香港中文大學
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Spin dynamics of a millisecond pulsar around a massive black hole

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Outline

Theory

Mathisson–Papapetrou–Dixon (MPD) formulation

↓

Simulation and interesting results

spin precession | orbital precession

Pulsar observation

Gravitational wave

Motion of test particle

- Weak Equivalence Principle

  The world line of a freely falling test body is independent of its composition or structure.

- Non-spinning object – geodesic equation

$$\frac{Du^\mu}{d\tau} = 0$$

Motion of extended body

- Spinning object – Mathisson-Papapetrou-Dixon equations

$$\frac{\mathrm{D}P^{\lambda}}{\mathrm{d}s} = -\frac{1}{2}u^{\pi}S^{\rho\sigma}R^{\lambda}{}_{\pi\rho\sigma}$$

Spin-curvature coupling and spin-orbit coupling

$$\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}s} = P^{\mu}u^{\nu} - P^{\nu}u^{\mu}$$

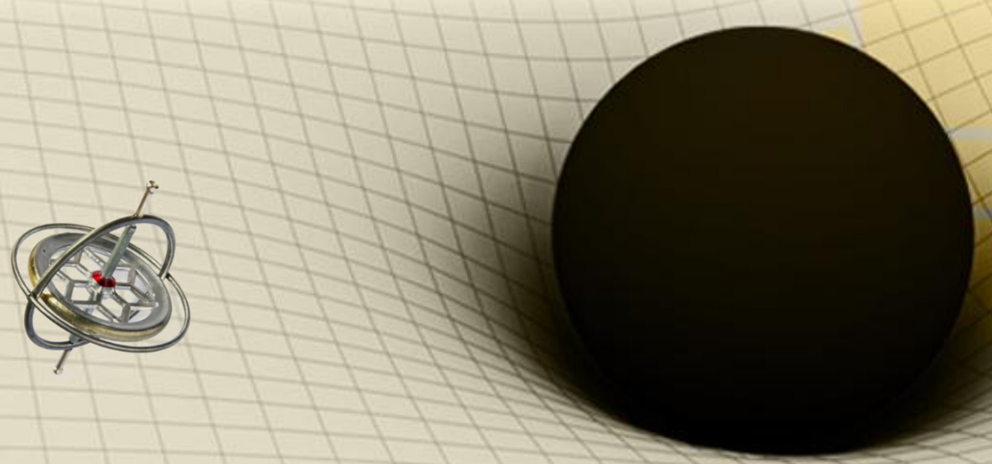
Precession of spin axis

$$P^{\lambda} = mu^{\lambda} + S^{\lambda\mu}\frac{\mathrm{D}u_{\mu}}{\mathrm{d}s}$$

Hidden momentum

EMRI binary system

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2$$



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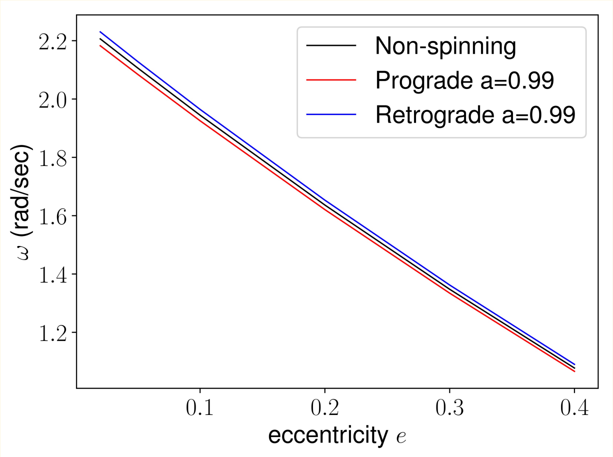
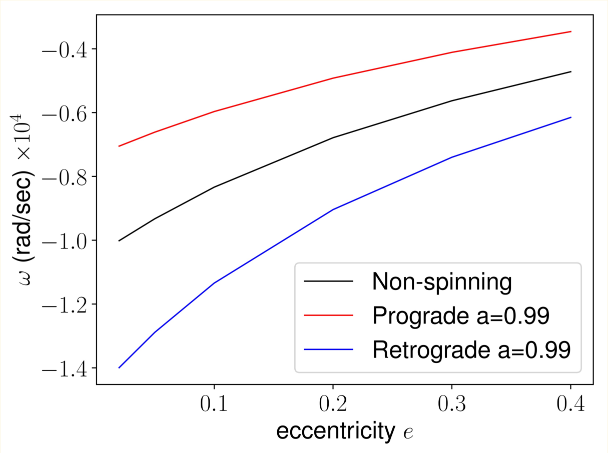
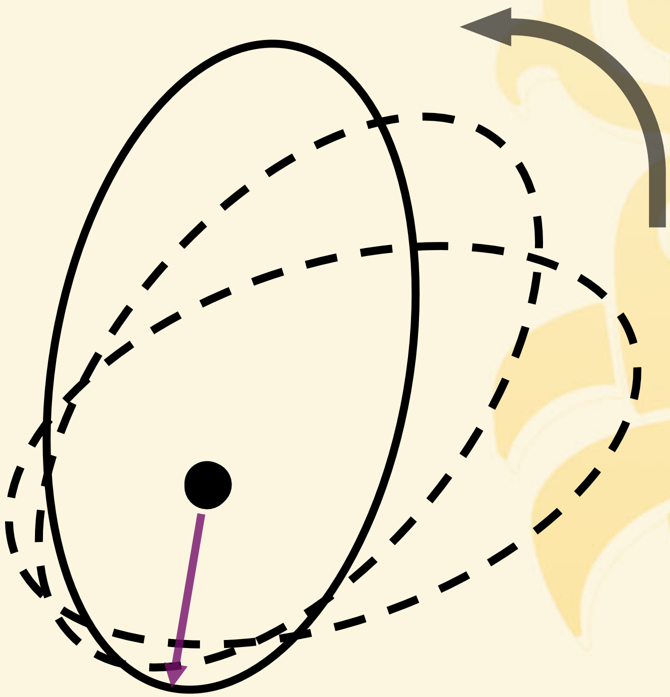
Precession of the NS orbit

