Constraining Lorentz Violation of Gravitational Waves with Lensing

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1. Introduction
2. Method
3. Parameter Estimation
4. GW In CUHK

Dispersion of Gravitational Waves

With $h = c = G = 1$

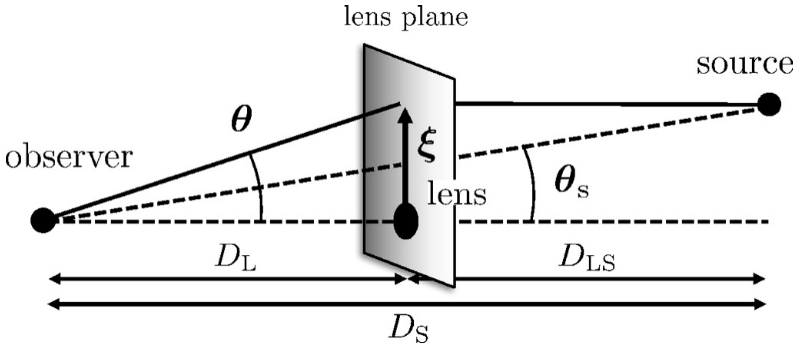
- Without Lorentz violation:

$$\omega = k \tag{1}$$

- Isotropic dispersion [1]:

$$\begin{aligned} &E^2 = p^2 + m_g^2 + Ap^\alpha \\ &\Rightarrow \omega^2 = k^2 + m_g^2 + Ak^\alpha \\ &\Rightarrow v_g(f) \approx 1 - \frac{1}{2}m^2 f^{-2} - \frac{1}{2}Af^{\alpha-2} \end{aligned} \tag{2}$$

Geometry of the Lensing[1]



[1] Figure 1. from R. Takahashi et al. "*Arrival time differences between gravitational waves and electromagnetic signals due to gravitational lensing*". ApJ 835 (Jan. 2017), arXiv:1606.00458

Lensing (Diffraction) of Gravitational Waves

- Lensed waveform

$$\tilde{h}_L(f) = F(f; \text{lensing parameters})\tilde{h}(f) \quad (3)$$

- Amplification function [2]:

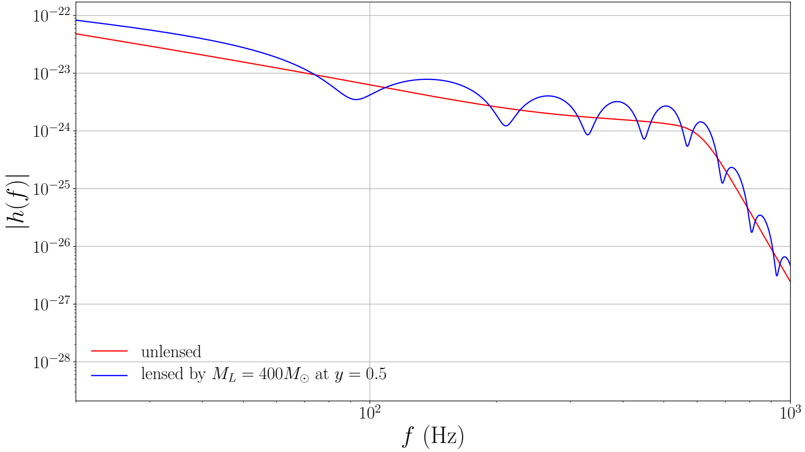
$$F(f; \vec{\theta}_s) \propto \frac{(1+z_L)f}{i} \int d^2\theta \exp(2\pi i f t_d(\vec{\theta}, \vec{\theta}_s)) \quad (4)$$

where t_d is the arrival time delay between lensed and unlensed rays.

- Time delay:

$$t_d(\vec{\theta}, \vec{\theta}_s) = \frac{(1+z_L)}{c}\left[\frac{D_L D_S}{2D_{LS}}|\vec{\theta}_s - \vec{\theta}|^2 - \psi(\vec{\theta}_s)\right] \quad (5)$$

Effect due to lensing



The Central Question

How would the lensing pattern look like if gravitational waves are with dispersions?

Arrival Time Delay

- With dispersion

$$t_d \rightarrow \frac{c}{v_g(f)} t_d \tag{6}$$

  From now on $\beta(f) = c/v_g(f)$
- Dispersion changes the phase differences along the rays. $\Rightarrow$ lensing pattern is changed.

