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What is a thought process? categorical sheaf-theoretic approach

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Abstract. Categorical formulations for thought processes and communications are provided. In order to treat the changing state of an individual over some time period, we introduce a state controlling "variable" in terms of objects (called generalized time) in a site (i.e., a category with a Grothendieck topology). We also capture communications (information flows) as observation morphisms between individuals. The interplay among the change of the state of an individual, the communication between individuals, and thought processes of those individuals is unified as the commutativities of the fundamental diagram in section 3. Finally, we show that the thought process (thinking) coincides with "thinking nothing" (process) when the thought process morphism coincides with the induced morphism from the identity.

Keywords: Topos, category, sheaf, thought process

1. Introduction

In the past, human attitudes have long been discussed in relation to personalities. An integral part of an attitude seems to be acquired through the influences during a person's childhood, e.g., family influences. Conditions such as: whether they have brothers or sisters, whether they are youngest or eldest, and how they are raised affect how children develop personalities. Sometimes an extroverted person may become more introverted after being frustrated or defeated in life. In order to objectively study such highly complex properties as personalties, we need to compare something inherent in human beings not subject to control by acquired environmental factors.

For that purpose, we focus on "thinking patterns" as basic structures of personalities which, with some minor exceptions, can be identified even in adulthood.

We classify subjects basically into two groups: Type S who can stop thinking and Type N who cannot. For rationale of this classification, consult (Nishimura, 2008) and (Nishimura, 2012).

Type S subjects (can) suspend using the mind to stop thinking when it is not necessary. For example, while on a train, looking at scenery through a window, they can switch off their minds. Meanwhile, Type N subjects continue thinking without having any explicit goals in mind.

Type S subjects can rest the mind if they do not need to think. Thus they are not pressured by anticipations or depressive thoughts. Type S people can be excellent managers of mental stresses, and if necessary, they can concentrate on a goal. As they do not create unnecessary thoughts, they can be direct and straightforward. Being so direct in many ways, Type S individuals do not have much anxiety before taking actions. From a negative perspective, this group of people could be considered to be more selfish or self-centered, inconsiderate, or even cold-hearted, but in any case very decisive.

2. Information and Understanding

The notion of a sheaf has recently been applied to theoretical physics, especially to quantum gravity by (Butterfield, 2001; Mallios, 2004; Kato, 2004; Kato, 2005; Kato, 2006), and (Kato, 2010; Kato, 2014). For general activity reports in theoretical physics, see (Penrose, 2005; Green, 1999) and (Smolin, 2001).

In terms of categorical concepts, e.g., functors, natural transformations, direct and inverse limits, brain activities like understanding and thinking have been formulated. See (Kato, 2005; Kato, 2006), (Kato, 2010; Penrose, 2005; Kato, 2002a; Kato, 2004; Kato, 2002b) for consciousness study in terms of category theory.

Let us recall several definitions from Category Theory. For details, see (Gelfand, 1996; Kato, 2006; Kashiwara, 2006; Grauent, 1984; Kato, 1999) .

A category C consists of objects and morphisms. For example, the category of sets has a set as an object, a morphisms in the category of sets is a (set theoretic) mapping.

In the category of topological spaces, an object is a topological space and a morphism in this category is a continuous map between topological spaces.

Let C and C' be categories. A functor F from C to C' is an assignment: (1) For an object A in C, FA is an object in C'. (2) For a morphism f from object $\boldsymbol{A}$ to object $\boldsymbol{B}$ in $\boldsymbol{C}$, i.e.,

$$A \xrightarrow{f} B$$

we get a morphism of objects in C' as

$$FA \xrightarrow{Ff} FB$$

satisfying $F(gof) = FgoFf$, where

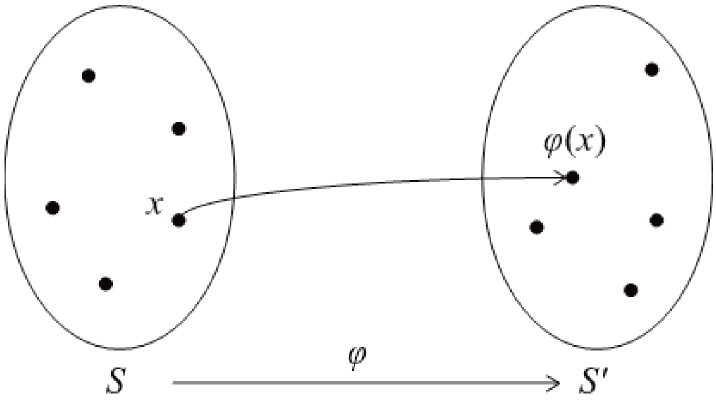
$$A \xrightarrow{f} B \quad \text{and} \quad B \xrightarrow{g} C.$$

