# THE STABILITY CONDITIONS OF MODIFIED GRAVITY MODELS AND THEIR APPLICATIONS

Gravity and Cosmology 2018
YITP, Kyoto University
13 February 2018

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Based on:
N. Frusciante, G.P. and A. Silvestri, JCAP 1607, no. 07, 018 (2016) [arXiv:1601.04064 [gr-qc]]
A. De Felice, N.Frusciante and G.P., JCAP03(2017)027, [arXiv:1609.03599 [gr-qc]]
N.Frusciante, G.P., S.Peirone and A.Silvestri, Ongoing work

# THE ERA OF HIGH PRECISION COSMOLOGY

Current and future surveys will probe the expansion of the universe and the formation of cosmic structure to unprecendent detail!

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Test Gravity at cosmological scales!

# WHY TEST GRAVITY?

- General Relativity not yet fully tested at cosmological scales
- Cosmological constant matches observations but theoretically unsatisfying for many
- Straightforward way to test the validity of any theory, deform it!

# SCALAR FIELD DE/MG



- Extensions of General Relativity $\longrightarrow$ additional degrees of freedom
- Wealth of scalar field extensions, e.g. Quintessence, Galileon theories
- Efficient methods required

# CONSTRUCTING THE EFT OF DE/MG: THE IDEA

We are looking for *"the most general action describing the linear perturbations of gravity models with one tensor and one scalar d.o.f. around an FRW background"*.

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# CONSTRUCTING THE EFT OF DE/MG: THE IDEA

We are looking for "*the most general action describing the linear perturbations of gravity models with one tensor and one scalar d.o.f. around an FRW background*".

- Construct the action in the Jordan frame by using general symmetry arguments. The residual symmetries one needs to satisfy are the 3D spatial diffeomorphisms.

- Natural time foliation, uniform field hypersurfaces correspond to equal time hypersurfaces. This sets the "*Unitary gauge*"

$$n_\mu \equiv \frac{\partial_\mu \phi}{\sqrt{(\partial_\mu \phi)^2}}$$

# THE EFT OF DE/MG

$$\mathcal{S}_{EFT} = \int d^4x\sqrt{-g}\Big[\frac{m_0^2}{2}(1+\Omega(t))R + \Lambda(t) - c(t)\delta g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K - \frac{\bar{M}_2^2(t)}{2}(\delta K)^2$$

$$-\frac{\bar{M}_3^2(t)}{2}\delta K^\mu_\nu \delta K^\nu_\mu + \hat{M}^2(t)2\delta g^{00}\delta\mathcal{R} + m_2^2(t)h^{\mu\nu}\partial_\mu g^{00}\partial_\nu g^{00} + \frac{\bar{m}_5(t)}{2}\delta\mathcal{R}\delta K$$

[Gubitosi et al. JCAP 1302 (2013) 032
Bloomfield et al. JCAP 1308 (2013) 010]

# STABILITY ANALYSIS OF A SCALAR FIELD

- After solving the momentum and Hamiltonian constraint the action for the scalar d.o.f. in an FRW background is obtained :

$$S_{EFT}^{(2)} = \frac{1}{(2\pi)^3}\int d^3k\,dt\,a^3\left(\mathcal{L}_{\dot\zeta\dot\zeta}\dot\zeta^2 - (G(t,k)k^2 + \bar{M}(t,k))\zeta^2\right)$$

- Theoretical stability guaranteed by avoiding:

  - Ghost Instability: $\mathcal{L}_{\dot\zeta\dot\zeta}(t,k) > 0$
  - Gradient Instability: $c_s^2 > 0$
  - Unbounded Hamiltonian from below

# THE EFT IN THE PRESENCE OF TWO FLUIDS

- For the matter fluids we choose CDM and radiation
- The action now becomes :

$$S^{(2)} = \frac{1}{(2\pi)^3}\int d^3k\,dt\,a^3\left(\dot{\vec{\chi}}^t \mathbf{A}\dot{\vec{\chi}} - k^2\vec{\chi}^t\mathbf{G}\vec{\chi} - \dot{\vec{\chi}}^t\mathbf{B}\vec{\chi} - \vec{\chi}^t\mathbf{M}\vec{\chi}\right)$$

with $\vec{\chi}^t = (\zeta, \delta_d, \delta_r)$

# MASS AND JEANS INSTABILITY

Due to the complicated nature of the problem we limit ourselves to just one matter fluid, CDM. By going to canonical kinetic terms we arrive at the following Lagrangian:

$$\mathcal{L}^{(2)} = \frac{a^3}{2}\left[\dot{\bar{\Psi}}_1^2 + \dot{\bar{\Psi}}_2^2 + \bar{B}(t,k)\left(\dot{\bar{\Psi}}_1\bar{\Psi}_2 - \dot{\bar{\Psi}}_2\bar{\Psi}_1\right) - \bar{C}_{ij}(t,k)\bar{\Psi}_i\bar{\Psi}_j\right]$$

which reduces to the following Hamiltonian:

$$H(\Phi_i, \dot{\Phi}_i) = \frac{a^3}{2}\left[\dot{\Phi}_1^2 + \dot{\Phi}_2^2 + \mu_1(t,k)\,\Phi_1^2 + \mu_2(t,k)\,\Phi_2^2\right]$$

