GSR approach to Horndeski inflation

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Based on: **"Reconciling tensor and scalar observables in G-inflation"**
with
S. Passaglia, H. Motohashi, W. Hu, and O. Mena.

Outline:

1. Horndeski inflation.
2. Generalized slow-roll (GSR) techniques.
3. G-inflation.
4. G-step model.

"The most general scalar-tensor theory, in curved spacetime, which leads to second-order equations of motion":

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g} \mathcal{L}_i$$

$$\longrightarrow \quad \mathcal{L}_2 = K(\phi, X)$$

$$\boxed{X \equiv -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi}$$

$$\longrightarrow \quad \mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\longrightarrow \quad \mathcal{L}_4 = G_4(\phi, X)\mathcal{R} + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2\right]$$

$$\longrightarrow \quad \mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3\right]$$

Horndeski inflation

- Kobayashi et al.; 1105.5723
- Motohashi, Hu; 1704.01128

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$S_\gamma^{(2)} = \sum_{\gamma=+,\times} \int \mathrm{d}^4x \frac{a^3 b_t}{4c_t^2} \left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2} \gamma_\lambda^2 \right)$$

In canonical inflation:

$$b_s = b_t = c_s = c_t = 1$$

Scalar parameters:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

$$\longrightarrow \quad b_s = \frac{c_s^2}{\epsilon_H}\left[\frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}\right]$$

$$\longrightarrow \quad c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

Tensor parameters:

$$\longrightarrow \quad b_t = w_1c_t^2 \qquad \longrightarrow \quad c_t^2 = \frac{w_4}{w_1}$$

$$w_1 = M_{\rm pl}^2 - 2\left(3G_4 + 2HG_5\dot{\phi}\right) + 2G_{5,\phi}X\,,$$

$$w_2 = 2M_{\rm pl}^2H - 2G_3\dot{\phi} - 2\left(30HG_4 - 5G_{4,\phi}\dot{\phi} + 14H^2G_5\dot{\phi}\right) + 28HG_{5,\phi}X\,,$$

$$w_3 = -9M_{\rm pl}^2H^2 + 3\left(X + 12HG_3\dot{\phi}\right) + 6\left(135H^2G_4 - 2G_{3,\phi}X - 45HG_{4,\phi}\dot{\phi} + 56H^3G_5\dot{\phi}\right) - 504H^2G_{5,\phi}X\,,$$

$$w_4 = M_{\rm pl}^2 + 2\left(G_4 - 2G_5\ddot{\phi}\right) - 2G_{5,\phi}X\,.$$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \, \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

Mukhanov - Sasaki equation

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\tau^2} + \left(c_s^2 k^2 - \frac{1}{z} \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} \right) v = 0$$

$$v = z\zeta$$

$$z = a\sqrt{\frac{2 b_s \epsilon_H}{c_s^2}} \quad ,$$

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Either,

- Assume slow-roll approximation.
- Solve numerically.
- Use GSR techniques.

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- Solve numerically.
- **Use GSR techniques.**

Generalized slow-roll recipe:

- Stewart; 0110322
- Dvorkin, Hu; 0910.2237
- Hu; 1104.4500
- Hu; 1405.2020
- Motohashi, Hu; 1503.04810
- Motohashi, Hu; 1704.01128

… and others.

Generalized slow-roll recipe:

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2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

$\longrightarrow$ $y \equiv \sqrt{2c_s k}\, v$ $\longrightarrow$ $x \equiv k s_s$ $\longrightarrow$ $s_s \equiv \int c_s \mathrm{d}\tau$

$\longrightarrow$ $f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2} \frac{a H s_s}{c_s}}$ $\longrightarrow$ $\Delta_\zeta^2(k) = \lim_{x \to 0} \left| \frac{xy}{f} \right|^2$

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3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

$\longrightarrow$ $$G = -2\ln f + \frac{2}{3}\left(\ln f\right)'$$

$\longrightarrow$ $$W(u) = \frac{3\sin(2u)}{2u^3} - \frac{3\cos(2u)}{u^2} - \frac{3\sin(2u)}{2u}$$

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4. Taylor expand GSR formula and write down analytic equations (OSR).

Optimized SR for Horndeski (leading order):

- Motohashi, Hu; 1704.01128

$$\ln\,\Delta^2 \simeq G\,(\ln\,x_f) + \sum_{p=1}^{\infty} q_p\,(\ln\,x_f)\,G^{(p)}\,(\ln\,x_f)$$

$$G = -2\ln\,f + \frac{2}{3}\,(\ln\,f)'$$

$$f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2} \frac{aHs_s}{c_s}}$$

$$\ln\,\Delta^2_\zeta \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\Big|_{x=x_1}$$

Scalars

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\Big|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\Big|_{x=x_1}$$

$$\ln\,\Delta^2_\gamma \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\Big|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\Big|_{x=x_1}$$

Tensors

$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\Big|_{x=x_1}$$

$$\ln\,x_1 \equiv \frac{7}{3} - \ln\,2 - \gamma_{\mathrm{E}}$$

$\boxed{\text{Optimized SR for Horndeski (leading order):}}$

$$r \equiv \frac{4\Delta_\gamma^2}{\Delta_\zeta^2} \approx 16\epsilon_H \frac{b_s c_s}{b_t c_t} \approx -\frac{8 b_s c_s}{b_t c_t} n_t,$$

Deviations from the standard consistency relation in the observations could be checked in this context.