Such a functor F is to be a covariant functor from C to C'. If a functor G from C to C' takes $f : A \to B$ in C into category C' as

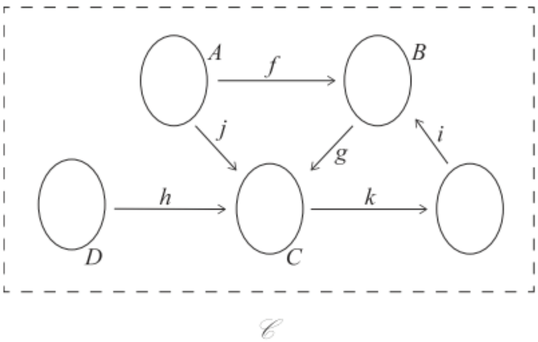
$$GA \xleftarrow{Gf} GB$$

i.e., reversing the direction, then G is said to be a contravariant functor.

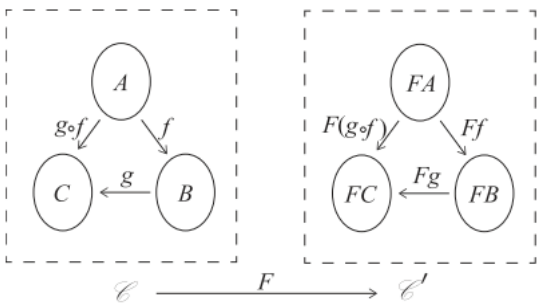
Schematic descriptions of sets and mappings, categories and morphisms and functors are as follow: Let S and S' be sets, and let $\varphi : S \to S'$ be a mapping. Namely, for an element x in S, there corresponds an element $\varphi(x)$ in S':



On the other hand, let $\mathscr{C}$ and $\mathscr{C}'$ be categories as above, let A, B, $\cdots$ be objects of C , and let f, g, h, be morphisms among objects, then we have:



For a covariant functor F from category $\mathscr{C}$ to category $\mathscr{C}'$, we may express as:



Since over different times the state of an entity changes, we need to keep track of various states by introducing a "variable," called a generalized time period. That is, in order to describe a state of an entity, we have to have not only a presheaf *P* associated to the entity, but also a "varying" object. Namely, for a presheaf *P*, representing an entity and with an object *V* of *S*, a state *P(V)* of *P* over *V* is determined.

By definition, a presheaf is a contravariant functor from a category *S* to another category, e.g., the category ((sets)) of sets. We take such a domain category as a site *S*, i.e., a category with a Grothendieck topology. See (Gelfand, 1996; Kato, 2006; Kashiwara 2006) for the notion of a site. Let $\hat{S}$ be the category of presheaves (contravariant functors) from the site *S* to the category ((sets)) of sets. For objects *m* and P in $\hat{S}$, we define an observation morphisms (or a measurement morphisms) from *m* to *P* over an object *V* in the site *S* as a set theoretic map (i.e., a morphisms of ((sets)))

$$s_V : m(V) \longrightarrow P(V), \tag{1_m}$$

where *m* need not be the presheaf associated with a human entity; i.e., *m* can be a presheaf associated with a particle.

When *m* happens to be an individual *Q* (a human being), then such an observation morphism should be interpreted as an information flow (i.e., communication) from individual *Q* to individual *P* over *V* .

Such an object *V* in S is called a generalized time (period). Then the image Im S_V of S_V is a subset of *P(V)* which is the information (the measurement) which the observer ***P*** receives from observed *m* over the generalized time period *V* .

In (l_m), let *m* = *P*. Namely, we get a self-observation map

$$p_V : P(V) \longrightarrow P(V). \tag{1_P}$$

See (Kato, 2002a; Kato, 2002b; Kato, 2004) for consciousness related topics. The elements in the set *P(V)* are regarded as"thoughts" over a generalized time *V*, and a map (endomorphism) from *P(V)* to *P(V)* is regarded as "thinking," i.e., a thought process over *V*.

