Minimal theory of quasidilaton massive gravity

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Based on

Minimal theory of quasidilaton massive gravity

arXiv 1701.01581, *with A. De Felice and S. Mukohyama*

Horndeski extension of the minimal theory of quasidilaton massive gravity

arXiv 1709.03108, *with A. De Felice and S. Mukohyama*

Outline

1. Overview, motivations
2. Construction
   i. From dRGT…
   ii. …via the precursor theory…
   iii. …to the minimal quasidilaton
3. Solutions and some cosmology
4. Future prospects

Minimal quasidilaton – overview

- 2 massive tensor modes + 1 scalar
- Free of Boulware-Deser ghost
- Quasidilatation global symmetry
- Breaks Lorentz invariance (LI) to propagate 3 instead of 6 expected d.o.f.
- Modifies gravity at cosmological scales

Graviton mass: $m \sim H_0 \sim 10^{-33} eV$

Motivations

- IR modification of gravity
- Explore viable massive gravity theories

Why the minimal quasidilaton?

Some advantages:

see e.g. Gümrükçüoglu et al. arXiv:1707.02004

I. From the point of view of **quasidilaton theories**, this has the **least number of degrees of freedom**, and thus is more tractable.

II. In contrast to **the MTMG**, the minimal quasidilaton theory allows to use a **Minkowski fiducial metric**

III. Passes the LIGO/Virgo tests

MTMG in De Felice et al. arXiv:1506.01594

Construction

i. Start from dRGT massive gravity
ii. Break LI and add the quasidilaton.
iii. Switch to Hamiltonian and analyse "*à la Dirac*"
iv. Add constraints so that the final number of degrees of freedom is 3 "*à la MTMG*".
v. This defines the minimal theory.

dRGT theory (arXiv:1011.1232)

de Rham,
Gabadadze,
Tolley

Contract the physical metric $g_{\mu\nu}$ with a new **fiducial metric** $f_{\mu\nu}$

$$\mathcal{K}^{(4)\mu}{}_{\rho}\mathcal{K}^{(4)\rho}{}_{\nu} = f^{\mu\rho}g_{\rho\nu}\,, \quad \mathcal{K}^{(4)\mu}{}_{\rho}\mathfrak{K}^{(4)\rho}{}_{\nu} = \delta^{\mu}_{\nu}$$

Thanks to the special form of the potential, **no Boulware-Deser ghost**.

$$\mathcal{L}_m = -\sqrt{-g}\frac{M_{\mathrm{P}}^2 m^2}{2}\sum_{i=0}^{4} c_i e_i\left[\mathcal{K}^{(4)}\right]$$

Propagates 5 d.o.f.

Quasidilaton (D'Amico et al. arXiv 1206.4253)

The Stückelberg scalar fields ϕ^a can be introduced to recover covariance

$$f_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

e.g. for Minkowski fiduciual metric

Choose: Stückelberg sector is shift- and SO(3) symmetric.

Add an **additional global symmetry** in the action. It acts on the Stückelberg fields as

$$\sigma \to \sigma + \sigma_0\,, \quad \phi^i \to \phi^i e^{-\sigma_0/M_{\rm P}}\,, \quad \phi^0 \to \phi^0 e^{-(1+\alpha)\sigma_0/M_{\rm P}}$$

quasidilaton scalar!

LI breaking

LI breaking potential

(De Felice & Mukohyama, arXiv 1506.01594)

Use ADM decomposition

$$f_{\mu\nu} \to M,\ M_i,\ \tilde{\gamma}_{ij}$$

$$g_{\mu\nu} \to N,\ N_i,\ \gamma_{ij}$$

And ADM Vierbein… as in MTMG !

The resulting potential:

$$\mathfrak{K}^i{}_k \mathfrak{K}^k{}_j = \gamma^{ik} \tilde{\gamma}_{kj}$$

$$\mathcal{L}_m = \sqrt{-g} \frac{M_{\rm P}^2 m^2}{2} \sum_{i=0}^{4} c_i \mathcal{L}_i$$

$$\mathcal{L}_i = -e^{(4-i)\sigma/M_{\rm P}} \left[\frac{M e^{\alpha\sigma/M_{\rm P}}}{N} e_{3-i}(\mathfrak{K}) + e_{4-i}(\mathfrak{K}) \right]$$

Precursor action

Defining a precursor action is the first step in constructing the minimal theory.

$$\boxed{\mathcal{L}_{\text{pre}} = \mathcal{L}_{\text{E-H}} + \mathcal{L}_m + \mathcal{L}_\sigma}$$

$$\mathcal{L}_{\text{E-H}} = \frac{M_{\text{P}}^2}{2}\sqrt{-g}R[g]$$

$$\mathcal{L}_\sigma = \sqrt{-g}\left[F(X,S) + \chi\left(X - \mathfrak{X}\right) + \theta S + g^{\mu\nu}\partial_\mu\theta\partial_\nu\sigma\right]$$

We can include a cubic Horndeski structure !

Auxiliary fields X, S, χ, θ

$$F(X,S) = P(X) - G(X)S$$

$$\mathfrak{X} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma$$

Degrees of freedom in the precursor theory

$$\bar{H}_{\rm pre}^{(T)} = \int d^3x \Big[-N\tilde{\mathcal{R}}_0 - N^i\tilde{\mathcal{R}}_i + \frac{M_{\rm P}^2}{2} m^2 M \mathcal{H}_1 + \xi_X P_X + \xi_\chi P_\chi + \xi_S P_S$$
$$+\sqrt{\gamma}\left(\lambda_X S_X + \lambda_\chi S_\chi + \lambda_S S_S + \lambda_T \tilde{T}\right) + \lambda^\tau \tilde{\mathcal{C}}_\tau \Big]$$

$\tau \in \{1, 2\}$

Linear in N, N^i, and M

11 d.o.f.

