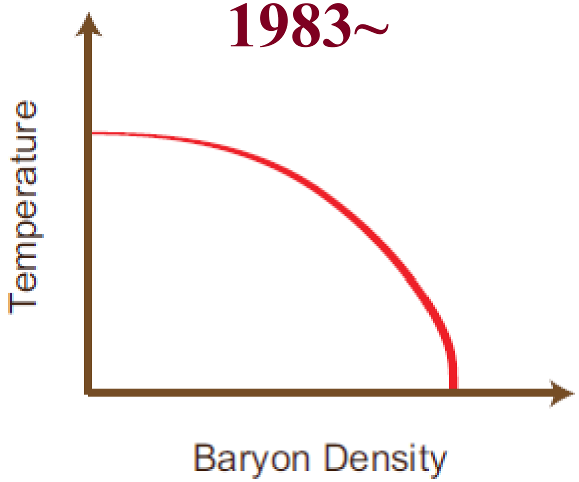
~1982

Baym

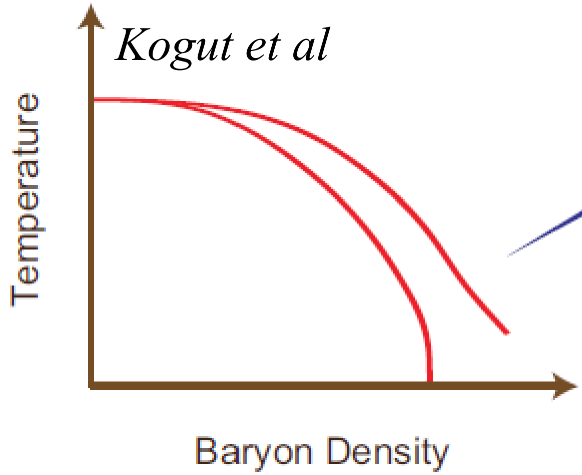


Assumption

1983~



Kogut et al



Serious possibility
(in PNJL, PQM)
2006~

Redlich, McLerran, Sasaki
Fukushima, Weise et al...

? **Model uncertainty**
Many discussions
2007~

Pawlowski, Schaefer
Wambach et al...

Various Theory Attempts

Lattice QCD Simulation

- ⊕ – First principle approach
- ⊖ – Not applicable to finite chemical potential

Phenomenological Analysis

- ⊕ – Guided by experimental data
- ⊖ – Not capable of drawing the phase boundaries

Effective Model Calculation

- ⊕ – Handy tool to test the idea on the phase diagram
- ⊖ – Uncertainties in model assumptions

And more...

Caveats

Lattice Simulation

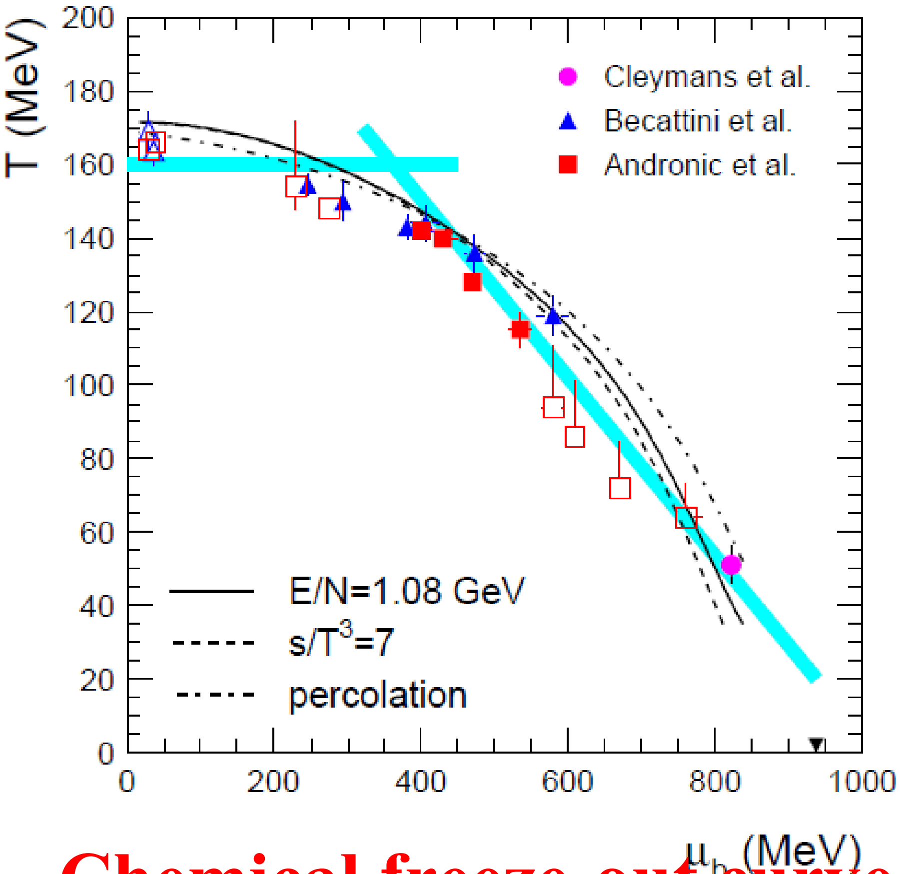
Sign Problem

- Multi-parameter reweighting method
- Taylor expansion
- Imaginary μ_B method
- Complex Langevin

Continuum Limit?

