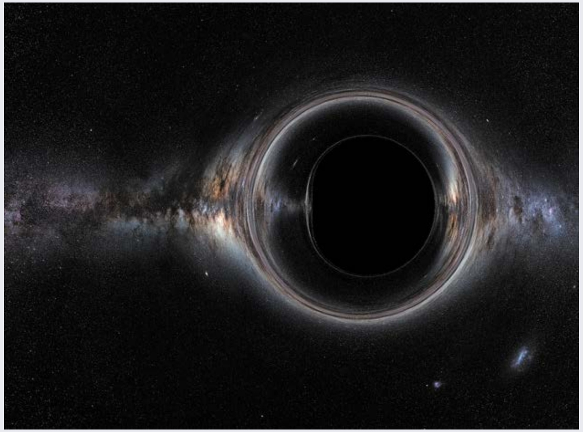
Massive Graviton Geons:
self-gravitating massive gravitational waves

Katsuki Aoki, Waseda University

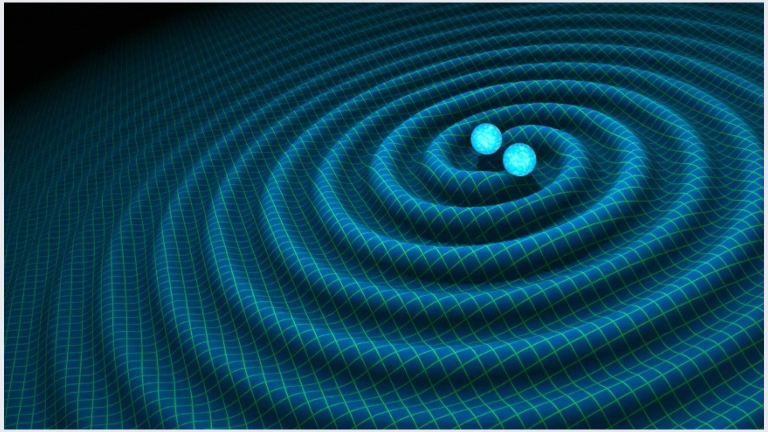
KA, K. Maeda, Y. Misonoh, and H. Okawa,
PRD 97, 044005 (2018), [arXiv: 1710.05606].

Introduction

Vacuum solutions to the Einstein equation?



Black Holes



Gravitational Waves

LIGO and Virgo observed both of them!

GW150914

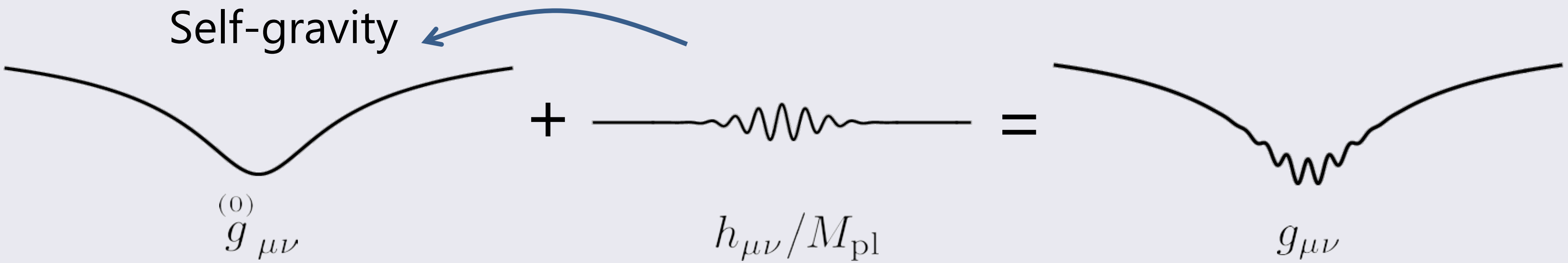
Initial mass: $65.3M_{\odot} = 36.2M_{\odot} + 29.1M_{\odot}$ $\rightarrow$ Final mass: $62.3M_{\odot}$

The energy is radiated by GWs!

GWs have their gravitational energy!

Due to the nonlinearities of the Einstein equation,
GWs (=perturbations) themselves change the background geometry.

Is it possible to realize self-gravitating gravitational waves?



Gravitational "Geons"

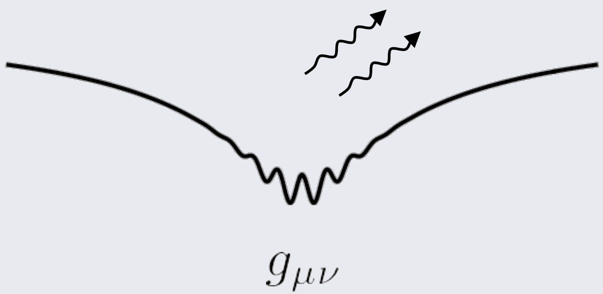
The original idea of "geon" is a gravitational electromagnetic entity.
= a realization of classical "body" by gravitational attraction.

