Effective Field Theory of Anisotroic Inflation

Asieh Karami

IPM - School of Astronomy

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In collaboration with T. Rostami and H. Firouzjahi
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Why effective field theory?

- There are many scenarios of inflation which are compatible with cosmological observations. So, it seems unrealistic if one can single out a particular model as the true realization of inflation in early universe. This naturally raises the question if one can classify the various inflationary scenarios either based on their main predictions or based on their theoretical constructions.

Why anisotropic inflation?

- Vector fields and gauge fields appear in abundant in Standard Model of particle physics and in quantum field theory. It is natural to expect that they play some roles during inflation.
  It is conceivable that they play the role of isocurvature light fields which may also be coupled to inflaton field. This brings the interesting possibilities that light gauge fields may affect the cosmological observations by generating some observable amount of statistical anisotropies.

Outline

Effective Field Theory

- There is a theory for whole energy scale
  - Use EFT to simplify calculations in some energy scales.

- Constructing EFT by considering symmetries of the model.

Anisotropic Inflation

- In the setup of anisotropic inflation we have the scalar field ϕ as the inflaton field and a $U(1)$ gauge field A_μ which is the source of electric field energy density during inflation.
- Because of the conformal symmetry associated with the Maxwell theory, the background electric field energy density is diluted if the gauge field is not coupled to inflaton. Therefore, in order for the background electric field energy density to survive the exponential expansion, the gauge field is coupled to the inflaton field as $-f(\phi)^2 F_{\mu\nu}F^{\mu\nu}/4$.
- The next goal is to choose the functional form of $f(\phi)$ such that the background electric field energy density to be a nearly constant but sub-leading fraction of the total energy density. For a given inflaton potential $V(\phi)$ the form of $f(\phi)$ can be obtained. In terms of scale factor $a(t)$, it takes the time-dependent value $f(\phi) \propto a(t)^{-2}$. At the perturbation level, this choice of $f(\phi)$ also yields a scale invariant power spectrum for the gauge field fluctuations.

Anisotropic Inflation

The imprints of the gauge fields fluctuations in primordial curvature perturbation power spectrum $P_{\mathcal{R}}$ has the form of quadrupole anisotropy which

$$P_{\mathcal{R}}(\mathbf{k}) = P_{\mathcal{R}}^{(0)}\left(1 + g_*(\widehat{\mathbf{n}} \cdot \widehat{\mathbf{k}})^2\right) ,$$

in which $P_{\mathcal{R}}^{(0)}$ is the isotropic power spectrum in the absence of gauge field, $\mathbf{k}$ is the mode of interest in Fourier space and $\widehat{\mathbf{n}}$ indicates the direction of anisotropy. In this way of parameterization, the parameter g_* measures the amplitude of statistical anisotropy. Observational constraints from Planck data implies $|g_*| \lesssim 10^{-2}$.

Effective Action of Anisotropic Inflation

Building blocks are

$$\begin{cases} \delta g^{00}, \\ X \equiv F_{\mu\nu} F^{\mu\nu}, \\ Y \equiv F_{\mu\nu} \tilde{F}^{\mu\nu}, \\ Z \equiv F^{0}_{\mu} F^{0\mu}, \end{cases}$$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ is its dual field. The most general action in unitary gauge in the decoupling limit is given by

$$\begin{aligned} S = \int d^4x \sqrt{-g} \quad \Big[\quad & \alpha(t) + B_1(t)\delta g^{00} + \frac{B_2^2(t)}{4}(\delta g^{00})^2 - \frac{M_1(t)}{4}\delta X \\ & + \frac{M_2(t)}{2}\delta g^{00}\delta X - \frac{M_3(t)}{4}(\delta g^{00})\delta(X)^2 - \frac{M_4(t)}{4}(\delta g^{00})^2\delta X \\ & - \frac{N_1(t)}{4}\delta Y + \frac{N_2(t)}{2}\delta g^{00}\delta Y - \frac{N_3(t)}{4}(\delta g^{00})\delta(Y)^2 \\ & - \frac{N_4(t)}{4}(\delta g^{00})^2\delta Y - \frac{P_1(t)}{4}\delta Z + \frac{P_2(t)}{2}\delta g^{00}\delta Z \\ & - \frac{P_3(t)}{4}(\delta g^{00})\delta(Z)^2 - \frac{P_4(t)}{4}(\delta g^{00})^2\delta Z \Big] + \ldots . \end{aligned}$$