- Improved action
- Extrapolation

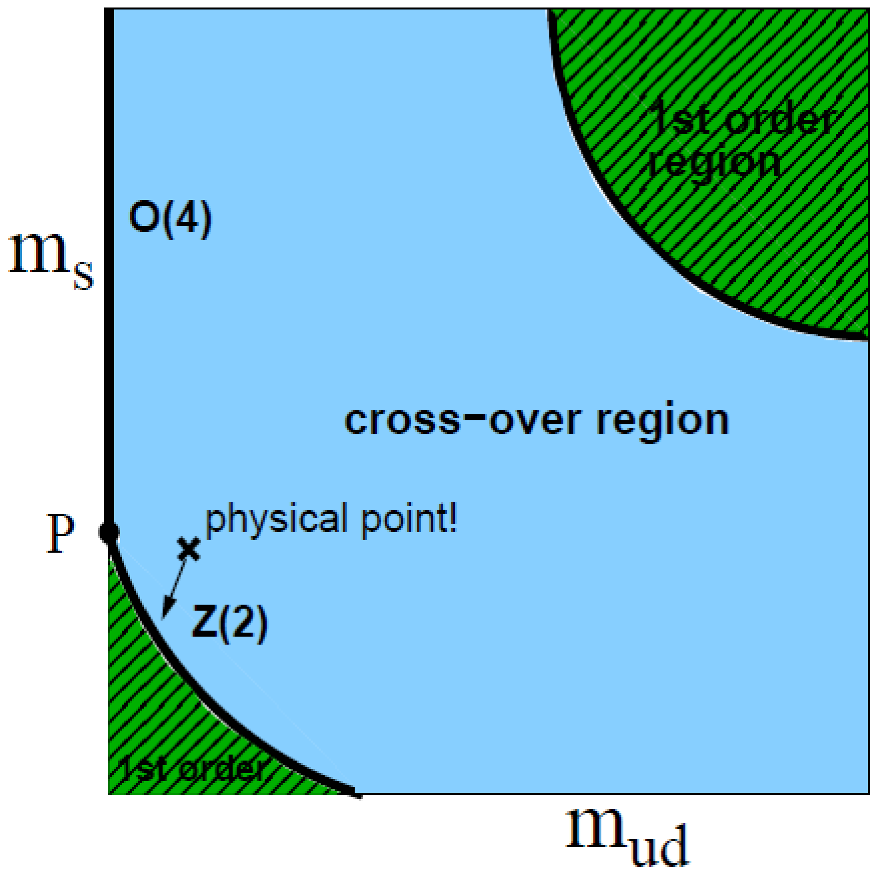
Statistical Model

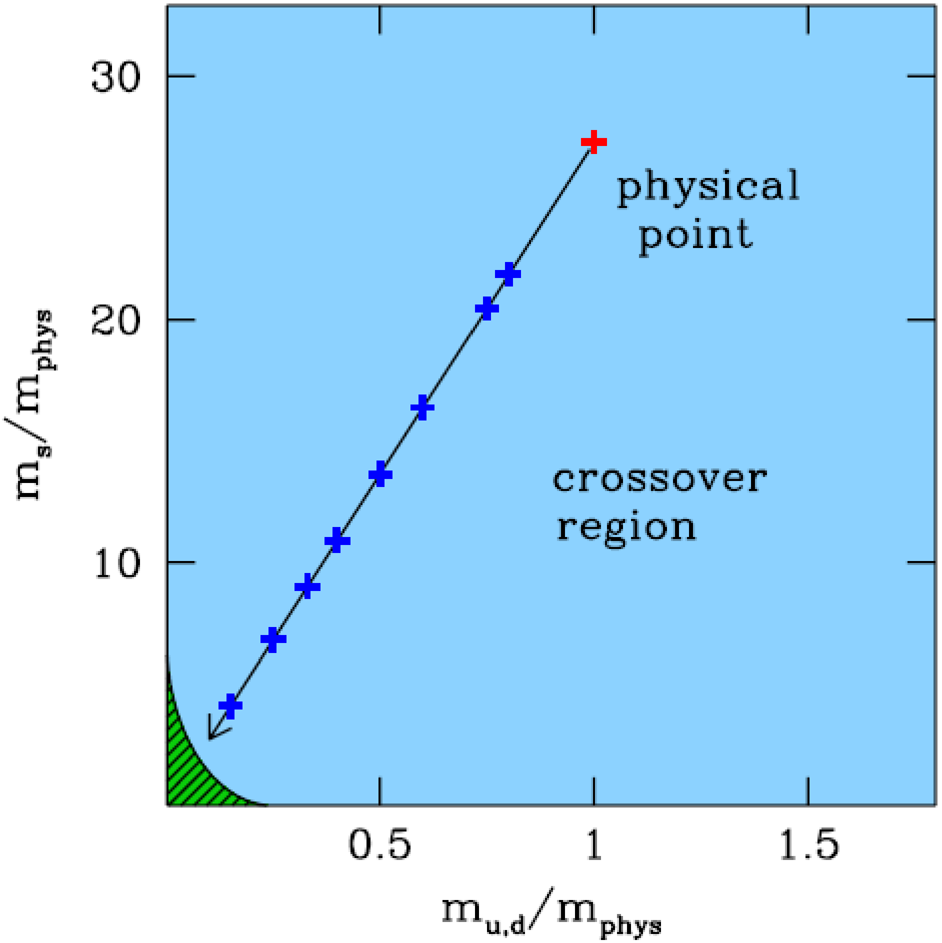


Chemical freeze-out curve is not the phase boundary

Pessimistic News about Lattice QCD

Endrodi-Fodor-Katz-Szado (2007)





c.f. de Forcrand-Kim-Philipsen

Pragmatist's Solution

Lattice QCD Simulation

- ⊕ – First principle approach
- ⊖ – Not applicable to finite chemical potential

Phenomenological Analysis

- ⊕ – Guided by experimental data
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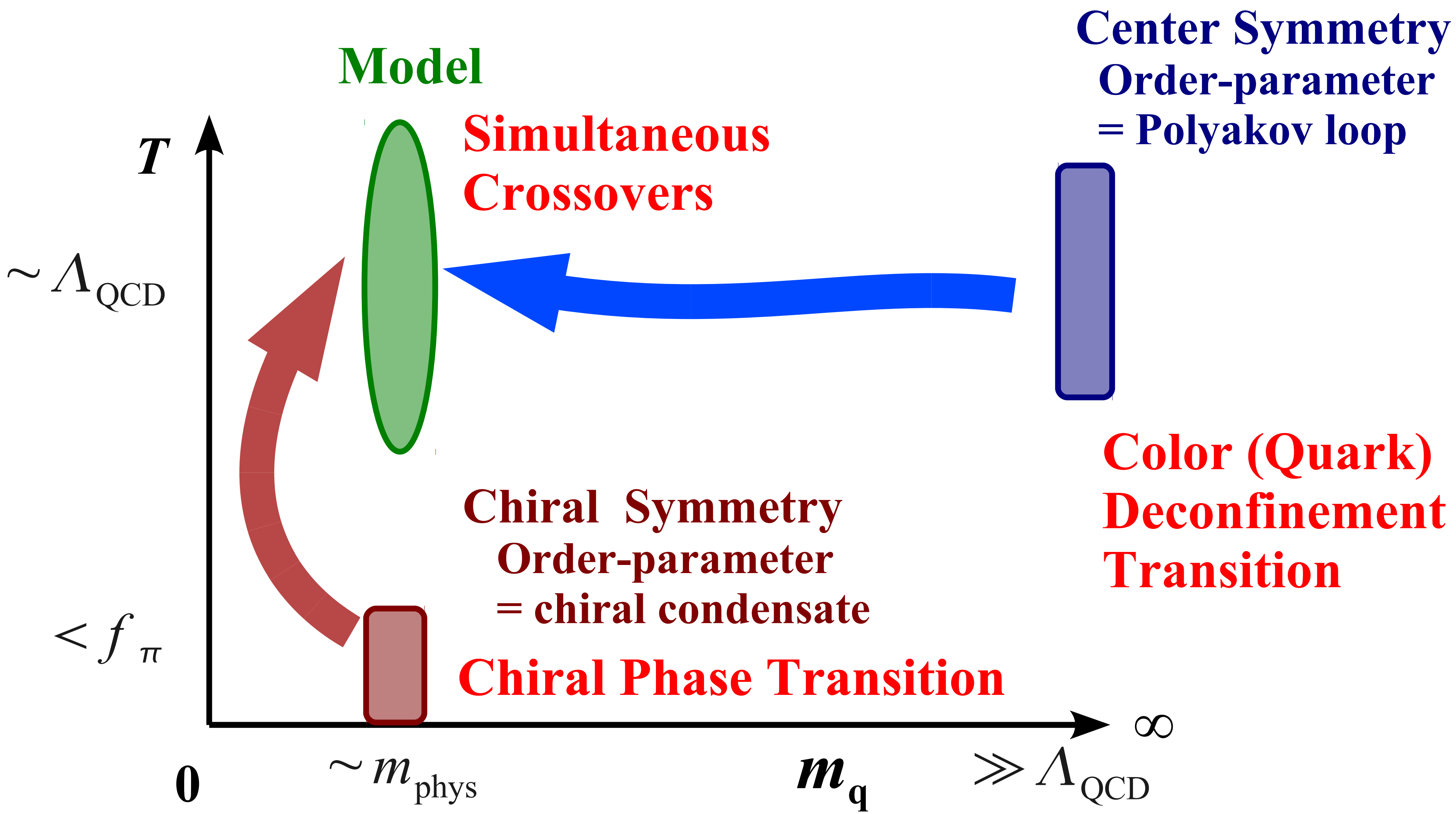
Effective Model Calculation

- ⊕ – Handy tool to test the idea on the phase diagram
- ⊖ – Uncertainties in model assumptions

can be complemented by other approaches

Model Construction



PNJL Model

Successful interpolation between the pure-gluonic and the chiral models

- **Deconfinement (*pure-gluonic* $m_q \gg \Lambda_{\rm QCD}$) Physics**
  - Order parameter – Polyakov loop $\langle \mathrm{tr}\, L \rangle \sim e^{-f_q/T}$ **Pisarski**
- **Chiral (*fermionic* $m_q \simeq m_{\rm phys}$) Physics**
  - Order parameter – Chiral condensate $\langle \bar{\psi} \psi \rangle$ **Nambu-Jona-Lasinio Hatsuda-Kunihiro**

Phase structure revealed by the PNJL model

- **Deconfinement Physics**
  - Smoother (larger center breaking) at higher baryon density
- **Chiral Physics**
  - Steeper (or first-order) at higher baryon density

Fukushima
Sasaki-Friman-Redlich
Hell-Ratti-Roessner-
-Thaler-Weise
Kyushu, Bari, etc

Pure-gluonic Model $(m_q \gg \Lambda_{QCD})$

Ansatz (Polyakov loop ~ A_0)

$$V(\ell) = -\frac{1}{2} a(T) \ell \bar{\ell} + b(T) \log\left[1 - 6\ell\bar{\ell} + 4(\ell^3 + \bar{\ell}^3) - 3(\ell\bar{\ell})^2\right]$$

Vandermonde determinant
= SU(3) Haar measure

Parameters

$$a(T) = T^4 (3.51 - 2.47 t^{-1} + 15.2 t^{-2})$$

$$b(T) = -1.75 t^{-3} \cdot T^4$$

$$t = T / T_0$$

3 (+1) parameters

Ratti-Thaler-Weise
Ratti-Roessner-Weise

$\ell(T)$
$p(T)$

c.f. strong-coupling ansatz

$$a(T) = T \cdot b \cdot 54 e^{-a/T}$$

$$b(T) = T \cdot b$$

2 parameters

Fukushima

Chiral Model ($m_q = m_{phys}$)

Nambu–Jona-Lasinio (NJL) Model

$$L = \bar{\psi}(i\gamma\cdot\partial - m_f) + \frac{g_S}{2}[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma_5\lambda\psi)^2] + g_D[\det\bar{\psi}(1-\gamma_5)\psi + \text{h.c.}]$$

Parameters

$$\Lambda = 631.4\,\text{MeV}$$
$$m_{ud} = 5.5\,\text{MeV}$$
$$m_s = 135.7\,\text{MeV}$$
$$g_S\Lambda^2 = 3.67$$
$$g_D\Lambda^5 = -9.29$$

