Theoretical Consistency of Stochastic Approach

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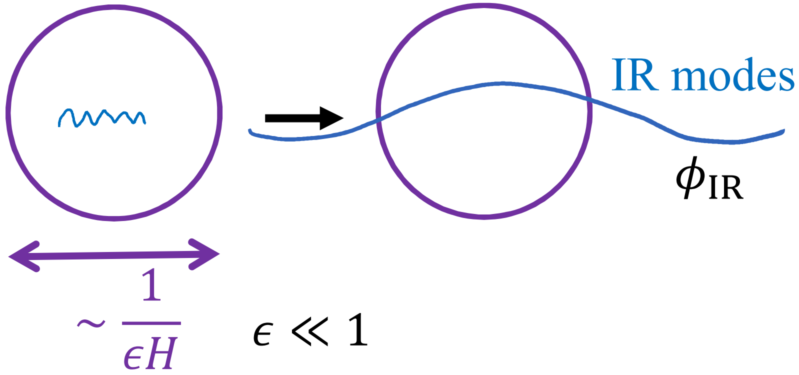
Observables = Fluctuations of infrared (IR) modes.

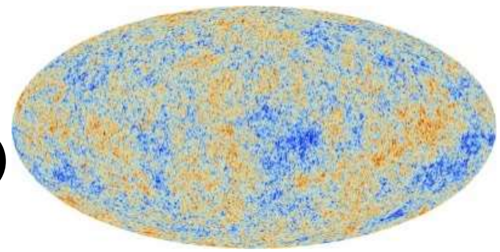
Quantum fluctuations of an inflaton ϕ

$\downarrow$

long-wavelength (IR) modes
well outside the Hubble scale.



Cosmic Microwave
Background (CMB)



ESA, Planck Collaboration (2013)

We observe IR Fluctuations.

Observables = QFT expectation values $< \phi_{IR}\phi_{IR} >$

?

IR divergences

IR divergences

QFT expectation values contain IR divergent terms.

ϕ: a minimally coupled massless scalar in de Sitter space

e.g. $\lambda\phi^4$ theory $(\lambda \ll 1)$ $(a(t) \propto e^{Ht})$

$$\frac{<\phi_{\rm IR}^2(x)>}{H^2} \sim \ln\frac{a}{a_0} + 1 + \lambda\left[\left(\ln\frac{a}{a_0}\right)^3 + \left(\ln\frac{a}{a_0}\right)^2 + \left(\ln\frac{a}{a_0}\right) + 1\right]$$
$$+\lambda^2\left[\left(\ln\frac{a}{a_0}\right)^5 + \left(\ln\frac{a}{a_0}\right)^4 + \left(\ln\frac{a}{a_0}\right)^3 + \cdots\right]$$
$$+ \cdots$$

$k \geq a_o H$: an IR cutoff

IR loops $\gg$ Tree level amplitudes.

Contributions from deep IR modes beyond the current observable scale.

IR loops affect observables for local observers (us)?

In Classical theory

Deep IR modes cannot affect observables, because for local observers, deep IR modes = homogeneous background.

In Quantum theory

Deep IR modes = Environmental degrees of freedom.

⇒ Deep IR modes should be integrated out.
⇔ Taking into account IR loops !!

Clarify whether IR loops affect observables for us.

⇒ Need to

- reconsider **what are observables for us.**
- understand **the physical meaning of IR loops.**

Stochastic approach may give us the consistent physical interpretation of IR loops.

If one treats UV modes as harmonic oscillators,

Stochastic Formalism

A. A. Starobinsky (1986) A. A. Starobinsky and J. Yokoyama(1994)

IR dynamics of $\phi_{\rm IR}$=Brownian motion with an external force.