Wheeler, 1955.

Gravitational Geons

Gravitational geons are singular-free time periodic vacuum solutions to GR.

Brill and Hartle, 1964, Anderson and Brill, 1997.



Gravitational geons

not stable and decay in time.

Gibbons and Stewart, 1984.

This may not be the case in modified gravity.

Geons can be a proof of beyond GR? or Geons can be dark matter?

We consider gravitational geons in modified gravity.

Massive gravitons?

GR is the theory of a massless graviton but
there could be several gravitons as other gauge theories.

Two dynamical tensors: $g_{\mu\nu}$ and $f_{\mu\nu}$ (Hassan and Rosen, 2011)

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f) - \frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f)$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdot}\epsilon_{\cdots\cdot}(\gamma^\mu{}_\nu)^n \qquad \gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu} \qquad \kappa^2 = \kappa_g^2 + \kappa_f^2$$

Free parameters: $\kappa_g, \kappa_f, m, b_i \; (i = 0, 1, 2, 3, 4)$

Bigravity contains one massless graviton and one massive graviton.

We do not assume any particular value of the graviton mass.

We consider self-gravitating massive gravitational waves.

Graviton $T^{\mu\nu}$ in Bigravity

Assuming $|\partial^2 g_{\mu\nu}| \ll m^2$ (no Vainshtein effect) and taking Isaacson average,

we find the Einstein and Klein-Gordon equations

$$G^{\mu\nu}[\overset{(0)}{g}] \simeq \frac{1}{M_{\rm pl}^2}\left(\langle T_{\rm gw}^{\mu\nu}\rangle_{\rm low} + \langle T_G^{\mu\nu}\rangle_{\rm low}\right)$$

$$\Box h_{\mu\nu} \simeq 0\,, \quad (\Box - m^2)\varphi_{\mu\nu} \simeq 0$$ + TT conditions

where $T_{\rm gw}^{\mu\nu} \sim (\partial h_{\mu\nu})^2\,, \quad T_G^{\mu\nu} \sim (\partial\varphi_{\mu\nu})^2 + m^2\varphi_{\mu\nu}^2$

$$M_{\rm pl} = \frac{\kappa}{\kappa_g \kappa_f}\,, M_G = \frac{\kappa}{\kappa_g^2}$$

The metrics are given by

$$g_{\mu\nu} \simeq \overset{(0)}{g}{}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} + \frac{\varphi_{\mu\nu}}{M_G}\,, \quad f_{\mu\nu} \simeq \overset{(0)}{g}{}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} - \frac{\varphi_{\mu\nu}}{\alpha M_G}\,;$$

$(\alpha = M_{\rm pl}^2/M_G^2)$

We shall ignore the massless gravitational waves $h_{\mu\nu}$.

Newtonian limit of bigravity

We then assume that the massive gravitons are non-relativistic.

$$\overset{(0)}{g}_{\mu\nu} dx^\mu dx^\nu = -(1+2\Phi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j$$

$$\varphi_{\mu\nu} = \begin{pmatrix} \psi_{00} & \psi_{0i} \\ * & \frac{\psi_{\rm tr}}{3}\delta_{ij} + \psi_{ij} \end{pmatrix} e^{-imt} + \text{c.c.}\,,$$

↑ traceless, $\psi^i{}_i = 0$

where $\Phi, \psi_{..}$ are slowly varying functions.

The transverse-traceless condition leads to $|\psi_{00}|, |\psi_{\rm tr}| \ll |\psi_{0i}| \ll |\psi_{ij}|$

Finally, we obtain the Poisson-Schrodinger equations

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \quad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,.$$

Self-gravitating bound state

The bound state of the Poisson-Schrodinger eqs. with intrinsic spin.

$$\psi_{ij}(t,\mathbf{x}) = \psi_{ij}(\mathbf{x})e^{-iEt}\,, \qquad i\frac{\partial}{\partial t} \to E$$

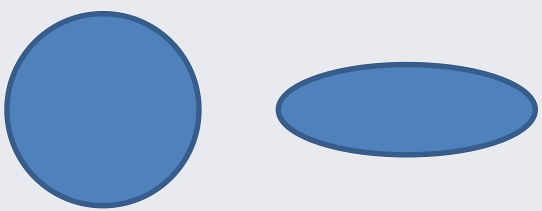
$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \qquad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,; \qquad \text{Spin-2}$$

Cf. $$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*\psi\,, \qquad i\frac{\partial}{\partial t}\psi = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi\,; \qquad \text{Spin-0}$$

Only difference is the intrinsic spin

ψ_{ij} : symmetric traceless tensor $\qquad$ ψ : scalar

What is the most stable configuration?