Geodesics precession



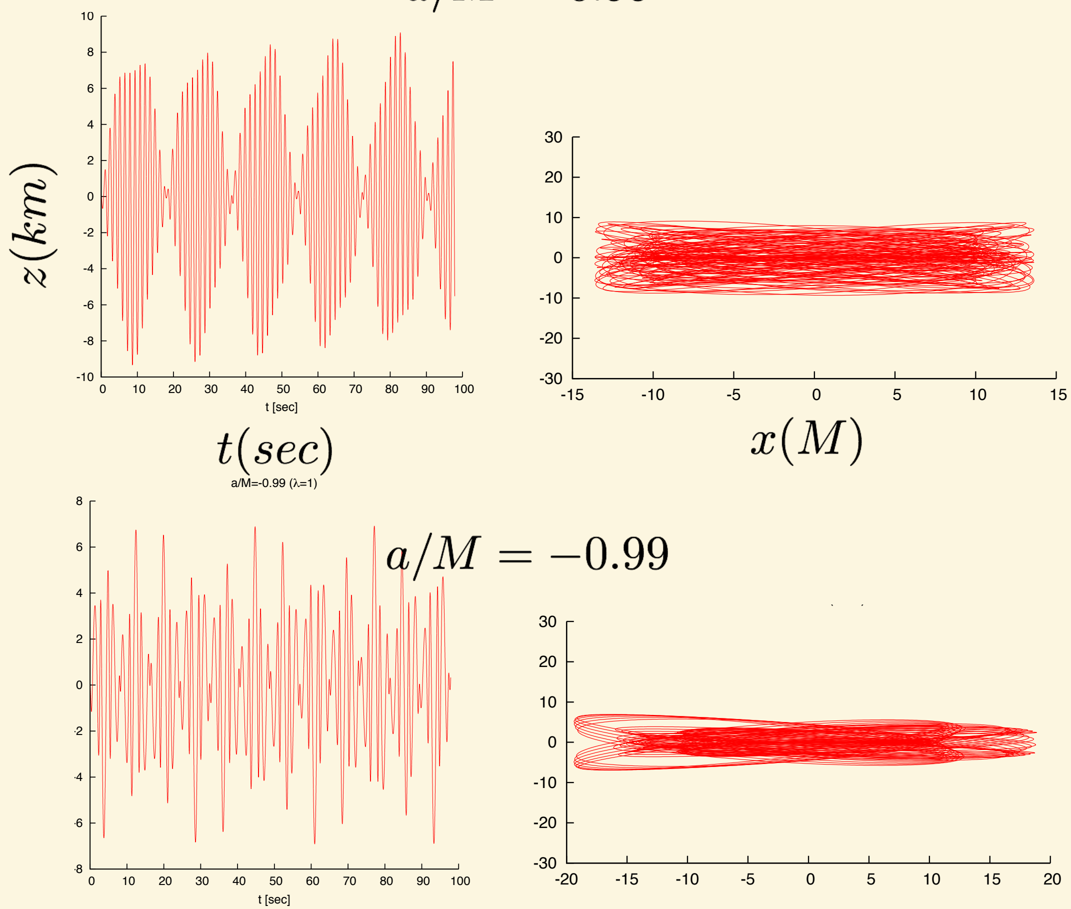
Corrections due to spin



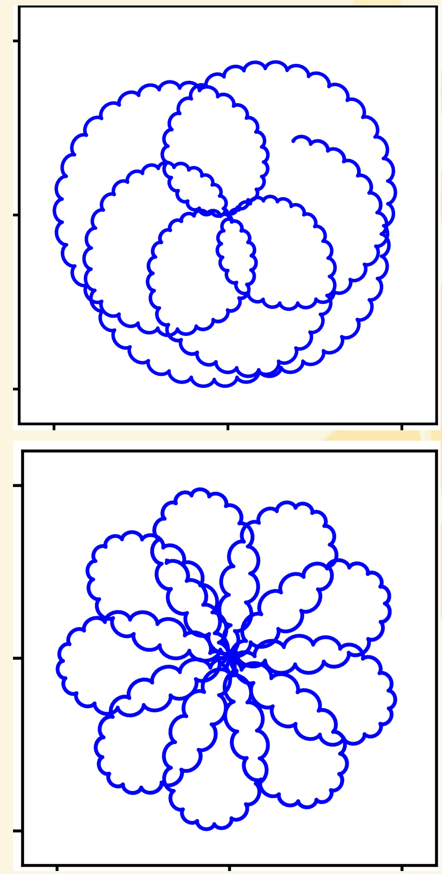


Non-planar orbital motion

$a/M = 0.99$



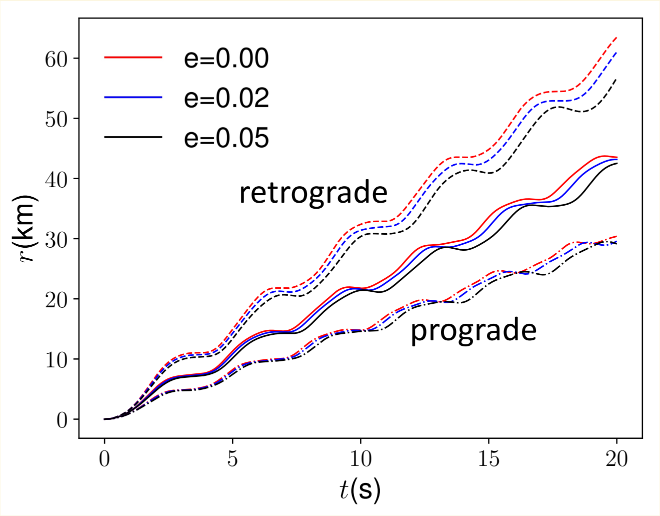