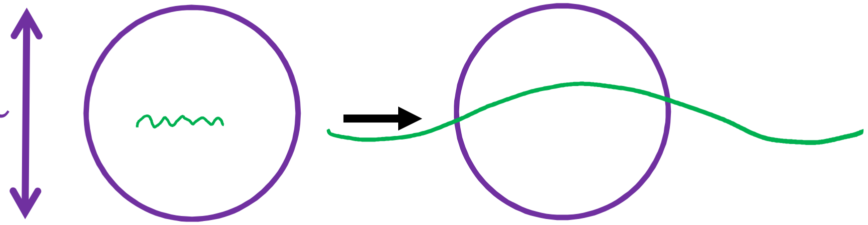
$$\dot{\phi}_{\rm IR} = \underbrace{-\frac{1}{3H}V'(\phi_{\rm IR})}_{\text{deterministic}} + \underbrace{\xi}_{\text{stochastic}}$$

$$< \xi(x_1)\xi(x_2) > = \frac{H^3}{4\pi^2}\delta(t_1 - t_2)\frac{\sin(\epsilon a(t_1)H|\vec{x_1} - \vec{x_2}|)}{\epsilon a(t_1)H|\vec{x_1} - \vec{x_2}|}$$

coarse-graining scale

Physical origin of ξ

$\frac{1}{\epsilon H} \sim$



Short-wavelength (UV) modes

are transferred to

IR modes (phase is random)

Stochastic force

- ✓ This eq. can be solved non-perturbatively.
- ✓ This eq. can correctly recover parts of IR loops.

N. C. Tsamis and R. P. Woodard (2005)

Classical *Stochastic Picture of the inflationary universe*

A. Linde(1986) A. A. Starobinsky (1986) Y. Nambu and M. Sasaki (1989)

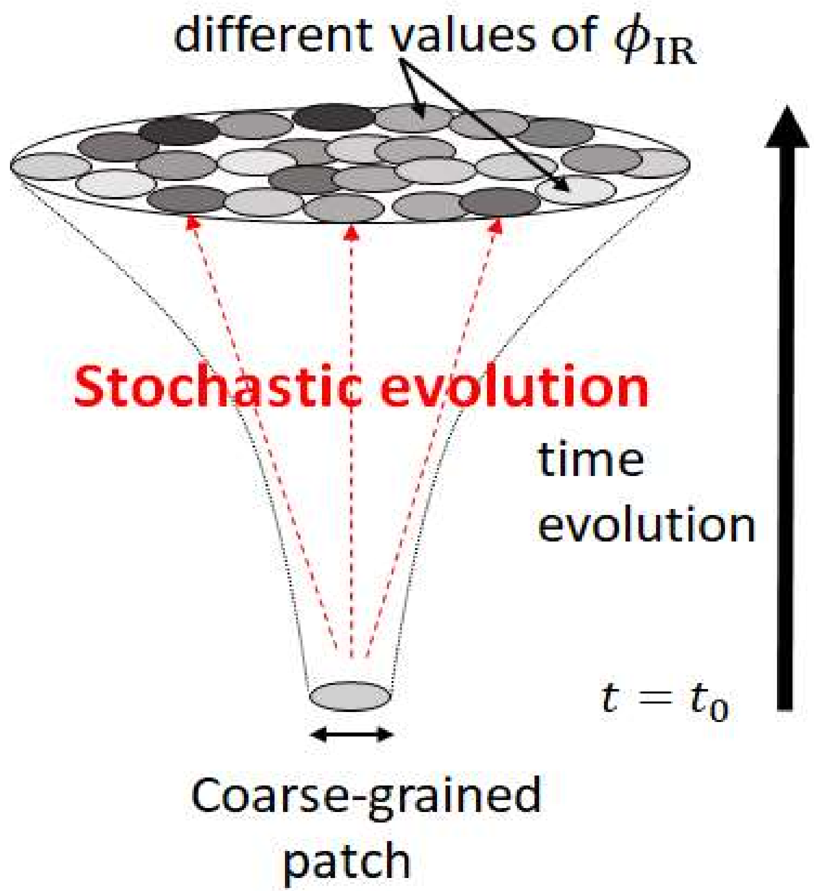
The time evolution of the inflationary universe = **classical stochastic process**

Brownian Motion

In this picture, one assumes that

a certain value of ϕ_{IR} is **classically realized** at each patch.

