YITP

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Darkness without dark matter and energy – generalized unimodular gravity

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Introduction: darkness in cosmology

DE and DM – rich playground for modifications of GR

DE and DM from pure gravity sector
– not local DoF but global (mechanical) DoF

Reduction of local gauge (diffeomorphism) invariance
(Horava-Lifshitz gravity – UV consistent renormalizable QG?)

Unimodular gravity (UMG): # local DoF = 2 + one global DoF (¤),
$p = w\varepsilon$, $w = -1 \neq w(z)$

Lorentz violation + UMG = Generalized UMG: $w = w(z) \neq const$

Plan

Intro: darkness in cosmology

Unimodular gravity

Generalized unimodular gravity (GUMG) -- source of dark fluid

Reduced diffeomorphism invariance and fluid dynamics in a comoving frame

Canonical formalism of GUMG

Discussion: GUMG, cosmological initial conditions and curvature density parameter Ω_K

Unimodular gravity

$$\det g_{\mu\nu} = -1$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu}$$
$$\partial_\mu \Lambda = 0$$

¤ = *const* -- spacetime constant of integration of EoM

$$p = -\varepsilon, \qquad \varepsilon = M_P^2 \Lambda$$

$$M_P^2 = (8\pi G)^{-1}$$

Local invariance – volume preserving diffeomorphisms:

$$x^\mu \to x^\mu + \xi^\mu(x), \qquad \partial_\mu \xi^\mu = 0$$

Violation of Lorentz invariance – Horava-Lifshitz gravity theory:

Search for UV consistent quantum gravity – unitary and perturbatively renormalizable

ADM *(3+1)*-decomposition of 4-metric:

$$g_{\mu\nu}dx^\mu dx^\nu = (N_i\,N^i - N^2)\,dt^2 + 2N_i\,dt\,dx^i + \gamma_{ij}dx^i\,dx^j$$

$$N = \frac{1}{\sqrt{-g^{00}}}, \qquad N_i = g_{0i}, \qquad N^i = \gamma^{ij}N_j$$

Lapse and shift functions

Generalized unimodular gravity (GUMG) -- source of dark fluid

$$\det g_{\mu\nu} = -1$$

⇓

$$N = N(\gamma), \qquad \gamma \equiv \det \gamma_{ij}$$

O(3)-invariant

$$S = \int d^4x \left\{ \frac{M_P^2}{2} g^{1/2} R(g) - \lambda \Big((-g^{00})^{-1/2} - N(\gamma) \Big) \right\}$$

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{1}{M_P^2} T^{\mu\nu},$$

$$T^{\mu\nu} \equiv -\frac{2}{g^{1/2}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x \, \lambda \Big((-g^{00})^{-1/2} - N(\gamma) \Big)$$

$$= \varepsilon \, u^\mu u^\nu + p \left(g^{\mu\nu} + u^\mu u^\nu \right)$$

$$u^\mu = -g^{0\mu} N$$

Perfect fluid

$$\varepsilon = \frac{\lambda}{2\sqrt{\gamma}}, \qquad p = \frac{\lambda}{\sqrt{\gamma}} \left(\frac{\gamma}{N} \frac{dN}{d\gamma} \right)$$

$$p = w\varepsilon, \qquad w = 2 \frac{\gamma}{N} \frac{dN}{d\gamma} = 2 \frac{d \ln N}{d \ln \gamma}$$

Equation of state

Reduced diffeomorphism invariance

UMG: *w = -1,*

$$\partial_\mu \boldsymbol{\xi}^\mu = 0$$

$$\delta_\xi g^{\mu\nu} = -\nabla^\mu \xi^\nu - \nabla^\nu \xi^\mu, \qquad \xi^\mu = \begin{bmatrix} \xi^0 \\ \xi^i \end{bmatrix}$$

$$\delta_{\boldsymbol{\xi}}\Big(N - N(\gamma)\Big)\Big|_{N=N(\gamma)} = N\Big[\partial_t \boldsymbol{\xi}^0 - (1+w)\, N^i \partial_i \boldsymbol{\xi}^0 - w\,\partial_i \boldsymbol{\xi}^i\Big] = 0$$

$$\boldsymbol{\xi}^\mu_\perp = \begin{bmatrix} 0 \\ \\ \xi^i_\perp \end{bmatrix}$$

two purely spatial diffeos with a transverse 3-vector parameter $\partial_i \xi^i_\perp(t, \mathbf{x}) = 0$

$$\boldsymbol{\xi}^\mu_{||} = \begin{bmatrix} \xi^0 \\ \\ \xi^i_{||}[\,\xi^0] \end{bmatrix}$$