Next, consider a covering in the site *S*

$$\{V \longleftarrow V_i\}_{i\in I}\,, \tag{2}$$

which would be $V = \underset{i\in I}{U} V_i$ if category *S* were a topological space. Next, we will introduce a special presheaf. *A* presheaf *P* is said to be a sheaf if the following glueing condition holds. For $s_i \in P(V_i)$ and $s_j \in P(V_j)$, $i, j \in I$, satisfying

$$s_{iV_i\times V_j} = s_j \left|_{V_i\times V_j}\right. , \tag{3}$$

there exists a unique $s \in P(V)$ to satisfy

$$s\left|_{V_i}\right. = s_i \qquad \text{for all } i \in I. \tag{4}$$

Note that a presheaf associated with a (functioning) brain is a sheaf. See also our forthcoming paper (Kato, to appear). This is because a (functioning) brain has an ability to form a global object from a given local data by glueing local information. See (Kato, 2005; Kato, 2006; Kato, 2010; Penrose, 2005) .

We can also formulate the notion of understanding (recognizing) in terms of categorical concepts as follows. For $s \in P(V)$, if there exist *W* in *S* and a morphism $V \xrightarrow{\varphi} W$ to satisfy

$$s = P(\varphi)(s') \tag{5}$$

For $s' \in P(W)$, then $s' \in P(W)$, is said to be an understanding of $s \in P(V)$. (Note that the map *P*(*φ*) would correspond to the the restriction map if *S* were a topological space.) Or $s \in P(V)$ is recognized as $s' \in P(W)$.

As examples of understanding (or recognizing), we give the following Examples 1 and 2.

Example 1: When one is allowed to see an only 10cm × 10cm area of an entire large painting, by glueing these local data, i.e., Eq.(3), we obtain global information $s \in P(V)$. Then each local datum

$s_i \in P(V_i)$ is the restriction of $s \in P(V)$ to V_i, i.e., Eq.(4). When such a large painting is well known, partial glueing as in Eq.(3) is enough due to memory to obtain the global $s \in P(V)$.
Example 2: When one hears only a few notes of a well known music piece, one may not be able to recognize (understand) the music piece (the global information). However, when enough notes are heard, one can recognize the music piece. Then the previously shown few notes are the image of the understanding map, i.e., Eq.(5), $s = P(\varphi)(s')$.

3. Fundamental Diagram

In this section, we are going to treat entities as human beings.

Hence an observation (measurement) morphism in Eq. (1_m) may be interpreted as an information flow by communication between two individuals P and Q.

In Eq. (1_m) in the previous section, we let $m = Q$. That is, Eq. (1_m) becomes

$$t_V : Q(V) \longrightarrow P(V). \tag{1'}$$

For a morphism in S

$$\psi : V \longrightarrow V', \tag{6}$$

we have the functionally induced morphism

$$P(\psi) : P(V') \longrightarrow P(V). \tag{7}$$

The morphism in Eq.(7) has a canonical nature. Namely, the morphism $\psi : V \longrightarrow V'$ induces the functorial (natural) change of the states of P from the period V to the period V'. Mathematically speaking, the contravariant functor P takes $\psi : V \longrightarrow V'$ in S into $P(\psi) : P(V') \longrightarrow P(V)$ in ((sets)).

When individual P communicates with individual Q over V, we have:

$$Q(V) \xrightarrow{t_V} P(V)$$

i.e., an information flow from $Q(V)$ to $P(V)$. And the image of this information flow $\mathrm{Im}t_V$ is the information that $P(V)$ received from $Q(V)$.
Similarly, for Q in $\hat{S}$ we have

$$Q(\psi) : Q(V') \longrightarrow Q(V). \tag{8}$$

Let the self observation map for Q be

$$q_V : Q(V) \longrightarrow Q(V). \tag{1_Q}$$

An interpretation of the composition $p_V \circ t_V$ of

$$Q(V) \xrightarrow{t_V} P(V)$$

with the thought process (thinking)

$$P(V) \xrightarrow{p_V} P(V)$$

is the thought process influenced by the communication t_V.
Hence, the commutativity

$$t_V \circ q_V = p_V \circ t_V$$

indicates that individual $P(V)$ listens as $Q(V)$ speaks; the communication t_v influences the thinking p_V of $P(V)$, and the thought process q_V of $Q(V)$ influences the communication t_V. That is, the communication between $P(V)$ and $Q(V)$, (in this case, $P(V)$ is the listener) is well engaged, i.e., a good communication between P and Q over the state-determining variable V.