$$\dot{\phi}_{\mathrm{IR}} = -\frac{1}{3H} V'(\phi_{IR}) + \xi$$

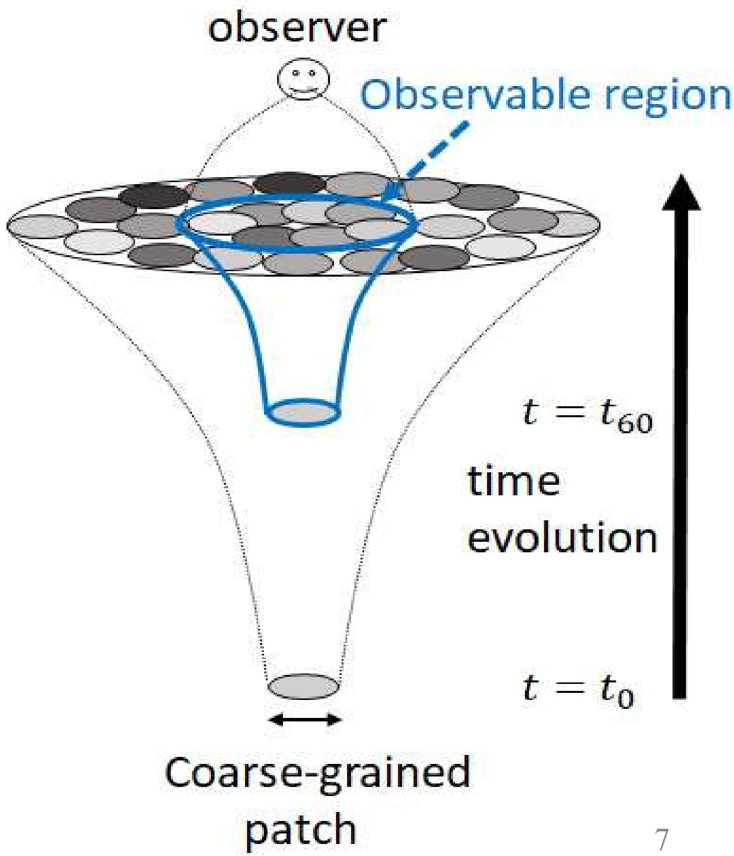


Observables in the stochastic picture

✓ The prescription of calculating adiabatic perturbations based on this classical picture is proposed.

T. Fujita, M. Kawasaki, Y. Tada, and T. Takesako (2013)
V. Vennin and A. A. Starobinsky (2015)

Observables $\neq \langle \hat{\phi}_{\rm IR} \cdots \hat{\phi}_{\rm IR} \rangle$



Loops from deep-IR modes beyond observable scale do not affect observables.

Brownian motion cannot capture correctly all the IR loops.

$$\frac{<\phi_{\rm IR}^2(x)>}{H^2} \sim \ln\frac{a}{a_0} + 1$$
$$+\lambda\left[\left(\ln\frac{a}{a_0}\right)^3 + \left(\ln\frac{a}{a_0}\right)^2 + \left(\ln\frac{a}{a_0}\right) + 1\right]$$
$$+\lambda^2\left[\left(\ln\frac{a}{a_0}\right)^5 + \left(\ln\frac{a}{a_0}\right)^4 + \left(\ln\frac{a}{a_0}\right)^3 + \cdots\right]$$
$$+ \cdots$$

These terms are not correctly recovered.

∵ UV modes are treated as harmonic oscillators.

IR dynamics = Classical stochastic process ?

If it fails seriously, IR loops may modify the current predictions drastically.

Our work: Setup

JT and T. Tanaka (2017)

- Model : $\mathcal{H} = \frac{1}{2}v^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi, v)$ on de Sitter background.

- Defs. of UV modes and IR modes

$v \equiv \dot{\phi}$

$k = \epsilon a H$

IR modes

$\phi^{\mathrm{IR}}(\vec{x}, t), v^{\mathrm{IR}}(\vec{x}, t)$

UV modes

$\phi^{\mathrm{UV}}(\vec{x}, t), v^{\mathrm{UV}}(\vec{x}, t)$

k

Time evolution

- Assumptions

1. No IR mode initially (at $t = t_0$).
2. $\mathrm{V}(\phi, v)$ is turned on at $t = t_0$, and we take the Bunch-Davies vacuum states for a free field at $t = t_0$.

