QUINTESSENTIAL INFLATION

Relic gravity waves

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GC-2018

BRIEF OVER VIEW

- Acceleration—Generic feature of cosmic history
- Late time acceleration: Dark energy, large scale modification of gravity.
- Quintessential inflation
- Building Blocks: Non-interference with thermal history and independence of initial conditions.
- Model independent Estimates.
- Brane world models.
- Reheating Mechanisms: gravitational particle production, Instant preheating, Curvaton mechanism.
- Unification in standard FRW : successful model. Prediction of the paradigm.

Inconsistencies of standard model

EARLY TIMES:

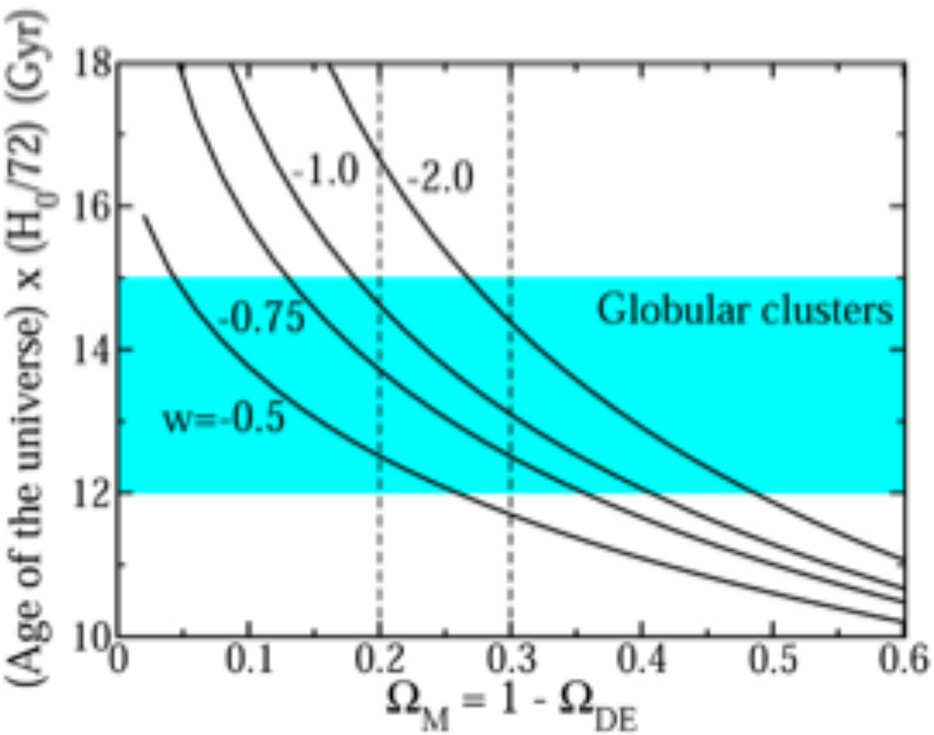
Flatness problem

Primordial inhomogeneities **INFLATION**

Horizon problem

LATE TIMES: Age crisis

AGE CRISIS AND ITS RESOLUTION WITH Λ



$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_b(t) + \frac{\Lambda}{3}$$

Late time acceleration

- DARK ENERGY: Fluid with large negative pressure
- Cosmological constant
- Slowly Rolling scalar field

- LARGE MODIFICATION OF GRAVITY
- Based upon massive extra degrees of freedom
- (standard) Scalar tensor theories
- Mass less extra degrees of freedom
- Galileon modified theories.
- Horndeski Theories

Modified theories

Massive degrees of freedom: Chameleon mechanism

S. Capozziello, V. F. Cardone, S.Carloni, A. Troisi, astro-ph/0307018

A. A. Starobinsky, JETP. Lett.
86, 157 (2007), [arXiv:0706.2041]; A. V. Frolov, PRL101, 061103 (2008)
T.Thongkool, Gannouji, MS, S. Jhingan Phys.Rev. D80 (2009) 043523;
PRD8,083515(2008)

Generic modification: No Go
Acceleration in Jordan frame.
No acceleration in Einstein frame.
Proper screening leaves no scope for late time acceleration

J.Wang,L. Hui, J. Khoury, PRL109,241301(2012)
K. Bamba, R. Gannouji, M.Kamijo, S. Nojiri, MS
JCAP1307,017(2013)

Modified theories

Massless degrees of freedom: Vainstein Mechanism Galileon theory.

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D79(2009) 064036

C. Deffayet, G. Esposito-Farese and A. Vikman, Phys. RevD79(2009) 084003

Horndeski, G.W. Int. J. Theor. Phys. 10, 363 (1974).

Strong constraints from GW

T.Baker et al, arXiv:1710.06394

Horndeski

$$L_2 = G_2(\phi, X); \quad L_3 = G_3(\phi, X)\Box\phi$$

$$L_4 = G_4 R + G_{4,X}\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$

$$L_5 = G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi + 2\phi^{\nu}_{;\mu}\phi^{\alpha}_{;\nu}\phi^{\mu}_{;\alpha}\right]$$

$$\ddot{h}_{ij} + 2H\dot{h}_{ij} + (1+\alpha_T)k^2 h_{ij} = 0 \quad \alpha_T \simeq \Delta t/d_s \sim 10^{-15}$$

$$\alpha_T \sim \left[G_{4,X} - G_{5,\phi} - (\ddot{\phi} - \dot{\phi}H)G_{5,X}\right]$$

Scalar field dynamics

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \,,$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \;\; P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

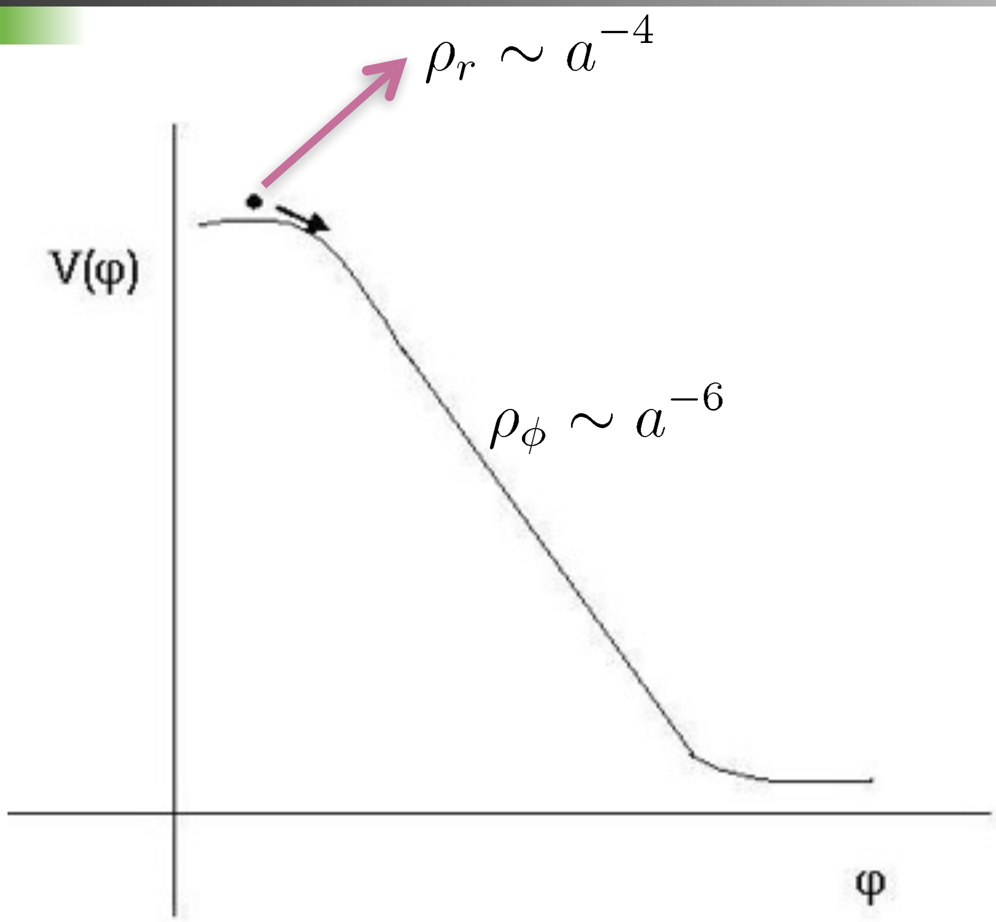
$$\rho_\phi = \rho_\phi^0 \exp\left(-\int 3(1+w(\phi))\frac{da}{a}\right) \,, \qquad w(\phi) = P_\phi/\rho_\phi$$

$$-1 \leq \omega_\phi \leq 1$$

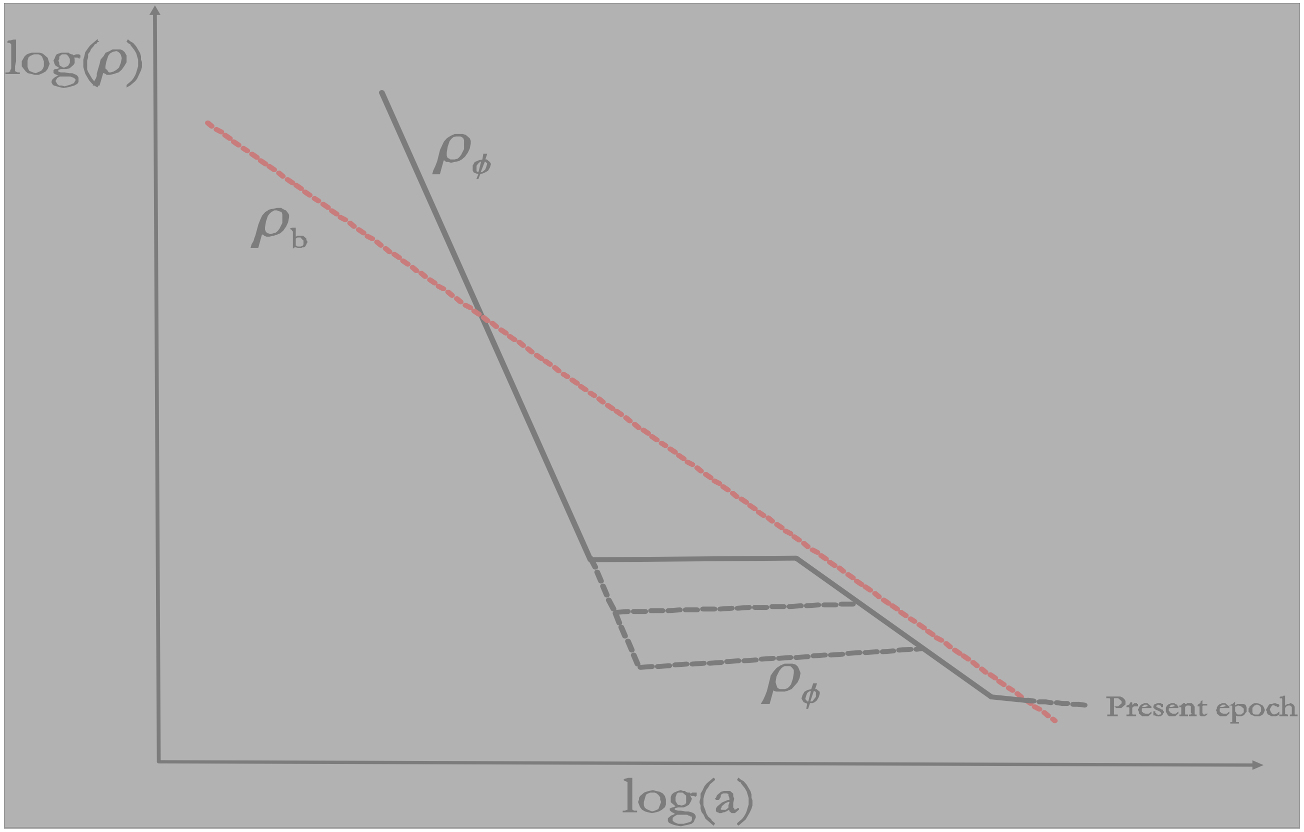
$$Steep\ Pot: \quad \omega_\phi \to 1\ (Kinetic\ Regime) \qquad \rho_\phi \sim 1/a^6$$