Stable? $\qquad$ Unstable?

Angular momentum of bound state

Maybe… spherically symmetric configuration (monopole)?

However, it is NOT because of the intrinsic spin!

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \quad E\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,;$$

The most stable = The lowest energy eigenvalue

= The lowest angular momentum

$$\Delta = \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2}$$

There are total angular momentum j and orbital angular momentum ℓ.

The monopole configuration: $j = 0$ but $\ell = 2$

A quadrupole configuration: $j = 2$ but $\ell = 0$ **Lowest energy**

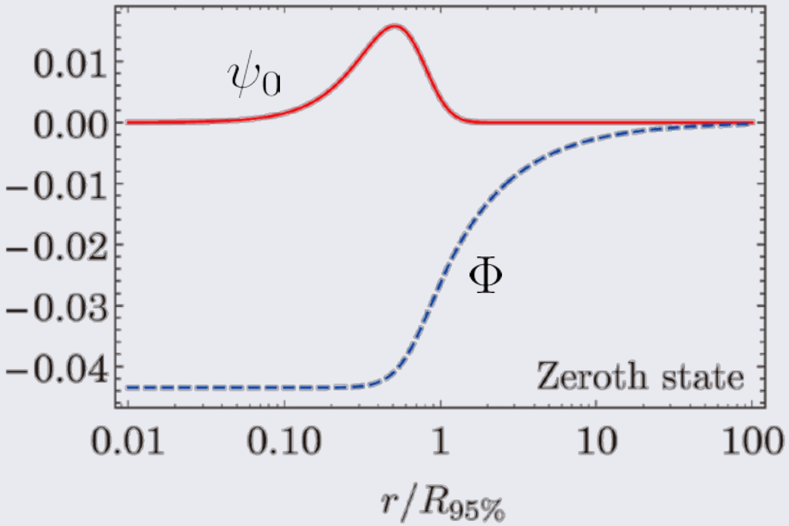
(cf. The monopole configuration in spin-0 case: $j = 0$ and $\ell = 0$)

Monopole geon and Quadrupole geon

The monopole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_0(r)e^{-iEt}(T^{-2}_{0,0})_{ij} \,,$$

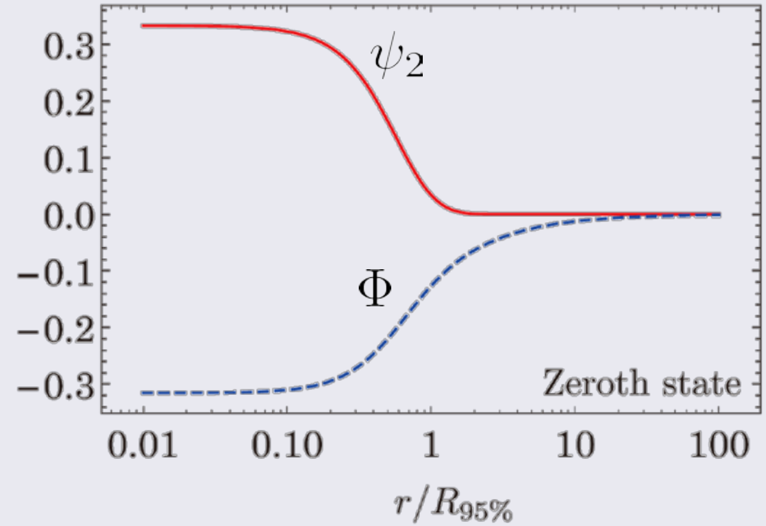
$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.027$$