γ_{ij} σ θ χ X S

(22 phase space d.o.f.)

1 first-class and **12 second-class** constraints

(8 phase space d.o.f.)

4 d.o.f.

Minimal theory: new constraints

We replace **2 precursor constraints** by **4 new constraints**

$$\tilde{\mathcal{C}}_\tau\,, \quad \tau \in \{1,2\}$$

Careful about keeping SO(3)

$$\{\tilde{\mathcal{R}}_i^{\rm GR}, H_1\} \approx \frac{M_{\rm P}^2}{2}\mathcal{C}_i\,, \quad \{\tilde{\mathcal{R}}_0^{\rm GR}, H_1\} \approx \frac{M_{\rm P}^2}{2}\mathcal{C}_0$$

(8 phase space d.o.f.)

4 d.o.f.

2 second-class constraints

(6 phase space d.o.f.)

3 d.o.f.

[In practice, 2 tensor modes and the quasidilaton σ]

Minimal theory, action

$$
\begin{aligned}
\mathcal{L} =& N\sqrt{\gamma}\left\{\frac{M_{\rm P}^2}{2}\left[{}^{(3)}R + K_{ij}K^{ij} - K^2\right] + P + G_{,X}\, g^{\mu\nu}\partial_\mu X\,\partial_\nu\sigma\right\} \\
&+ \lambda_\chi N\sqrt{\gamma}\left[\frac{\lambda_T}{N}\left(\partial_\perp\sigma + \frac{\lambda_T}{N}\right) - \frac{\lambda_T^2}{2N^2} - \frac{1}{2}\left(2X + g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right)\right] \\
&+ \sqrt{\gamma}\, G_{,X}\,\lambda_T^{;i}\sigma_{;i}\left(\partial_\perp\sigma + \frac{\lambda_T}{N}\right) - \frac{m^2 M_{\rm P}^2}{2}\left[\mathcal{H}_1 + N\mathcal{H}_0 + \frac{\partial\mathcal{H}_1}{\partial\sigma}\,\sigma_{;i}\lambda^i + \frac{1}{2}\sqrt{\gamma}\,\Theta^{jk}\gamma_{ki}\lambda^i{}_{;j}\right] \\
&+ \frac{m^4 M_{\rm P}^2\lambda^2\sqrt{\gamma}}{64N}\left(2\Theta_{ij}\Theta^{ij} - \Theta^2\right) - \frac{m^2 M_{\rm P}^2\lambda}{4}\left[2\left(\partial_\perp\sigma + \frac{\lambda_T}{N}\right)\frac{\partial\mathcal{H}_1}{\partial\sigma} + \sqrt{\gamma}K_{ij}\Theta^{ij}\right] \\
&- \frac{\lambda\lambda_T m^2}{4N}\sqrt{\gamma}\, G_{,X}\left(X\,\Theta + \Theta^{ij}\sigma_{;i}\sigma_{;j}\right) + \lambda_T\sqrt{\gamma}\,\Big\{G_{,X}\big(K_{ij}\sigma^{;i}\sigma^{;j} - K\sigma_{;i}\sigma^{;i} - 2X\,K\big) \\
&+ \left(\partial_\perp\sigma + \frac{\lambda_T}{N}\right)\left(G_{,XX}X^{;i}\sigma_{;i} + 2G_{,X}\sigma^{;i}{}_{;i} - P_{,X}\right) - G_{,X}\partial_\perp X \\
&- \frac{1}{M_{\rm P}^2}\frac{\lambda_T}{N}\, X\, G_{,X}^2\left(2\sigma_{;i}\sigma^{;i} + 3X\right)\Big\}
\end{aligned}
$$

There are still some Lagrange multipliers λ, λ_T

Luckily there is a unique mini-superspace solution: $\lambda = \lambda_T = 0$

Mini-superspace solutions

de Sitter attractor

The equation from λ is rewritten in a nice form.

$$\frac{d}{dt}\left[a^{4+\alpha}\mathcal{X}^{1+\alpha}\,J(\mathcal{X})\right]=0$$

$$\mathcal{X}\equiv\frac{e^{\sigma/M_{\rm P}}}{a}$$

$$J\equiv c_0\mathcal{X}^3+3c_1\mathcal{X}^2+3c_2\mathcal{X}+c_3$$

where a is the scale factor. This implies that there exists a de Sitter attractor where either

$\mathcal{X}$ is constant ($\alpha=-4$) or $J(\mathcal{X})=0$ ($\alpha\neq-4$).

Stability of de Sitter solution

Study the quadratic action for linear perturbations, and obtain the no-ghost conditions.

It is nice and stable ! ☺

Gravitational modes in the minimal quasidilaton

Constraints

$$m_g \lesssim 1.2 \times 10^{-22}\, eV\,, \quad |1 - c_g/c| \lesssim 10^{-15}$$

GW 150914
GW170817/GRB170817A

The minimal theory of quasidilaton massive gravity successfully passes the tests of both GW and multimessenger detections.

- The sound speed of the tensor modes in the subhorizon limit **coincides with the speed of light**.
- Small graviton **mass of order** $H_0 \sim 10^{-33}\ eV$.

Future prospects

i. Cosmology with matter and general FLRW.
ii. Small scale behaviour – Vainshtein screening and astrophysics
iii. Minimal… other theories
iv. Theoretical consistency checks

…keep in touch! ☺

Thank you for your attention !