Quintessential inflation

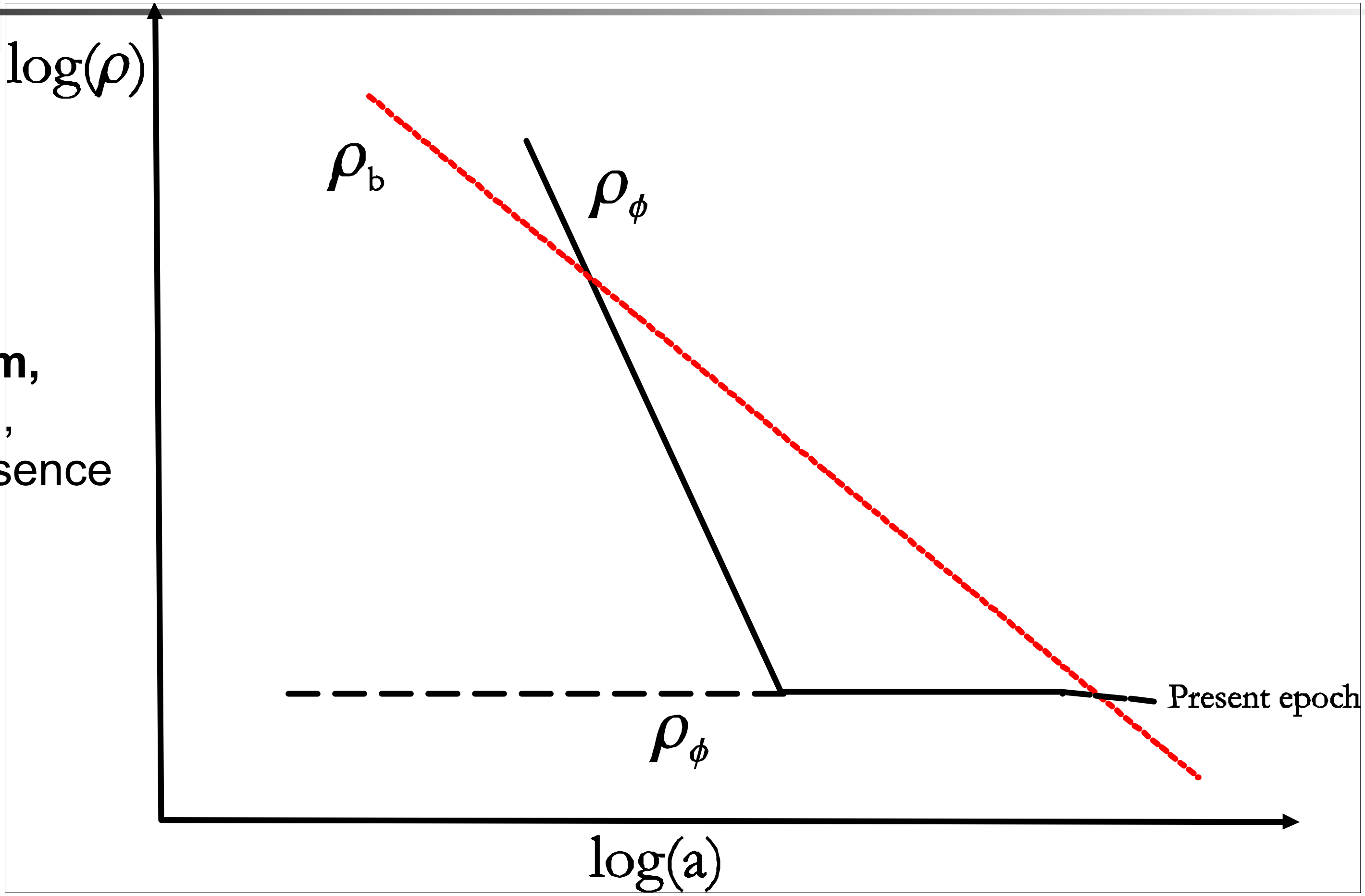
Model of reincarnation



Tracker



Absence of trackers: Thawing Models



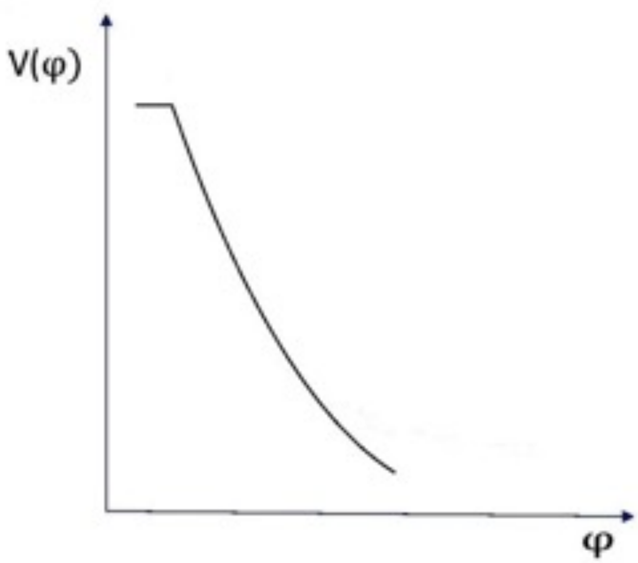
Phantom,
Tachyon,
Quintessence

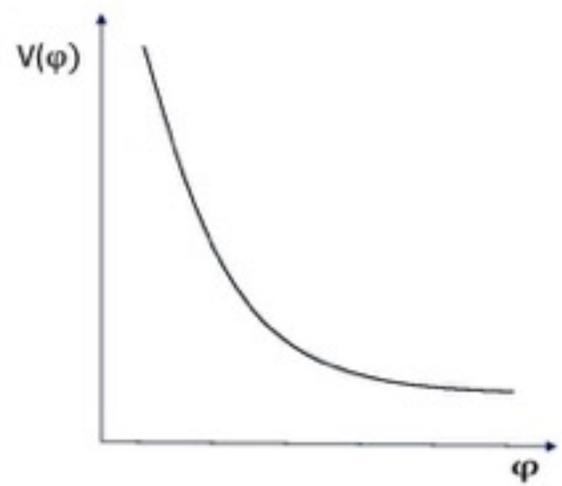
Desirable Post Inflationary Evolution

INITIAL CONSDITIONS INDEPENDENCE -TRACKER

SCLAMING SOLUTION
LATE TIME EXIT FROM SCALING REGIME

Field potential shallow at late stages.
Triggering minimum at late times in the potential.





SCALING BEHAVIOUR

$$x = \frac{\dot{\phi}}{\sqrt{6}HM_p}; \quad y = \frac{\sqrt{V}}{\sqrt{3}HM_p}$$

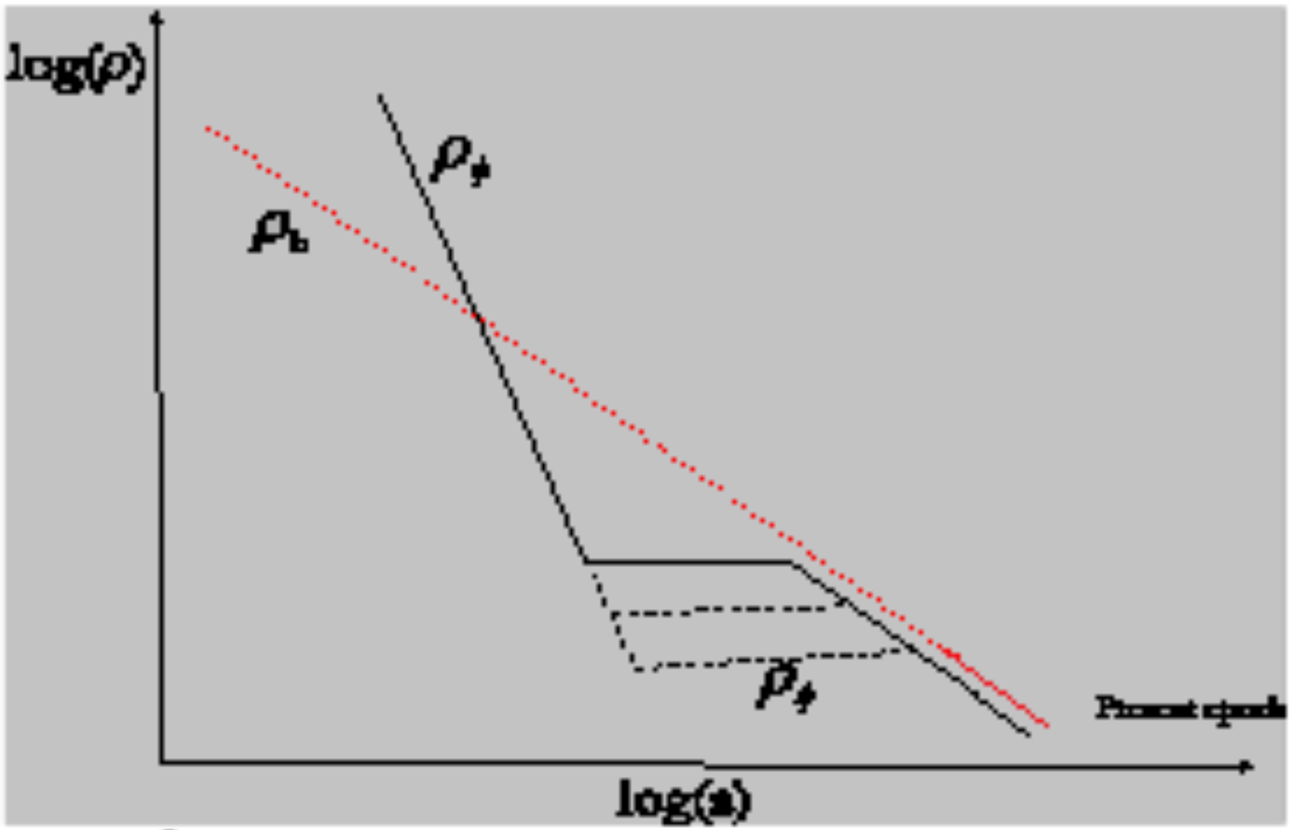
$$V = V_0 e^{-\lambda\phi/M_p}; \quad \lambda_s = -M_p \frac{V_\phi}{V} = \lambda$$

$$\frac{dx}{dN} = f(x, y, \omega_b); \quad \frac{dy}{dN} = g(x, y, \omega_b); \quad x^2 + y^2 + \frac{\rho_b}{3M_p^2 H^2} = 1$$

$$Scaling\ solution: \ \omega_\phi = \omega_b$$

$$\Omega_\phi = \frac{3(1+w_b)}{\lambda^2}; \ \lambda^2 > 3(1+w_b) \qquad NSC: \ \Omega_\phi \lesssim 0.01$$