← m_π, f_π, m_K, $m_{\eta'}$, one more? M_{ud} or $\langle\bar{\psi}\psi\rangle$

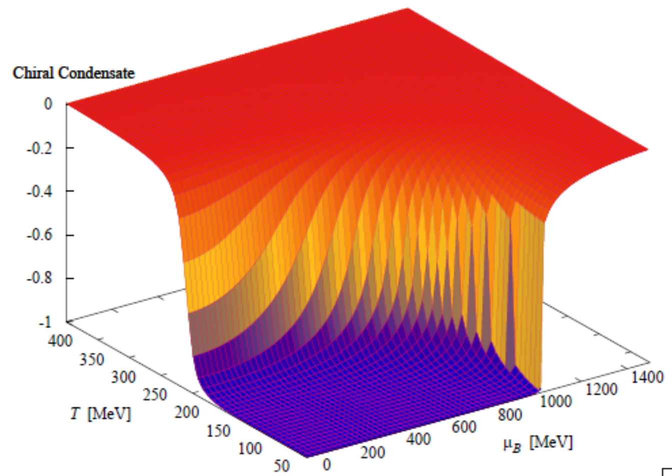
c.f. PQM model
Schaefer, Wambach

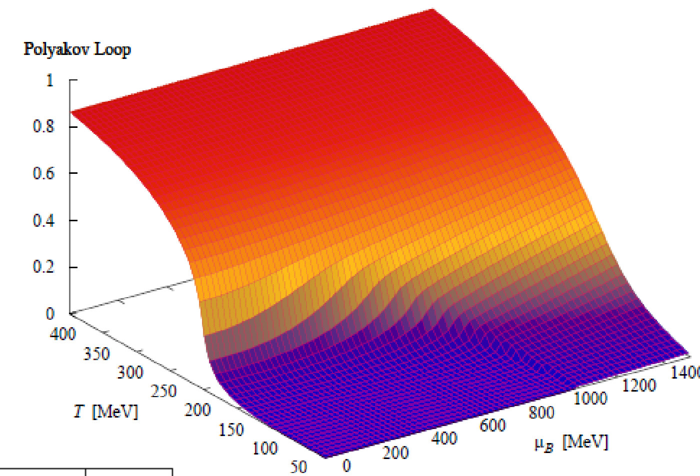
Coupling (covariant derivative for A_0)

$$2N_f\int\frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\left[\log\left(1+L\,\mathrm{e}^{-(E-\mu)/T}\right)+\log\left(1+L^\dagger\mathrm{e}^{-(E+\mu)/T}\right)\right]$$

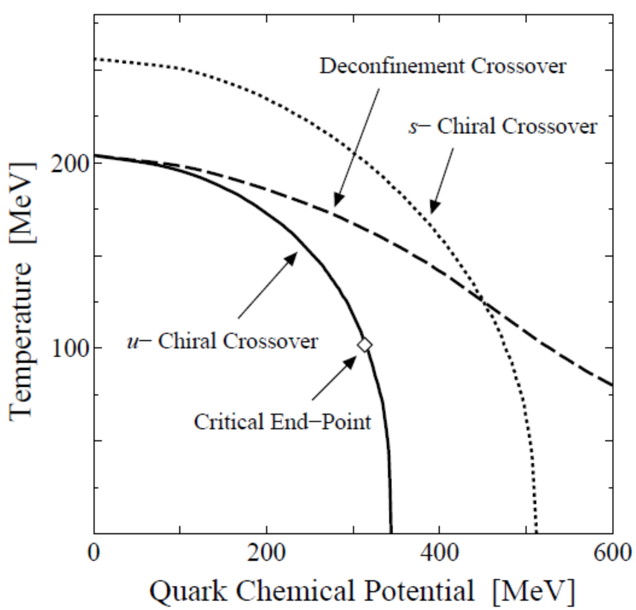
Typical Phase Structure from PNJL

Chiral Condensate and Polyakov Loop





Unique tool to deal with two order parameters on the equal footing!



2-flavors – KF (2004)
3-flavors – KF (2008)

Why $T_{dec} > T_{ch}$ possible in PNJL?

Remember Casher's argument

Spatially *local* confinement is essential

PNJL describes only *global* confinement

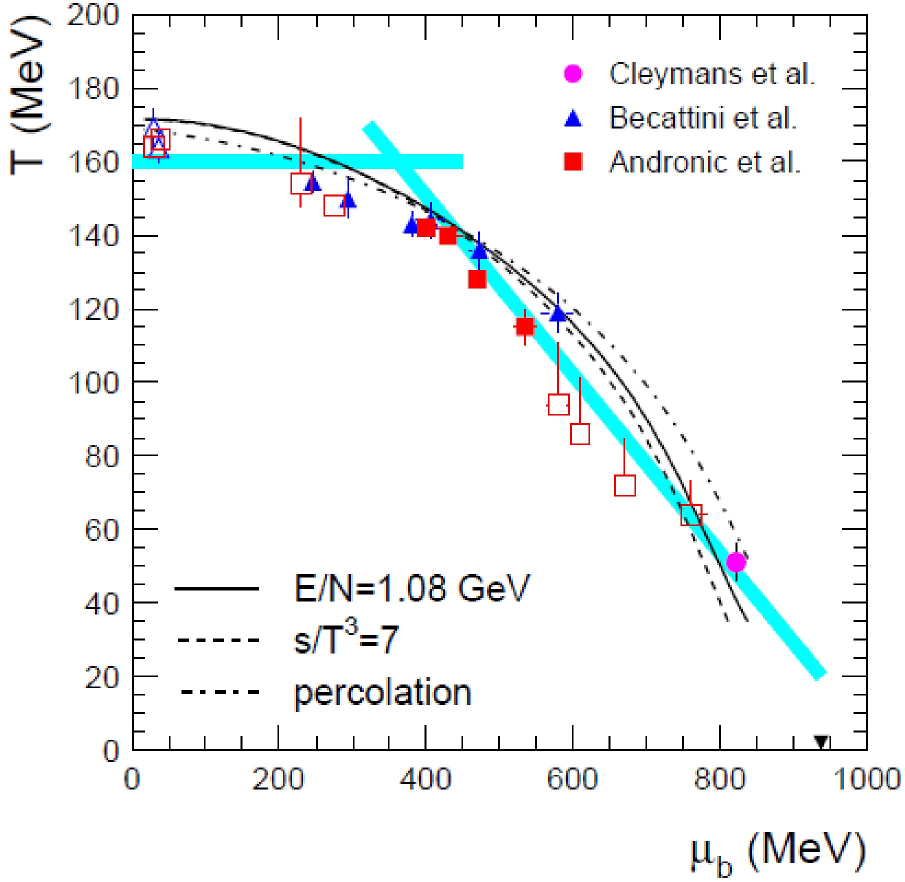
$$Z_q = \prod_{i,p} \det\left(1 + L\,\mathrm{e}^{-(E_i(p)-\mu)/T}\right)\left(1 + L^\dagger\,\mathrm{e}^{-(E_i(p)+\mu)/T}\right)$$
$$= \prod_{i,p}\left(1 + \mathrm{tr}\,L\,\mathrm{e}^{-(E_i-\mu)/T} + \mathrm{tr}\,L^\dagger\,\mathrm{e}^{-2(E_i-\mu)/T} + \mathrm{e}^{-3(E_i-\mu)/T}\right)$$
$$\times\left(1 + \mathrm{tr}\,L^\dagger\,\mathrm{e}^{-(E_i+\mu)/T} + \mathrm{tr}\,L\,\mathrm{e}^{-2(E_i+\mu)/T} + \mathrm{e}^{-3(E_i+\mu)/T}\right)$$

The excitation of a baryon is NOT the excitation of three quarks

$T_{dec} > T_{ch}$ might be allowed only in the model

Idea to “Cure” the Model

Chemical Freeze-out Points from the Statistical Model



arXiv: 0911.4806
Andronic, Blaschke, Braun-Munzinger, Cleymans, KF, McLerran, Oeschler, Pisarski, Redlich, Sasaki, Satz, Stachel