The background Friedmann equations are given by

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[B_1(t)+\alpha(t)\right], \quad \dot{H}+H^2 \;=\; -\frac{1}{3M_{\rm Pl}^2}\left[2B_1(t)-\alpha(t)\right].$$

Solving for α and B_1, in the small anisotropy limit we have

$$\alpha(t)\simeq 3M_P^2H^2, \quad B_1(t)\simeq -\epsilon H^2 M_P^2 .$$

The fraction of gauge field energy density to total energy is

$$R = \frac{\frac{1}{2}(M_1-\frac{P_1}{2})a^{-2}\dot{A}_x^2}{3M_P^2H^2} .$$

The background Maxwell equation is

$$\partial_t\left(\left(M_1-\frac{P_1}{2}\right)a(t)\dot{A}_x\right)=0.$$

From above equations we can conclude that

$$A_x(t)\propto a^3(t), \quad M_1-\frac{P_1}{2}\propto a^{-4}(t) .$$

The Second Order Action

One can restore the inflaton fluctuations by performing the transformation

$$x^0 \to x^{0\prime} = x^0 + \pi \,,$$

π plays the role of Goldstone boson and encodes the fluctuations of inflaton in an arbitrary coordinate system. g_{00} transforms as

$$\begin{aligned} g'^{00}(x') &= \frac{\partial x'^0}{\partial x^\mu}\frac{\partial x'^0}{\partial x^\nu} g^{\mu\nu}(x) \\ &= g^{00} - 2\dot{\pi} g^{00} + \partial_i \pi \partial_j \pi g^{ij} + \dot{\pi}^2 g^{00} \,, \end{aligned}$$

and

$$\delta g^{00} \to 2\dot{\pi} + a^{-2}(\pi_{,i})^2 - \dot{\pi}^2 .$$

The Second Order Action

By going to Coulomb-radiation gauge, the full second order action is given by

$$S = \int d^4x\sqrt{-g} \quad \Big[\quad B_1\left(-\left(\dot{\pi}\right)^2 + a^{-2}\left(\pi_{,i}\right)^2\right) + B_2\dot{\pi}^2 - \frac{1}{4}M_1\delta X^{(2)} + M_2\dot{\pi}\delta X^{(1)} +$$

$$-\frac{1}{4}P_1\delta Z^{(2)} + P_2\dot{\pi}\delta Z^{(1)} + \dot{P}_1\pi\delta Z^{(1)}\Big],$$

where

$$\delta X^{(1)} = -\frac{4}{a^2}\dot{A}_x\delta\dot{A}_x, \quad \delta Z^{(1)} = 2a^{-2}\dot{A}_x\delta\dot{A}_x\,,$$

and

$$\delta X^{(2)} = \frac{2}{a^2}\Big[-\delta\dot{A}_x^2 - \delta\dot{A}_y^2 + \frac{1}{a^2}\delta A_{x,y}^2 + \frac{1}{a^2}\delta A_{y,x}^2 - \frac{2}{a^2}\delta A_{x,y}\delta A_{y,x}\Big],$$

$$\delta Z^{(2)} = \frac{1}{a^2}[\delta\dot{A}_x^2 + \delta\dot{A}_y^2].$$

The Free Fields

Scalar Field

The free action of π is given by

$$S_2^{(\pi)} = \int d^4x\sqrt{-g}\,(-B_1)\left[\left(1-\frac{B_2}{B_1}\right)\dot{\pi}^2 - a^{-2}\left(\pi_{,i}\right)^2\right].$$

Note that $B_1 \propto \dot{H} < 0$ so the kinetic energy has the correct sign.
The free wave function of π with the Minkowski initial conditions deep inside the horizon is

$$\pi(k) = \frac{H}{2k^{3/2}\sqrt{c_s|B_1|}}(1+ikc_s\tau)e^{-ikc_s\tau}.$$

The free field action for δA_i fluctuations is given by

$$S_2^{(\delta A)} = \int d^4x\sqrt{-g}\frac{1}{2a^2}\left[(M_1 - \frac{P_1}{2})\left(\delta\dot{A}_i\right)^2 - M_1 a^{-2}\left(\epsilon_{ijk}\delta A^{i,j}\right)^2\right].$$