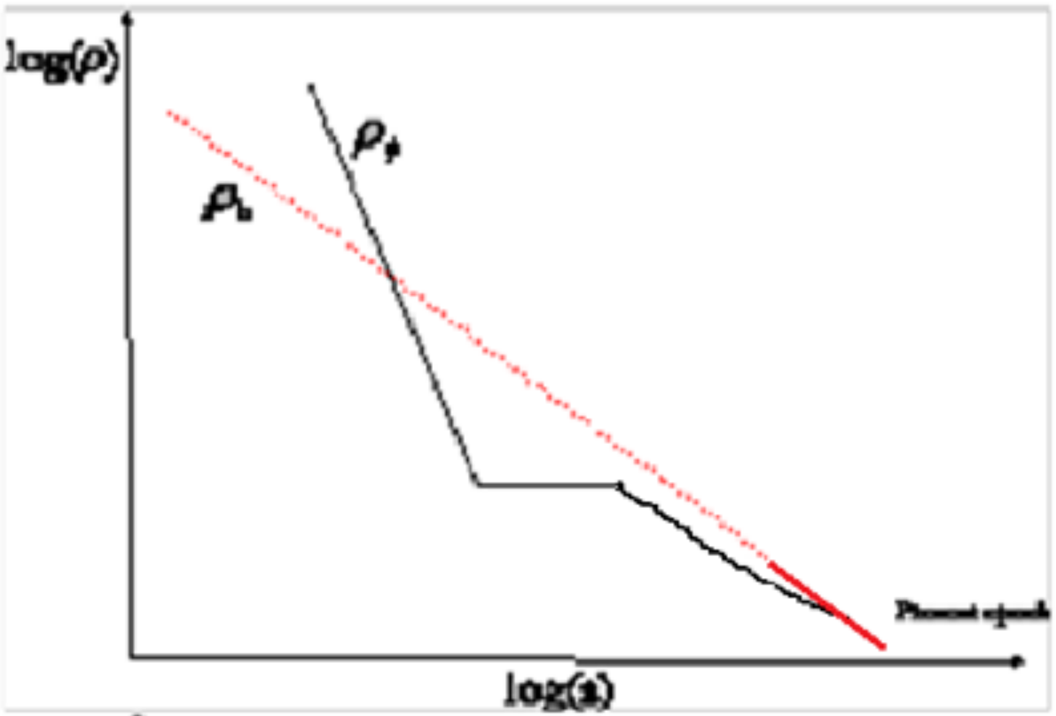
SCALING BEHAVIOUR



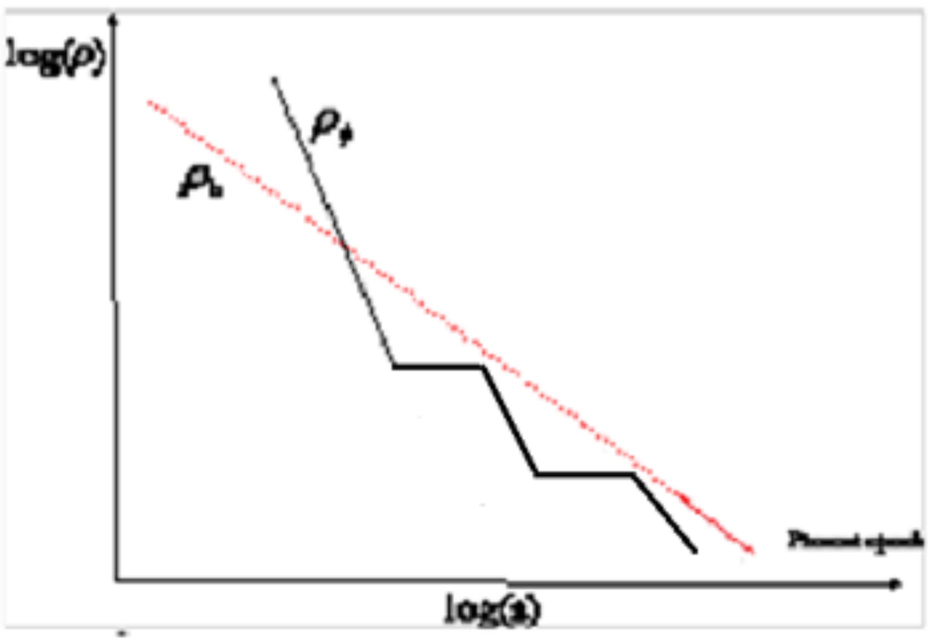
$$V = V_0 e^{-\lambda\phi/M_p}$$

$$\lambda^2 > 3(1+\omega_b)$$

Potential less steeper than exponential

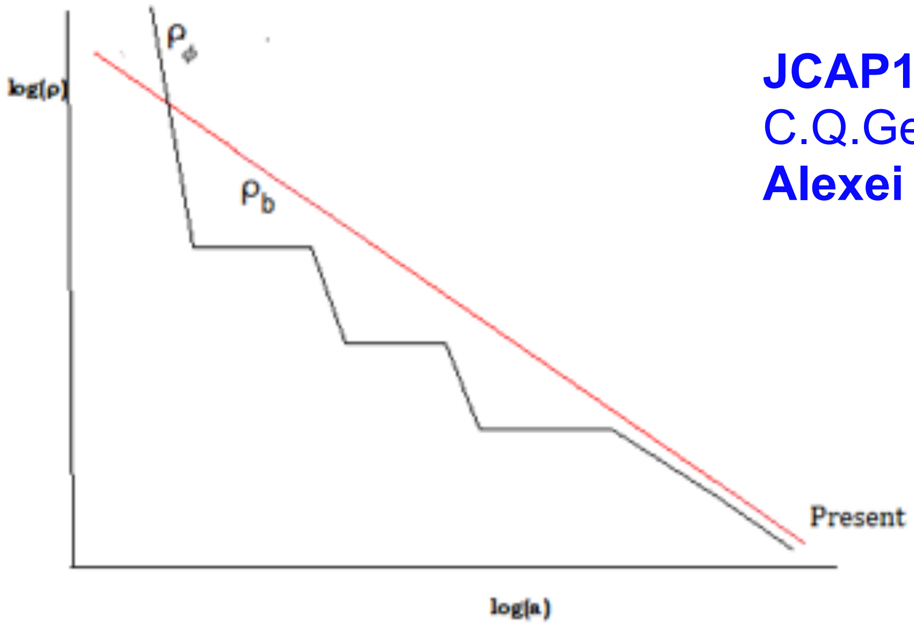


Potential more steeper than exponential
General case



Potential more steeper than exponential specific case

$$V = V_0 e^{-\lambda\phi^n/M_p^n}; \quad n > 1; \quad \lambda_s = -M_p \frac{V_\phi}{V} \sim \lambda\phi^{n-1}$$



JCAP1706(2017),06,011
C.Q.Geng, C.C. Lee, MS, E.N.Saridakis,
Alexei A. Starobinsky

Post Inflationary Evolution

$$V = V_0 e^{-\lambda \phi^n / M_p^n}; \quad n > 1; \quad \lambda_s = -M_p \frac{V_\phi}{V} \sim \lambda \phi^{n-1}$$

$$x = \frac{\dot{\phi}}{\sqrt{6} H M_p}; \quad y = \frac{\sqrt{V}}{\sqrt{3} H M_p}$$

$$\Gamma = \frac{V V_{\phi\phi}}{V_\phi^2} = 1 - \frac{(n-1)}{n\lambda} \frac{M_p^n}{\phi^n}$$

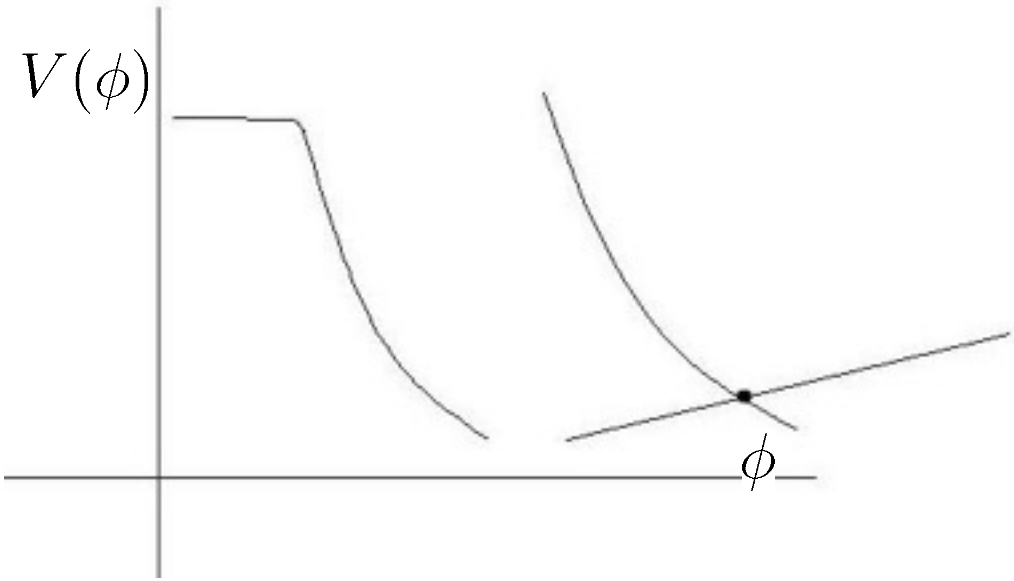
$$\frac{dx}{dN} = f(x,y) : \quad \frac{dy}{dN} = g(x,y)$$

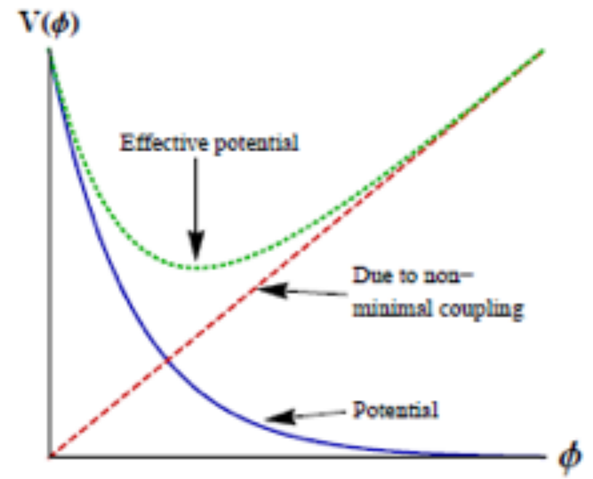
$$\frac{d\lambda_s}{dN} = -\sqrt{6} \lambda_s^2 (\Gamma - 1)$$

Coupling to matter

$$V_{eff} = V(\phi) - \frac{Q}{M_P} T\phi; \quad V_s = V_0 e^{-\lambda\phi/M_{pl}}; \quad T = 3P_m - \rho_m$$

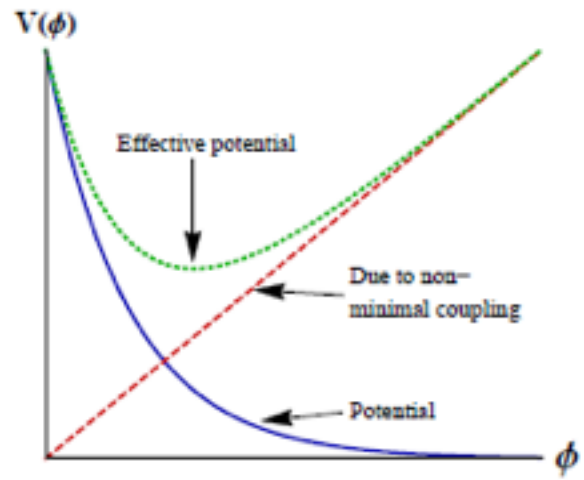
$$V_{eff} = V(\phi) + \frac{Q}{M_P} \rho_m \phi; \quad V_s = V_0 e^{-\lambda\phi/M_{pl}}$$

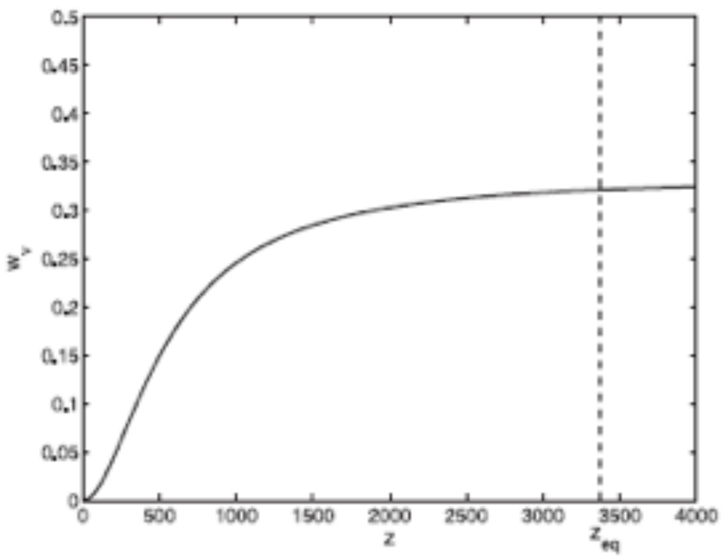




Late time attractor

$$\omega_{DE} = \left(\frac{\lambda \omega_m - Q}{Q + \lambda} \right)$$





B. Gumjudpai , T. Naskar, MS, S. Tsujikawa JCAP0506(2005)

Late Time Exit to acceleration

coupling to massive neutrino matter

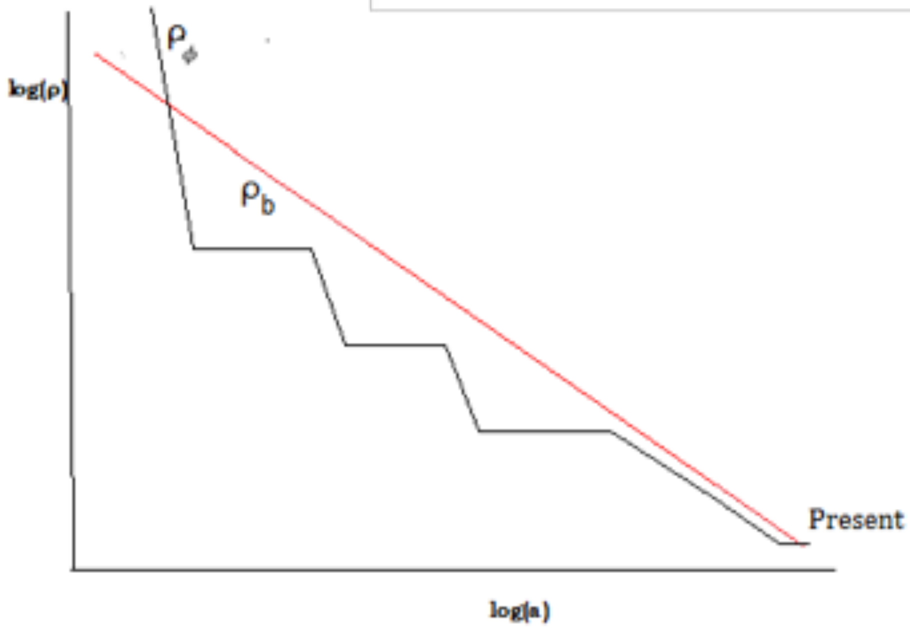
$$V_{eff} = V(\phi) - \frac{Q}{M_P} T_{m\nu}\phi; \quad V_s = V_0 e^{-\alpha\phi/M_{pl}}; \quad T_\nu = 3P_{m\nu} - \rho_{m\nu}$$

$$V_{eff} = V(\phi) + \frac{Q}{M_P}\rho_\nu\phi; \quad V_s = V_0 e^{-\alpha\phi/M_{pl}}$$

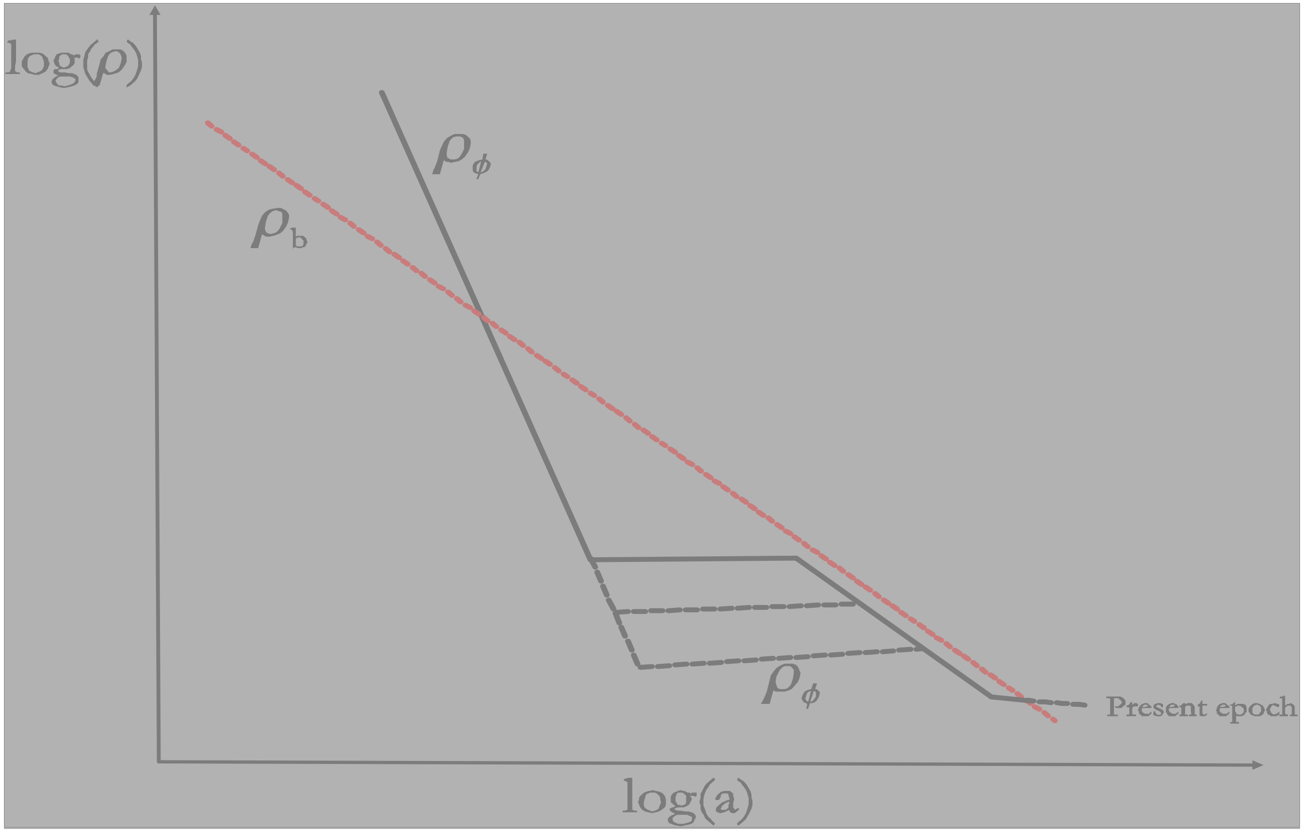
$$\omega_{DE} = \left(\frac{\alpha\omega_\nu - Q}{Q+\alpha}\right)$$