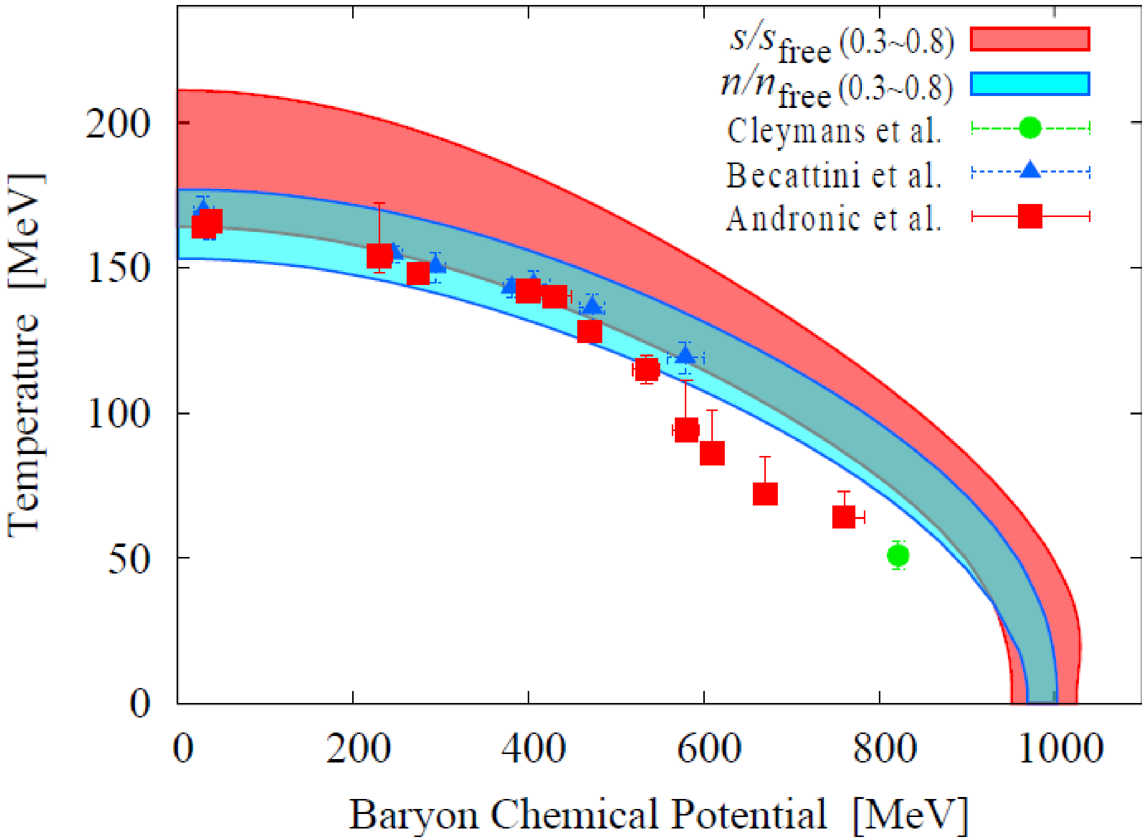
Only available information at finite baryon density

Not the phase boundary

How to connect this to the QCD phase boundaries (PNJL)?

Thermodynamics from the SM

Results using THERMUS (Wheaton and Cleymans) KF (2010)



$$s_{\text{free}} = \left((N_c^2 - 1) + \frac{7}{4} N_c N_f \right) \frac{4\pi^2}{45} T^3 + \frac{N_c N_f}{3} \mu^2 T$$

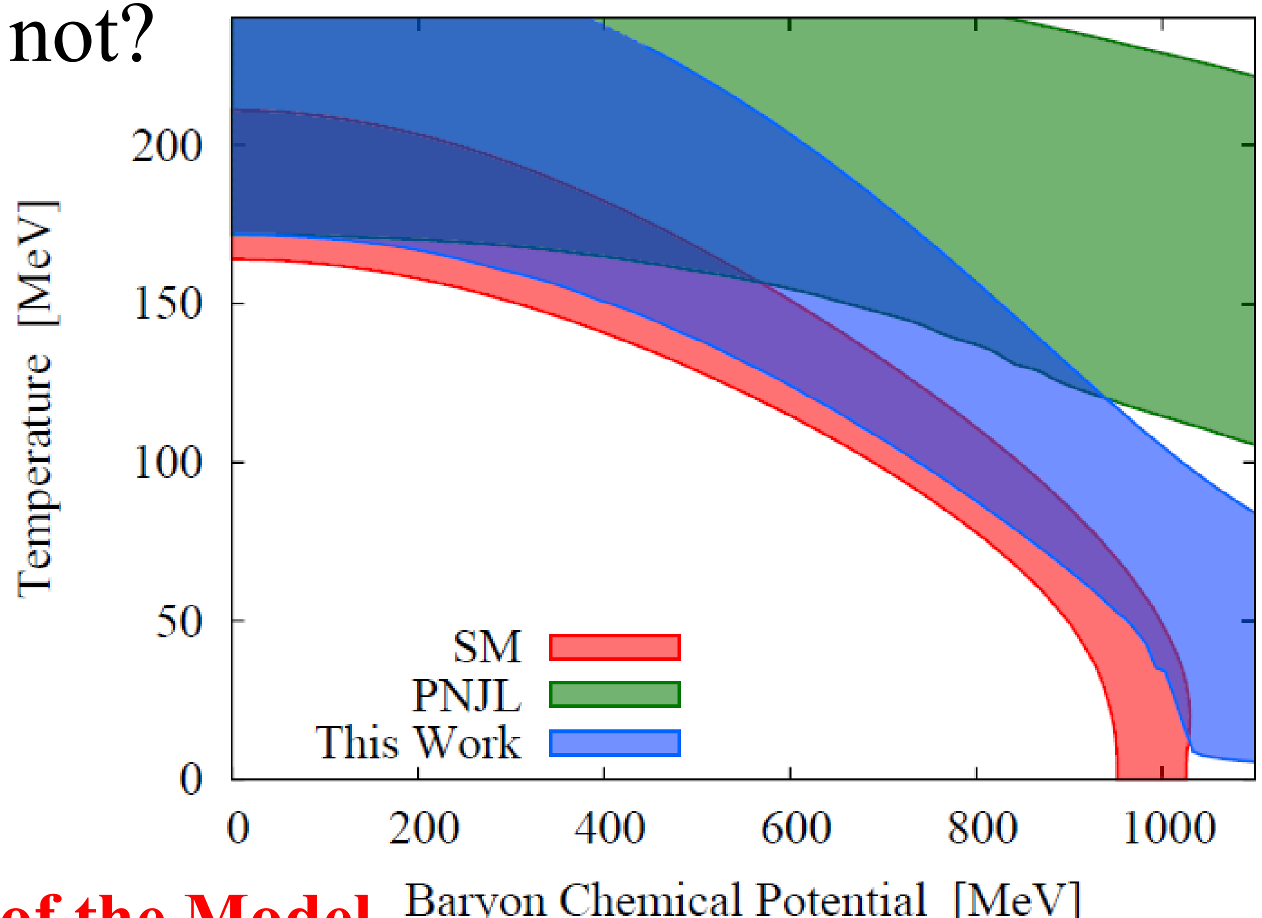
$$n_{\text{free}} = N_f \left(\frac{\mu^3}{3\pi^2} + \frac{\mu T^2}{3} \right)$$

To discuss the phase diagram, the neutrality conditions are intentionally relaxed (w.r.t. strangeness and electric charge), which are minor effects to the entropy.

Blow-up in entropy should be regarded as "*deconfinement*" intuitively.

Entropy Comparison

Energy density, baryon density, etc may exceed the free quark-gluon (SB) limit, but the entropy density may not?



Discrepancy from loss of confinement (baryons)

Failure of the Model

$T_o(\mu_B)$ – *Simplest Remedy*

Parametrization for Chemical Freezeout Curve

$$T_f(\mu_B) = a - b\,\mu_B^2 - c\,\mu_B^4$$

$$a = 166(2)\,\mathrm{MeV} \quad b = 1.39(16)\times 10^{-4}\,\mathrm{MeV}^{-1} \quad c = 5.3(21)\times 10^{-11}\,\mathrm{MeV}^{-3}$$

Cleymans-Oeschler-Redlich-Wheaton (2006)

Assumed Parametrization for $T_0(\mu)$

$$\frac{T_0(\mu_B)}{T_0(0)} = 1 - (b\,T_0)\left(\frac{\mu_B}{T_0}\right)^2 = 1 - 2.78\times 10^{-2}\left(\frac{\mu_B}{T_0}\right)^2$$

c.f Perturbative Matching

$$T_0(\mu_B) = T_\tau e^{-1/(\alpha_0 b(\mu))} \qquad \frac{T_0(\mu_B)}{T_0(0)} \simeq 1 - 2.1\times 10^{-2}\left(\frac{\mu_B}{T_0}\right)^2$$

Schaefer-Pawlowski-Wambach

Lattice QCD

$$\frac{T_c(\mu_B)}{T_c(0)} \simeq 1 - 6.8\times 10^{-3}\left(\frac{\mu_B}{T_0}\right)^2$$