Combining (1'), (7), (8) and (1_P), (1_Q), we obtain the following fundamental diagram.

$$\begin{tikzcd}
& Q(V) \arrow[dl, "t_V"'] \arrow[dd, "q_V"] & & Q(V') \arrow[ll, "Q(\psi)"'] \arrow[dl, "t_{V'}"] \arrow[dd, "q_{V'}"] \\
P(V) \arrow[dd, "p_V"] & & P(V') \arrow[ll, "P(\psi)"] \arrow[dd, "p_{V'}"] & \\
& Q(V) \arrow[dl, "t_V"'] & & Q(V') \arrow[ll, "Q(\psi)"'] \arrow[dl, "t_{V'}"] \\
P(V) & & P(V') \arrow[ll, "P(\psi)"] &
\end{tikzcd} \tag{9}$$

The commutativity $P(\psi) \circ p_{V'} = p_V \circ P(\psi)$ of the vertical front square diagram indicates the coherency of thoughts of individual *P*. This is because p_V (and $p_{V'}$) is a thought map, i.e., thinking as in (1*p*), and the canonical map $P(\psi)$ indicates the change of the state from *P(V)* to the latter state *P(V')* (and similarly for *Q*). The commutativity $t_V \circ q_V = p_V \circ t_V$ of the vertical side square diagram corresponding to *V* (and *V'*) indicates a (good) communication between individuals *P* and *Q* associated with their thought maps p_V and q_V.

The commutativity $P(\psi) \circ t_{V'} = t_V \circ Q(\psi)$ of the horizontal top square indicates the consistency of the communication morphisms t_V and $t_{V'}$ corresponding to two states over *V* and *V'* between individuals *P* and *Q*. Finally, all the commutativities (among the six paths) from *Q(V')* and to *P(V)* indicate the (most) general functional communication between individuals P and *Q* over two different states associated with *V* and *V'*.

4. Thinking Nothing

As a categorical sheaf theoretic formation of thinking nothing, which Type S subjects are capable of, will be given as a corollary of the fundamental communicative diagram in section 3.

Recall the definition of an identity morphism of a category. For an object *A* in a category, there exists a morphism I_A, called an identity morphism to satisfy

$$I_A \circ f = f \quad \text{and} \quad g \circ I_A = g, \tag{10}$$

where *f* and *g* are arbitrary morphisms $f : B \longrightarrow A$ and $g : A \longrightarrow C$.
In terms of a presheaf entity *P* in $\hat{S}$, such an identity morphism characterized by (10) becomes

$$I_{P(V)} : P(V) \longrightarrow P(V) \tag{11}$$

over a generalized time *V* in *S*. The process of thinking nothing corresponds to this identity morphism $I_{P(V)}$. In particular, when the entity *P* thinks nothing over two corresponding states *P(V)* and *P(V')* to generalized time periods *V* and *V'* respectively, from the fundamental diagram for $p_v = I_{P(V)}$ and $p_{v'} = I_{P(V')}$ we get only the canonical change of states from *V* to

$$P(\psi) \circ I_{P(V')} = I_{P(V)} \circ P(\psi)$$

in the front vertical commutativity. That is, the thinking process $I_{P(V)}$ indicates simply self-awareness with thinking nothing. Namely, when the thinking process morphism P_V coincides with the canonically induced morphism $P(1_V) = I_{P(V)}$ from the identity $V \xrightarrow{1_V} V$, thinking nothing occurs.

5. Conclusion

The fundamental process of thinking can be formulated with the methods of category theory and sheaf theory. A morphism is interpreted as communication. When such a morphism is an endomorphism, the induced endomorphism is a thought process of the entity. We have obtained the fundamental commutative diagram (9) indicating the most general communication between two entities over two different states. The formulation given in this paper provides a mathematical aspect of a thought process.

Acknowledgments

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