Probing the Universe through the Stochastic GW Background

Towards optimal detection

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Stochastic GW background

- random phase & no directional dependence
- Overlapped astrophysical GWs $\Delta T < f^{-1}$
- GWs from the early Universe
Sensitivity curves for GW background

\[ \Omega_{GW}(f) = \frac{f \, d\rho_{GW}}{\rho_c \, df} \]

- Planck
- LiteBIRD
- SKA
- LISA
- Adv-LIGO
- KAGRA
- DECIGO
- DECIGO\times2

Frequency [Hz]
Astrophysical GW background

![Graph showing various sources of gravitational waves with frequency and amplitude axes. Labels include CMB, Pulsar timing, Direct detection, Adv-LIGO, KAGRA, SKA, LISA, DECIGO, DECIGO x2, WD binaries, NS binaries, BH binaries, POPIII supernovae.]

- Inflation
- Astrophysical GW background
- WD binaries
- NS binaries
- BH binaries
- POPIII supernovae
Cosmological GW background

- Inflation
- Preheating $T \approx 10^9 \text{GeV}$
- Electroweak phase transition $T \approx 100 \text{GeV}$
- Cosmic strings $G\mu \approx 10^{-12}$
How to detect a stochastic background

Cross Correlation

Detector 1
\[ s_1(t) = h(t) + n_1(t) \]

Detector 2
\[ s_2(t) = h(t) + n_2(t) \]

\[ \langle S \rangle = \int_{-T/2}^{T/2} dt \langle s_1(t)s_2(t) \rangle \]
\[ = \int_{-T/2}^{T/2} dt \langle h^2(t) + h(t)n_2(t) + n_1(t)h(t) + n_1(t)n_2(t) \rangle \]
\[ \text{no correlations} \rightarrow 0 \]
\[ = \int_{-T/2}^{T/2} dt \langle h^2(t) \rangle \]

GW signal

s: observed signal
h: gravitational waves
n: noise

(for detector at the same location)
Optimal filtering


\[
S := \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' s_1(t)s_2(t') Q(t,t')
\]

Signal in Fourier space

\[
\mu := \langle S \rangle = \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \delta_T(f-f') \langle \tilde{h}_1^*(f) \tilde{h}_2(f') \rangle \tilde{Q}(f')
\]

Noise in Fourier space

\[
\sigma^2 := \langle S^2 \rangle - \langle S \rangle^2 \approx \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} dx'' \delta_T(f-f') \delta_T(k-k') \langle \tilde{n}_1^*(f) \tilde{n}_1(-k) \rangle \langle \tilde{n}_2^*(-f') \tilde{n}_2(k') \rangle \tilde{Q}(f') \tilde{Q}(k')
\]

Signal-to-noise ratio

\[
\text{SNR}^2 = \frac{\mu^2}{\sigma^2} \approx \left( \frac{3H_0^2}{10\pi^2} \right) T \frac{\gamma(|f|) \Omega_{gw}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)} \left( \frac{\tilde{Q}}{\langle \tilde{Q}, \tilde{Q} \rangle} \right)^2
\]

Maximized when

\[
\tilde{Q}(f) = \lambda \frac{\gamma(|f|) \Omega_{gw}(|f|)}{|f|^3 P_1(|f|) P_2(|f|)}
\]

\(P_i(|f|)\) : noise spectrum

\(\langle \tilde{n}_i^*(f) \tilde{n}_i(f') \rangle = \frac{1}{2} \delta(f-f') P_i(|f|)\)

\(\gamma(|f|)\) : overlap reduction function (determined by detector positions)
We need template = spectral shape

“We Upper Limits on the Stochastic Gravitational-Wave Background from Advanced LIGO's First Observing Run”, LIGO & Virgo Collaboration, PRL. 118, 121101 (2017)

parametrized by a single power law

$$\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{f_{\text{ref}}}\right)^\alpha$$

$$f_{\text{ref}} = 25\text{Hz}$$
Idea

Many models of stochastic background predict a peaked shape. Is broken-power law better for fitting?