diffeo with a timelike parameter $\xi^0(t, \mathbf{x})$

$$\xi^i_{||}[\,\xi^0] = \partial^i \frac{1}{\Delta} \frac{\partial_t \xi^0 - (1+w) N^l \partial_l \xi^0}{w}$$

spatially nonlocal dependence of »$^{i}_{||}$ on »^{0}

Dynamics of dark fluid in a comoving frame

Three diffeos $\Rightarrow \quad N^i \sim u^i = 0$

$$\nabla^\mu T_{\mu\nu} = 0,\ w \neq 0 \quad \Longrightarrow \quad \begin{cases} \partial_i (N w \varepsilon \sqrt{\gamma}) = 0,\ N w \varepsilon \sqrt{\gamma} = S(\mathbf{x}) \\ \partial_t (N \varepsilon \sqrt{\gamma}) = 0,\ N \varepsilon \sqrt{\gamma} = T(t) \end{cases}$$

$$\Downarrow$$

$$w = \frac{T(t)}{S(\mathbf{x})}$$

$$N = \mathrm{const}\, \gamma^{w/2},\ w = \mathrm{const} \neq 0$$

$$\Downarrow$$

$$T(t), S(\mathbf{x}) = \mathrm{const}, \qquad \varepsilon = \frac{\mathrm{const}}{\gamma^{(w+1)/2}}$$

Global mode

Special case of dark dust – "projectable" GR

$$N = \text{const}, \;\; w = 0$$

$$\Downarrow$$

initial conditions function ***S(x)*** survives:

$$\varepsilon = \frac{S(\mathbf{x})}{N\sqrt{\gamma}} \equiv \frac{\tilde{S}(\mathbf{x})}{\sqrt{\gamma}}$$

DM?

Analogue of mimetic model.

Non-clustering DE = global (mechanical) mode
Initial conditions for it and its quantization?

$$\Downarrow$$

Need a physical sector of GUMG – canonical formalism

Lagrangian quantization? Problem of open algebra of gauge generators and BV formalism (Faddeev-Popov technique does not apply beyond one loop):

$$\delta_{\boldsymbol{\xi}} g^a = R^a_\mu \xi^\mu, \qquad R^b_\mu \frac{\delta R^a_\nu}{\delta g^b} - R^b_\nu \frac{\delta R^a_\mu}{\delta g^b} = C^\lambda_{\mu\nu} R^a_\lambda,$$

$$C^\lambda_{\mu\nu} = C^\lambda_{\mu\nu}[\,g\,] \neq \text{const}$$

field-dependent and nonlocal

Canonical formalism of GUMG

$$S[\gamma_{ij},\pi^{ij};N^i] = \int d^4x\left(\pi^{ij}\dot{\gamma}_{ij} - N(\gamma)H - N^iH_i\right)$$

Hamiltonian and momentum constraint functions

What are the **canonical** generators of diffeos?

$$\xi^i_{||} = \partial^i\frac{1}{\Delta}\frac{\partial_t\xi^0}{w} + \ldots \sim \partial_t\xi^0 + \ldots$$

$$\left.\begin{array}{l}\delta_{||}\gamma_{ij} \sim \partial_t\xi^0 + \ldots \\ \delta_{||}N^i \sim \partial_t^2\xi^0 + \ldots\end{array}\right\}$$

Not canonical!

$\Downarrow$

Include N^i and p_i into phase space

$$S[\gamma_{ij},\pi^{ij};N^i,p_i;v^i] = \int d^4x\left(\pi^{ij}\dot{\gamma}_{ij} + p_i\dot{N}^i - N(\gamma)H - N^iH_i - v^ip_i\right)$$

primary constraints with Lagrange multiplies

Hamiltonian $$\mathcal{H} = \int d^3x \left(N(\gamma) H + N^i H_i + v^i p_i \right)$$

$$\frac{\delta S}{\delta v^i} = -p_i = 0$$
$$\dot{p}_i = \{p_i, \mathcal{H}\} = -H_i = 0$$
$$\dot{H}_i = \{H_i, \mathcal{H}\} = T_i \equiv \partial_i (N w H) = 0$$
$$\dot{T}_i = S_i \equiv \partial_i \Big(\Omega \partial_k N^k + X \Big) = 0$$

all constraints

$$\dot{S}_i = \partial_i \Big(\Omega \partial_k v^k + \ldots \Big) = 0$$

equations on Lagrange multipliers

$$\Omega \equiv 1 + w + 2 \frac{d \ln w}{d \ln \gamma}$$

$N^k_\perp, v^k_\perp$ ***4* transverse components are not constrained – gauge fixing**

GUMG: $$T_i \equiv \partial_i (NwH) = 0 \;\Rightarrow\; NwH = E(t)$$

global (mechanical) mode

Compare:

GR: $$H \equiv \frac{\pi_{ij}^2 - \frac{1}{2}\pi^2}{\sqrt{\gamma}} - \sqrt{\gamma}\,{}^3R = 0$$

UMG: $$N = \frac{1}{\sqrt{\gamma}},\; w = -1 \;\Rightarrow\; \frac{H}{\sqrt{\gamma}} = \Lambda = \mathrm{const}$$

Algebra of constraints and # of DoF

eight constraints

$$p_i = 0,\ H_i = 0,\ T_i = 0,\ S_i = 0$$

four 1st class constraint functions

$$p_i^{\perp},\ H_i^{\perp}$$

four 2nd class constraint functions

$$p_i^{||},\ H_i^{||},\ T_i^{||},\ S_i^{||}$$

2£ # of DoF =

# of phase space variables
- 2£ # of 1st class constraints
- # of 2nd class constraints = 18 – 8 - 4 = 2£ 3

Discussion

- Coupled field theory and mechanical (finite-dimensional) system – the problem of disentangling the global mode in the Hamiltonian and symplectic form

- Breakdown of the 3D diffeo invariance -- local invariance is too ugly. Recovery of 3D invariance (like in Horava gravity)

$$N - N(\gamma) = 0 \Rightarrow \Delta N - \kappa\, {}^3R = 0$$

dimensionless parameter
-- to avoid extra scale

Anisotropic stress tensor:

$$\varepsilon = -\Delta\lambda$$
$$T_{ij} = -\frac{2}{N}\Big(\kappa G_{ij} - \kappa(\nabla_i\nabla_j - \gamma_{ij}\Delta) + \nabla_{(i}N\nabla_{j)} - \frac{1}{2}\gamma_{ij}\nabla^n N\nabla_n\Big)\lambda$$

Perfect fluid on homogeneous FRW: $p = \frac{\kappa\lambda}{3N}, \quad \varepsilon = 0$

violation of energy conditions!

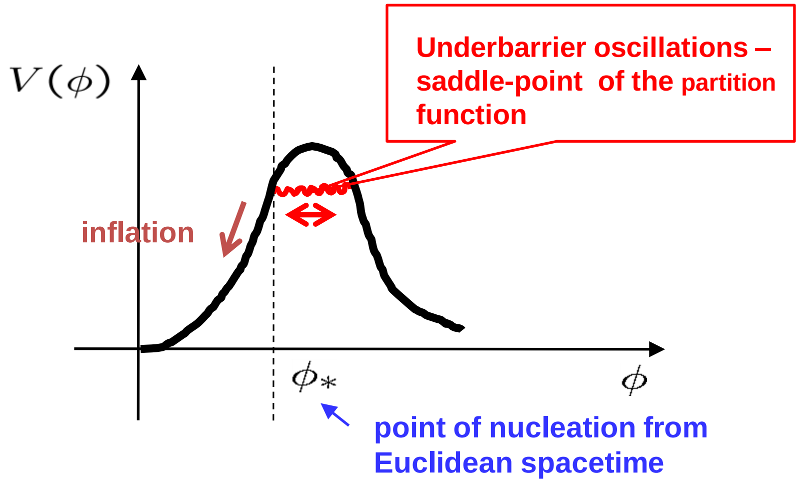
- **Motivation – model of microcanonical initial conditions for CFT driven cosmology**

a) Density matrix generalization of the Hartle-Hawking state

b) Initial conditions for inflation at the top of inflaton potential

c) Quasi-thermal primordial power spectrum -- revival of a mild "big bang" preceding inflation

d) New type of hill-top inflation as a source of Higgs or R^2-inflation model



e) Origin of the Universe is a semiclassical subplanckian phenomenon – intrinsic mechanism of suppression for high energy scales due to hidden sector of conformal fields

BUT ONLY FOR ***K=+1*** !

$$\Omega_K = -\frac{K}{H^2 a^2} < 0, \qquad \Omega_K + \Omega_\Lambda + \Omega_m = 1$$

Planck, lensing and BAO data:

$$\Omega_K = 0.000 \pm 0.005$$

Observational tendency: $\Omega_K \to 0$

Imitation of *K≠0* by GUMG in flat space Friedmann equation

$$H^2 = \frac{\Omega_m + \Omega_\Lambda + \Omega_K}{3M_P^2}$$

$$w = -\frac{1}{3}, \quad \varepsilon_K = -\frac{3M_P^2 K}{a^2} \sim \frac{1}{\gamma^{(1+w)/2}}, \quad \Omega_K = \frac{\varepsilon_K}{3M_P^2 H^2}$$

THANK YOU!