Exit from scaling regime to late time acceleration

$$V = V_0 e^{-\lambda\phi^n/M_p^n}; \quad n > 1; \quad \lambda_s = -M_p \frac{V_\phi}{V} \sim \lambda\phi^{n-1}$$

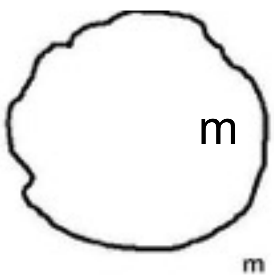


VIABLE EVOLUTION



Cosmological constant

$$\rho_v \simeq \frac{m^4}{64\pi^2} \ln\left(\frac{m^2}{\mu^2}\right)$$

Σ 

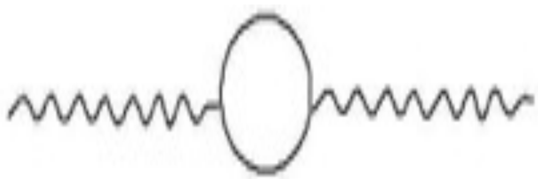
NATURALNESS OR DECOUPLING

A parameter in the Lagrangian of a field theory is said to be natural If by switching it off at the classical level leads to the enhancement Of symmetry which is also respected at the quantum level.

$$L = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m_s\right)\Psi + g\phi\bar{\Psi}\Psi$$

$$m = 0 \qquad enhances \quad symmetry$$

m_s



$$\delta m^2 \sim g^2 m_s^2 \ln(\mu^2/m_s^2)$$

Electrodynamics

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m_e]\psi$$

$$m_e \to 0 : \quad enhanced\ symmetry$$

$$\boxed{(\delta m_e)_{one\ \ loop} \sim e^2 m_e \ln(\Lambda_c/m_e)} \ \ +O(M_H^2/\Lambda_c^2)$$

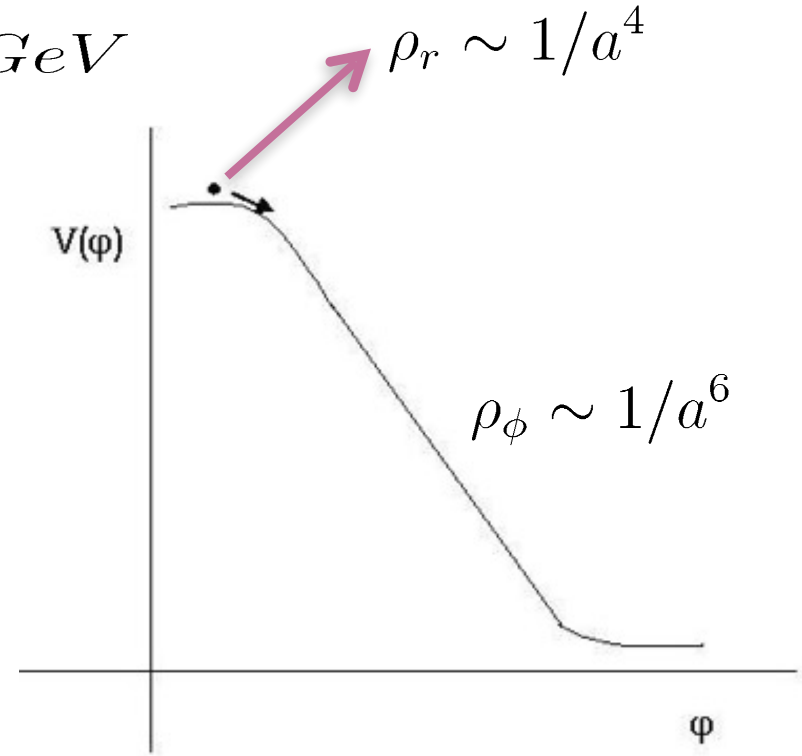
Inflation

$$H_{inf} \simeq 2.37 \times 10^{14} r^{1/2} GeV$$

$$\boxed{h^2_{GW} = \frac{H^2_{inf}}{8\pi M_p^2};}$$

$$H_{end} \lesssim H_{inf}$$

$$\rho_{r,end} = 0.01 \times g_p H^4_{end}$$



Model independent estimates

$$\boxed{\left(\frac{\rho_\phi}{\rho_r}\right)_{end} = \frac{3\pi}{64}\left(\frac{\rho_g}{\rho_r}\right)_{eq}\frac{1}{h^2_{GW}}}$$

$$\boxed{h^2_{GW} = \frac{H^2_{inf}}{8\pi M_p^2}};\quad H_{inf} \simeq 2.37 \times 10^{14} r^{1/2} GeV$$

$$\boxed{\left(\frac{\rho_\phi}{\rho_r}\right)_{end} = \frac{3\pi}{64}\left(\frac{\rho_g}{\rho_r}\right)_{eq}\frac{1}{h^2_{GW}} \lesssim \frac{4\times 10^6}{r}}$$

$$\boxed{Nucleosynthesis\ constraint(\mathbf{NC}): \quad (\rho_g/\rho_r)) \lesssim 0.01}$$

Gravitational Particle Production

$$\rho_{r,end} = 0.01 \times g_p H_{end}^4$$

B.Spokoiny, PLB315,40(1993)

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} \simeq \frac{\rho_{\phi,end}}{0.01 \times g_p H_{end}^4} \gtrsim \frac{2 \times 10^{10} g_p^{-1}}{r}$$

$$\mathbf{NC}: \quad \left(\frac{\rho_\phi}{\rho_r}\right)_{end} = \frac{3\pi}{64}\left(\frac{\rho_g}{\rho_r}\right)_{eq} \frac{1}{h_{GW}^2} \lesssim \frac{4 \times 10^6}{r}$$

Gravitational particle production misses the NC at least by two orders of magnitudes

INSTANT PREHEATING

$$Inflation\ ends,\ \phi = \phi_{end};\quad \phi' = \phi - \phi_{\rm end}$$

$$\mathcal{L}_{\rm int} = -\frac{1}{2}g^2\phi^2\chi^2 - h\bar{\psi}\psi\chi \qquad m_\chi = g\phi$$

$$\dot{m}_\chi \gtrsim m_\chi^2 : |\phi| \lesssim \left(V_{end}^{1/2}/g\right)^{1/2}$$

$$\Delta t_p \sim \frac{|\phi|}{|\dot{\phi}|} \sim \left(gV_{end}^{1/2}\right)^{1/2}$$

$$k_p \sim (\Delta t)^{-1} \sim g^{1/2}V_{end}^{1/4} \rightarrow n_k \simeq exp(-\pi k^2/gV_{end}^{1/4})$$

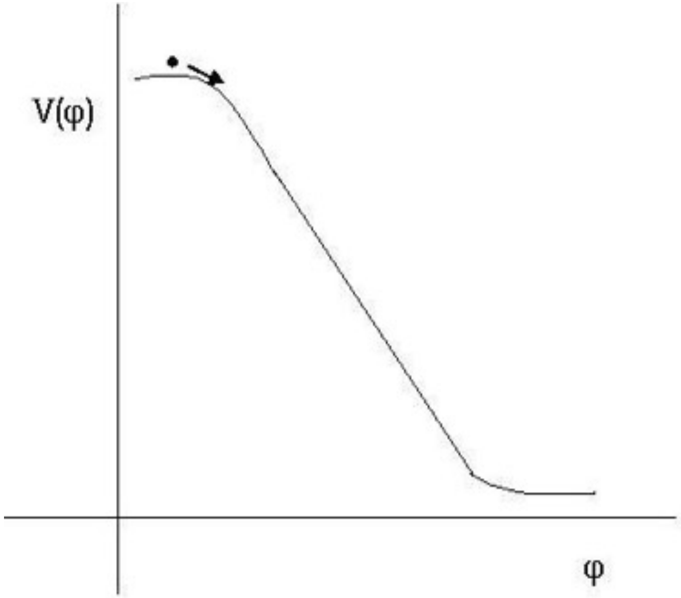
$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} = \frac{12\pi^3}{g^2}$$

INSTANT PREHEATING

$$m_\chi = g|\phi|$$

$$\Gamma_{\chi \to \bar{\Psi}\Psi} = \frac{h^2 m_\chi}{8\pi}$$

$$\Gamma_{\chi \to \bar{\Psi}\Psi} >> H_{end}$$



$$h \gtrsim 0.13 \times g^{-1/2} r^{1/4}$$

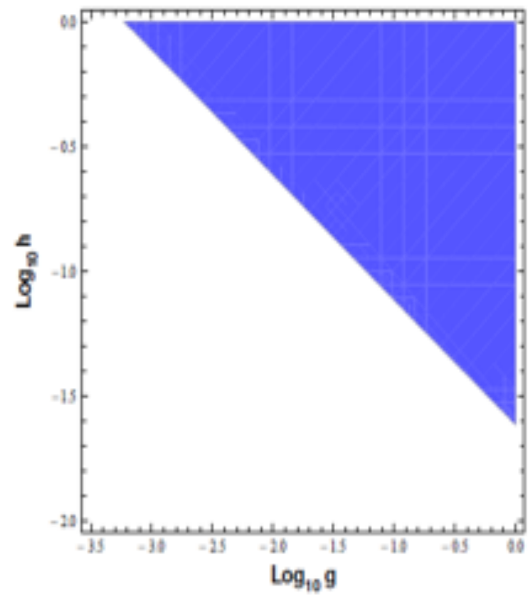
INSTANT PREHEATING

$$\left(\frac{\rho_\phi}{\rho_r}\right)_{end} = \frac{12\pi^3}{g^2}$$

$$\mathbf{NC:} \quad \left(\frac{\rho_\phi}{\rho_r}\right)_{end} \lesssim \frac{4\times 10^6}{r}$$