Now, the absence of a tachyonic instability is guaranteed by

$$\mu_i(t,0) > 0 \quad \text{or} \quad |\mu_i(t,0)| \ll H^2$$

# RESULTS

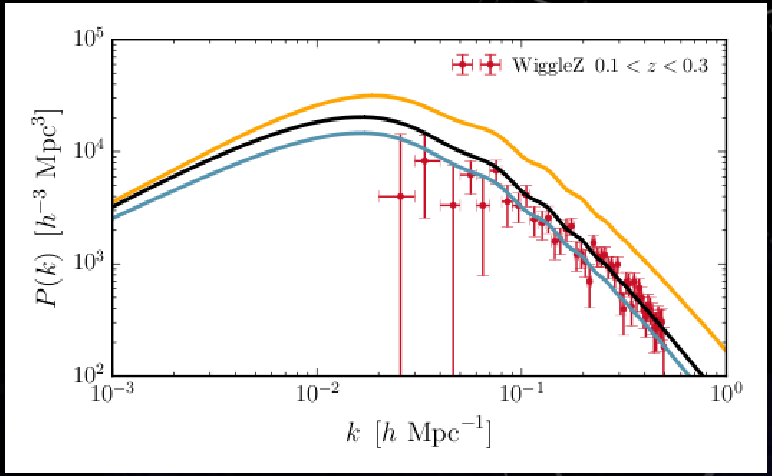
- We derived the conditions in a model independent way both in vacuum as well as in the presence of different matter fields
- Each one of the conditions can be made model specific
- As an example, beyond Horndeski in the presence of matter and radiation:

$$c_{s,d}^2 = 0$$

$$(3c_s^2 - 1)\bar{\rho}_r \left[\bar{\rho}_d \left(c_s^2(F_3 F_1^2 + 3F_2^2 F_1) - 2a^2 F_2^2 \mathcal{G}_{11}\right) - 4\mathcal{B}_{12}^2 {F_2}^2\right] - 16 c_s^2 \mathcal{B}_{13}^2 F_2^2 \bar{\rho}_d = 0$$