\[ \Omega_{GW}(f) = \begin{cases} \Omega_{GW*} \left( \frac{f}{f_*} \right)^{n_{GW1}} & \text{for } f < f_*, \\ \Omega_{GW*} \left( \frac{f}{f_*} \right)^{n_{GW2}} & \text{for } f > f_*, \end{cases} \]

example

- Phase transition
  \[ n_{GW1}=3, \ n_{GW2}=-2 \]
- Preheating
  \[ n_{GW1}=3, \ n_{GW2}: \text{exponential cutoff} \]
  \[ f^* \sim \text{energy scale of the event} \]

Spectral shape is important information to identify generation mechanism.
Example

GWB from superradiant instabilities
(Ultralight scalar fields around spinning black holes)

“Stochastic and resolvable gravitational waves from ultralight bosons”
Brito et al. PRL 119, 131101 (2017)
Template fitting

LIGO+ VIRGO+KAGRA design

single: SNR=70.7, $\delta \chi^2=1440$
broken: SNR=80.0, $\delta \chi^2=47.4$
(perfect template: SNR=80.3)

~10% loss of signal-to-noise ratio → $\delta \chi^2$ shows single is bad fit

$\Omega_{GW} = 1.43 \times 10^{-7}$
$n = 2.3$
$\Omega_{GW}$ (at 25Hz) = $1.25 \times 10^{-8}$
\[ \delta \chi^2_{\text{single}} - \delta \chi^2_{\text{broken}} \]

\[ \Omega_{GW^*} = 10^{-8} \]
\[ f^*_\text{=30Hz} \]

Broken power-law improves fitting → better measurement of shape
How accurately can we measure the tilt?

Prediction by Fisher analysis for

\[ n_{GW1} = 3 \]
\[ n_{GW2} = -2 \]

LIGO O1 constraint

LVK design

\[ n_{GW1} = 3.0 \pm ? \]
\[ n_{GW2} = -2.0 \pm ? \]

1. Large amplitude is necessary to measure the tilt
2. The error also depends on the peak position
How accurately can we measure the tilt?

\[ \sigma_{n_1,n_2} \propto \text{SNR}^{-1} \]

\[ \text{SNR} \approx \frac{3H_0^2}{10\pi^2} \sqrt{T} \left[ \int_{-\infty}^{\infty} df \frac{\gamma^2(|f|)\Omega_{gw}^2(|f|)}{f^6 P_1(|f|)P_2(|f|)} \right]^{1/2} \]

Sensitivity curve \( \propto \frac{10\pi^2}{3H_0^2} \left[ \frac{f^5 P_1(|f|)P_2(|f|)}{T\Delta \log f\gamma^2(|f|)} \right]^{1/2} \)

SNR>2 for in each frequency bin \( \Delta \log f = 0.1 \)

Ω\(_{GW1}\) is determined accurately

Ω\(_{GW2}\) is determined accurately
General expectation

\( \Omega_{GW} = 10^{-8} \)

\( f_* = 30 \text{Hz} \)

Larger amplitude increases the area

\[ \sigma_{n_1,n_2} \propto \text{SNR}^{-1} \propto \Omega_{GW}^{-1} \]
Peak frequency dependence

$$\Omega_{GW*} = 10^{-8}$$

- For $$f_s = 20 \text{ [Hz]}$$, \( n_{GW2} \) is determined accurately with 50% error.
- For $$f_s = 30 \text{ [Hz]}$$, \( n_{GW2} \) is determined accurately with 10% error and \( n_{GW1} \) is determined accurately with 50% error.
- For $$f_s = 50 \text{ [Hz]}$$, \( n_{GW2} \) is determined accurately with 50% error.

\( n_{GW2} \) is determined accurately \( n_{GW1} \) is determined accurately
Discussion

• Fitting by broken power-law is more time consuming
  single: 1 free parameter ($n_{GW}$)
  broken: 3 free parameter ($n_{GW1}$, $n_{GW2}$, $f^*$)

• Strategy?
  1. GW search by single power-law
  2. Fitting by broken power-law
     High SNR detection is necessary for the 2nd step

• Same discussion holds for DECIGO
  → More chance to detect GW background
Summary

• Detection of a stochastic GW background is the next challenging step for GW science

• It’s searched by matched filtering so we need to prepare templates (= spectral shape)

• We made quantitative estimations on broken-power law fitting and found that it dramatically improves $\delta \chi^2$

• We also made estimation on how accurately the spectral told can be determined. Precise fitting of spectral shape would help to identify the generation mechanism