Gravitational waves from a spinning particle orbiting a Kerr black hole

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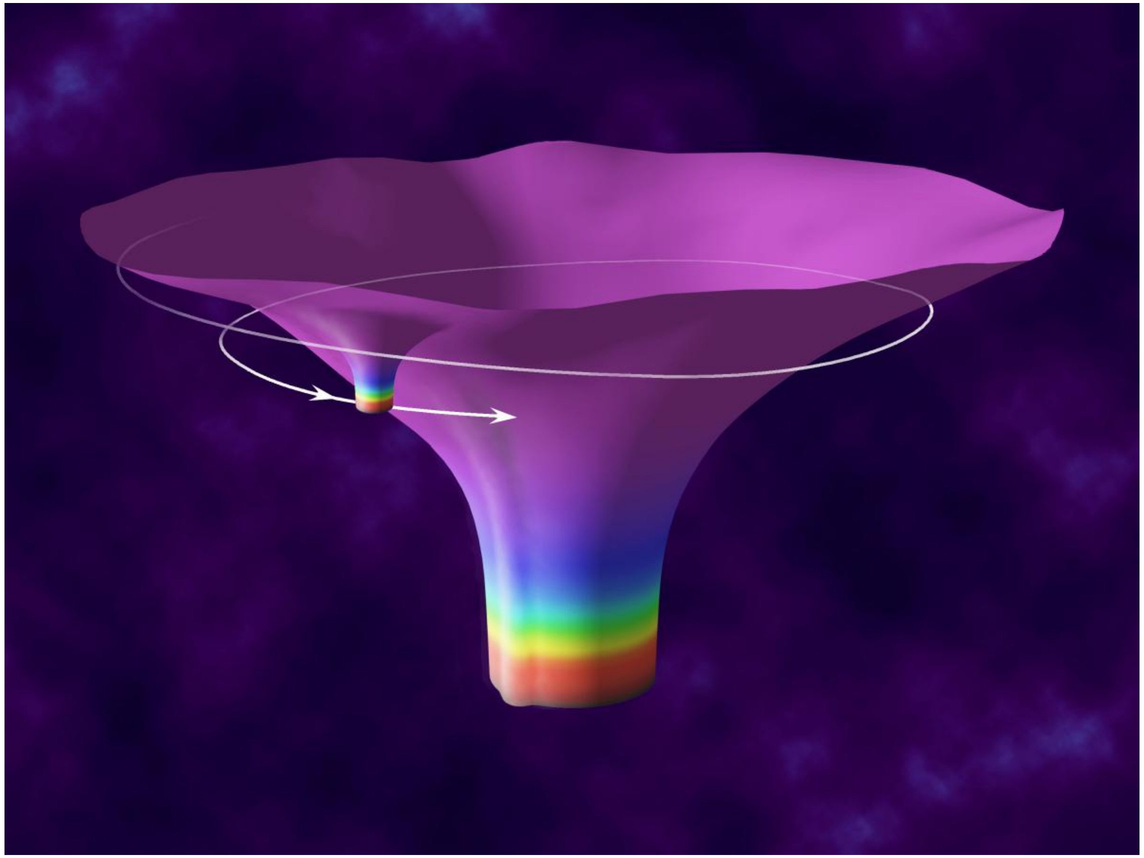
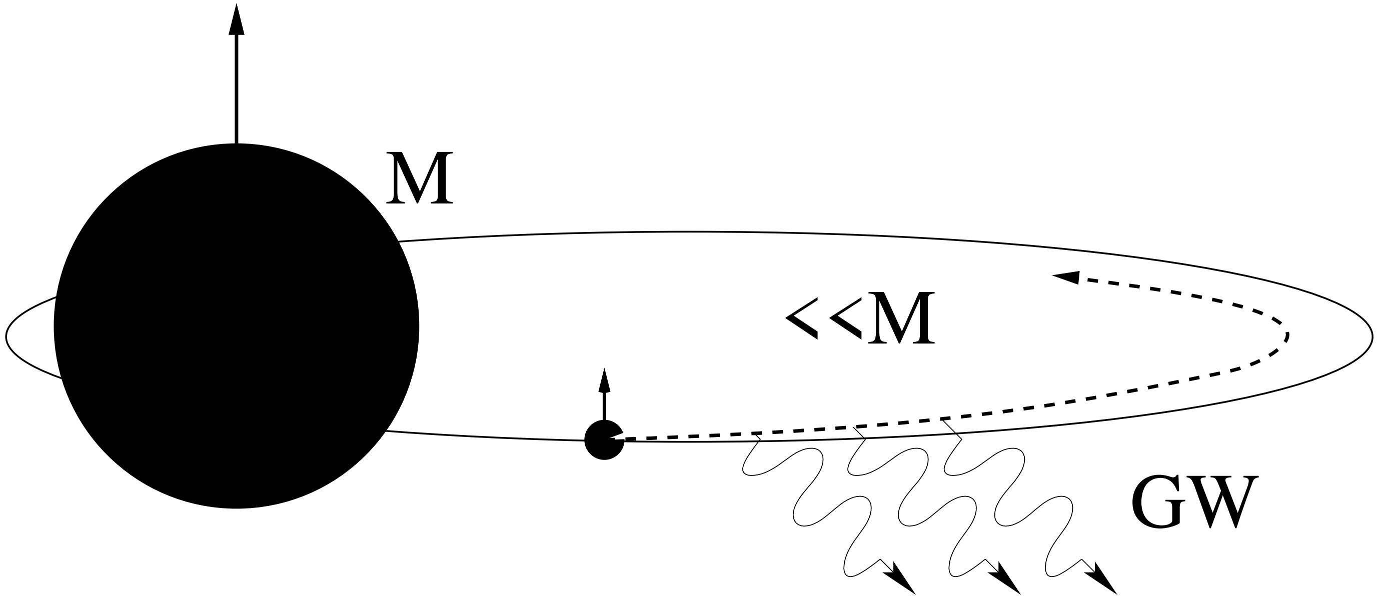
Motivation

