

*EFTCAMB: exploring Large Scale Structure
observables with viable dark energy and modified
gravity models*

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Test gravity on cosmological scales

- Observations: extra component \rightarrow Dark Energy
- Pletora of Dark Energy & Modified Gravity models
- focus on models with one extra scalar DoF

Model independent parametrizations to test gravity:

- Growth functions: μ and γ ,

[Silvestri *et al.* PRD 87, 104015 (2013)]

- Parametrized Post Friedmann framework,

[Baker *et al.*, PRD 87, 024015 (2013)]

- Effective Field Theory of Cosmic Acceleration,

[Gubitosi *et al.* JCAP 1302 (2013) 032
Bloomfield *et al.* JCAP 1308 (2013) 010]

- Horndeski and beyond parametrizations ,

[Bellini & Sawicki, JCAP 1407 (2014) 050
Gleyzes *et al.* JCAP 1502 (2015) 018
NF *et al.* JCAP 1607 (2016) no.07, 018]

$\{\mu, \gamma\}$, Horndeski and bH \Rightarrow EFT

EFT for dark energy and modified gravity: the action

- Operators are time-dependent spatial diffeomorphisms invariants;
- Unitary gauge: the extra scalar d.o.f. does not appear directly;

The action:

$$\mathcal{S}_{EFT} = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} (1 + \Omega(t)) R + \Lambda(t) - c(t) \delta g^{00} \right. \\ \left. + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2(t)}{2} (\delta K)^2 \right. \\ \left. - \frac{\bar{M}_3^2(t)}{2} \delta K_\nu^\mu \delta K_\mu^\nu + \frac{\hat{M}^2(t)}{2} \delta g^{00} \delta \mathcal{R} + m_2^2(t) h^{\mu\nu} \partial_\mu g^{00} \partial_\nu g^{00} \right\} + S_m[\chi_i, g_{\mu\nu}],$$

where e.g. $\delta A = A - A^{(0)}$, $A^{(0)}$ background value in FLRW

- $M_2^2 = -\bar{M}_3^2 = 2\hat{M}^2$ and $m_2^2 = 0$: Horndeski (and all the models belonging to them);
- $M_2^2 + \bar{M}_3^2 = 0$ and $m_2^2 = 0$: Beyond Horndeski class of models;
- $m_2^2 \neq 0$: Lorentz violating theories (e.g. low-energy Hořava gravity).

Extensions

- Additional linear operators

$$\begin{aligned} & \frac{\bar{m}_5(t)}{2} \delta \mathcal{R} \delta K, \quad \lambda_1(t) (\delta \mathcal{R})^2, \quad \lambda_2(t) \delta \mathcal{R}_\nu^\mu \delta \mathcal{R}_\mu^\nu, \\ & \lambda_3(t) \delta \mathcal{R} h^{\mu\nu} \nabla_\mu \partial_\nu g^{00}, \quad \lambda_4(t) h^{\mu\nu} \partial_\mu g^{00} \nabla^2 \partial_\nu g^{00}, \\ & \lambda_5(t) h^{\mu\nu} \nabla_\mu \mathcal{R} \nabla_\nu \mathcal{R}, \quad \lambda_6(t) h^{\mu\nu} \nabla_\mu \mathcal{R}_{ij} \nabla_\nu \mathcal{R}^{ij}, \\ & \lambda_7(t) h^{\mu\nu} \partial_\mu g^{00} \nabla^4 \partial_\nu g^{00}, \quad \lambda_8(t) h^{\mu\nu} \nabla^2 \mathcal{R} \nabla_\mu \partial_\nu g^{00} \end{aligned}$$

[J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, JCAP 1308, 025 (2013)
NF, G. Papadomanolakis and A. Silvestri, JCAP 1607 (2016) no.07, 018]

- beyond the linear order

$$\begin{aligned} & M_3^4(t) (\delta g^{00})^3, \quad M_1(t) (\delta K)^3, \quad M_1^3(t) (\delta g^{00})^2 \delta K, \\ & M_4^2(t) \delta g^{00} (\delta K)^2, \quad M_5^2(t) (\delta g^{00})^2 \delta \mathcal{R}, \quad M_6^2(t) \delta g^{00} \delta K_\nu^\mu \delta K_\mu^\nu, \\ & M_2(t) \delta K_\mu^\nu \delta K_\lambda^\mu \delta K_\nu^\lambda, \quad M_3(t) \delta K \delta K_\mu^\nu \delta K_\nu^\mu, \quad M_4(t) \delta g^{00} \delta \mathcal{R} \delta K, \\ & M_5(t) \delta g^{00} \delta K_\nu^\mu \delta \mathcal{R}_\mu^\nu, \quad m_3^2(t) h^{\mu\nu} (\partial_\mu g^{00} \partial_\nu g^{00}) \delta g^{00} \end{aligned}$$