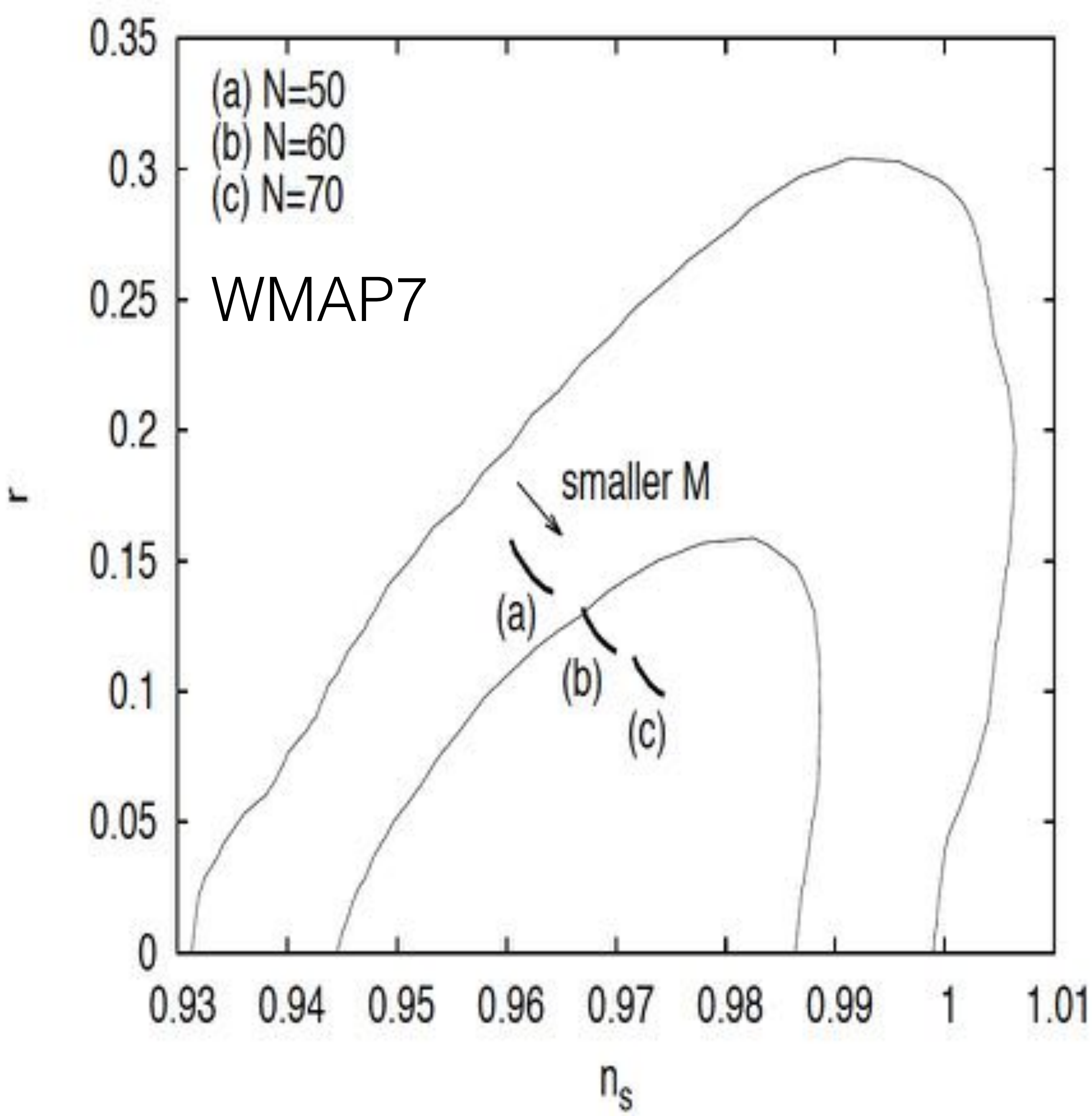
$$\mathcal{L}_2 = X - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_3 = M^{-3} X \Box \phi$$

$$\mathcal{L}_4 = \frac{1}{2} M_{\rm pl}^2 \mathcal{R}$$

G_3 + chaotic inflation = G-inflation

Ohashi, Tsujikawa; 1207.4879



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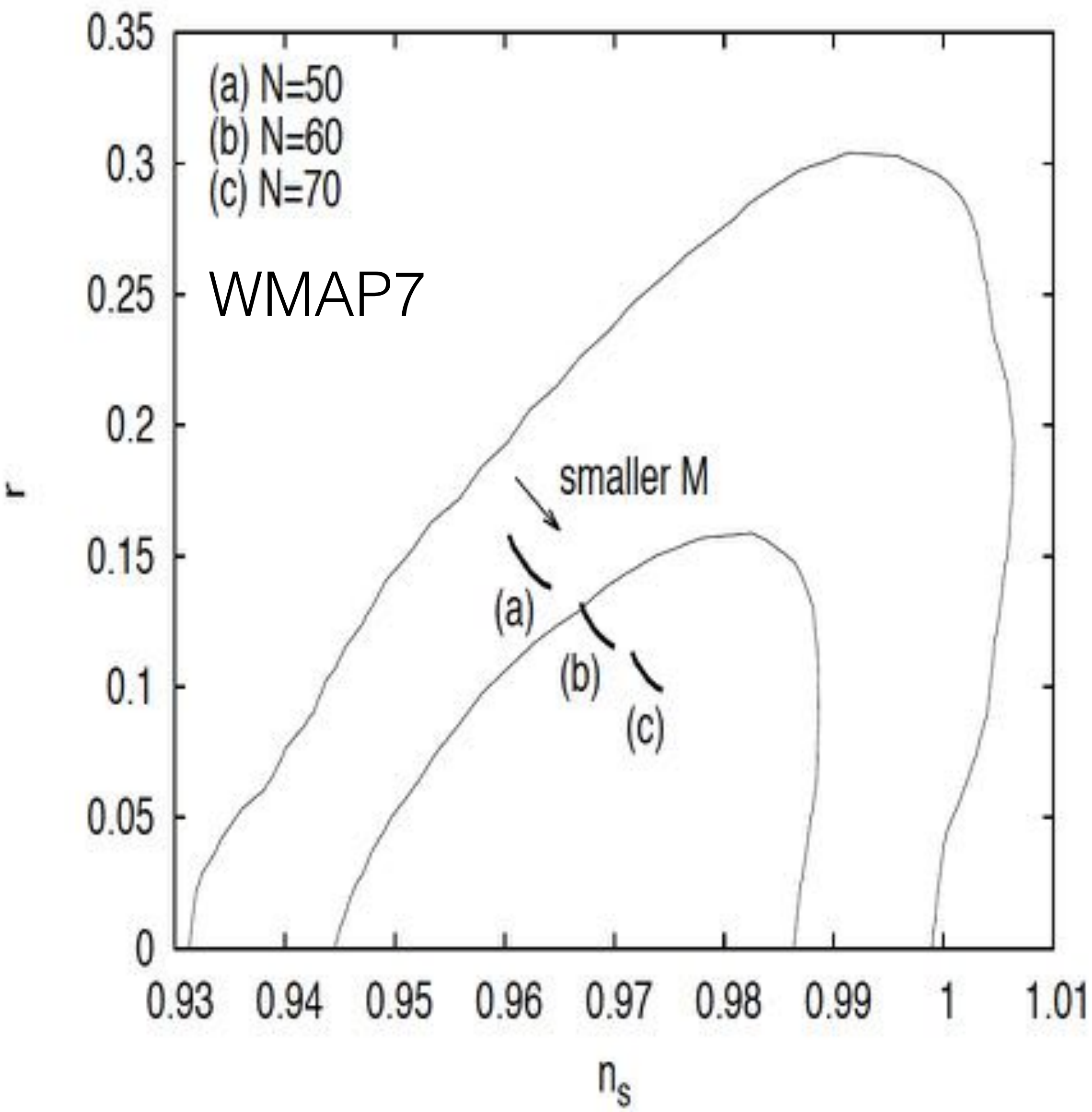