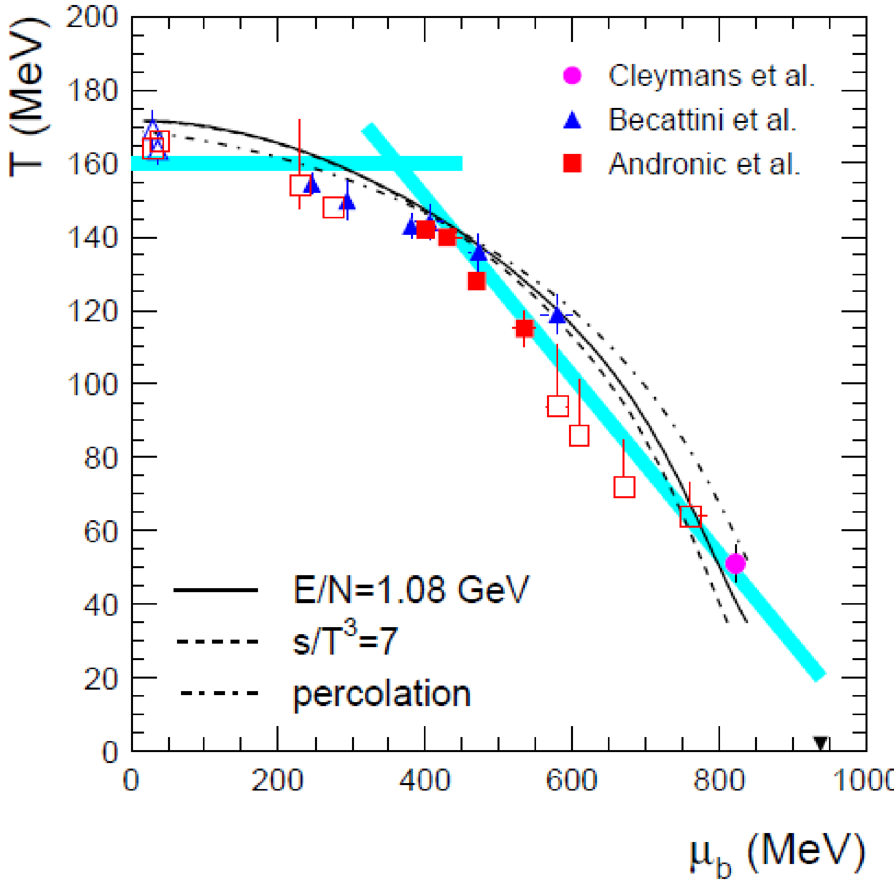
Curvature ~ 1/3

de Forcrand

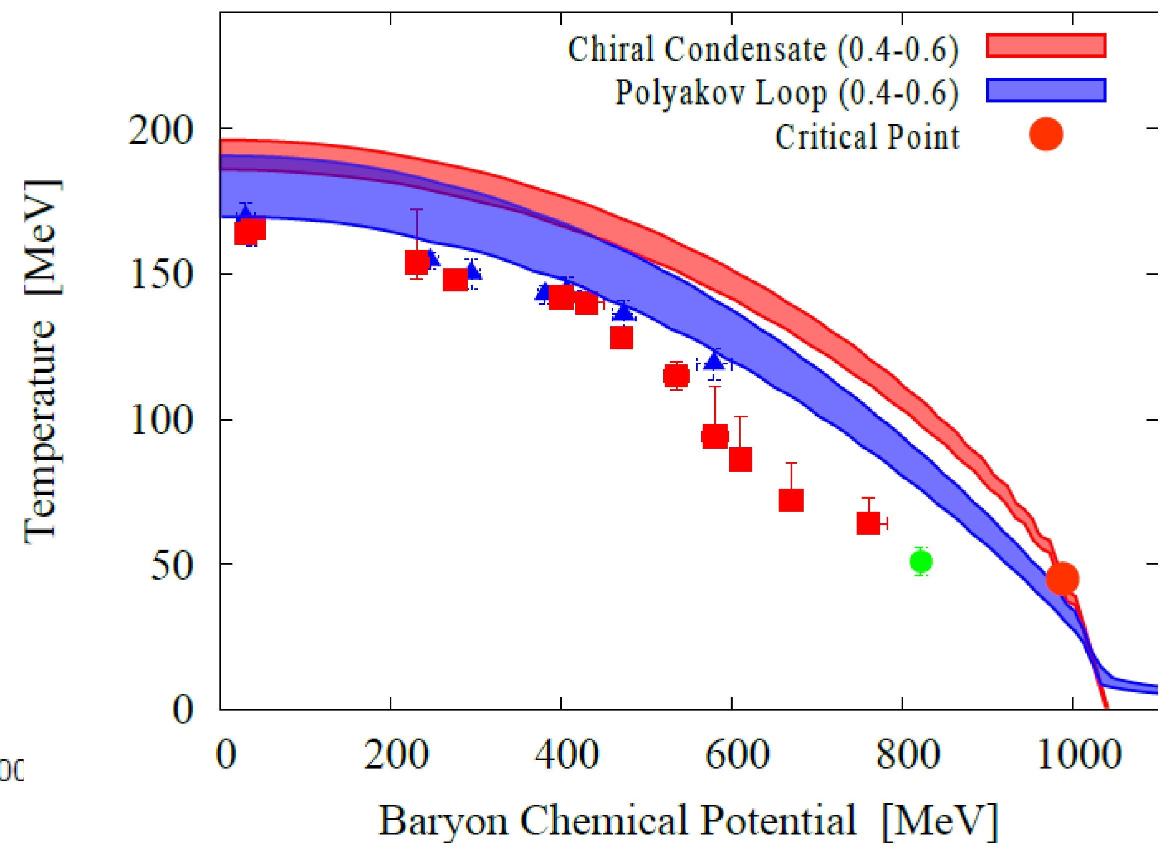
Phase Boundaries

Matching in Thermodynamics

Chemical freeze-out curve



Phase boundaries from a model that is consistent with SM



Fukushima (2010)

Inhomogeneity (questions and work in progress)

Inhomogeneous Chiral Condensate

- Confining potential may induce for $N_c = 3$
- First-order phase boundary may induce generally

Hidaka-Kojo--McLerran-Pisarski
Nakano-Tatsumi
Nickel

Inhomogeneous Diquark Condensate

- Chromomagnetic instability induce FFLO
- Crystallography in the GL expansion

Huang-Shovkovy
Giannakis-Ren
Fukushima
Rajagopal-Sharma

Inhomogeneous Polyakov Loop

- Why not???
- Implication from perturbative calculations

Fukushima-Ohta

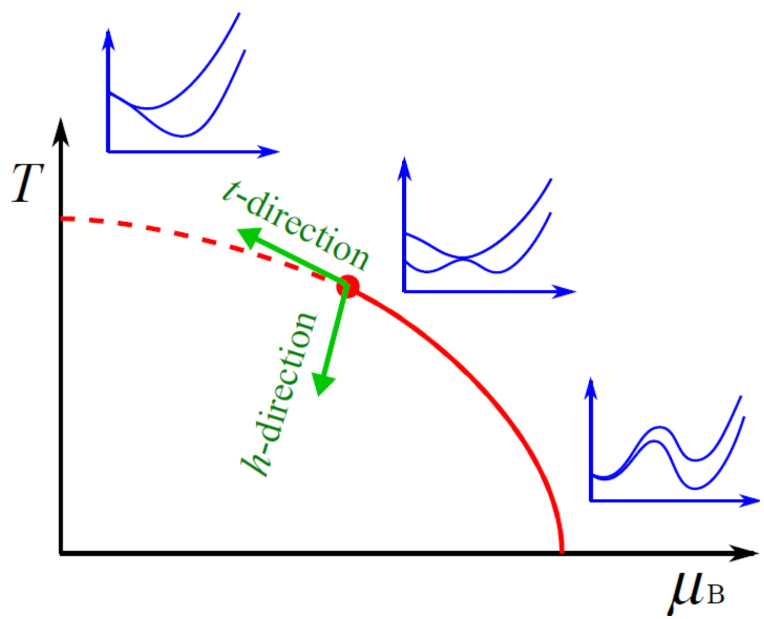
Inhomogeneity from GL

GL+derivative expansion **(Nickel)**

$$\Omega = c_2 M^2 + c_4 M^4 + c_4{}'(\nabla M)^2 + c_6 M^6 + c_6{}'(\nabla M)^2 M^2 + c_6{}''(\Delta M)^2$$

$$\Omega = \alpha_2 M^2 + \alpha_4\left[M^4 + (\nabla M)^2\right] + \alpha_6\left[M^6 + 5(\nabla M)^2 M^2 + \frac{1}{2}(\Delta M)^2\right]$$