[NF, G. Papadomanolakis, JCAP 1712 (2017) no.12, 014]

General Mapping

Let us introduce the ADM metric:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

a general Lagrangian can be written as follows:

$$L = L(N, \mathcal{R}, \mathcal{S}, K, \mathcal{Z}, \mathcal{U}, \mathcal{Z}_1, \mathcal{Z}_2, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5; t),$$

where

$$\mathcal{S} = K_{\mu\nu}K^{\mu\nu}, \quad \mathcal{Z} = \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}, \quad \mathcal{U} = \mathcal{R}_{\mu\nu}K^{\mu\nu},$$

$$\mathcal{Z}_1 = \nabla_i \mathcal{R} \nabla^i \mathcal{R}, \quad \mathcal{Z}_2 = \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk},$$

$$\alpha_1 = a^i a_i, \quad \alpha_2 = a^i \Delta a_i, \quad \alpha_3 = \mathcal{R} \nabla_i a^i, \quad \alpha_4 = a_i \Delta^2 a^i, \quad \alpha_5 = \Delta \mathcal{R} \nabla_i a^i,$$

[R. Kase and S. Tsujikawa, Int. J. Mod. Phys. D **23**, no. 13, 1443008 (2015)]

- Write the general action $\int d^4x \sqrt{-g} L$ in unitary gauge and expand it up to second order in perturbations;
- Write the EFT action in ADM notation;
- Compare the two actions.

General Mapping

$$\Omega(t) = \frac{2}{m_0^2} \mathcal{E} - 1, \quad c(t) = \frac{1}{2}(\dot{\mathcal{F}} - L_N) + (H\dot{\mathcal{E}} - \ddot{\mathcal{E}} - 2\mathcal{E}\dot{H}),$$

$$\Lambda(t) = \bar{L} + \dot{\mathcal{F}} + 3H\mathcal{F} - (6H^2\mathcal{E} + 2\ddot{\mathcal{E}} + 4H\dot{\mathcal{E}} + 4\dot{H}\mathcal{E}), \quad \bar{M}_2^2(t) = -\mathcal{A} - 2\mathcal{E},$$

$$M_2^4(t) = \frac{1}{2} \left(L_N + \frac{L_{NN}}{2} \right) - \frac{c}{2}, \quad \bar{M}_1^3(t) = -\mathcal{B} - 2\dot{\mathcal{E}}, \quad \bar{M}_3^2(t) = -2L_S + 2\mathcal{E},$$

$$m_2^2(t) = \frac{L_{\alpha_1}}{4}, \quad \bar{m}_5(t) = 2\mathcal{C}, \quad \hat{M}^2(t) = \mathcal{D}, \quad \lambda_1(t) = \frac{\mathcal{G}}{2},$$

$$\lambda_2(t) = L_{\mathcal{Z}}, \quad \lambda_3(t) = \frac{L_{\alpha_3}}{2}, \quad \lambda_4(t) = \frac{L_{\alpha_2}}{4}, \quad \lambda_5(t) = L_{\mathcal{Z}_1},$$

$$\lambda_6(t) = L_{\mathcal{Z}_2}, \quad \lambda_7(t) = \frac{L_{\alpha_4}}{4}, \quad \lambda_8(t) = \frac{L_{\alpha_5}}{2}.$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}$ are combinations of terms obtained by deriving the Lagrangian w.r.t. the main variables.

[NF, G. Papadomanolakis and A. Silvestri, JCAP 1607 (2016) no.07, 018]

Example: Minimally coupled quintessence

The action with the scalar field ϕ :

$$\mathcal{S}_\phi = \int d^4x \sqrt{-g} \left[\frac{m_0^2}{2} R - \frac{1}{2} \partial^\nu \phi \partial_\nu \phi - V(\phi) \right],$$

Apply unitary gauge and ADM formalism



$$\mathcal{S}_\phi = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} [\mathcal{R} + \mathcal{S} - K^2] + \frac{1}{N^2} \frac{\dot{\phi}_0^2(t)}{2} - V(\phi_0) \right\},$$

Apply the general mapping recipe



$$\Omega(t) = 0, \quad c(t) = \frac{\dot{\phi}_0^2}{2}, \quad \Lambda(t) = \frac{\dot{\phi}_0^2}{2} - V(\phi_0).$$

[NF, G. Papadomanolakis and A. Silvestri, JCAP 1607 (2016) no.07, 018]

Stability conditions

Let us consider the following second order action for more than one scalar fields

$$S^{(2)} = \frac{1}{(2\pi)^3} \int d^3k dt a^3 \left(\dot{\vec{\chi}}^t \mathbf{A} \dot{\vec{\chi}} - k^2 \vec{\chi}^t \mathbf{G} \vec{\chi} - \dot{\vec{\chi}}^t \mathbf{B} \vec{\chi} - \vec{\chi}^t \mathbf{M} \vec{\chi} \right),$$

where $\vec{\chi}^t = (\phi_1, \phi_2, \dots)$.