Derive an effective IR dynamics by integrating out UV modes

Decompose the path integral into UV parts and IR parts.

$$\int \mathcal{D}\phi_+ \mathcal{D}v_+ \mathcal{D}\phi_- \mathcal{D}v_- \, e^{i(S^+ - S^-)} \qquad S = \int \mathrm{d}^4x \, a^3 \left(v\dot{\phi} - \mathcal{H}(v,\phi)\right)$$

$$\to \int \mathcal{D}\phi_+^{\mathrm{IR}} \mathcal{D}v_+^{\mathrm{IR}} \mathcal{D}\phi_-^{\mathrm{IR}} \mathcal{D}v_-^{\mathrm{IR}} \, e^{iS_{\mathrm{IR}}} \boxed{\int \mathcal{D}\phi_+^{\mathrm{UV}} \mathcal{D}v_+^{\mathrm{UV}} \mathcal{D}\phi_-^{\mathrm{UV}} \mathcal{D}v_-^{\mathrm{UV}} \, e^{iS_{\mathrm{UV}}} e^{iS_{\mathrm{int}}^{\mathrm{UV-IR}}}}$$

JT and T. Tanaka (2017)

An effective IR dynamics

Stochastic noises

$$\dot{\phi}_c^{\mathrm{IR}} = v_c^{\mathrm{IR}} + \mu(\phi_c^{\mathrm{IR}}, v_c^{\mathrm{IR}}) + \xi_\phi$$

$$\dot{v}_c^{\mathrm{IR}} = -3Hv_c^{\mathrm{IR}} - \partial_\phi V_{\mathrm{eff}}(\phi_c^{\mathrm{IR}}, v_c^{\mathrm{IR}}) + \xi_v$$

$$\phi_c \equiv \frac{\phi_+ + \phi_-}{2}, \quad \phi_\Delta \equiv \phi_+ - \phi_-$$

Probability distribution of ξ_ϕ and ξ_v: $P\left[\xi_\phi, \xi_v; \phi_c^{\mathrm{IR}}, v_c^{\mathrm{IR}}\right]$

$$P = \int Dv_\Delta^{\mathrm{IR}} D\phi_\Delta^{\mathrm{IR}} \boxed{\left(e^{-A_1 {v_\Delta^{\mathrm{IR}}}^2 - iA_2 {v_\Delta^{\mathrm{IR}}}^3 - \cdots}\right)\left(e^{-B_1 {\phi_\Delta^{\mathrm{IR}}}^2 - \cdots}\right)\left(e^{-C_1 v_\Delta^{\mathrm{IR}} \phi_\Delta^{\mathrm{IR}} - \cdots}\right)} e^{i\xi_\phi v_\Delta^{\mathrm{IR}} - i\xi_v \phi_\Delta^{\mathrm{IR}}}$$

An effective IR dynamics=Classical process?

$$P = \int Dv_\Delta^{\rm IR} D\phi_\Delta^{\rm IR} \left(e^{-A_1 {v_\Delta^{\rm IR}}^2 - iA_2 {v_\Delta^{\rm IR}}^3 - \cdots}\right)\left(e^{-B_1 {\phi_\Delta^{\rm IR}}^2 - \cdots}\right)\left(e^{-C_1 v_\Delta^{\rm IR}\phi_\Delta^{\rm IR} - \cdots}\right) e^{i\xi_\phi v_\Delta^{\rm IR} - i\xi_v \phi_\Delta^{\rm IR}}$$

$= \exp[i\Gamma_{(s)}]$

$$\frac{A_1}{H^4} \sim \frac{<\xi_\phi\xi_\phi>}{H^4} \sim O(1), \quad \frac{B_1}{H^2} \sim \frac{<\xi_v\xi_v>}{H^2} \sim O(\lambda^2) \quad \frac{C_1}{H^3} \sim \frac{<\xi_\phi\xi_v>}{H^3} \sim O(\lambda).$$

Gaussian part of $\phi_\Delta^{\rm IR}$: suppressed by λ.

Regarding $\phi_\Delta^{\rm IR}$, non-Gaussian parts contribute at the same order.

$P[\xi_\phi, \xi_v; \phi_c^{\rm IR}, v_c^{\rm IR}]$ can be negative.