The precession of the orbital plane



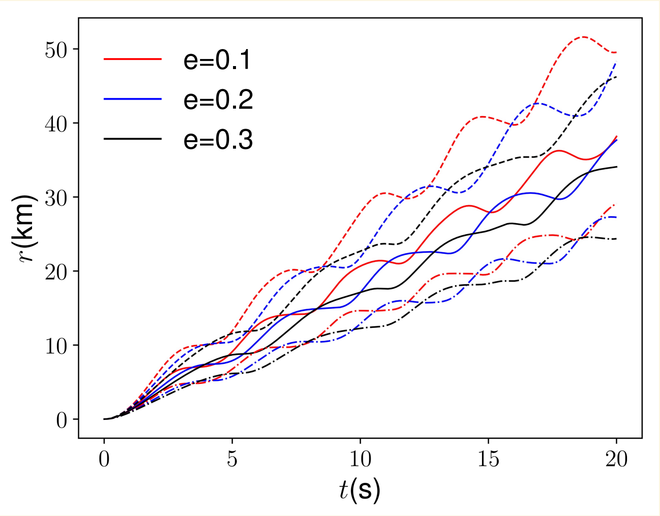
(Singh, Wu, and Sarty, 2014)

Deviation from the geodesics

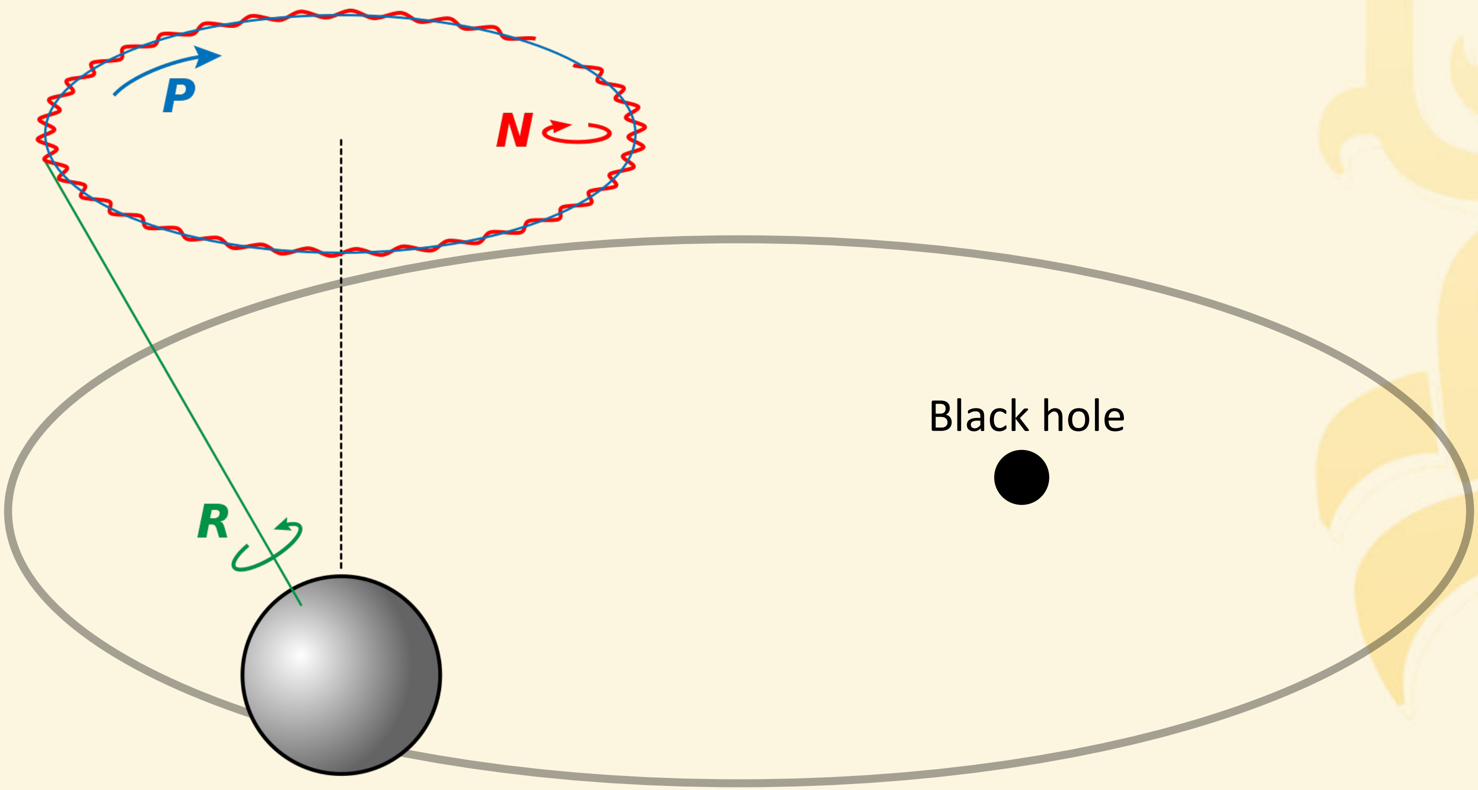
low eccentricity
(quasi-circular)