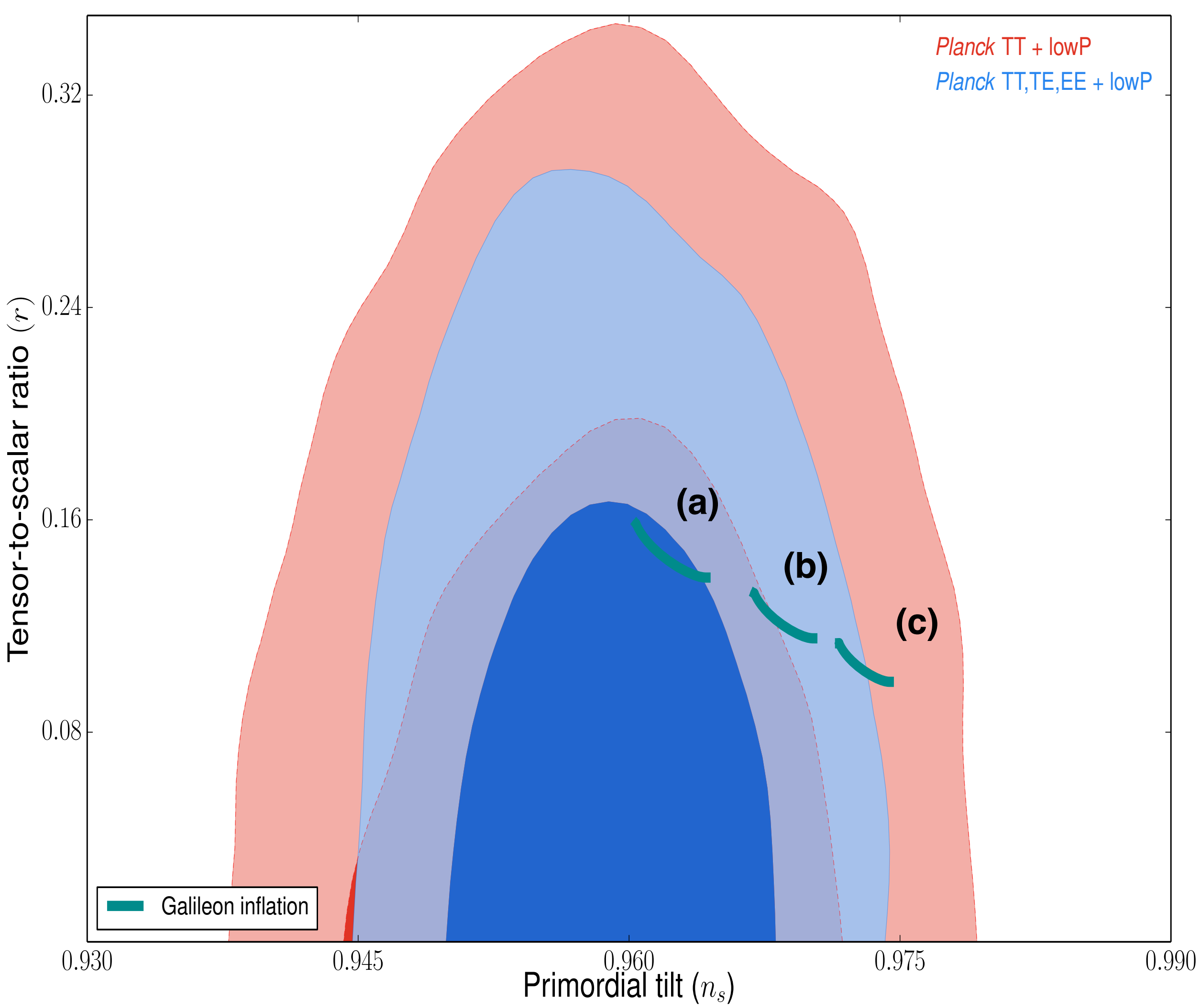

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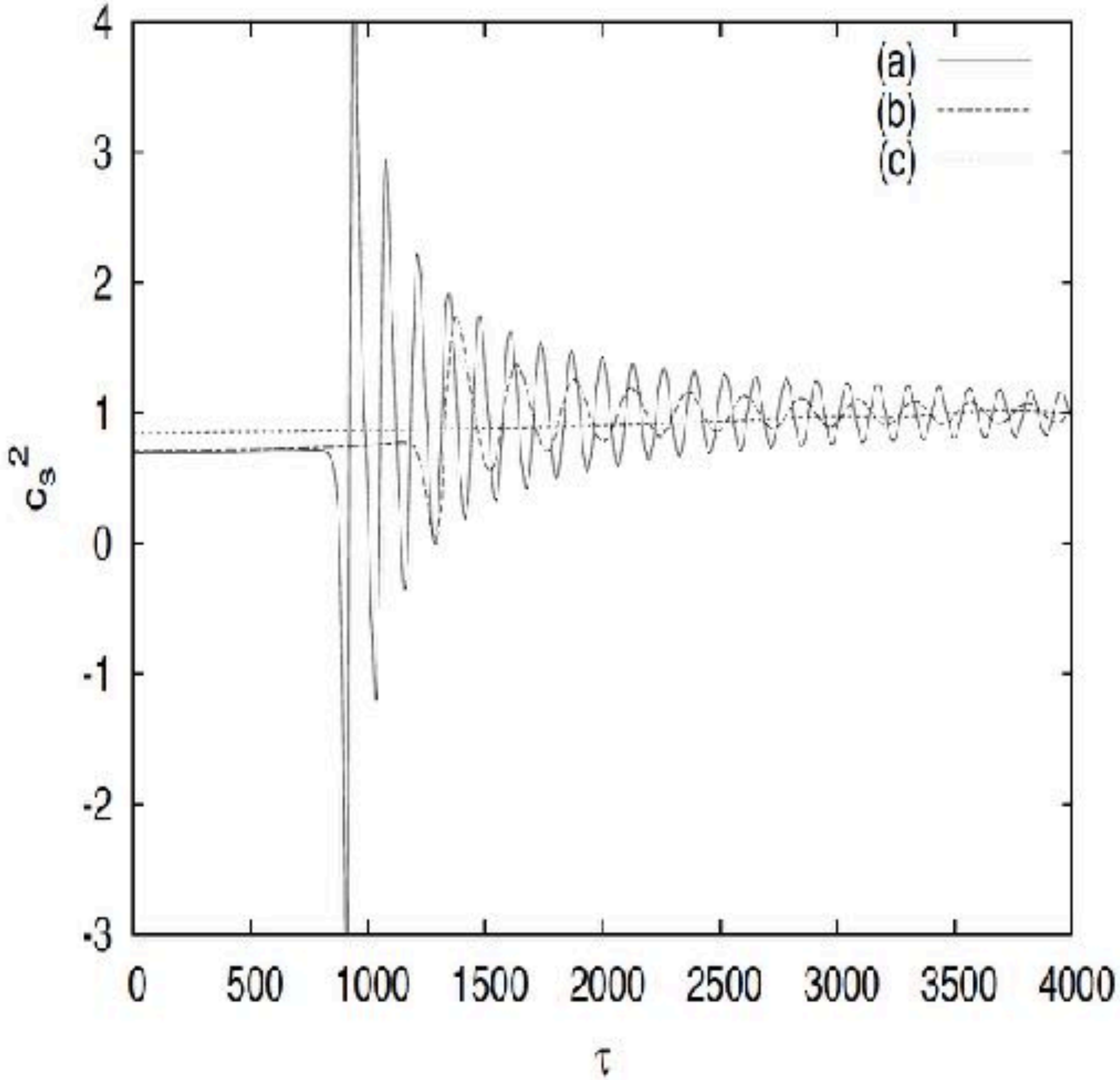