Path integral for IR modes can be written as

$$\int \mathcal{D}\phi_c^{\mathrm{IR}}\mathcal{D}v_c^{\mathrm{IR}}\mathcal{D}\phi_\Delta^{\mathrm{IR}}\mathcal{D}v_\Delta^{\mathrm{IR}} e^{i\int \mathrm{d}^4x\, a^3 v_\Delta^{\mathrm{IR}}(\dot\phi_c^{\mathrm{IR}}-v_c^{\mathrm{IR}}-\mu)} e^{i\Gamma_{(\mathrm{s})}\left[v_\Delta^{\mathrm{IR}},\phi_\Delta^{\mathrm{IR}},v_c^{\mathrm{IR}},\phi_c^{\mathrm{IR}}\right]} e^{i\int \mathrm{d}^4x\, a^3 \phi_\Delta^{\mathrm{IR}}(-\dot v_c^{\mathrm{IR}}-3Hv_c^{\mathrm{IR}}-\partial_\phi V_{\mathrm{eff}})}$$

Partial integration over v_c^{IR}. $\phi_\Delta^{\mathrm{IR}}(t) \to -\hat F \equiv i\int_t^{t_f}\frac{\mathrm{d}t'}{a^3(t')}\frac{\delta}{\delta v_c^{\mathrm{IR}}(t')}(1+\cdots)$

$$\int \mathcal{D}\phi_c^{\mathrm{IR}}\mathcal{D}v_\Delta^{\mathrm{IR}}\mathcal{D}v_c^{\mathrm{IR}}\ \underbrace{e^{i\Gamma_{(\mathrm{s})}}\Big|_{\phi_\Delta^{\mathrm{IR}}\to\hat F}\left[e^{i\int \mathrm{d}^4x\, a^3 v_\Delta^{\mathrm{IR}}(\dot\phi_c^{\mathrm{IR}}-v_c^{\mathrm{IR}}-\mu)}\right]}_{\simeq\, e^{i\Gamma_{(\mathrm{s})}\left[v_\Delta^{\mathrm{IR}},\phi_\Delta^{\mathrm{IR}}=0,v_c^{\mathrm{IR}},\phi_c^{\mathrm{IR}}\right]} e^{i\int \mathrm{d}^4x\, a^3 v_\Delta^{\mathrm{IR}}(\dot\phi_c^{\mathrm{IR}}-v_c^{\mathrm{IR}}-\mu)}} \int \mathcal{D}\phi_\Delta^{\mathrm{IR}} e^{i\int \mathrm{d}^4x\, a^3 \phi_\Delta^{\mathrm{IR}}(-\dot v_c^{\mathrm{IR}}-3Hv_c^{\mathrm{IR}}-\partial_\phi V_{\mathrm{eff}})}$$

$$\dot\phi_c^{\mathrm{IR}} = v_c^{\mathrm{IR}} + \mu + \xi$$ Approximately Gaussian noise

$$\dot v_c^{\mathrm{IR}} = -3Hv_c^{\mathrm{IR}} - \partial_\phi V_{\mathrm{eff}}$$

→Positive probability！

※ v_c^{IR} no longer corresponds to conjugate momentum.

IR dynamics of ϕ_c^{IR} = a classical stochastic process.

Classical Aspects of Stochastic Inflation

By changing the definition of v_c^{IR} appropriately, we can rewrite the IR dynamics as

$$\dot{\phi}_c^{\mathrm{IR}} = \tilde{v}_c^{\mathrm{IR}} + \mu + \xi$$

Approximately Gaussian noise

$$\dot{\tilde{v}}_c^{\mathrm{IR}} = -3H\tilde{v}_c^{\mathrm{IR}} - \partial_\phi V_{\mathrm{eff}}$$

※ $\tilde{v}_c^{\mathrm{IR}} \neq$ conjugate momentum

→Positive probability！

IR dynamics of ϕ_c^{IR} = a classical stochastic process.

Summary

◆ **Motivation**

Light scalar $\phi_{\rm IR}$: $\langle\hat{\phi}_{\rm IR}\cdots\hat{\phi}_{\rm IR}\rangle$ ⋯IR div.→ What are observables for us?

If IR dynamics = classical stochastic process,

classical stochastic picture allows us to propose observables.

An IR dynamics of $\phi_{\rm IR}$ = **classical** stochastic process?

◆ **Conclusions and Discussions**

1. Yes! IR loop contributions = increase of statistical variance.

2. In this picture, classicalization of IR modes is assumed. However, it is known that decoherence during inflation is not exact. We have to define observables within a framework of QFT with taking into account the effects of decoherence.