eccentric

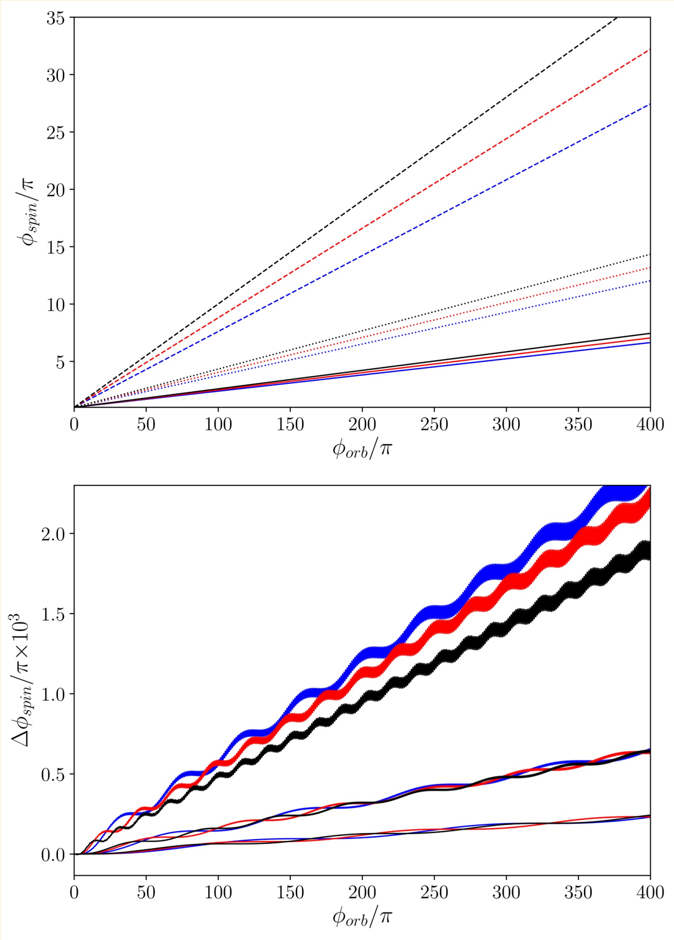


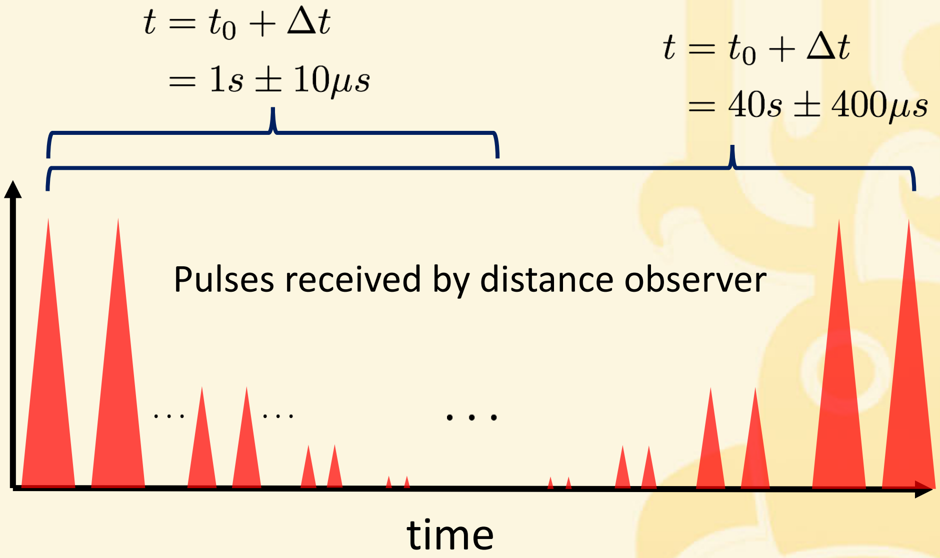
Precession of spinning-axis



(image credit: H. Sulzer)