$$g \gtrsim r^{1/2} \times 10^{-2}$$

$$h \gtrsim 0.13 \times g^{-1/2} r^{1/4}$$

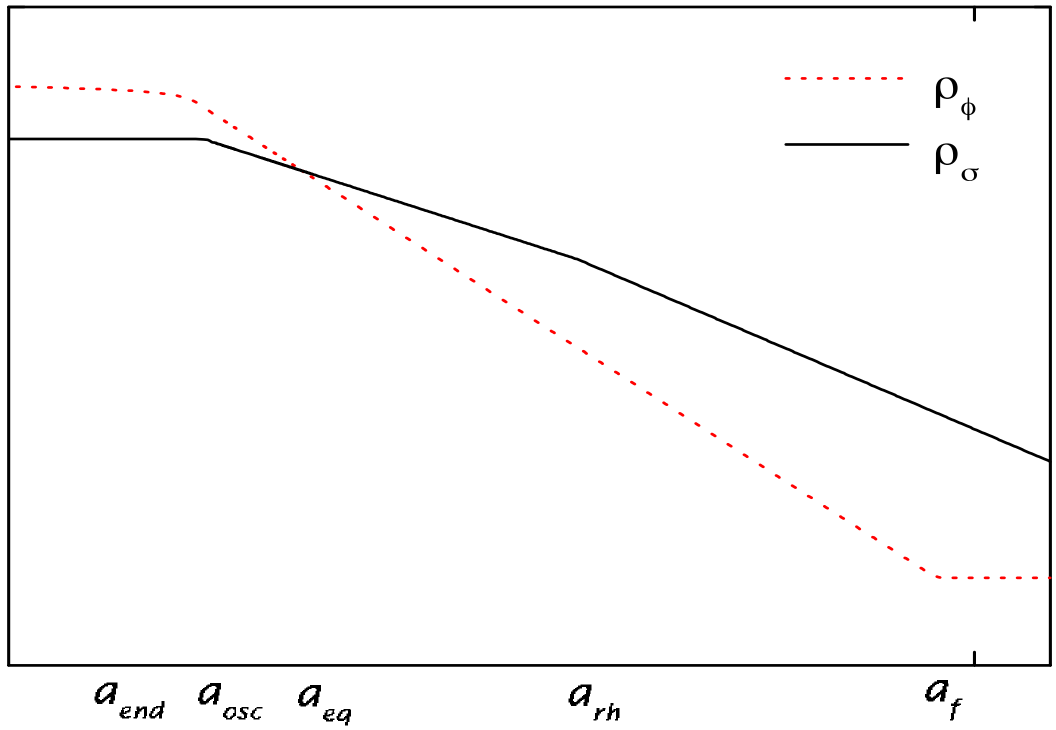


CURVATON PREHEATING

$$\text{Inflaton :} \quad \phi$$

$$\text{Curvaton :} \quad U(\sigma) = \frac{1}{2} m^2 \sigma^2$$

$$\text{Parameters :} \quad m, \sigma_i, \Gamma_{\sigma \to \bar{\psi}\psi}$$



ρ_ϕ
ρ_σ
a_{end}
a_{osc}
a_{eq}
a_{rh}
a_f

Model Building

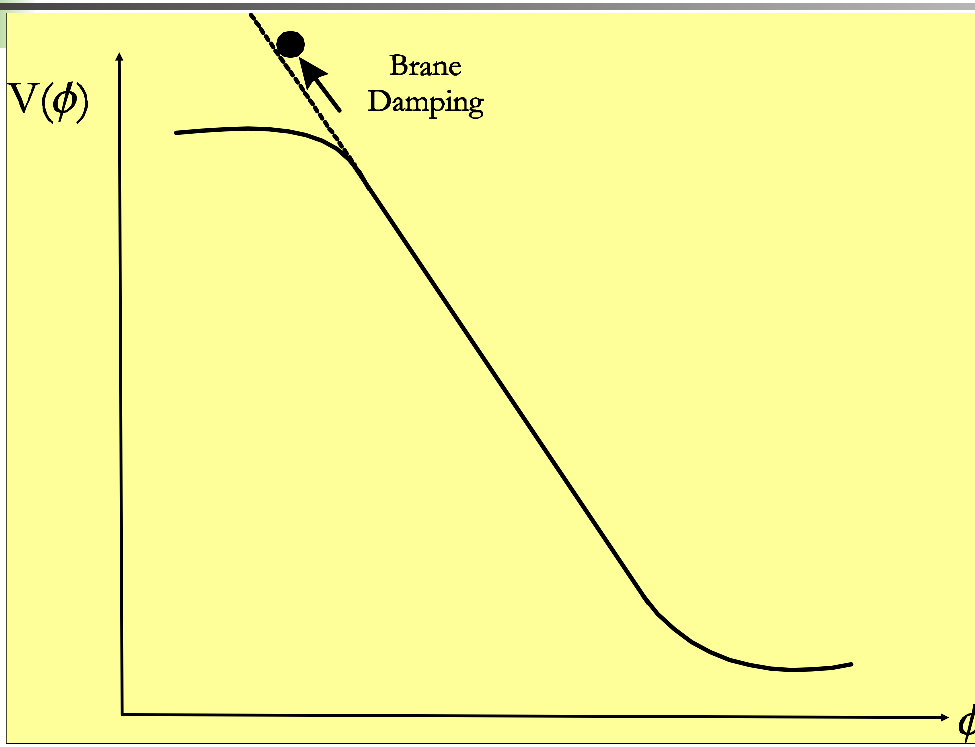
I. Potentials steep at early times and shallow at late stages

II. Potentials shallow at early stages and steep at late times

B.Spokoiny, PLB315,40(1993)
P.J.E. Peebles, A. Vilenkin, PRD 60(1999)
V.Sahni, MS, T. Souradeep, PRD65(2001)
K.Dimopoulos, F. Valle,astro-ph/0111417
W. Hossain et al, IJMPD24(2015)

MS, V. Sahni, Phys.Rev. D70 (2004) 083513

Quintessential inflation Brane world model



$$H^2 = \frac{8\pi G}{3}\rho_b\left(1 + \frac{\rho_b}{2\lambda_B}\right)$$

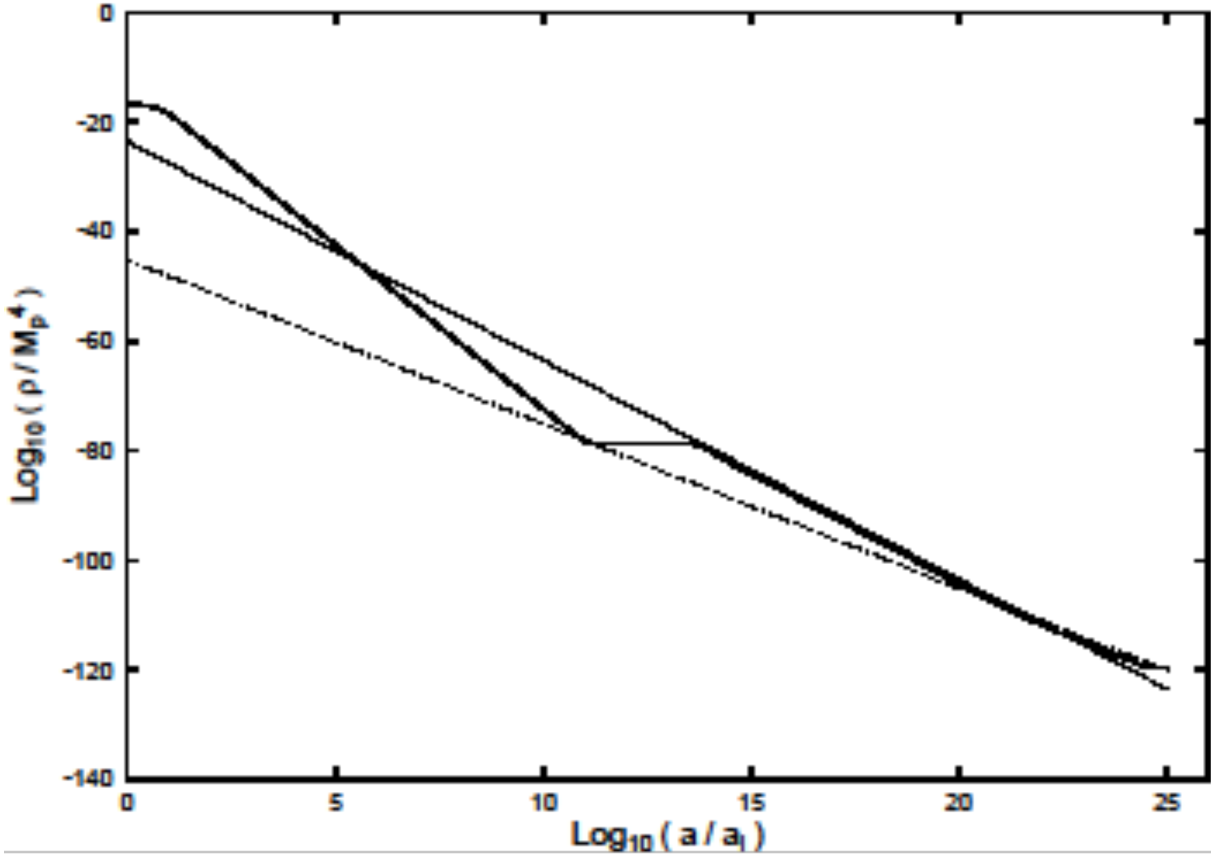
$$\epsilon = \epsilon_{FRW}\frac{1 + V/\lambda_B}{(1 + V/2\lambda_B)^2}$$

$$\eta = \eta_{FRW}(1 + V/2\lambda_B)^{-1}$$

$$V = V_0(\cosh\alpha\phi/M_p - 1)^p;\ p > 0$$

***MS*, *V. Sahni, Phys. Rev. D* 70, 083513 (2004);**
***Phys. Rev. D* 65, 023518 (2002); MS, N. Dadhich, T.Shiromizu, PLB568, 118(2003).**
MS, V. Sahni, Phys.Rev. D70 (2004) 083513

Quintessential Inflation



OBSERVATIONAL CONSTRAINTS

$$n_S = 1 - \frac{4}{N}$$

$$r = \frac{24}{N}$$

$$N = 70, \quad n_S = 0.94, \quad r = 0.34$$

GAUSS BONNET IN THE BULK

effort to rescue the model

$$\boxed{\rho >> M_{GB}^4 \;\Rightarrow H^2 \simeq \rho^{2/3} \quad (GB)}$$

$$\boxed{M_{GB}^4 >> \rho >> \lambda \;\Rightarrow H^2 \simeq \rho^2 \quad (RS)}$$

$$\boxed{\rho << \lambda \;\Rightarrow H^2 \simeq \rho \qquad (GR)}$$

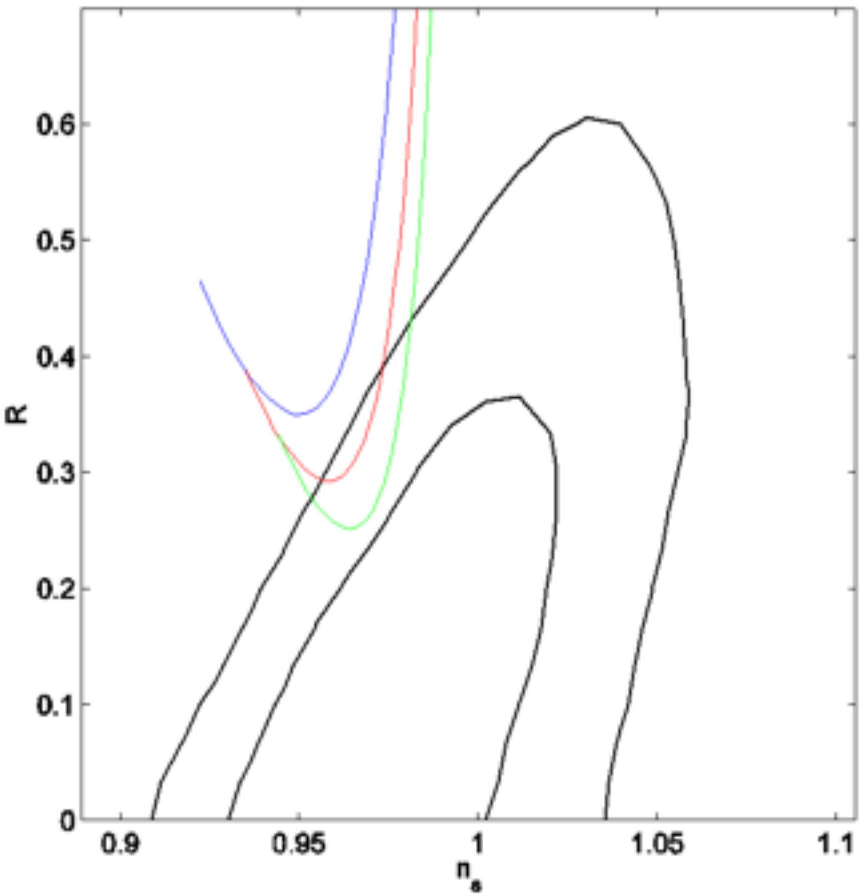
J-F. Dufaux, J. Lidsey,
Roy Maartens, MS,
PR D70 (2004) 083525

Gauss-Bonnet term in the bulk

J-F. Dufaux, J. Lidsey,
Roy Maartens, MS,
PR D70 (2004) 083525

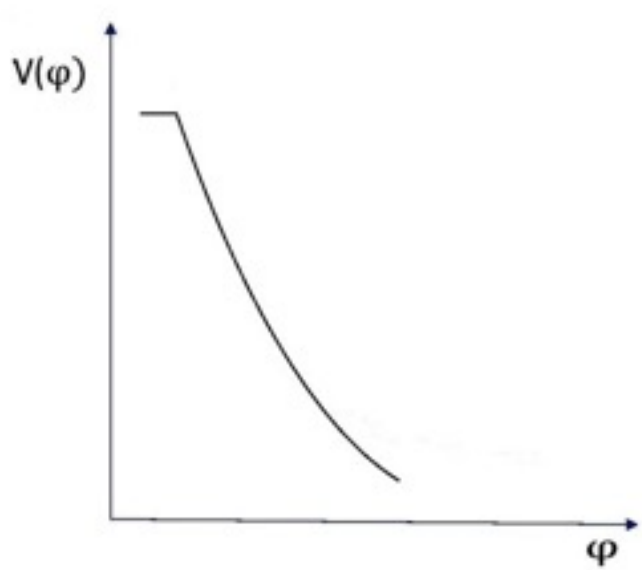
S. Tsujikawa, MS,
Roy Maartens,
PRD70 (2004) 063525

MS, V. Shani,
PR D 70, 083513 (2004)



Quintessential inflation

Standard FRW Framework



C.Wetterich, arXiv:1308.1019

A. Agarwal , MS, N. Singh, PL B770 (2017) 200-208

Wali Hossain, R. Myrzakulov, MS, E. Saridakas, arXiv:1402:6661.