$$\boxed{\mathcal{L}_3 = M^{-3} X \Box \phi}$$

a) $M = 3 \times 10^{-4} M_{\rm pl}$

b) $M = 4.2 \times 10^{-4} M_{\rm pl}$

c) $M = 1 \times 10^{-3} M_{\rm pl}$

For $c_s^2 > 0$, $M > 4.2 \times 10^{-4} M_{\rm pl}$

G_3+*tanh* + chaotic inflation= G-step

HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$

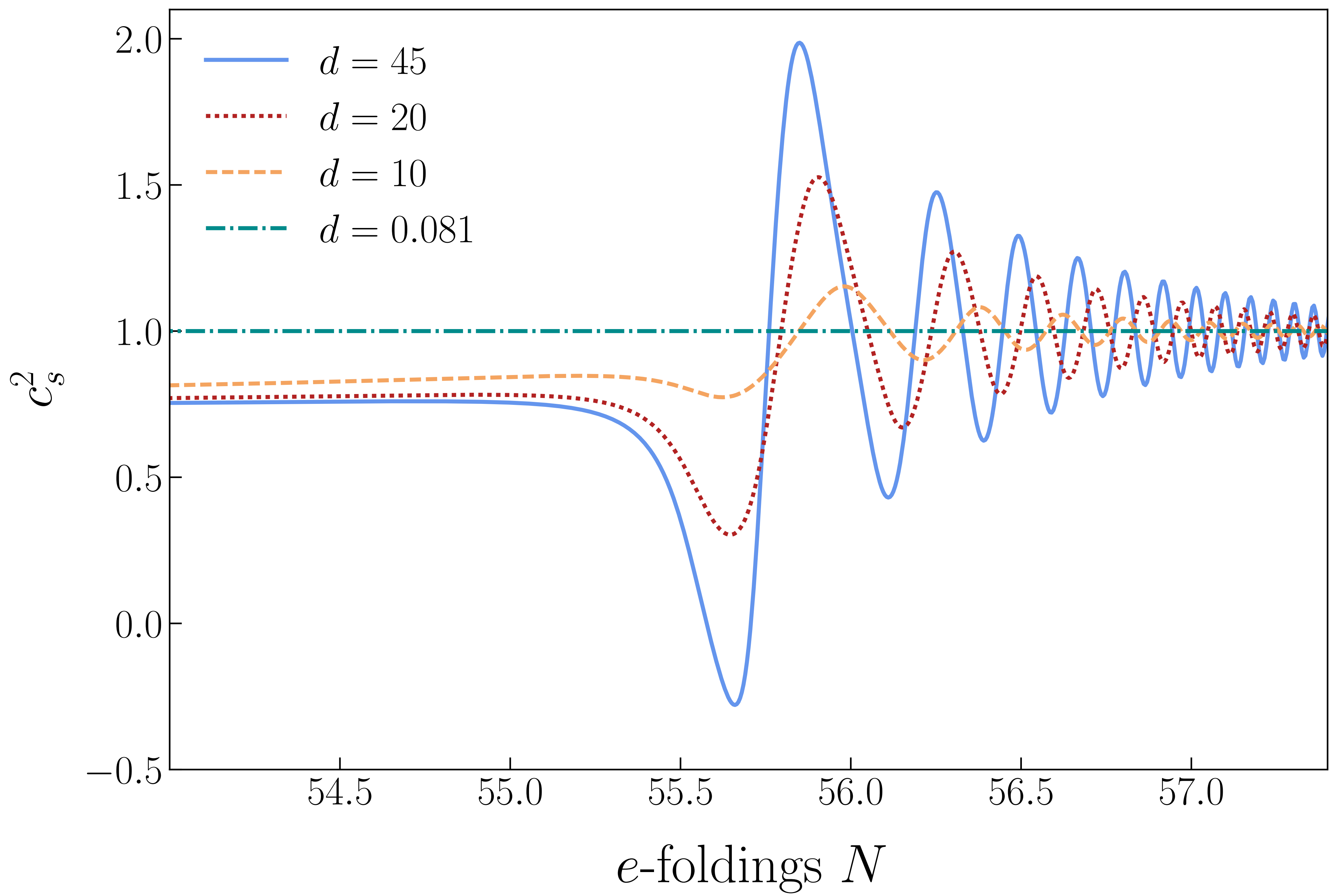
G-step

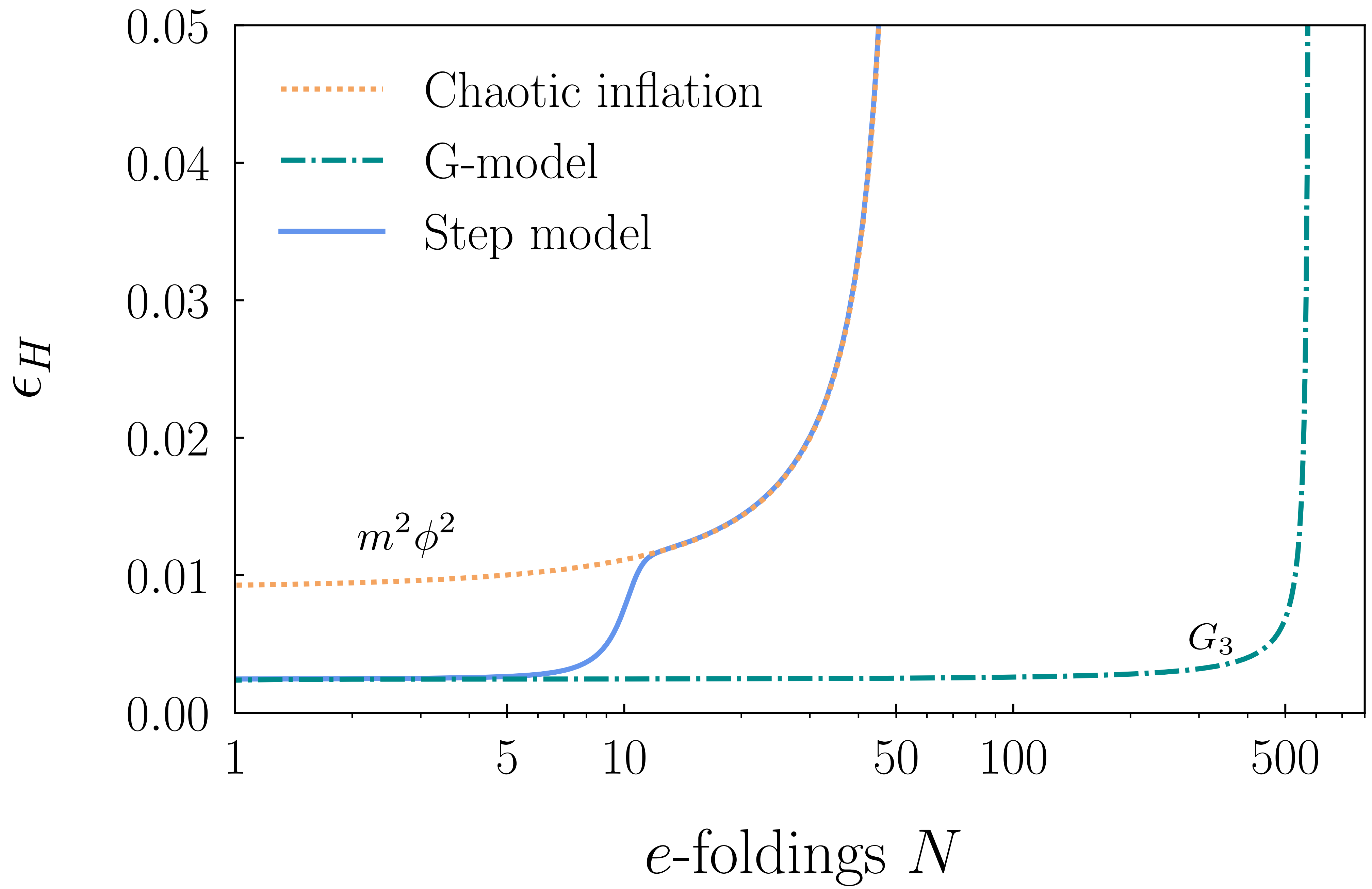
HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$