in 2-flavor NJL



Lessons from CSC

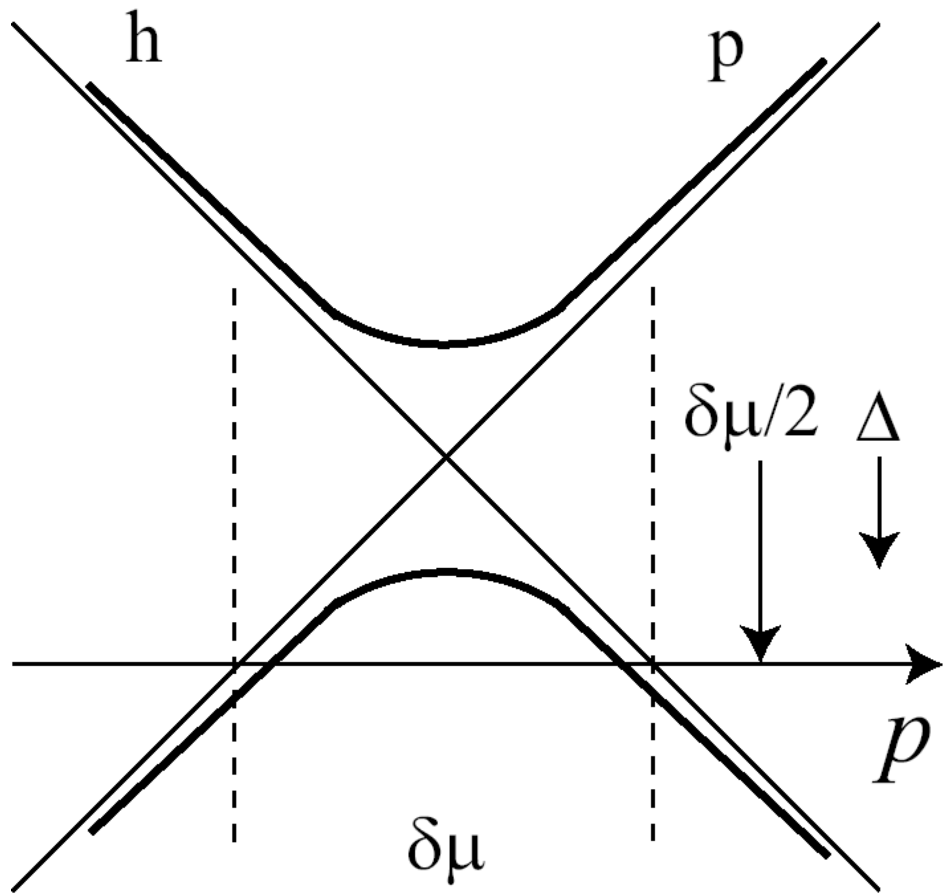
Quasi-particle energy is "gapless" for finite Δ

$$\delta\mu > 2\Delta$$

CFL

⬇

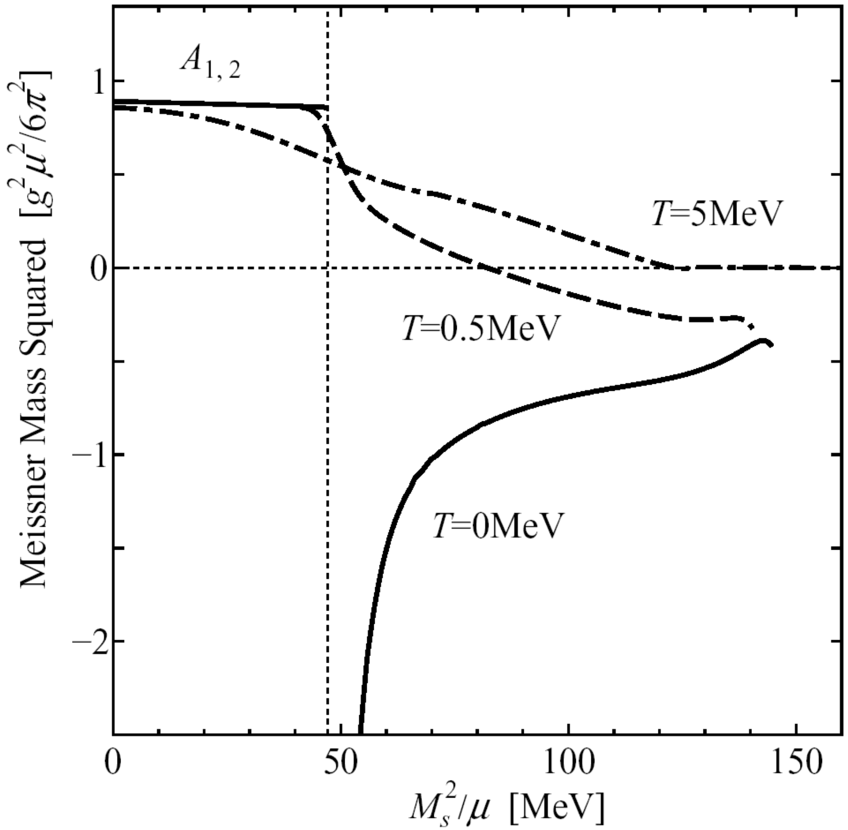
gapless CFL (gCFL)



Chromomagnetic Instability

Meissner Screening Mass

Screening mass for transverse gluons ← Meissner effect
c.f. Longitudinal gluons (or A_0) have Debye mass



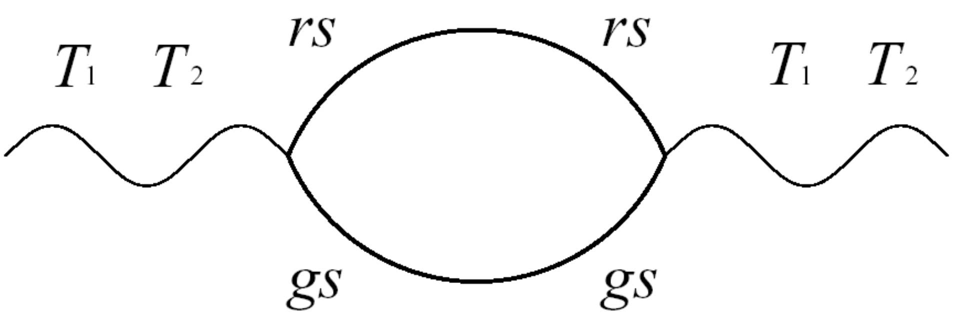
In (near) the gapless region
the mass becomes pure imaginary

Gluons have expectation value

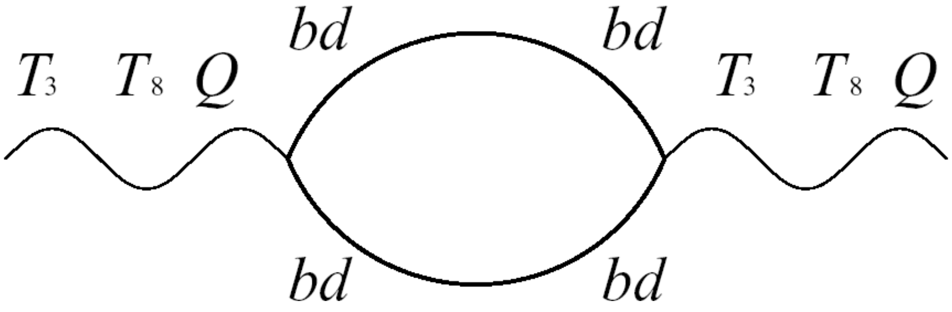
$$\partial^2 \Omega / \partial A_T^2 < 0$$

Relevant Diagrams

$A_{1,2}$ gluons τ_1 in color space red-green



$A_{3,8}$ gluons (photon) red-red, green-green, blue-blue



bd-gs (τ_1) gapless

rs-bu (τ_2) gapless (quadratic)

Destination from Instability

Instability for A_i = Instability for q_i

$$\langle \psi\, \psi \rangle \sim |\Delta| e^{iq\cdot x}$$ (Covariant Derivative $q_i + A_i$)

$$(\partial - igA)\Delta = e^{ig\,A\cdot x}\,\partial\left(\Delta\, e^{-ig\,A\cdot x}\right)$$