S.Ahmad,MS,Myrzakulov PRD 96, 063515 (2017)

JCAP1706(2017),06,011
C.Q.Geng,C.C. Lee, MS,E.N.Saridakis,
Alexei A. Starobinsky

INFLATION: MODEL

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{M_p^2}{2}R + \frac{k^2(\phi)}{2}\partial^\mu\phi\partial_\mu\phi + V(\phi)\right]$$

$$k^2(\phi) = \left(\frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2}\right)\frac{1}{1+\beta^2 e^{\alpha\phi/M_p}} + 1$$

$$\phi/M_p >> 1, \quad k(\phi) \to 1$$

Model

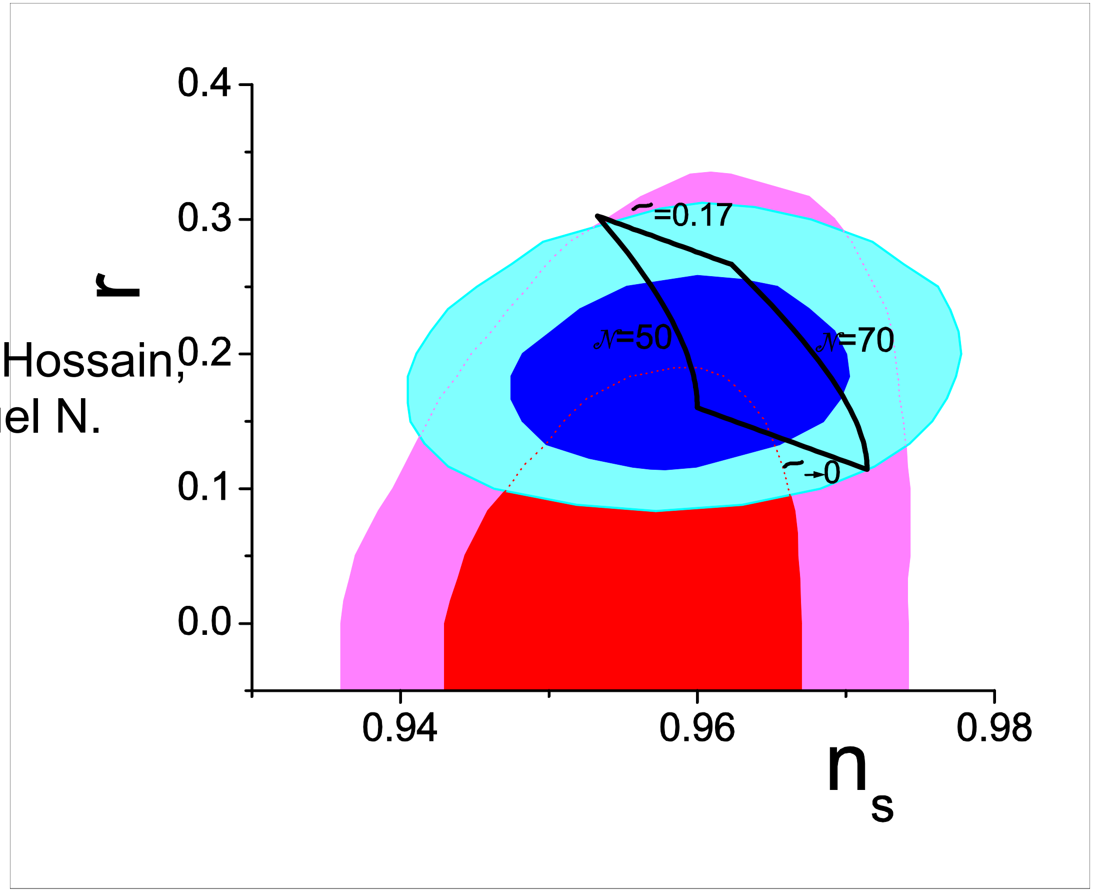
$$\mathcal{S}_E = \int d^4x \sqrt{g} \left[-\frac{M_p^2}{2} R + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + V(\mathbb{k}^{-1}(\sigma)) \right]$$

$$\sigma = \mathbb{k}(\phi); \quad k(\phi) = \frac{\partial \mathbb{k}}{\partial \phi}$$

$$V_s(\sigma) \propto e^{-\tilde{\alpha}\sigma/M_p} \quad ; \quad V_l(\sigma) \propto e^{-\alpha\sigma/M_p}$$

$$\alpha >> 1; \quad \tilde{\alpha} << 1$$

Model



Chao-Qiang Geng, Md. Wali Hossain, R. Myrzakulov, MS, Emmanuel N. Saridakis

PR D 92, 023522 (2015);

PR D 89, 123513 (2014)

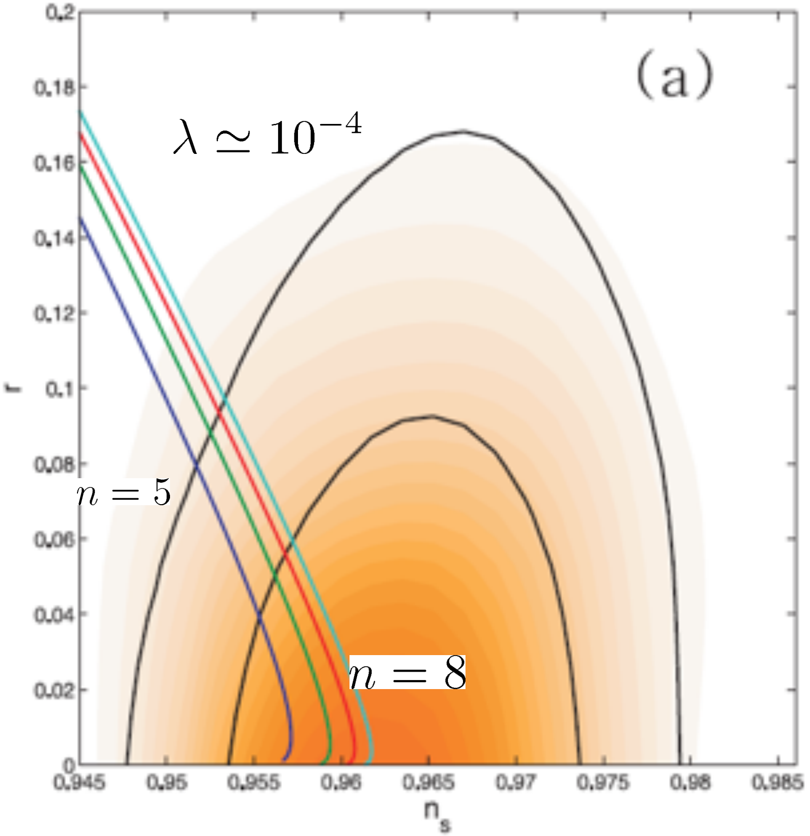
The Successful Model

$$V = V_0 e^{-\lambda\phi^n/M_p^n}; \quad n > 1$$

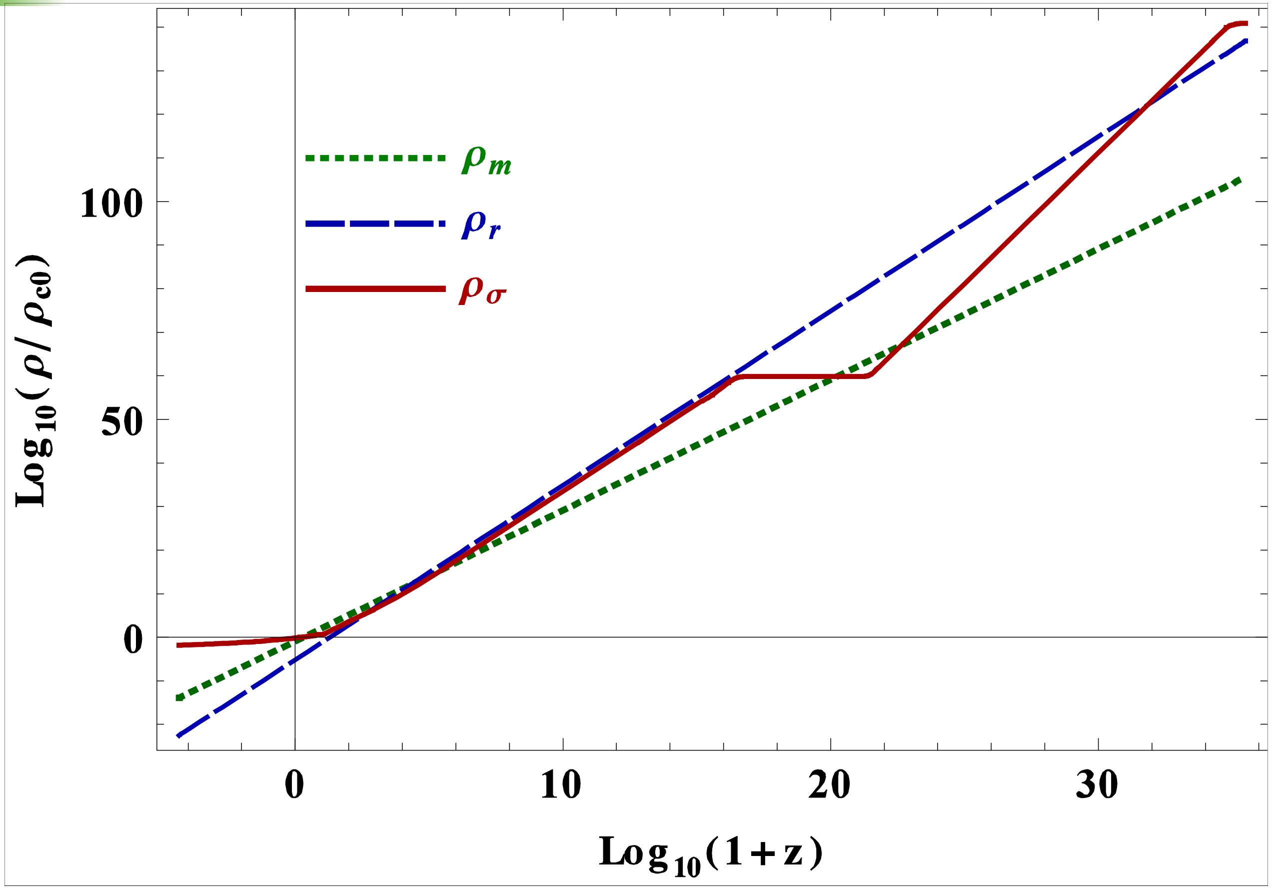
$$slope: \ \lambda\phi^{n-1} << 1(\lambda << 1)$$

$$\lambda\phi^{n-1} >> 1 \quad for\ large\ \phi$$

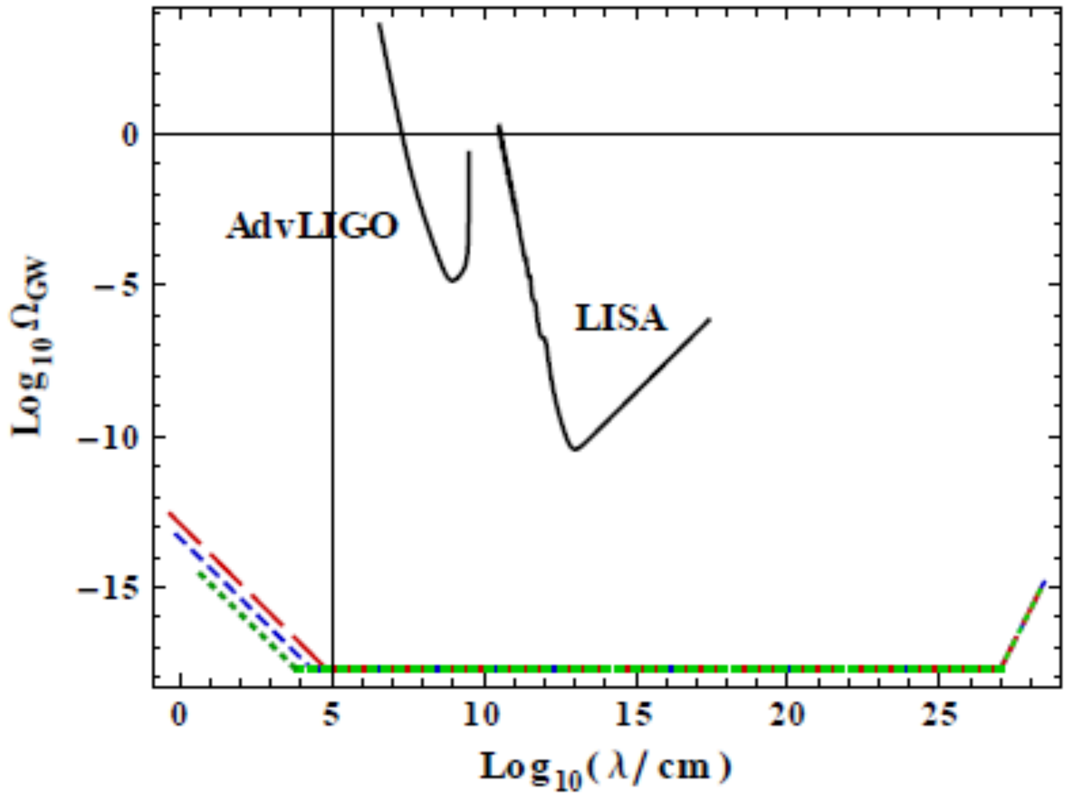
JCAP1706(2017),06,011,
C.Q.Geng,C.C. Lee, **MS**,E.N.Saridakis,
Alexei A Starobinsky



UNIVERSE HISTORY



PREDICTION



AdvLIGO
LISA
0
−5
−10
−15
0
5
10
15
20
25
$\mathrm{Log}_{10}\,\Omega_{GW}$
$\mathrm{Log}_{10}(\lambda/\mathrm{cm})$

THANKS

MODIFIED THEORIES OF GRAVITY

LARGE SCALE MODIFICATION

- Chameleon theories- f(R)
- Modified theories based upon Vainshtein screening
- Galileon theories
- Massive gravity

Extra degrees of freedom: Scalar degree(s) of freedom

Requirements

- Local physics be intact
- Cosmic acceleration

SCALARON: Requirements

DE:

$$m_\varphi^2 = V_{\varphi\varphi}(\varphi) \simeq H_0^2 \simeq (10^{-33} eV)^2$$

Solar Physics:

Chameleon Field

MASS SCREENING OR FIFTH FORCE SUPPRESSION
(Basic Idea)

$$L = -(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{\alpha}{M_p}\varphi T$$