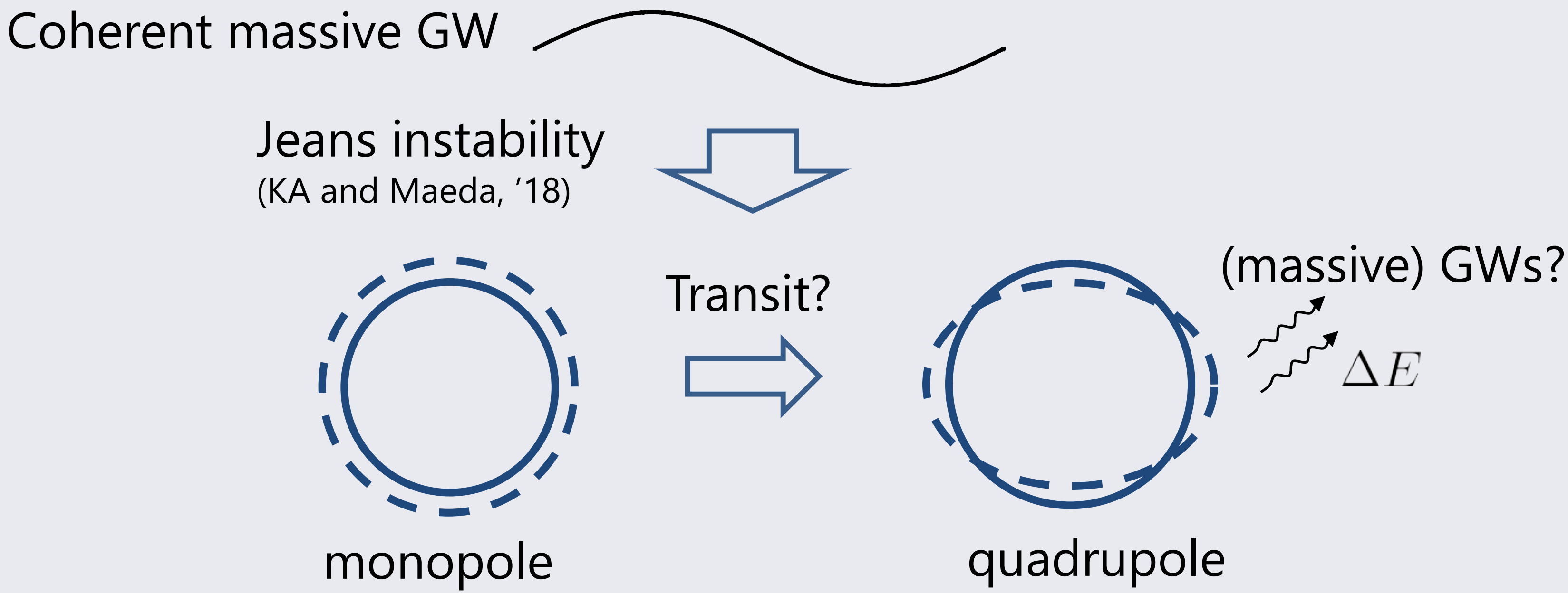
The quadrupole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_2(r)e^{-iEt}\sum_{j_z} a_{j_z}(T^{+2}_{2,j_z})_{ij} \,,$$

$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.16$$



Stability of geons



$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.027$$

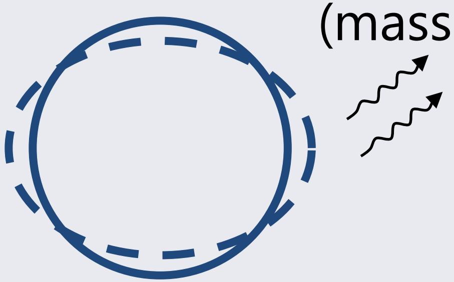
$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.16$$

The monopole geon is unstable against quadrupole mode perturbations.

Stability of geons

The final state must be the quadrupole geon.

It could emit GWs due to non-spherically symmetric oscillations.



But, the emission is small because of the large hierarchy between the time and the length scales.

Anisotropic pressure $\sim T^{\rm TT}_{G,ij}(\mathbf{x})e^{-2imt}$, $\quad \partial_k T^{\rm TT}_{G,ij}(\mathbf{x}) \ll m T^{\rm TT}_{G,ij}(\mathbf{x})$

(GWs are emitted if $\omega^2 = k^2$ or $\omega^2 = k^2 + m^2$)

$\rightarrow$ The non-relativistic quadrupole geon is an (approximately) stable object.

Geons as field dark matter

If a mass is $\sim 10^{-21}$eV, massive graviton can be a fuzzy dark matter.

Ultralight axion: spin-0 DM
Massive graviton: spin-2 DM

In FDM, the central part of DM halos is given by the "soliton" (=geon).

Although the field configuration is not spherically symmetric,
the energy distribution is spherically symmetric.

ψ_{ij} : not spherical $\quad\quad$ $\psi^*_{ij}\psi^{ij}$: spherical

and the energy distribution is exactly the same as that of spin-0 case.

Spin-2 FDM could shear successes of spin-0 FDM.

Summary

Massive graviton geons = self-gravitating massive GWs

New vacuum solutions to bigravity theory.

The ground state must be non-spherical.

Spin-0: ground state = monopole ⇒ $\ell = j = 0$

Spin-2: ground state = quadrupole ⇒ $\ell = 0, \ j = 2$

Ultralight massive graviton can be FDM as well.

Note that DM is not new “particle” but spacetime itself

$$g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{\varphi_{\mu\nu}}{M_G} ;$$

Possible prospects: Hairy BHs?, Geon as BE condensate? etc…