# NUMERICAL IMPLEMENTATION

- CAMB: takes the IC and provides the observables, e.g. Matter power spectrum

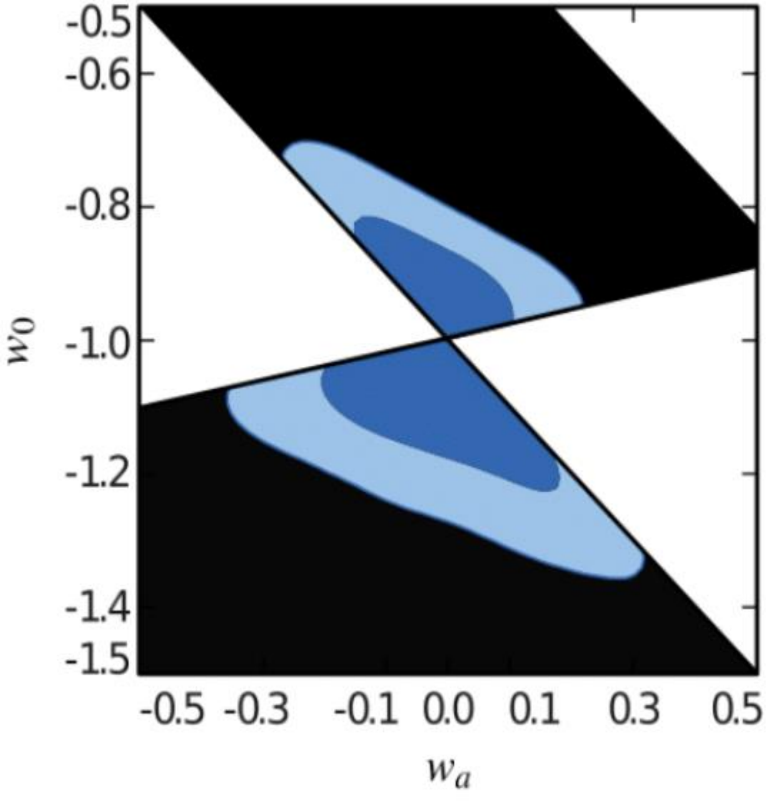


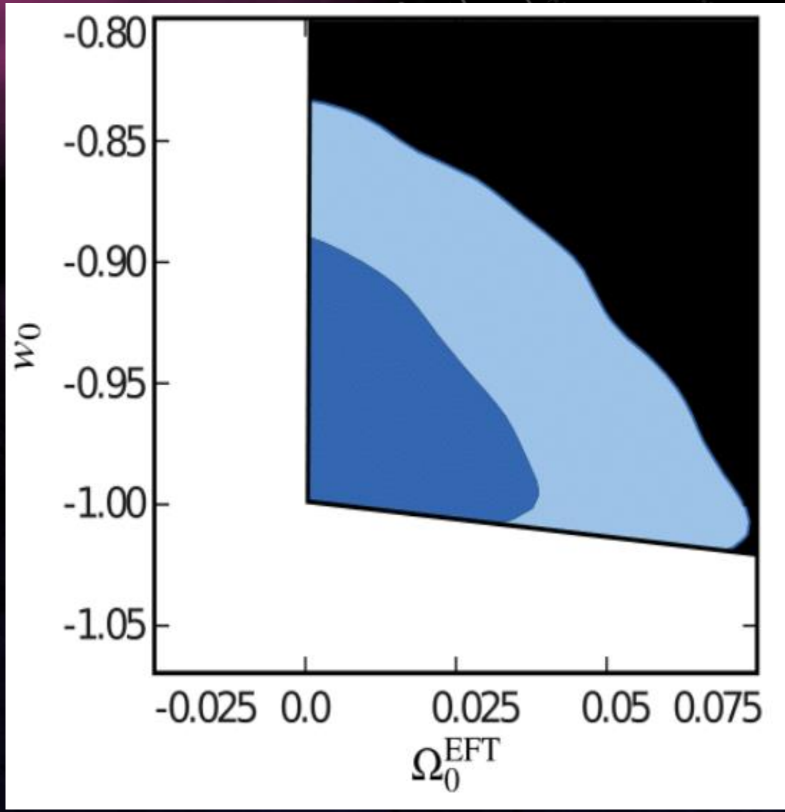

- EFTCAMB: Patch implementing the EFT of DE/MG

- Viability conditions implemented in a module allowing for
  - automatic application during data analysis making it more efficient
  - extensive parameter space sampling in order to study the viable regions

B. Hu, M. Raveri, Noemi Frusciante, A. Silvestri, PRD 89 (2014) 103530,
M. Raveri, B. Hu, Noemi Frusciante, A. Silvestri, PRD 90 (2014) 043513

# PARAMETER SPACE



# SUMMARY

- Tests of gravity at cosmological scales increasingly accurate due to upcoming data
- Many modifications of GR proposed, mainly to explain cosmic acceleration
- Stability conditions derived for the scalar field extensions in the presence of matter
- Numerical implemented into EFTCAMB and presently used heavily to study the viable parameter space within the language of the EFT.

# THANK YOU!


THE WORLD IS BEAUTIFUL
OUTSIDE WHEN THERE IS
STABILITY INSIDE.

# CONTRUCTING THE EFT OF DE/MG: THE BUILDING BLOCKS

The Arnowitt-Deser-Misner (ADM) 3+1 decomposition of spacetime is particularly suitable for this job

$$ds^2 = -\underbrace{N^2}_{\text{lapse}} dt^2 + h_{ij}(dx^i + \underbrace{N^i}_{\text{shift}} dt)(dx^j + N^j dt)$$

The building blocks consist of geometrical objects invariant under the time dependent spatial diffeomorphisms multiplied by time dependent unspecified functions, called *"EFT functions"*.

# MATTER ACTION

Sorkin Schutz Lagrangian for a barotropic fluid:

$$S_{\mathrm{m}} = -\int d^4x[\sqrt{-g}\,\rho(n) + J^\nu \partial_\nu \ell]\,,$$

where ρ and n the energy and number density respectively.

This Langrangian finite when w=0, in contrast to $P(\mathcal{X}) \equiv P(-\frac{(\partial\sigma)^2}{2})$

B.F. Schutz and R. Sorkin, Annals Phys. 107, 1 (1977). doi:10.1016/0003-4916(77)90200-7
J.D. Brown, Class. Quant. Grav. 10, 1579 (1993). doi:10.1088/0264-9381/10/8/017 [gr-qc/9304026]