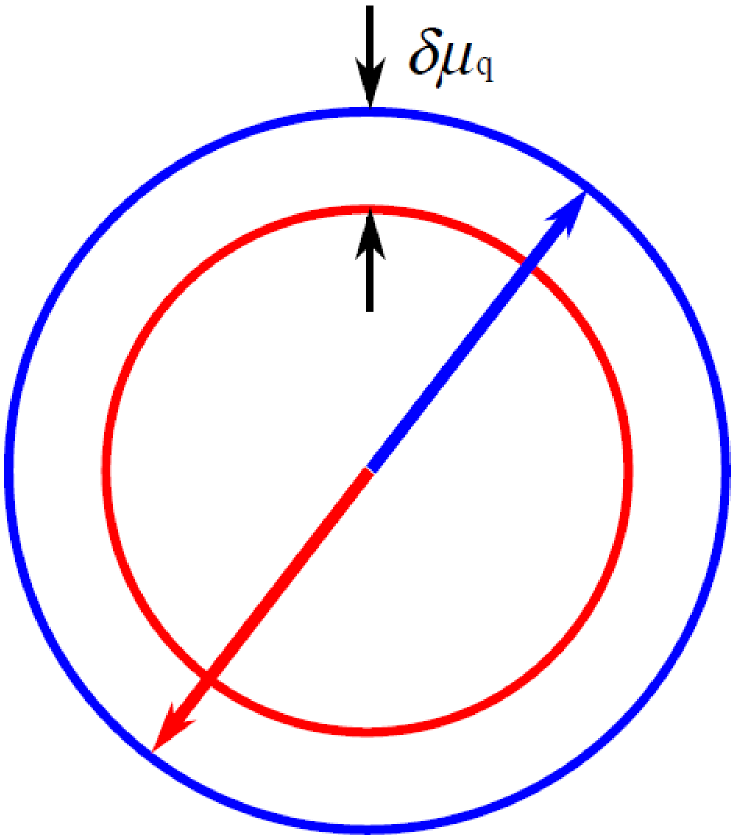
Giannakis-Ren
Fukushima

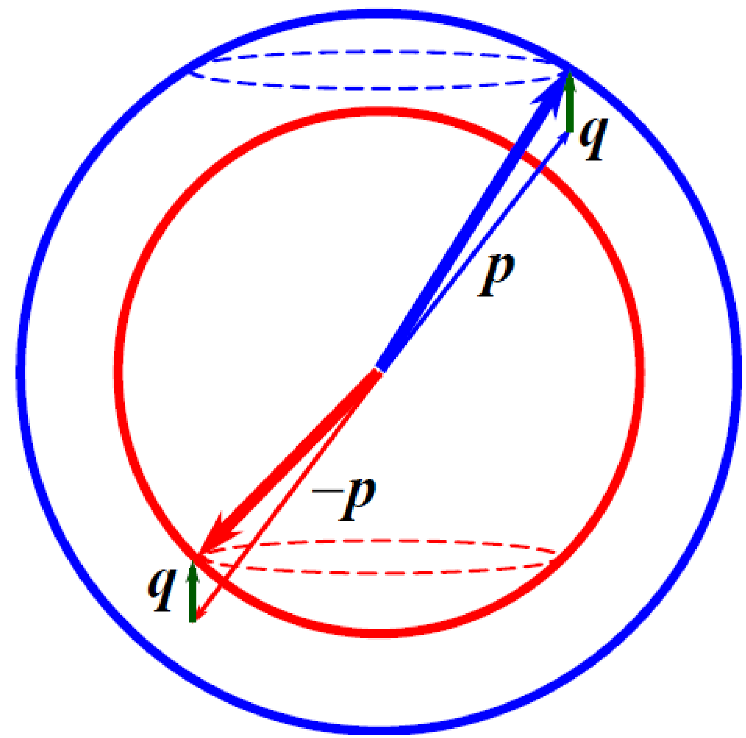
Rotational symmetry is broken

Spatially inhomogeneous!

Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state

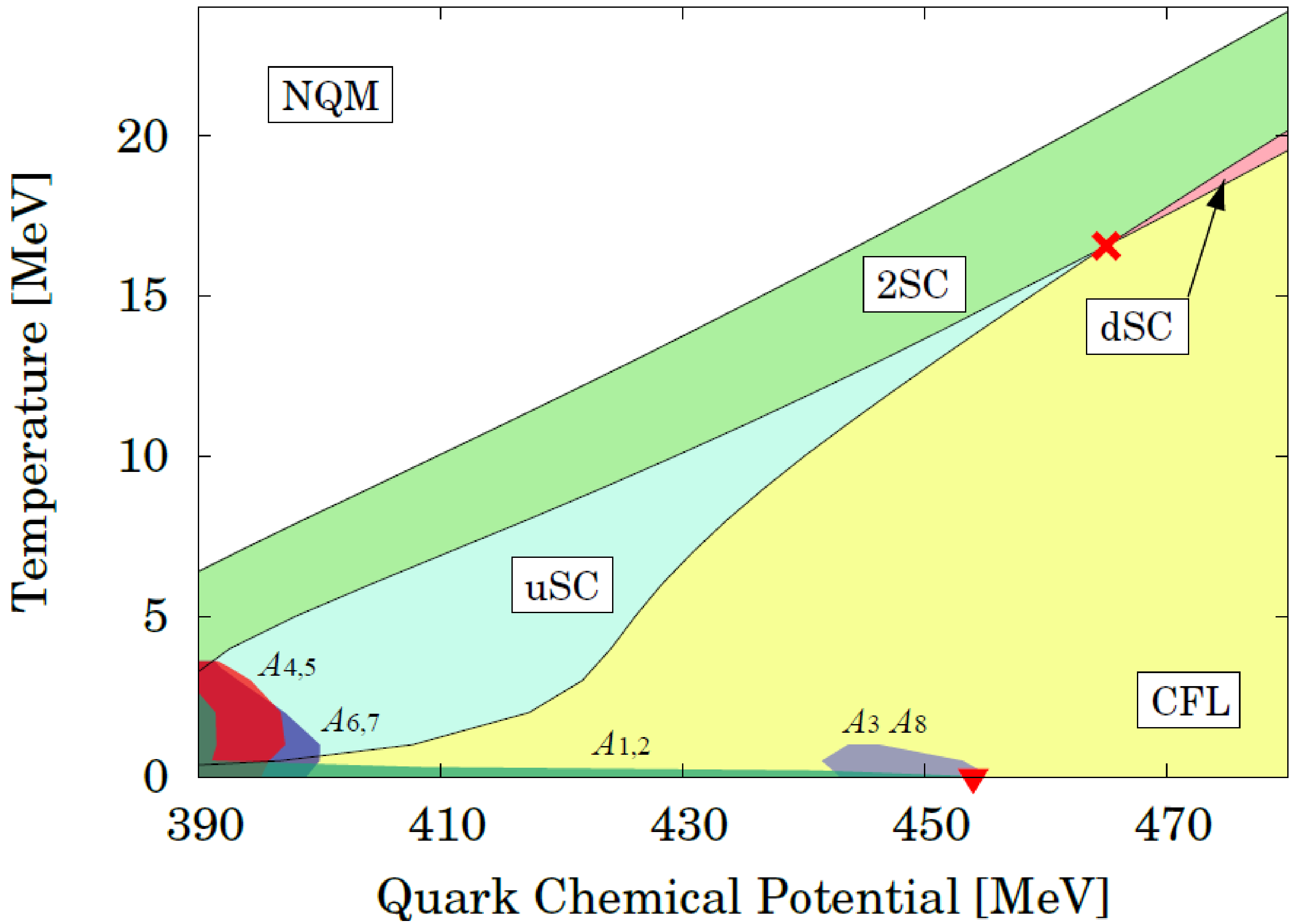
Pairing with a Net Momentum





Unstable Regions

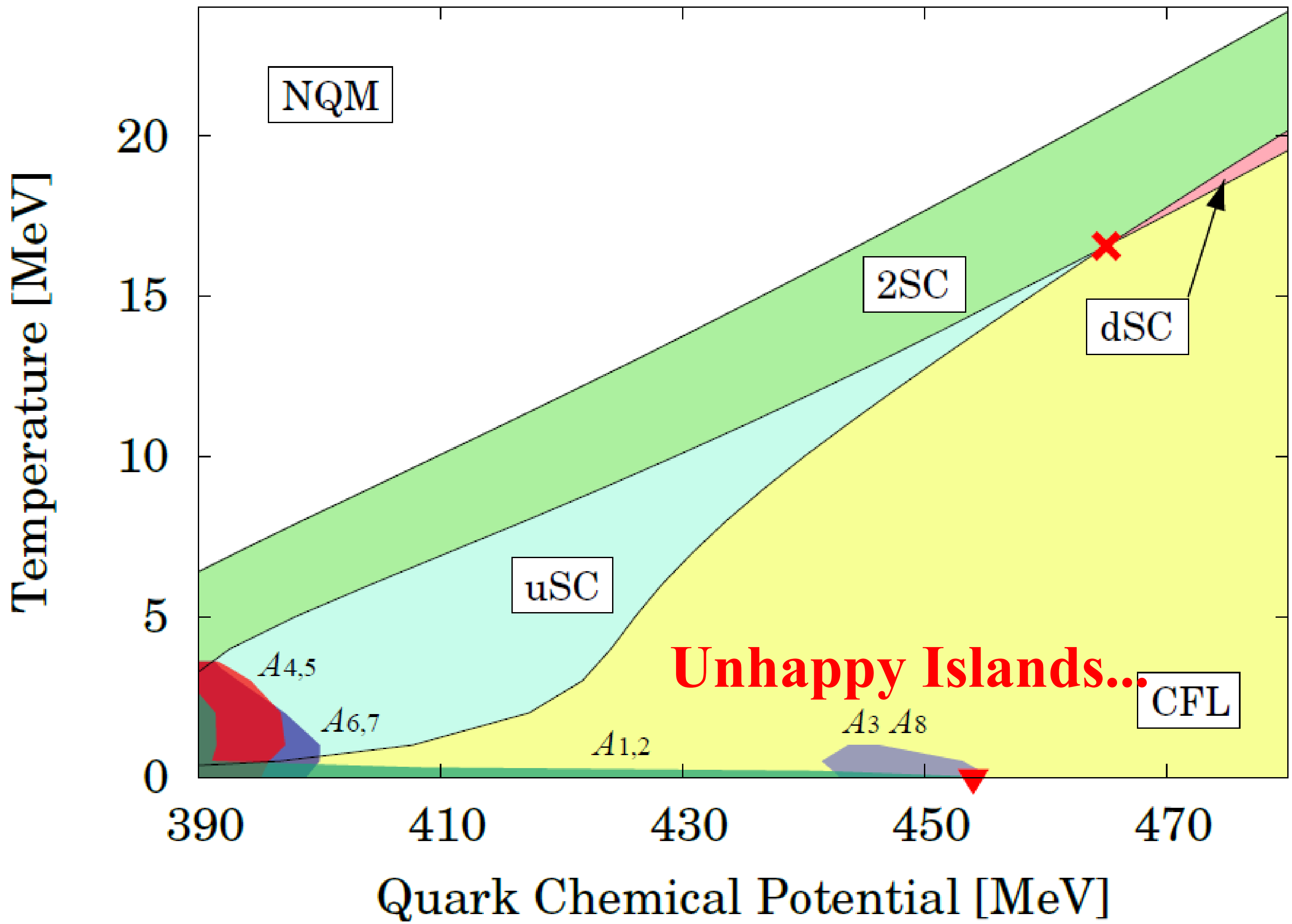
Single-Plane Wave FFLO (at least!)