For simplicity we define

$$\tilde{M}_1 \equiv M_1 - \frac{P_1}{2} \equiv \overline{M}_1 a^{-4}, \qquad \dot{A}_x(t) \equiv \overline{A} a^3\,,$$

in which $\overline{M}_1$ and $\overline{A}$ are constants. With these definitions

$$R = \frac{\overline{M}_1 \overline{A}^2}{6M_P^2 H^2}\,.$$

Decomposing the gauge field fluctuations in terms of its polarization base $\epsilon_i^s(k)$ in Fourier space

$$\delta A_i = \sum_s \delta A^{(s)}(k,t)\epsilon_i^s(k)\,.$$

The polarization vector can have either the linear polarization form with $s = 1, 2$ or the circular (helicity) polarization form with $s = \pm$.
Now imposing the Minkowski initial condition for the gauge field fluctuations deep inside the horizon we obtain

$$\delta A_i^{(s)} = \frac{1}{k^{3/2}\sqrt{2c_v\overline{M}_1}H\tau^3}(1+ikc_v\tau)e^{-ikc_v\tau}\,,$$

in which c_v represents the speed of gauge field fluctuations

$$c_v^2 = \frac{M_1}{\tilde{M}_1}\,,$$

Power Spectrum

The leading interactions involving π and δA_i fluctuations are given by

$$S^{(\pi\delta A)} = \int d\tau d^3x 4\left(H\overline{A}\,\overline{M}_1\pi\delta A'_x + \overline{A}a^{-1}\overline{M}_2\pi'\delta A'_x\right),$$

where $M_2 - P_2/2 \equiv \tilde{M}_2 \equiv \overline{M}_2 a^{-4}$ with $\overline{M}_2$ being a constant.
For simplicity we choose the wave number as

$$\mathbf{k} = k\left(\cos\theta, \sin\theta, 0\right)\,,$$

where θ is the angle between the wave number and the preferred direction $\hat{\mathbf{n}}$, i.e. $\cos\theta = \widehat{\mathbf{k}}\cdot\widehat{\mathbf{n}}$.

$$\frac{\delta P_{\rm total}}{P_\pi^{(0)}} = \frac{8H^2c_s^5\overline{M}_1\,\overline{A}^2}{B_1 c_v}\left(1+\frac{3\overline{M}_2}{\overline{M}_1}\right)^2 N^2\sin^2\theta\,.$$

Comparing the above expression with the amplitude of quadrupole anisotropy g_* yields

$$g_* = -\frac{48Rc_s^5N^2}{\epsilon\, c_v}\left(1+\frac{3\overline{M}_2}{\overline{M}_1}\right)^2\,,$$

Bispectrum

The leading cubic interactions in conformal time are

$$S^{(3)} = \int d\tau d^3x 2\left(a^{-4} H \overline{M}_1 \pi \delta A'^2 + a^{-5}\overline{M}_2 \pi' \delta A'^2\right),$$

The leading anisotropic contributions to bispectrum is given by

$$\Big\langle \pi(\mathbf{k}_1)\pi(\mathbf{k}_2)\pi(\mathbf{k}_3)\Big\rangle_{ijk} = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{ijk}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3),$$

where

$$\begin{aligned} B_{\text{tot}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \quad &= \quad 36\frac{c_s^6 H^5 R}{\epsilon^3 c_v^2 M_P^4} N_{k_1}N_{k_2}N_{k_3}\left(1+27\frac{\overline{M}_2^3}{\overline{M}_1^3}+45\frac{\overline{M}_2^2}{\overline{M}_1^2}+15\frac{\overline{M}_2}{\overline{M}_1}\right) \\ &\quad\quad \times\left(\frac{C(\mathbf{k}_2,\mathbf{k}_3)}{k_2^3k_3^3}+2\text{c.p.}\right), \end{aligned}$$

and

$$C(\mathbf{k}_2,\mathbf{k}_3) \equiv 1-(\widehat{\mathbf{n}}.\widehat{\mathbf{k}}_2)^2-(\widehat{\mathbf{n}}.\widehat{\mathbf{k}}_3)^2+(\widehat{\mathbf{n}}.\widehat{\mathbf{k}}_2)(\widehat{\mathbf{n}}.\widehat{\mathbf{k}}_3)(\widehat{\mathbf{k}}_2.\widehat{\mathbf{k}}_3).$$