Localized source: $r = 0, \quad T = -M\delta^3(r)$

$$\varphi = -\frac{\alpha M}{M_p}\frac{e^{-mr}}{4\pi r}$$

$$\frac{\alpha}{M_P}\varphi = -2\alpha^2 GM\frac{e^{-mr}}{r}$$

$m, \ \alpha \to m(\rho), \ \alpha(\rho)$ **Chameleon or Symmetron**

$Kinetic \ Suppression \ Galileon$

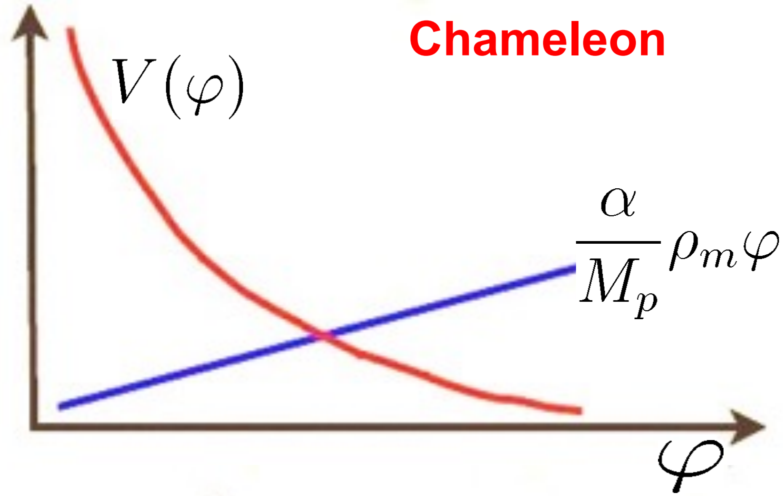
Kinetic suppression

$$\Box\varphi = \left(V(\varphi) + \frac{\alpha}{M_p}\rho_m\varphi\right)_{,\varphi}$$

$$\Box\varphi = +\frac{\alpha}{M_p}\rho_m$$

Chameleon versus Galileon(Vainshtein)

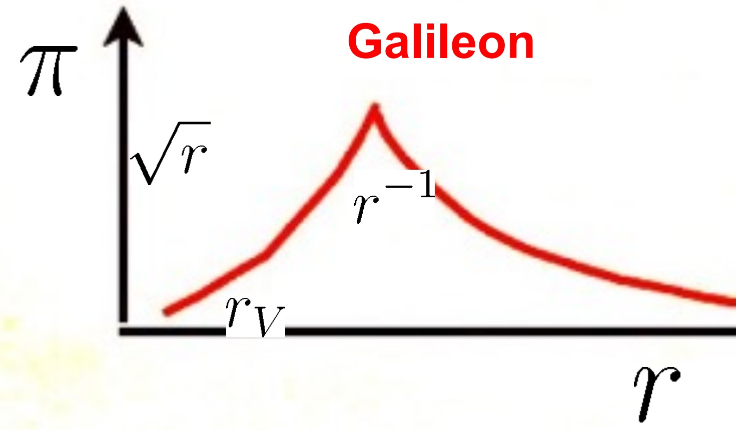
$$V_{eff} = V(\varphi) + \frac{\alpha}{M_p}\rho_m \varphi$$



Galileon

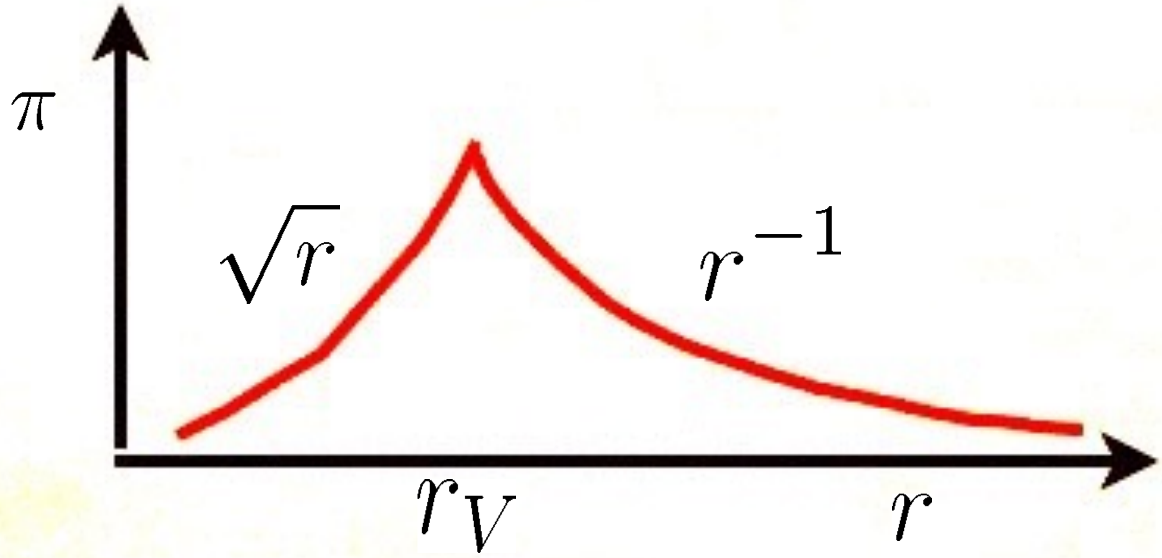
$$L_3 = -\frac{1}{2}(\partial_\mu \pi)^2 + \frac{1}{m^2}(\partial_\mu \pi)^2 \Box \pi$$

$$\Box\pi + \frac{1}{m^2}[(\Box\pi)^2 - \partial^\mu\partial^\nu\pi\partial_\mu\partial_\nu\pi] = \frac{\alpha}{M_p}\rho_m$$

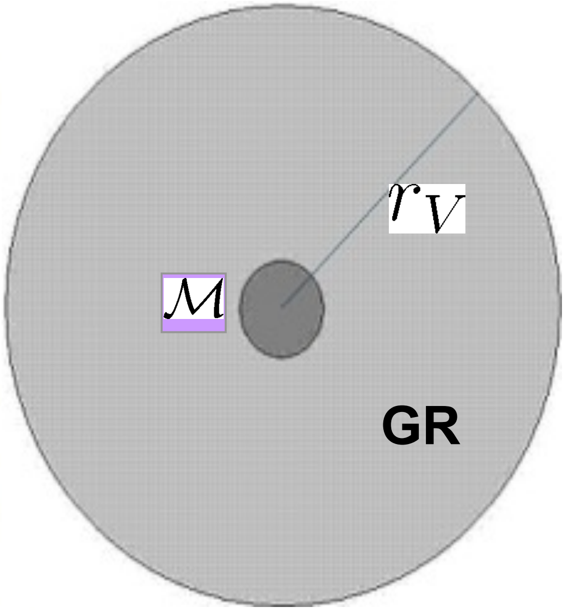


Small distances: $\pi(r) \sim \sqrt{r};$

Large distances: $\pi(r) \sim r^{-1}$

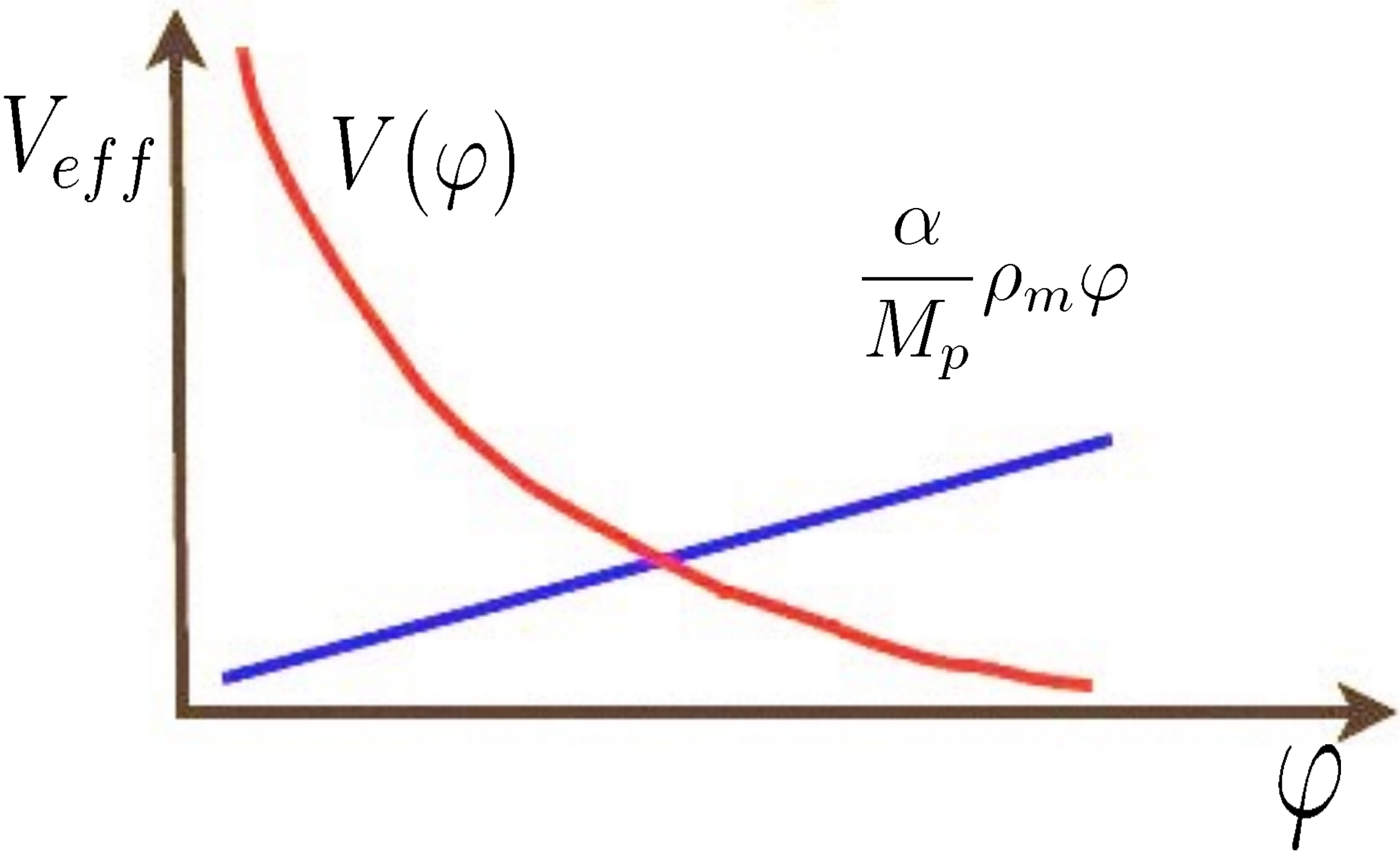


π
$\sqrt{r}$
r^{-1}
r_V
r

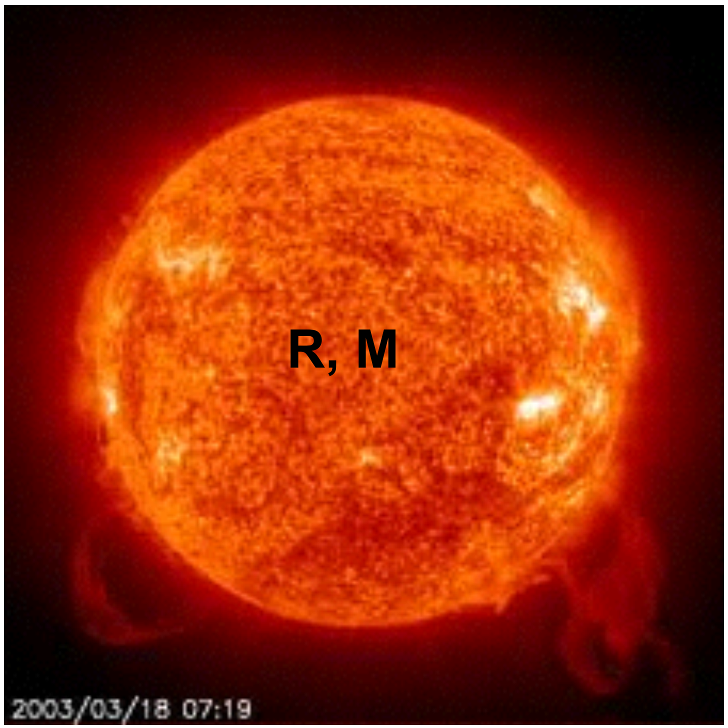


r_V
$\mathcal{M}$
GR

:


V_{eff}
$V(\varphi)$
$\frac{\alpha}{M_p}\rho_m\varphi$
φ

Chameleon at work

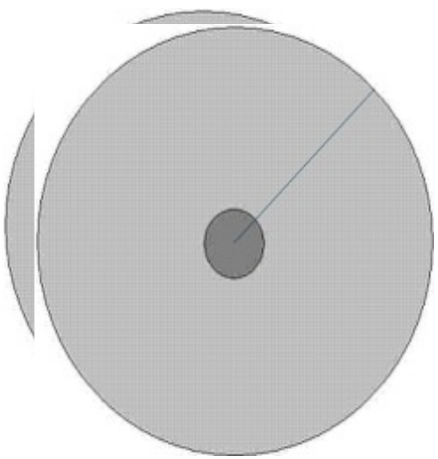


$$\phi_s = -\frac{GM}{r}\alpha^2 \epsilon_{thin}$$

$$\epsilon_{thin} \propto \frac{\varphi^{out}_{min}(\rho) - \varphi^{in}_{min}(\rho)}{\Phi_{\mathcal{M}}}$$