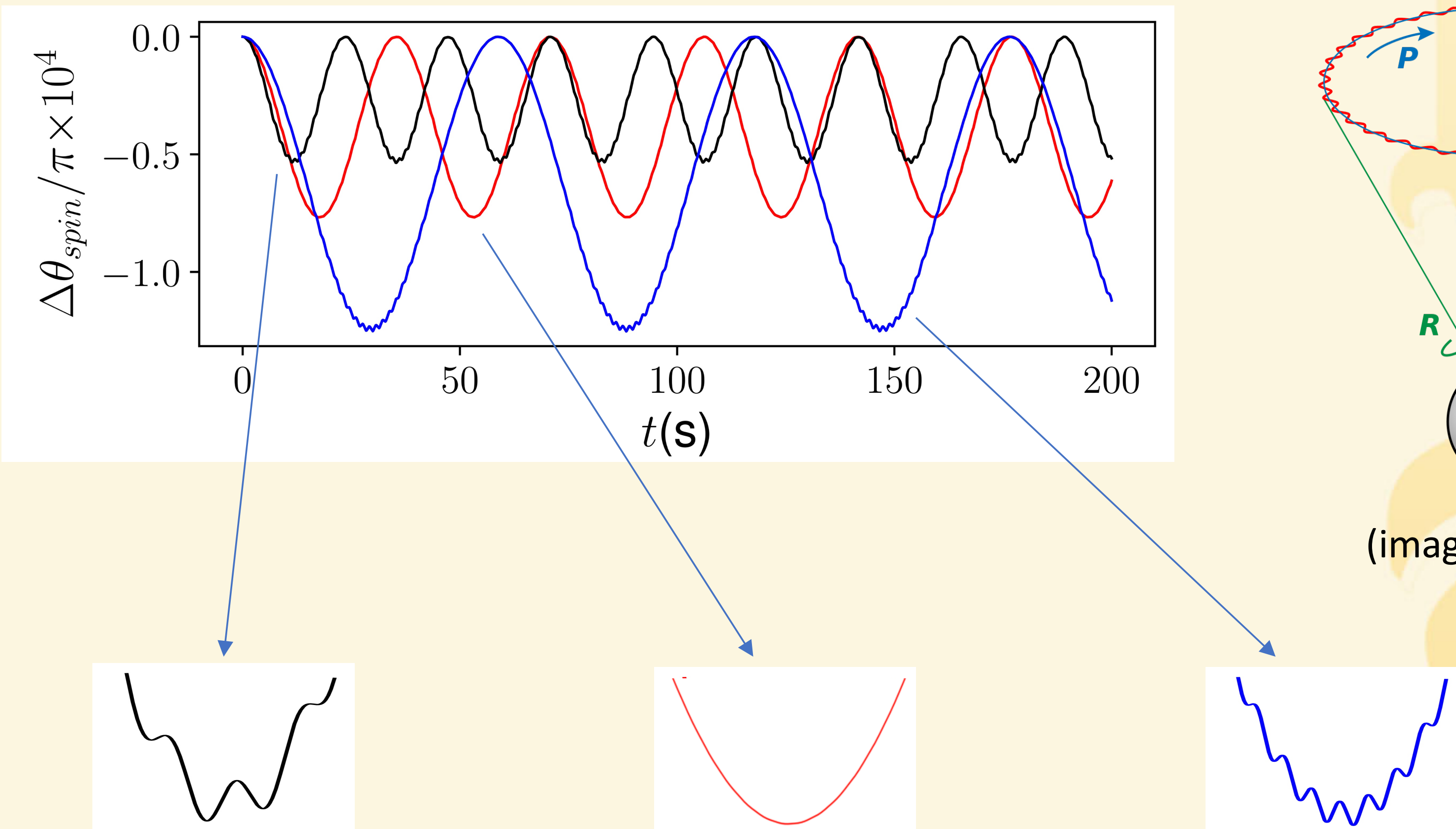
Precession of spinning-axis

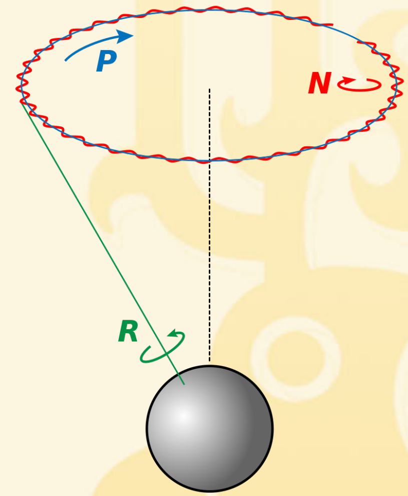




- ✓ Pulse profile changes or even disappears when the spinning-axis wobbles around
- ✓ Assume a conal emission ~10^o , the time shifts by about $10\mu s$ and $400\mu s$.

Nutation of spinning-axis

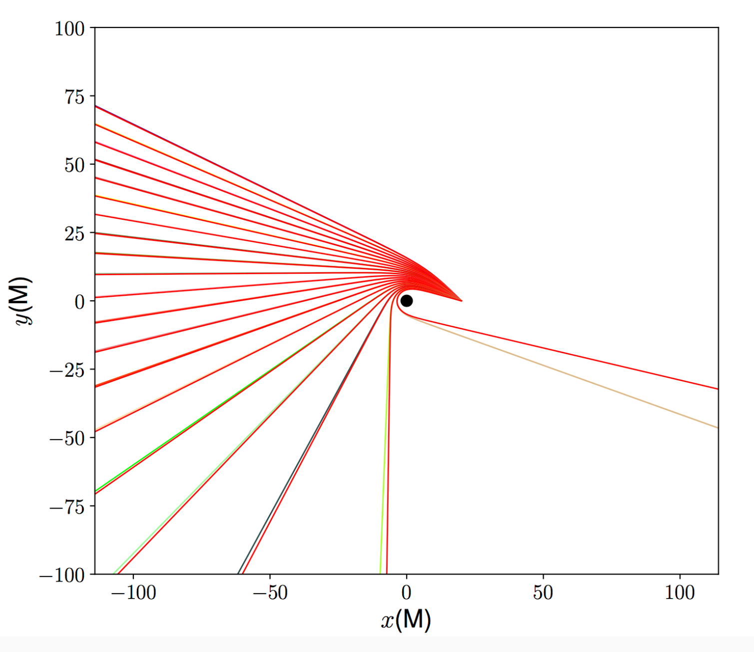


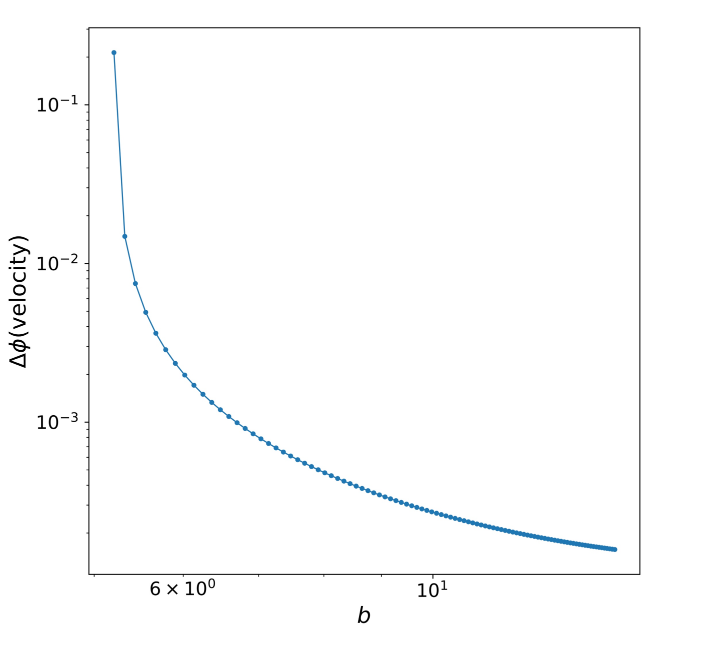


(image credit: H. Sulzer)