- To study extreme-mass ratio inspirals (EMRIs) as GW sources using black hole perturbation theory
  - ⋆ One of the main targets for LISA



Motion of a spinning particle in Kerr spacetime



- Zeroth order in the mass ratio $O[(\mu/M)^0]$:
  - ⋆ Geodesic orbits with (E, L_z, C)
- First order in the mass ratio $O[(\mu/M)^1]$:

  Deviation from the geodesic orbits because of
  - ⋆ Radiation reaction
  - ⋆ Spin of the particle, ...

  How important is the spin of the partcle for orbits and GWs?

Equations of motion of the spinning particle

⋆ Mathisson-Papapetrou-Pirani (MPP) equation:

- ∗ Neglect higher multipoles than quadrupole, accurate up to the linear order in the spin

$$
\begin{aligned}
\frac{D}{d\tau} p^{\mu}(\tau) &= -\frac{1}{2} R^{\mu}{}_{\nu\rho\sigma}(z(\tau)) v^{\nu}(\tau) S^{\rho\sigma}(\tau), \\
\frac{D}{d\tau} S^{\mu\nu}(\tau) &= 2p^{[\mu}(\tau) v^{\nu]}(\tau) (= 0),
\end{aligned}
$$

- ∗ $v^{\mu}(\tau) = dz^{\mu}(\tau)/d\tau$: four-velocity
- ∗ $p^{\mu}(\tau)$: four-momentum
- ∗ $S^{\mu\nu}(\tau)$: spin tensor

$\Rightarrow$ 14 degrees of freedom for 10 equations

⋆ Spin supplementary condition (4 equations):

$S^{\mu\nu}(\tau) p_{\nu}(\tau) = 0$ (determines COM of the particle)

Energy flux in the adiabatic approximation

- $T_{\rm orbit} \ll T_{\rm radiation}$ $[T_{\rm orbit} = O(M),\ T_{\rm radiation} = O(M^2/\mu)]$
  - ⋆ Energy balance argument

$$\left\langle \frac{dE}{dt} \right\rangle_t^{\rm GW} = \sum_{\ell m} \frac{\mu^2}{4\pi\omega^2} \left(\underbrace{\left| Z^{\infty}_{\ell m\omega} \right|^2}_{\rm Infinity\ \ part} + \underbrace{\alpha_{\ell m\omega} \left| Z^{\rm H}_{\ell m\omega} \right|^2}_{\rm Horizon\ \ part} \right),$$

where $\omega = m\Omega_\phi$, $\alpha_{\ell m\omega} \propto \omega - mq/(2r_+)$ and

$$Z^{\infty,{\rm H}}_{\ell m\omega} \sim \int d\tau R^{\rm in/up}_{\ell m\omega}(r) T_{\ell m\omega}(r),$$