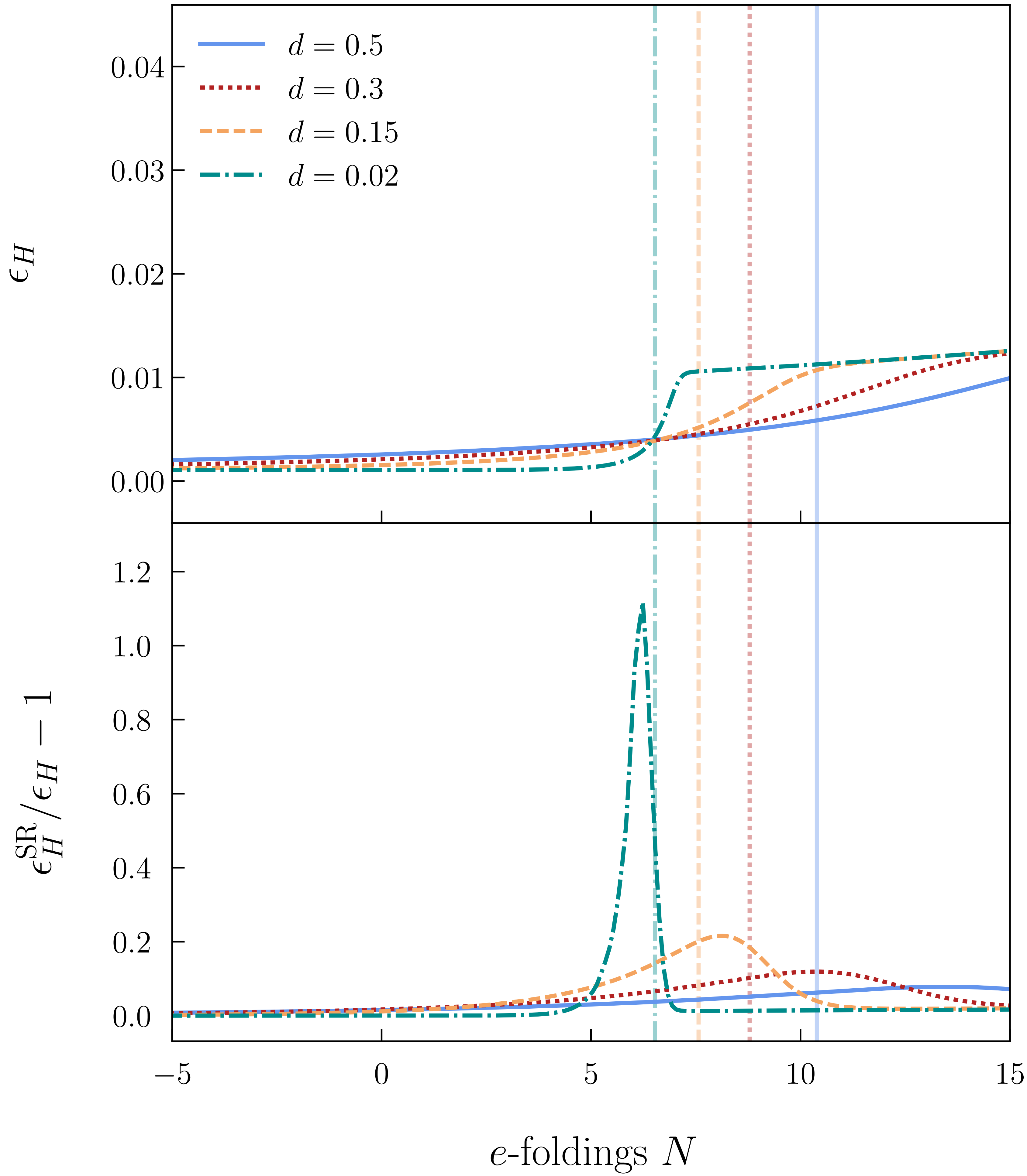
$$\phi_r = 13M_{\rm pl}$$

$$M = 1.3\times 10^{-4}$$



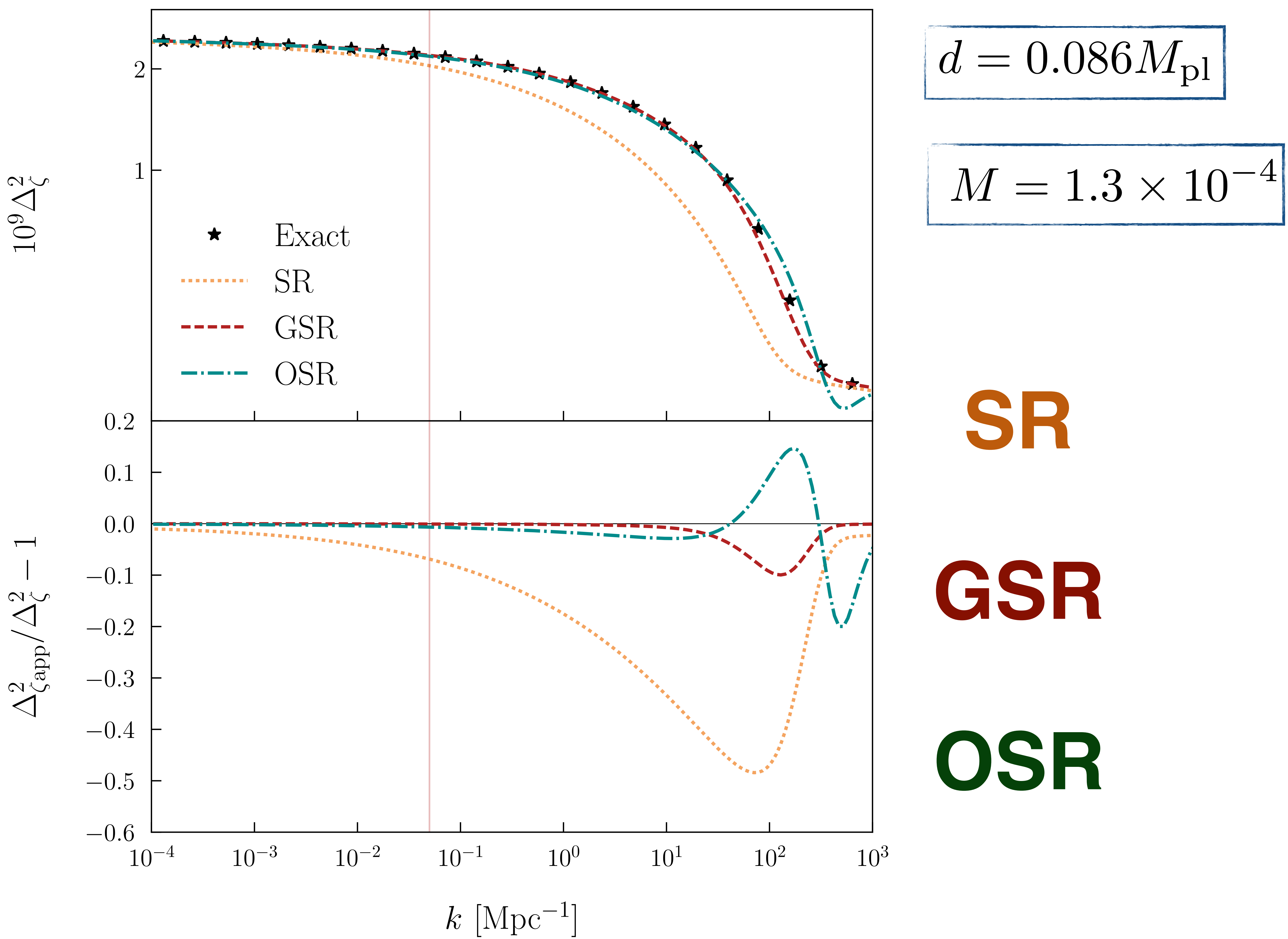


- G-step = a transition between the two regimes.
- Step size ~ 4 *e*-folds

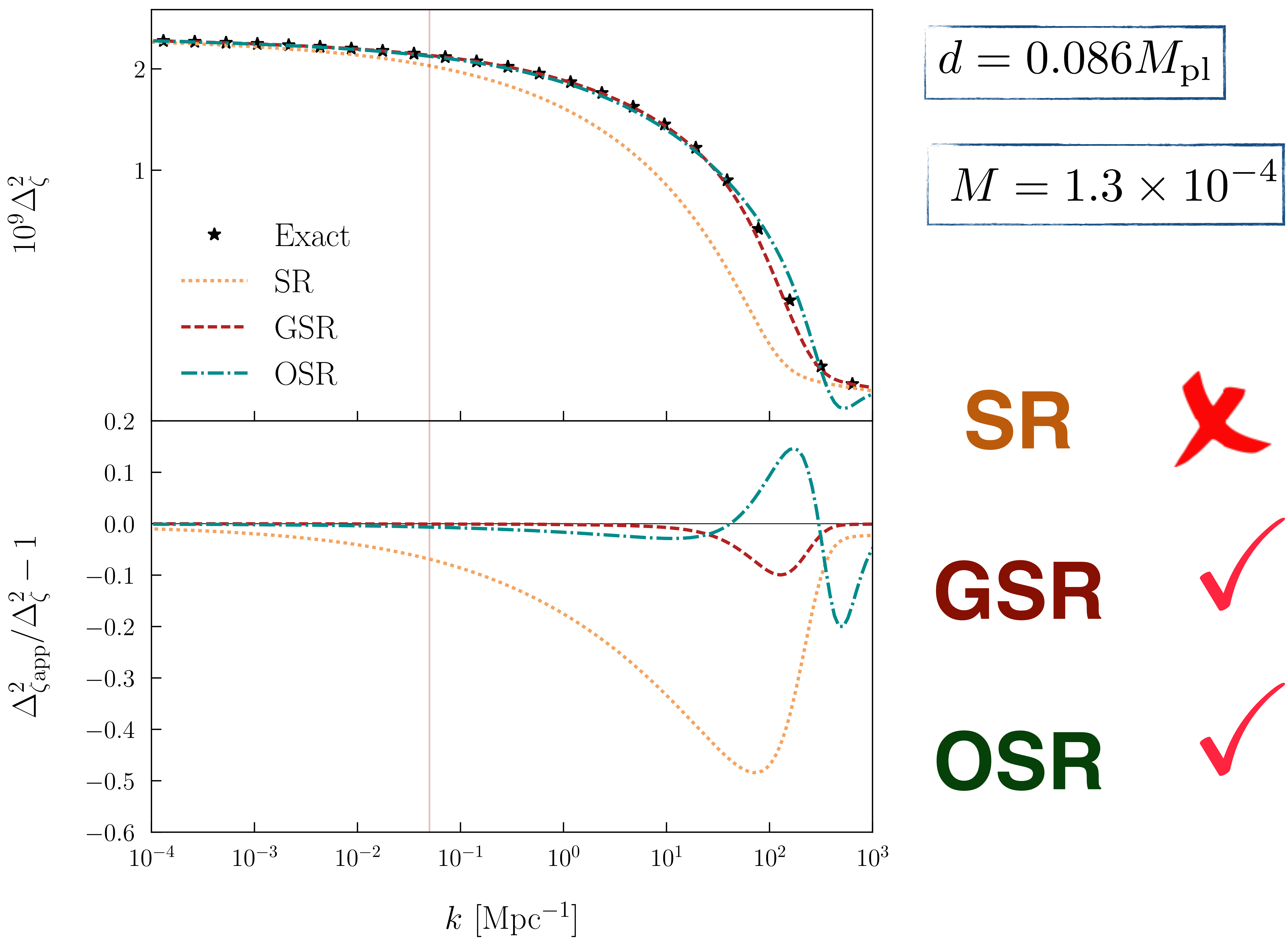


$$\boxed{\epsilon_H = -\frac{\dot{H}}{H^2}}$$

- $N = 0$: CMB scales.
- Vertical lines: where the transition occurs.
- SR violation is maximal around the transition.



$d = 0.086 M_{\rm pl}$
$M = 1.3 \times 10^{-4}$
Exact
SR
GSR
OSR
$10^9 \Delta_\zeta^2$
$\Delta^2_{\zeta\rm app}/\Delta^2_\zeta - 1$
k [Mpc^{-1}]
SR
GSR
OSR