Gravitational lensing effect

- A small perturbation $\sim 10^{-4}$ rad becomes up to ~ 0.1 rad due to lensing of the black hole





Red solid lines: photon paths with different impact parameter $b = \dfrac{Lz}{E}$
Colorful lines: photon paths with perturbed initial velocity

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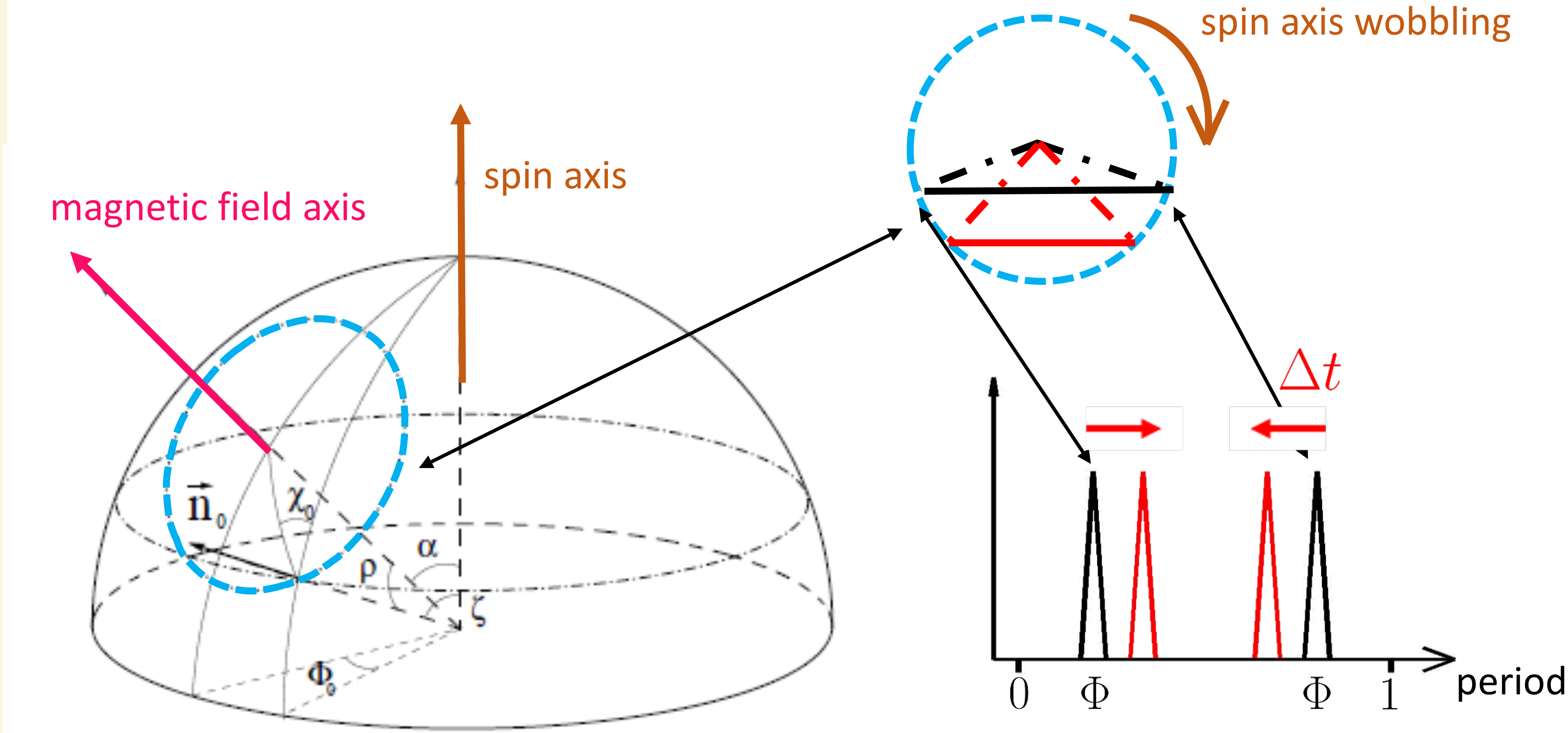
Spin-axis wobbling effect

✓ Pulse time shift: $\Delta t = \Delta t_A + \Delta t_L + \cdots$

✓ Pulse profile shift: $\Delta \xi = \Delta \xi_A + \Delta \xi_L + \cdots$

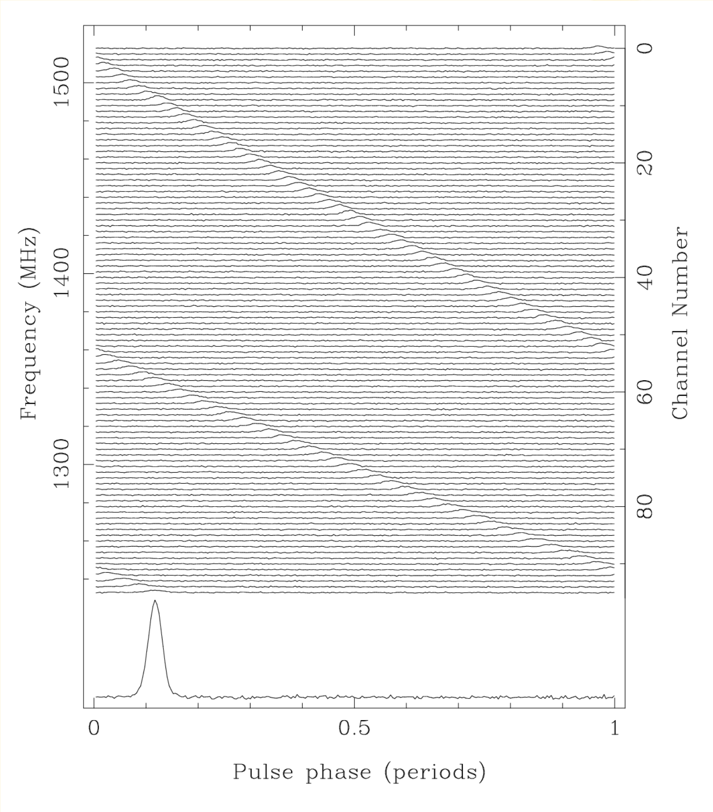
$\Rightarrow$ $\Delta t = \Delta t_{(A,L,etc)}$