$R^{\rm in/up}_{\ell m\omega}(r)$: Homogeneous solutions of the radial Teukolsky equation
$T_{\ell m\omega}(r)$: Source term constructed from energy-momuntum tensor

  - ⋆ Energy-momuntum tensor :

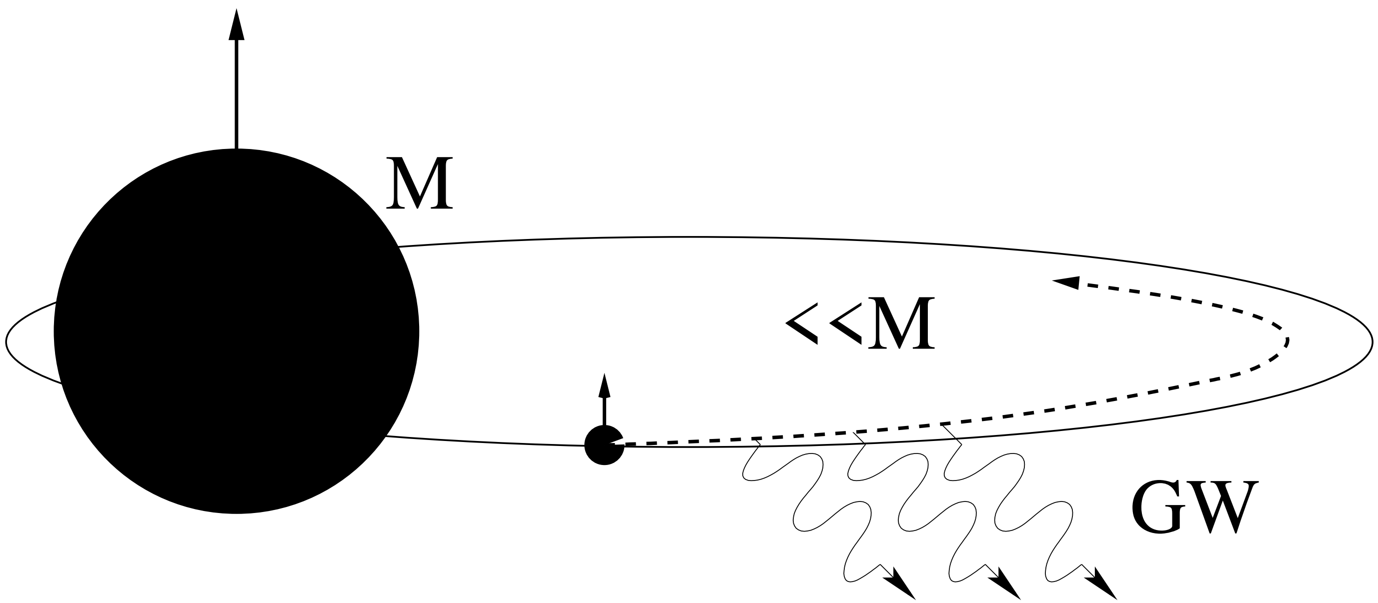
$$T^{\alpha\beta} = \int d\tau \left\{ p^{(\alpha} v^{\beta)} \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} - \nabla_\mu \left(S^{\mu(\alpha} v^{\beta)} \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} \right) \right\}.$$

How to calculate the energy flux

- Solve the EOM for a spinning particle: $v^\mu(\tau)$, $p^\mu(\tau)$, $S^{\mu\nu}(\tau)$
- Construct energy-momentum tensor
  - ⋆ The source term of the Teukolsky equation $T_{\ell m\omega}(r)$
- Solve the Teukolsky equation $R^{\mathrm{in/up}}_{\ell m\omega}(r)$
  - ⋆ The analytic method by Mano et al. (1995)
    - ∗ $R^{\mathrm{in/up}}_{\ell m\omega}(r) \sim \sum a^\nu_n F_{n+\nu}(r)$
    - ∗ $a^\nu_{n+1}\alpha^\nu_n + a^\nu_n\beta^\nu_n + a^\nu_{n-1}\gamma^\nu_n = 0$
- Calculate the amplitude of each mode $Z^{\infty/\mathrm{H}}_{\ell m\omega}$
- Sum over all modes $\displaystyle\left\langle \frac{dE}{dt} \right\rangle^{\mathrm{GW}} \sim \sum_{\ell m} \left| Z^{\infty/\mathrm{H}}_{\ell m\omega} \right|^2$