In order to avoid instabilities one has to demand:

- no-Ghost condition: positive kinetic term;
- no-Gradient condition: $c_{s,i}^2 > 0$,
- no-tachyonic instability: assure the Hamiltonian to be bounded from below, then, we demand $|\mu_i(t, 0)| \lesssim H^2$.

[A. De Felice, NF and G. Papadomanolakis, JCAP 1703 (2017) no.03, 027]

Stability conditions for the tensor modes

The EFT action for tensor modes can be written as

$$S_{EFT}^{T(2)} = \frac{1}{(2\pi)^3} \int d^3k dt a^3 \frac{A_T(t)}{8} \left[(\dot{h}_{ij}^T)^2 - \frac{c_T^2(t, k)}{a^2} k^2 (h_{ij}^T)^2 \right],$$

with

$$\begin{aligned} A_T(t) &= m_0^2(1 + \Omega) - \bar{M}_3^2, \\ c_T^2(t, k) &= \bar{c}_T^2(t) - 8 \frac{\lambda_2 \frac{k^2}{a^2} + \lambda_6 \frac{k^4}{a^4}}{m_0^2(1 + \Omega) - \bar{M}_3^2}, \\ \bar{c}_T^2(t) &= \frac{m_0^2(1 + \Omega)}{m_0^2(1 + \Omega) - \bar{M}_3^2}, \end{aligned}$$

Stability conditions

- no-Ghost instability: $A_T > 0$,
- No gradient instability: positive speed of propagation $c_T^2 > 0$.

[NF, G. Papadomanolakis and A. Silvestri, JCAP 1607 (2016) no.07, 018]

The parameters space

Matter fields:

- in general do not affect the no-ghost and speed conditions,
- only one exception: beyond Horndeski.

In matter the speeds of propagation of the three DoFs are:

$$\begin{aligned}c_{s,d}^2 &= 0, \\(3c_s^2 - 1)\rho_r &[\rho_d (c_s^2(F_3F_1^2 + 3F_2^2F_1) - 2a^2F_2^2\mathcal{G}_{11}) - 4\mathcal{B}_{12}^2F_2^2] \\&- 16c_s^2\mathcal{B}_{13}^2F_2^2\rho_d = 0\end{aligned}$$

for Horndeski: they completely decouple.

- they change the no-tachyonic conditions.

[(in vacuum) NF, G. Papadomanolakis and A. Silvestri, JCAP 1607 (2016) no.07, 018
(in matter) A. De Felice, NF, G. Papadomanolakis, JCAP 1703 (2017) no.03, 027]

EFTCAMB website:

<http://www.eftcamb.org/>



By **B. Hu, M. Raveri, M. Frusciante and A. Silvestri**

EFTCAMB is a patch of the public Einstein-Boltzmann solver CAMB, which implements the Effective Field Theory approach to cosmic acceleration. The code can be used to investigate the effect of different EFT operators on linear perturbations as well as to study perturbations in any specific DE/MG model that can be cast into EFT framework. To interface EFTCAMB with cosmological data sets, we equipped it with a modified version of CosmoMC, namely EFTCosmoMC, creating a bridge between the EFT parametrization of the dynamics of perturbations and observations.

B. Hu, M. Raveri, NF, A. Silvestri, PRD **89** (2014) 103530,
M. Raveri, B. Hu, NF, A. Silvestri, PRD **90** (2014) 043513

EFTCAMB & EFTCosmoMC

- EFTCAMB evolves the full scalar and tensor perturbative equations without relying on QSA;
- EFTCAMB is compatible with massive neutrinos;
- Built-in models: designer- $f(R)$, minimally couple quintessence, low-energy Hořava gravity, Covariant Galileon, $f(R)$ - Hu & Sawicki (soon), Reparametrized Horndeski (RPH);
- Built-in: several choices for EFT functions & $w_{DE}(a)$;
- Built-in: Stability requirements \rightarrow **viability priors** for EFTCosmoMC;
- EFTCosmoMC: exploration of the parameter space performing comparison with several cosmological data sets;
- Validated with other EB codes, agreement at sub-percent level