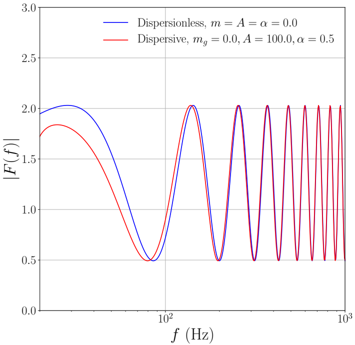
Amplification Functions

- For point mass lens

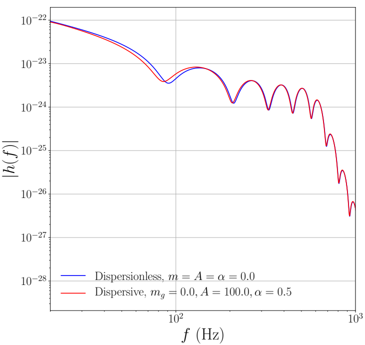
$$F(f;y) = \exp\left(\frac{\pi}{4}w\beta\right)\left(\frac{w}{2}\beta\right)^{i\frac{w}{2}\beta} \times \Gamma\left(1 - i\frac{w}{2}\beta\right){}_1F_1\left(i\frac{w}{2}\beta, 1; i\frac{w}{2}\beta y^2\right) \quad (7)$$

where $w = 8\pi M_L(1 + z_L)f$, (7) can be reduced to known case [3] when there is no dispersion.

Image Pattern



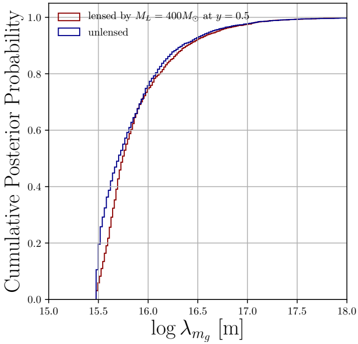
(a) $|F(f)|$



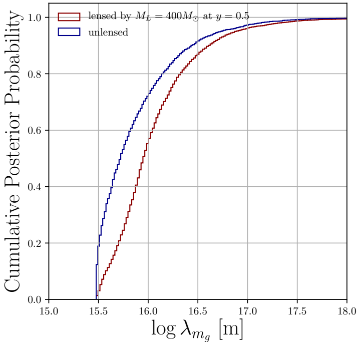
(b) $|h(f)|$

Parameter Estimations

$$\lambda_g = \hbar / m_g c$$



(c) $d_L = 200$ Mpc



(d) $d_L = 100$ Mpc

Advantages

- Relies solely on the lensed signals.
- SNR of signal is boost.
- Improved constraint on m_g

Summary

- Lensing pattern of gravitational waves with dispersions $\Rightarrow$ probe dispersion using lensing.
- Better constrains on m_g
- Systematic run is on going. Will have more complete results soon.
- Incorporating the SIS.

Our Awesome Group!

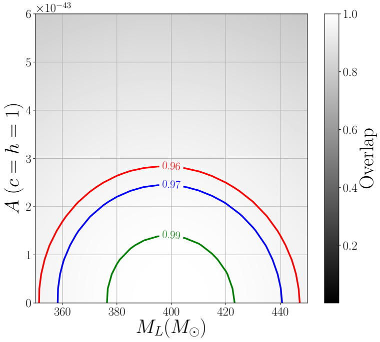
References

1 Saeed Mirshekari et al. "*Constraining Lorentz-violating, modified dispersion relations with gravitational waves*". Phys. Rev. D 85, 024041. (Jan. 2012)

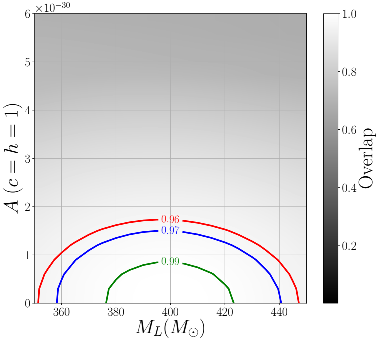
2 Schneider, et al (1992). "*Gravitational Lenses*".Springer's Publications. ISBN: 0941-7834. DOI: 10.1007/978-3-662-03758-4

3 R. Takahashi et al. "*Wave Effects in the Gravitational Lensing of Gravitational Waves from Chirping Binaries*". ApJ 595 (Oct. 2003), pp. 1039-1051. eprint: astro-ph/0305055.

Overlap Plots



(e) $\alpha = 0$



(f) $\alpha = 0.5$

Unlensed Dispersive GWs [1]

- Propagation time delay when $A = 0$

$$\Delta t = (1+z)\Big(\Delta t_e + \frac{m_g^2}{2} D_0 \big(\frac{1}{f_e^2} - \frac{1}{f'^2_e}\big)\Big) \tag{8}$$

- This leads to a phase difference,

$$\delta\Psi(f) = -\frac{\pi D_0 m_g^2}{(1+z)f} \tag{9}$$

such that

$$h_{\mathrm{disp}}(f) = h(f) e^{i\delta\Psi(f)} \tag{10}$$