spin-aligned binary in circular orbit

- As a first step:
  - ⋆ Circular and equatorial orbits
  - ⋆ Particle's spin is parallel to the BH spin



PN flux for spin-aligned binary in circular orbit

	$(\text{spin})^1$	$(\text{spin})^2$	$(\text{spin})^3$
PN	3.5PN (NNLO) [Bohé+(2013)]	3PN (NLO) [Bohé+(2015)]	3.5PN (LO) [Marsat (2015)]
BHP	2.5PN for μ's spin [Tanaka+(1996)]	$*$	$*$
This work	6PN for μ's spin [+ BH absorption]	$*$	$*$

[$*$: In BHP, PN fluxes are derived without expanding in M's spin]

- $\star$ nPN means $(M\Omega_\phi)^{2n/3}$ correction to leading order
- $\star$ 4PN means $(M\Omega_\phi)^{8/3}$ correction to leading order

  where $M\Omega_\phi$ is the orbital frequency

Phase shift due to the particle's spin

- Orbital phase $\Phi \equiv \int \Omega_\phi(t)dt = \int \Omega_\phi \frac{dE}{d\Omega_\phi}\left(\frac{dE}{dt}\right)^{-1} d\Omega_\phi$
- Phase shift due to μ's spin: $\delta\Phi = \Phi(\hat{s} \neq 0) - \Phi(\hat{s} = 0)$
  - ⋆ Early inspiral for one-year observation of LISA
    - ∗ $(M\Omega_\phi^{(i)})^{1/3} \sim 0.2$, $(M\Omega_\phi^{(f)})^{1/3} \sim 0.25$ and $(M, \mu) = (10^5, 10)M_\odot$
  - ⋆ Late inspiral for one-year observation of LISA
    - ∗ $(M\Omega_\phi^{(i)})^{1/3} \sim 0.3$, $(M\Omega_\phi^{(f)})^{1/3} \sim 0.4$ and $(M, \mu) = (10^6, 10)M_\odot$

$(a, s) = (0.9M, 0.9\mu)$	Early inspiral	Late inspiral
$\delta\Phi$ at 1.5PN	3.96	2.14
$\delta\Phi$ at 2PN	-1.25	-1.04
$\delta\Phi$ at 2.5PN	1.17	1.51
$\delta\Phi$ at 3PN	-0.64	-1.28
$\delta\Phi$ at 3.5PN	0.34	1.06
$\delta\Phi$ at 4PN	-0.14	-0.66
$\delta\Phi$ at 4.5PN	0.05	0.41
$\delta\Phi$ at 5PN	-0.01	-0.19
$\delta\Phi$ at 5.5PN	-0.0003	0.02
$\delta\Phi$ at 6PN	0.003	0.07
Φ up through 6PN	700472.95	1051632.90

Summary

- Gravitational waves from a spinning particle around a Kerr BH
  - ⋆ 6PN energy flux for circular and spin-aligned orbits
    - ∗ New terms beyond 3.5PN
    - ∗ BH absorption is also derived
  - ⋆ Phase shift due to the particle's spin $\delta\Phi$
    - ∗ $\delta\Phi \lesssim 1$ at 4PN and beyond for typical binaries in the LISA band
      $[\delta\Phi = \Phi(\hat{s} \neq 0) - \Phi(\hat{s} = 0)]$
- Future
  - ⋆ Circular and slightly inclined orbits
    - ∗ Spin-spin precessions
  - ⋆ Eccentric and spin-aligned orbits in the equatorial plane
    - ∗ Periastron shift
  - ⋆ More generic orbits? Coupling between orbit and spins
    - ∗ Ω_ϕ, Ω_r, Ω_θ and $\Omega_{\rm spin-prec}$?