Gravitational Waves

In the limit of small anisotropy the tensor perturbations in metric are

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + (\delta_{ij} + h_{ij})\, dx^i dx^j\right) \,.$$

The quantum operators $\widehat{h}_{ij}(\mathbf{k}, \tau)$ are decomposed in terms of the annihilation and creation operators as usual

$$\widehat{h}_{ij}(\mathbf{k}, \tau) = \sum_{s=+,\times} \widehat{h}_s(\mathbf{k}, \tau) e^{(s)}_{ij}(\mathbf{k}) \quad , \quad \widehat{h}_s(\mathbf{k}, \tau) = h_s(k, \tau) a_s(\mathbf{k}) + h^*_s(k, \tau) a^\dagger_s(-\mathbf{k}) \,,$$

where $e^{(s)}_{ij}(\mathbf{k})$ is polarization base in Fourier space and the tensor excitations has the standard profile

$$h_s(k, \tau) = \frac{2iH\tau}{M_P\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} \,,$$

the interaction Hamiltonians involving the mixing of tensor perturbations with the scalar and gauge field fluctuations have the following form

$$H_{int} \quad = H_{\pi h_+} + H_{\pi' h_+} + H_{\delta A_x h_+} + H_{\delta A_V h_\times}$$

in which

$$H_{\pi h_+} = 2\sqrt{2}\,\overline{M}_1 \overline{A}^2 H \sin^2\theta a^4 \pi h_+, \quad H_{\pi' h_+} = 2\sqrt{2}\,\overline{A}^2 \overline{M}_2 \sin^2\theta a^3 \pi' h_+$$

and

$$H_{h_+ \delta A_x} = -\frac{\overline{M}_1\,\overline{A}}{\sqrt{2}} \delta A'_x h_+, \quad H_{h_\times \delta A_V} = \frac{i\overline{M}_1 \quad \overline{A}}{\sqrt{2}} \sin\theta \delta A_V h_\times\,.$$

The total anisotropy in tensor power spectrum is given by

$$\delta\langle hh\rangle_{\mathrm{tot}} = -3\Big(c_s\left(c_v^2 - 3\right) - c_v^2 + 1\Big)^2 \frac{RH^2}{M_P^2 c_v k^3} N^2 \sin^2\theta\,.$$

Only the polarization $s = +$ contributes to the scalar-tensor cross correlation. This is because π couples only to $s = +$ polarization and not to $s = \times$ polarization.

The total scalar-tensor cross correlation is given by

$$\left\langle \pi_{\mathbf{k}_1}(\tau_e) h_{+\mathbf{k}_2}(\tau_e) \right\rangle_{\text{tot}} = \frac{\sqrt{2}\,\overline{A}^2 N}{3M_P^2 B_1 H k^3 c_s}\left[-6\overline{M}_1 c_s^2 - \overline{M}_2 \left(c_s+1\right)\left(\left(c_s-4\right)c_s \right.\right.$$

$$\left.\left. +1\right)\right] \sin^2\theta - \frac{\overline{A}^2 c_s^2 N^2}{4M_P^2 \sqrt{2} B_1^2 H k^3 c_v}\left[4B_1\left(2\overline{M}_1 + 3\overline{M}_2\right)\left(\left(c_s-1\right)c_v^2 - 3c_s + 1\right) \right.$$

$$\left. -9\overline{M}_2 c_s^2 \right] \sin^2\theta$$

Also one can calculate the corrections to the scalar power spectrum induced from the scalar-tensor mixing. However, these corrections are much suppressed compared to anisotropy we obtained before.

Summery

- The leading contribution in anisotropy power spectrum comes from the interactions sourced by operators involving M_1 and P_1. The power anisotropy is in the shape of quadrupole with its amplitude scaling like $RN^2c_s^5/c_v$ in which c_s is the sound speed of scalar perturbations and c_v is the speed of gauge field fluctuations.
- Our method can naturally incorporate the scenarios with non-trivial c_s and c_v.
- Our approach was particularly useful to calculate the bispectrum and is easily applicable to calculate the trispectrum in the general setup of anisotropic inflation.
- The scalar-tensor cross correlation induces non-trivial TB and EB cross correlations on CMB maps which do not exist in usual isotropic models.
- It will be interesting to study the effects of parity violating interactions within the setup of anisotropic inflation. In these cases, the two polarization of tensor perturbations behave quite differently and the tensor perturbations acquire handedness

Thank You!