(Rafikov and Lai 2008)

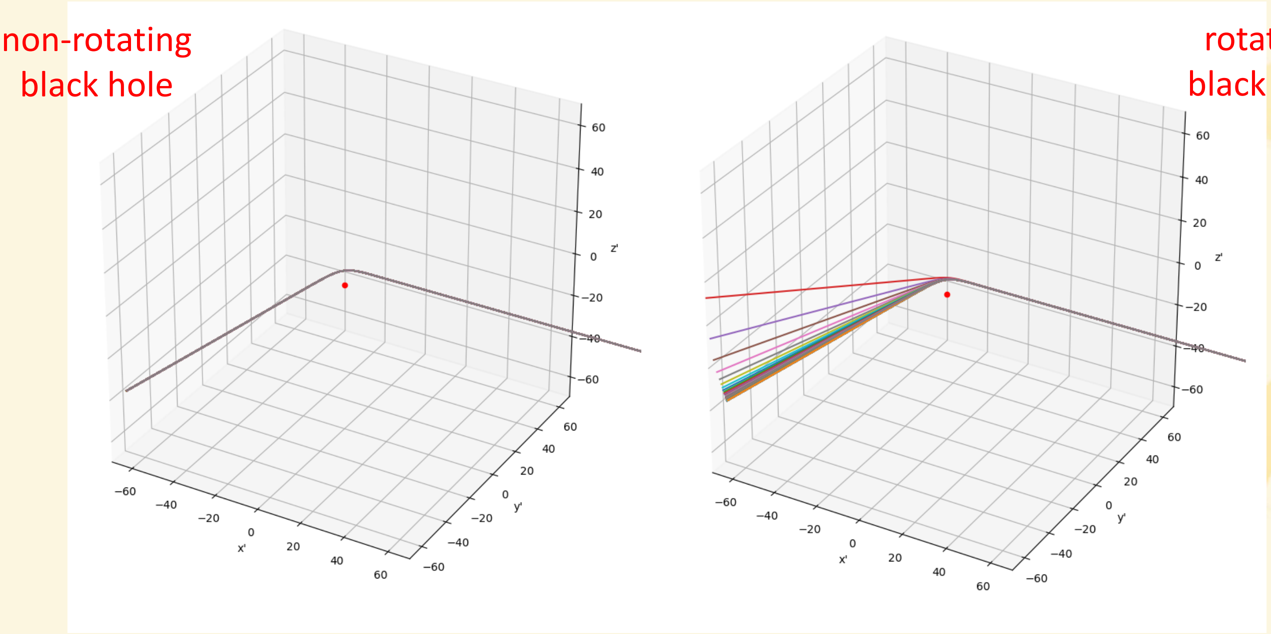
Temporal dispersion of pulse signals

- ✓ Pulse arrival time dispersion in the presence of line-of-sight plasma
- ✓ Pulse emission in all frequencies follow the same trajectory but will arrive at different time



(Lorimer, D Ross, and M Kramer)

Frequency dependent spatial dispersion of emission



(Kimpson, Wu and Zane 2018)

✓ Emission of different frequencies have different paths under the gravity of a rotating black hole