[Bellini et al., Phys.Rev. D97 (2018) no.2, 023520]

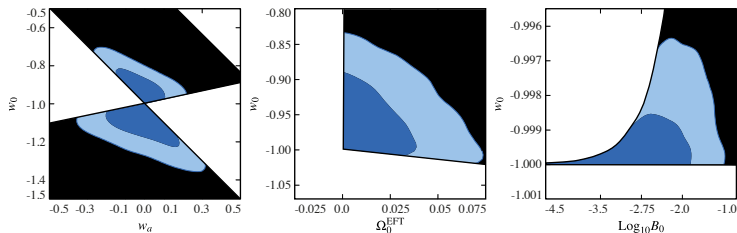
The threefold face of EFTCAMB

Model	Background	Mapping	Perturbations
PURE EFT	✓	✓/✗	✓
FULL MAPPING	✓/✗	✓/✗	✓
Other Parametrizations	✓/✗	✓/✗	✓

Built-in: ✓; To be implemented: ✗.

Numerical Notes: B. Hu, M. Raveri, NF, A. Silvestri, arXiv:1405.3590[astro-ph.IM]

Constraining power of viability conditions



Designer $f(R)$ on $w\text{CDM}$:

- $w_0 \in (-1, -0.94)$ 95% C.L.
Planck+WP+BAO,
- $w_0 \in (-1, -0.9997)$ 95% C.L.
Planck+WP+BAO+lensing.

[M. Raveri, B. Hu, NF, A. Silvestri, PRD 90 (2014) 4, 043513]

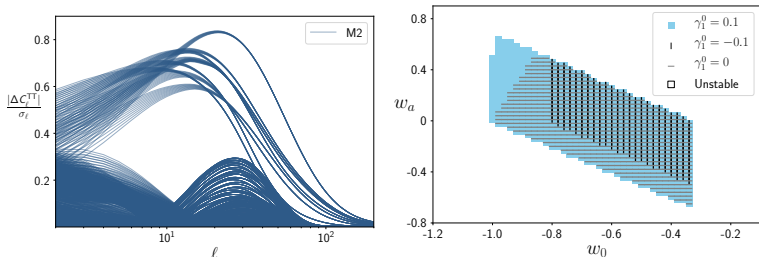
After GW170817

Horndeski action reduces to

$$S_{rH} = \int d^4x \sqrt{-g} [\mathcal{K}(\phi, X) + G_3(\phi, X)\square\phi + G_4(\phi)R],$$

[P. Creminelli and F. Vernizzi, Phys. Rev. Lett. 119, 251302 (2017)]

In terms of EFT functions we only have: Ω , \bar{M}_1^3 , M_2^4



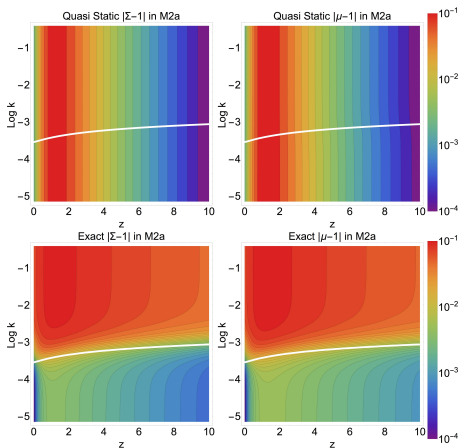
IF $\bar{M}_1^3 \neq 0 \rightarrow M_2^4 \neq 0$

[In preparation: NF, S. Peirone, N. Lima, S. Cansas]

Can we trust quasi static approximation?

QS approximation:

$$\mu(a, k) = \frac{1}{1 + \Omega} \frac{1 + M^2(a) \frac{a^2}{k^2}}{g_1(a) + M^2(a) \frac{a^2}{k^2}}, \quad \Sigma(a, k) = \frac{1}{2(1 + \Omega)} \frac{1 + g_2(a) + M^2(a) \frac{a^2}{k^2}}{g_1(a) + M^2(a) \frac{a^2}{k^2}}.$$



Conclusion

In the context of DE/MG models it is important to have:

- a model independent parametrization: EFT approach for cosmic acceleration;
- a model independent parametrization preserving a **DIRECT LINK** with the most popular models used in cosmology ($f(R)$, Galileon, Hořava, etc..);
- a general recipe for mapping any MG theory in EFT language;
- general stability requirements in vacuum and in matter (when exploring cosmological models);
- a Einstein-Boltzman code for DE/MG: EFTCAMB/EFTCosmoMC;
- built-in module which enforces the stability requirements \Rightarrow Viability Priors.