Vainshtein at work

$$r_V = \left(\frac{GM_s}{m^2}\right)^{1/3} = \left(\frac{M_s}{H_0^2 M_p^2}\right)^{1/3} \simeq 100pc$$

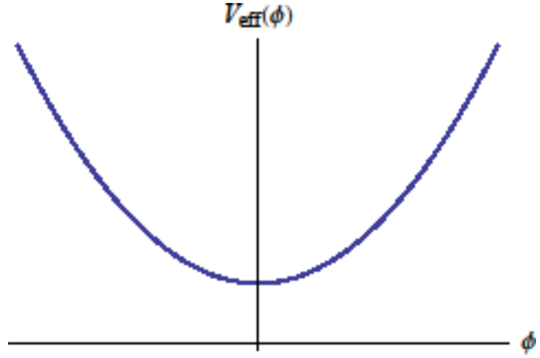


$$r_V(Gal) \simeq 1.2Mpc$$

Symmetron: Dark energy

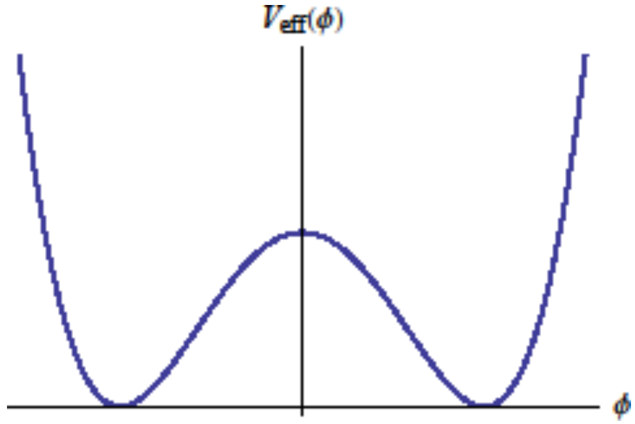
$$V_{\rm eff} = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4; \quad m_{eff} = \left(\frac{\rho}{M^2} - \mu^2\right)$$

$$m_{eff} \simeq \frac{\rho}{M^2} \rightarrow \phi_0 = 0$$



$$m_{eff} = -\mu^2 \rightarrow \phi_0 = \pm\sqrt{\frac{\mu^2 - \rho/M^2}{\lambda}};$$

$$m_s = \sqrt{2}\mu;$$



Symmetron: Dark energy

Symmetry should break:

$$\rho \sim \rho_{cr} = H_0^2 M_p^2 \to m_s \simeq \frac{H_0 M_p}{M}$$

Local gravity constraints

$$M < 10^{-4} M_p \to m_s \simeq 10^4 H_0$$

Chameleon/Symmetron theories-

No scope for self acceleration

arXiv:1284612: J. Wang, L. Hui and J. Khoury

arXiv: 1211.2289: K. Bamba, R. Gannouji, MS, To appear in JCAP

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Massive gravity---Motivation

1939 : **To write down the consistent relativistic equation for spin 2 field**

Now **Cosmic acceleration**

$$M : point\ source \qquad -\frac{GM}{r}e^{-mr};\ m \sim H_0$$

$$h_{\mu\nu} : h'_{\mu\nu} = \Lambda^{\alpha}_{\mu}\Lambda^{\beta}_{\nu}h_{\alpha\beta}$$

Massive gravity—Pauli-Fierz(1939)

$$h_{\mu\nu} : h'_{\mu\nu} = \Lambda_\mu^\alpha \Lambda_\nu^\beta h_{\alpha\beta}$$

$$\left(\Box + m^2\right) h_{\mu\nu} = 0;\ \partial_\mu h^{\mu\nu} = 0;\ h = 0$$

$$\mathcal{L}_{PF} = L_{m=0} - \frac{1}{2} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2\right) + \frac{1}{M_p} h_{\mu\nu} T^{\mu\nu}; (m^2_{ghost} \sim 1/(c-1))$$

$$\mathcal{L}_{m=0} = \frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\lambda h \partial^\lambda h$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

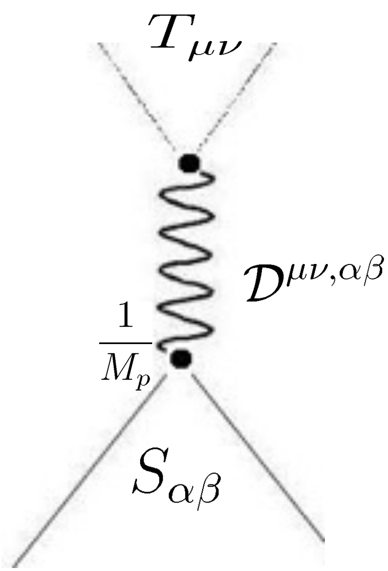
vDVZ discontinuity- 1970

$$\mathcal{D}^{0}_{\alpha\beta,\rho\sigma} = -\frac{1}{k^2}\left[\frac{1}{2}\left(\eta_{\alpha\rho}\eta_{\beta\sigma} + \eta_{\alpha\sigma}\eta_{\beta\rho}\right) - \frac{1}{2}\eta_{\alpha\beta}\eta_{\rho\sigma}\right]$$

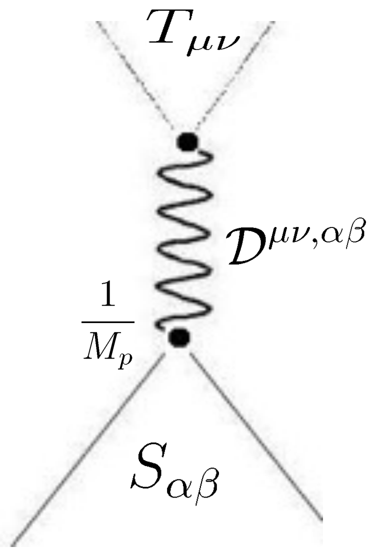
$$\mathcal{D}^{m}_{\alpha\beta,\rho\sigma} = -\frac{1}{k^2+m^2}\left[\frac{1}{2}\left(\eta_{\alpha\rho}\eta_{\beta\sigma} + \eta_{\alpha\sigma}\eta_{\beta\rho}\right) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\rho\sigma}\right]$$

$$A^{(0)} = -\frac{4\pi}{k^2} G M_1 M_2$$

$$A^{(m)} = -\frac{4\pi}{k^2+m^2}\left(\frac{4}{3}G\right) M_1 M_2$$

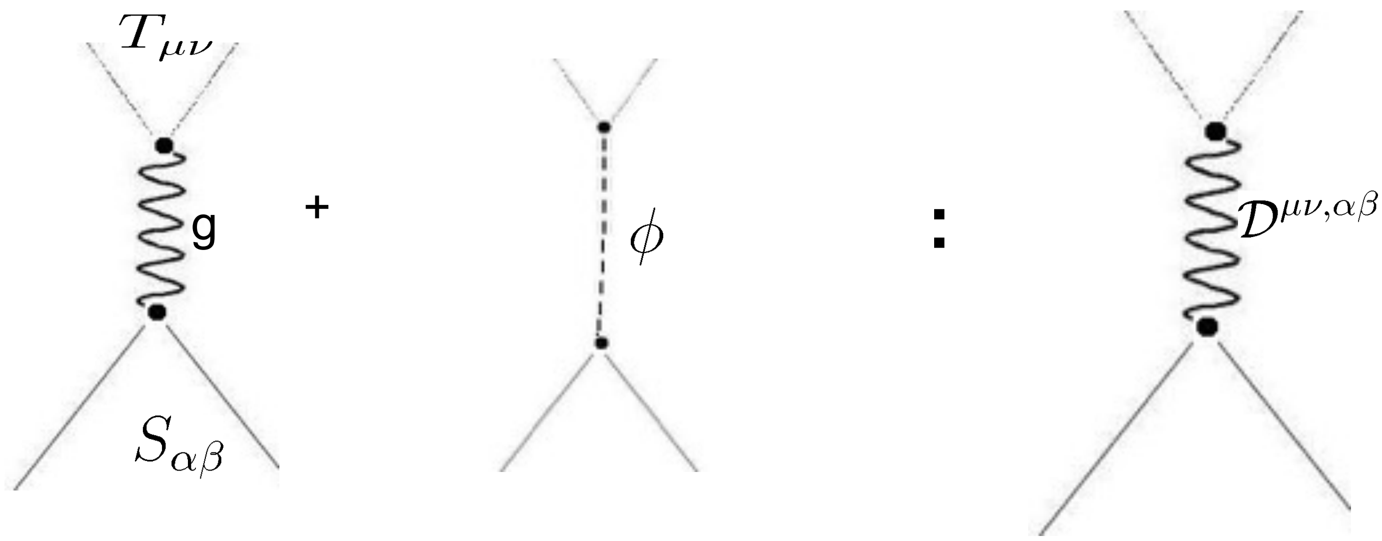


vDVZ discontinuity- 1970

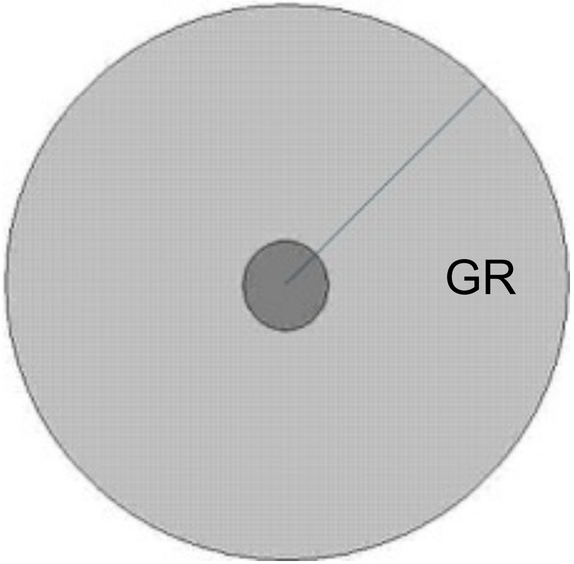


$$h_{\mu\nu} = h^T_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + \partial_\mu \partial_\nu \phi$$

$$\phi : \qquad \frac{1}{M_p}\phi T$$



VAINSHTEIN SCREENING(1972)



$$r_V = \left(\frac{M}{m^2 M_p^2}\right)^{1/3} \simeq 10^{20} cm$$

Ghost---Boulware-Deser

$$\boxed{G_{\mu\nu} - \frac{1}{2}m^2\Big[\left(h_{\mu\nu} - h\eta_{\mu\nu}\right) + \mathcal{O}(h^2_{\mu\nu})\Big] = 0}$$

$$\boxed{\nabla^\mu\left(h_{\mu\nu} - h\eta_{\mu\nu}\right) + \mathcal{O}(h^2_{\mu\nu}) = 0}$$

$$G^\mu_\mu(L) = 2\partial^\mu\partial^\nu\left(h_{\mu\nu} - h\eta_{\mu\nu}\right)$$

$$\boxed{\mathcal{O}(\partial^2 h^2_{\mu\nu}) - \frac{1}{2}m^2\left(-3h + \mathcal{O}(h^2_{\mu\nu})\right) = 0}$$

Ghost---Boulware-Deser

$$\boxed{-\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{\Lambda_3^2}(\Box\phi)^2; \quad \Lambda_3 = \left(m^2 M_p\right)^{1/3}}$$

$$\boxed{-\frac{1}{2}(\partial_\mu\phi)^2 - \partial_\mu\chi\partial^\mu\phi - \frac{1}{2}\Lambda_3^2\chi^2}$$

$$\phi' = \phi + \chi$$

$$\boxed{-\frac{1}{2}(\partial_\mu\phi')^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\phi - \frac{1}{2}\Lambda_3^2\chi^2}$$

Stuckelberg formalism

$$h_{\mu\nu}: \quad H_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

$$\mathcal{L}_m = -\frac{m^2 M_P^2}{2} g^{\mu\nu} g^{\rho\sigma} \left(H_{\mu\rho} H_{\nu\sigma} - H_\mu H_{\rho\sigma} \right)$$

$$\phi^a = x^a$$

$$\phi^a = x^a + \frac{A^a}{M_p} + \eta^{ab} \frac{\partial_b \phi}{m^2 M_p}$$

$$\mathcal{L} = R - \frac{1}{2} m^2 M_p^2 \mathcal{U}(H_{\mu\nu}, g_{\mu\nu})$$

dRGT

C. De Rham, G. Gabadadze, A. Tolley, PRL106, 2011

Decoupling limit

$$M_p \to \infty,\ m \to 0,\ T = \infty,\ \Lambda = fixed,\ \ \frac{T}{M_p} = fixed$$

$$\mathcal{U}_{M_p \to \infty} \equiv \mathcal{U}_{h_{\mu\nu} \to 0} = \mathcal{U}_0 + F_G(\phi) h_{\mu\nu} + \ldots$$

NO FRW cosmology

$$\mathcal{L} = R - \frac{1}{2} m^2 M_p^2 \mathcal{U}(H_{\mu\nu}, g_{\mu\nu})$$

$$\frac{\partial}{\partial t}\left(M_p^2 m^2 a(t)\right) = 0$$

Extended schemes

$$m \to m(\sigma)$$

G. Gabadadze et al, arXiv:1206.4253

K. Bamba, Wali Hossain, Myrzakulov, Nojiri, MS, arXiv:1309..6413(to appear in PRD)

R.Gannouji,,wali Hossain, MS, E. Saridakis, PRD87,123535(2013)

COSMIC ACCELERATION

$$\Lambda$$