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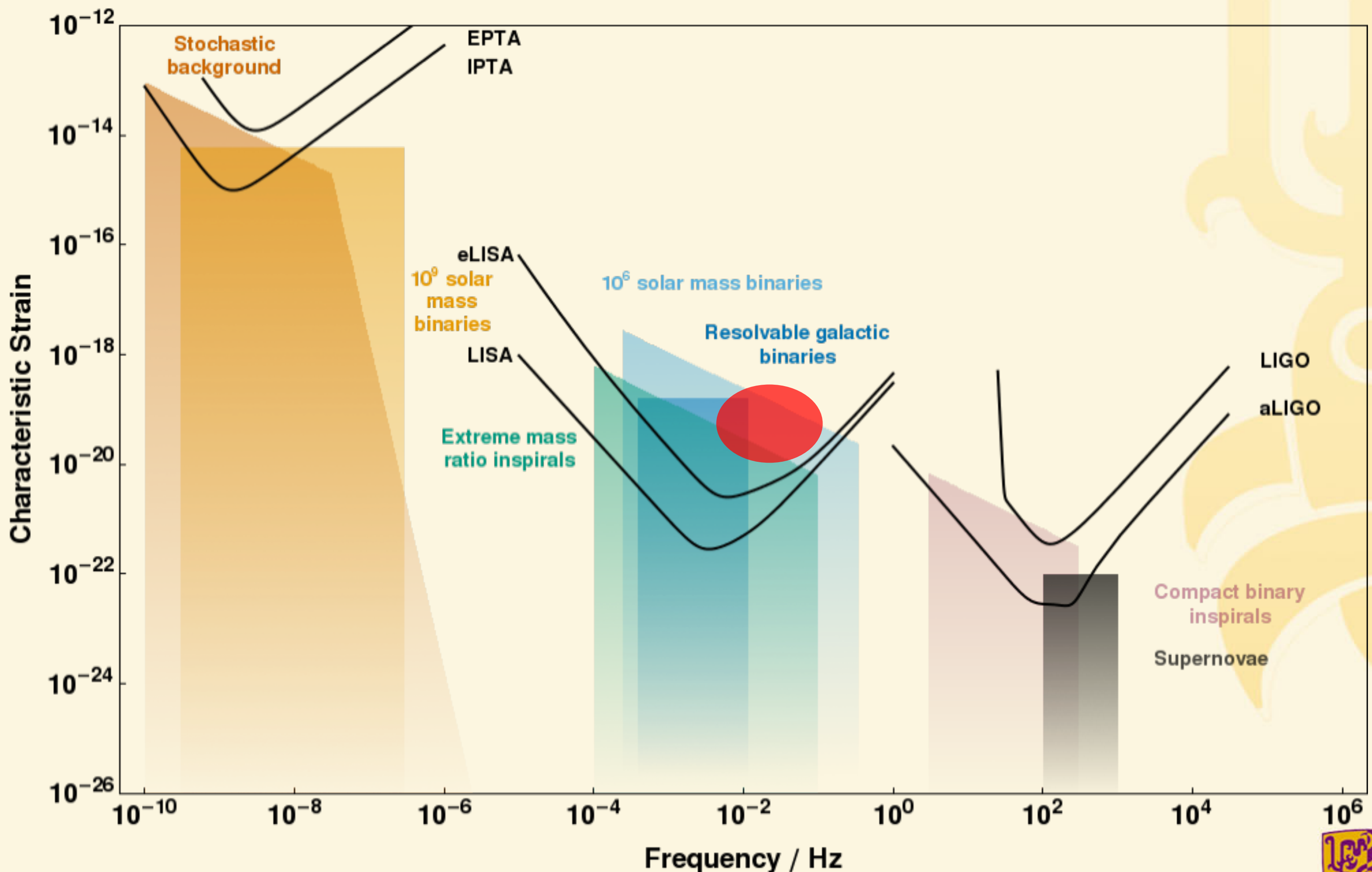
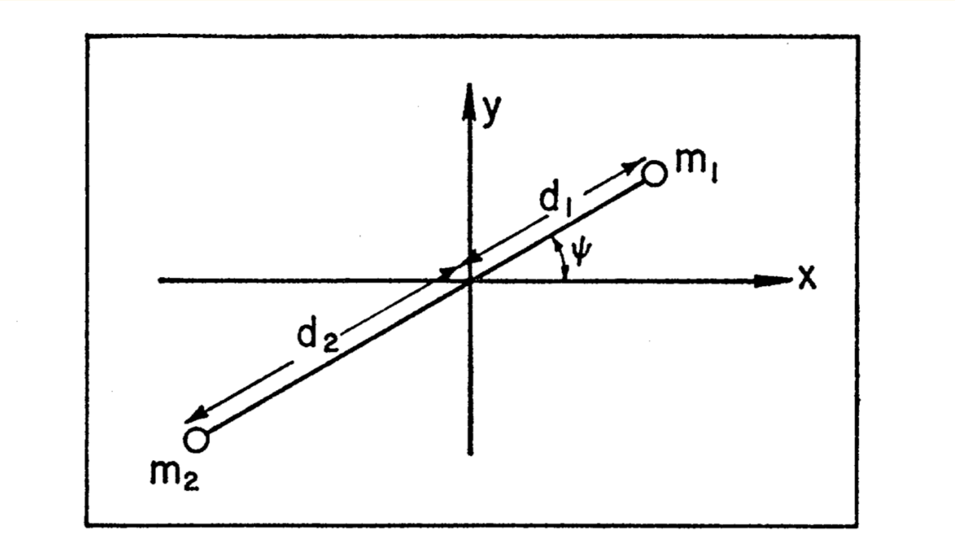
LISA band



(image credit: Moore, Cole and Berry)

LISA band

The definition of orbital plane, inclination angle, etc…



$$Q_{xx} = \mu d^2 \cos^2\psi,$$
$$Q_{yy} = \mu d^2 \sin^2\psi,$$
$$Q_{xy} = Q_{yx} = \mu d^2 \sin\psi \cos\psi$$

(Peters, P 1963)

Summary

1. Spin-curvature coupling:
   - Non-geodesic motion
   - Precession and nutation of pulsar's spin axis
2. Implication on pulsar observation:
   - Orbital precession would shift the arrival time of pulses
   - Spin precession would distort the pulse profile and even lead to the disappearance of pulses
   - Emission with different frequencies have spatial and temporary dispersion
3. Gravitational wave:
   - Corrections to phase to gravitational wave and distortion of the waveform

Reference

- ✓ Singh, Dinesh, Kinwah Wu, and Gordon E. Sarty. "Fast spinning pulsars as probes of massive black holes' gravity." *Monthly Notices of the Royal Astronomical Society* 441.1 (2014): 800-808.
- ✓ Peters, P. (n.d.). Gravitational Radiation from Point Masses in a Keplerian Orbit.
- ✓ Kramer, Michael, et al. "Tests of general relativity from timing the double pulsar." *Science* 314.5796 (2006): 97-102.
- ✓ Lorimer, Duncan R. "Binary and millisecond pulsars." *Living reviews in relativity* 11.1 (2008): 8.
- ✓ Rafikov, Roman R., and Dong Lai. "Effects of Pulsar Rotation on Timing Measurements of the Double Pulsar System J0737–3039." *The Astrophysical Journal* 641.1 (2006): 438.
- ✓ Lorimer, Duncan Ross, and Michael Kramer. *Handbook of pulsar astronomy*. Vol. 4. Cambridge university press, 2005.

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