Fukushima

Unstable Regions

Single-Plane Wave FFLO (at least!)



Fukushima

Crystallography

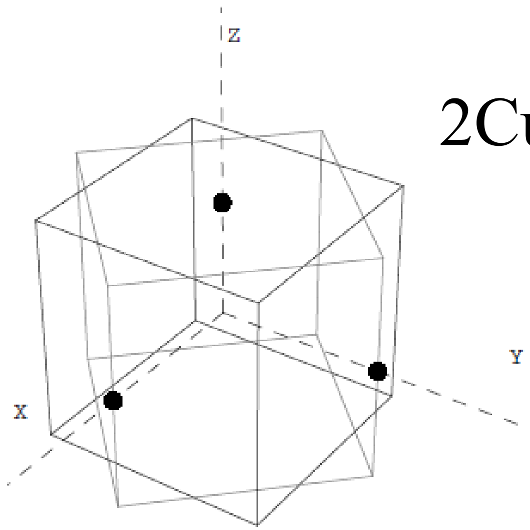
Rajagopal-Sharma

$$\langle u(x)d(x)\rangle \sim \Delta_{ud}\sum e^{2i\,\boldsymbol{q}_3^a\cdot\boldsymbol{x}}$$

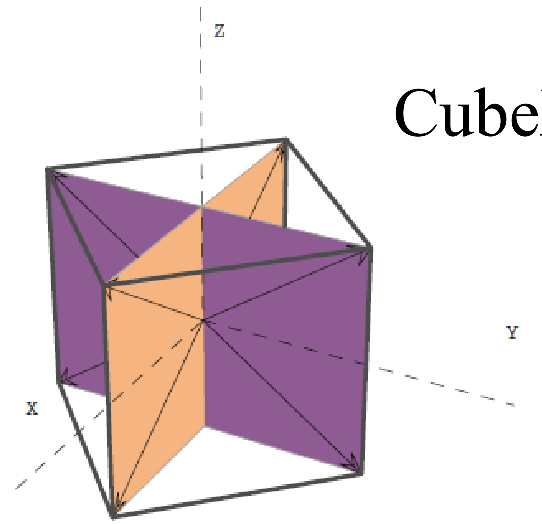
$$\langle s(x)u(x)\rangle \sim \Delta_{su}\sum e^{2i\,\boldsymbol{q}_2^a\cdot\boldsymbol{x}}$$

$$\langle d(x)s(x)\rangle \sim \Delta_{ds}\sum e^{2i\,\boldsymbol{q}_1^a\cdot\boldsymbol{x}}$$

Approximation $\Delta_{ud} \simeq \Delta_{su} \quad \Delta_{ds} \simeq 0$



2Cube45



CubeX

Interesting Questions

Crystallography using the GL expansion

$$\Omega = c_2(\{q\})\,M^2 + c_4(\{q\})\,M^4 + c_6(\{q\})\,M^6$$

In progress with Kamikado

Spiral structure not included naively...

Quark-hadron continuity including inhomogeneity

Nuclear Matter with a Superfluidity = Color-Flavor Locked State

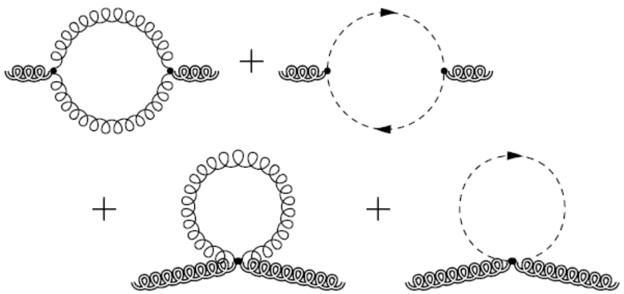
Schaefer-Wilczek
Alford-Berges-Rajagopal

Conjecture:
Quarkyonic or Chiral Spiral ~ Crystalline Color Superconductor ?
Symmetry breaking pattern, Mapping of collective excitations

Inhomogeneous Polyakov Loop

This is a possibility even in the pure YM

Phase of the Polyakov loop $a(x)=\frac{1}{\beta}(C+\varepsilon(x))$



Expand the perturbative potential in terms of ε

Fukushima-Ohta (2000)

$$\Gamma_{\text{tree}}[\epsilon] = -\frac{V}{\beta^3 N g^2}\sum_{i>j}\int\frac{\mathrm{d}^3 q}{(2\pi)^3}|\epsilon_{ij}(q)|^2 q^2$$

$$\Gamma^{(2)}_{\text{loop}}[C,\epsilon] = -\frac{V}{\beta^3}\sum_{i>j}\int\frac{\mathrm{d}^3 q}{(2\pi)^3}|\epsilon_{ij}(q)|^2\left\{\frac{11}{48\pi^2}q^2\ln\frac{q^2}{M^2}+\frac{1}{4\pi}f(q,C_{ij})\right\}$$

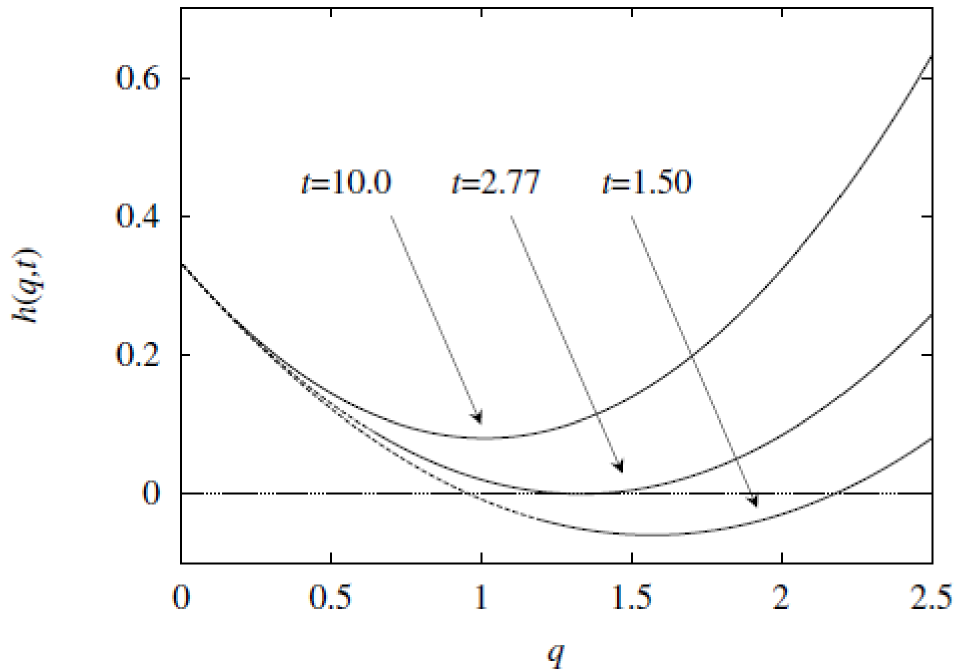
$$f(q,C) = \frac{2}{\pi}\int_0^\infty \mathrm{d}p\left\{p+q\left(1-\frac{p^2}{q^2}\right)\ln\left|\frac{2p-q}{2p+q}\right|\right\}N(p,C_{ij})$$

$$N(p,C_{ij}) = \frac{1}{\mathrm{e}^{p+\mathrm{i}C_{ij}}-1}+\frac{1}{\mathrm{e}^{p-\mathrm{i}C_{ij}}-1}$$

Instability

Coefficient for inhomogeneity on top of $C = 0$

$$h(q,t) = \frac{1}{4\pi}\left\{\frac{q^2}{N\alpha(q,t)} + f(q, C=0)\right\}$$



$$g^2(q,t) = \frac{24\pi^2}{11N\ln(qt)}$$

Corresponding α_s for unstable q and t is too big to justify perturbation

Still, suggestive results...

Extensions

So far, there is no investigation of inhomogeneous Polyakov loop with dynamical fermions.

So far, there is no investigation of the interplay between the inhomogeneous chiral condensate and the inhomogeneous Polyakov loop.

Chiral-density wave state

Polyakov-loop wave sate?

Concluding Remarks

All the chiral-based models are inconsistent with the thermodynamics from the Statistical Model. One possible and reasonable prescription is proposed.

Deconfinement and chiral transitions stay close to each other and to the chemical freeze-out curve.

Inhomogeneity must be paid more attention to. Crystallography like CSC may be possible. Suggestion about the inhomogeneous Polyakov loop.