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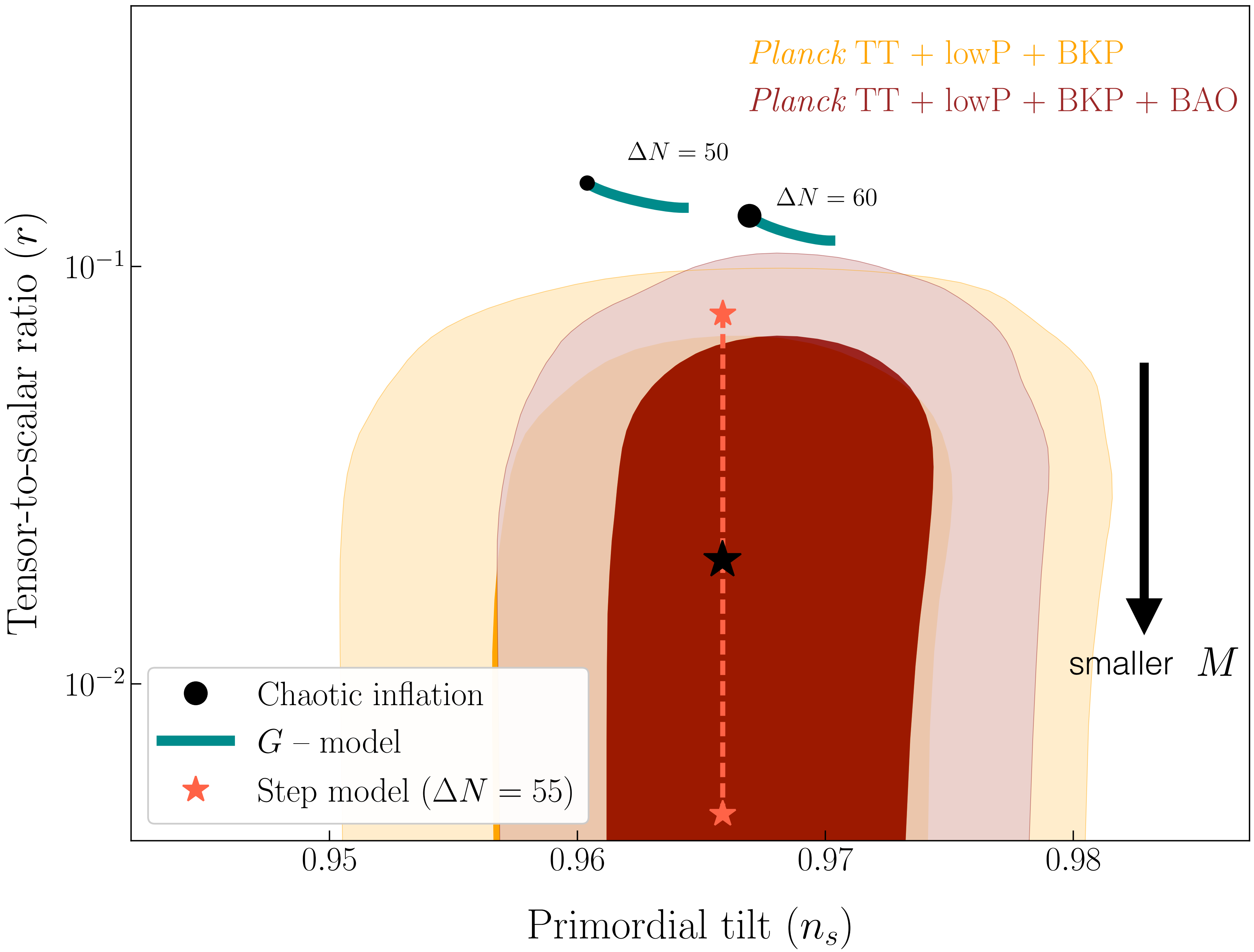
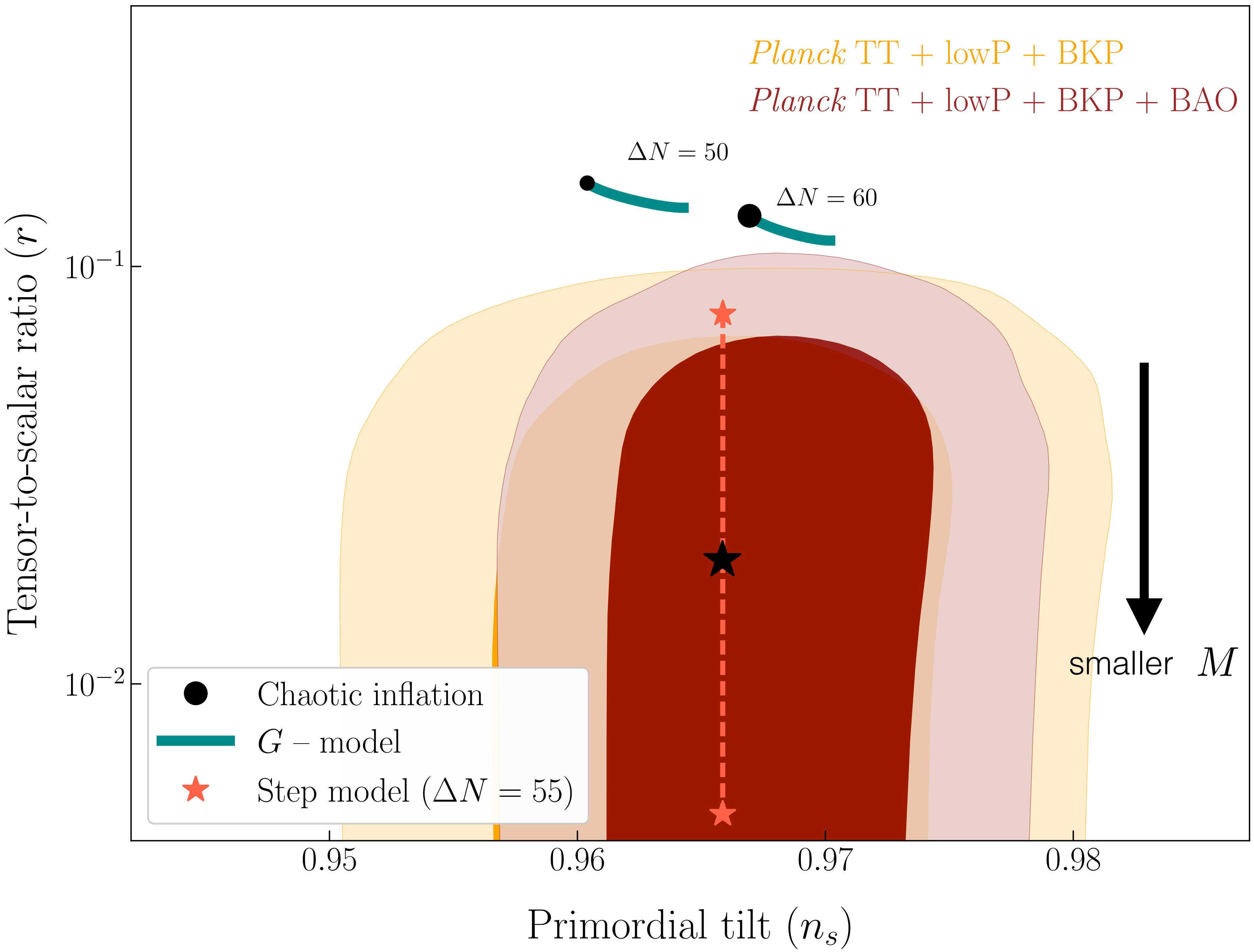


$$\boxed{n_s = 9.66}$$

$$\boxed{\alpha_s = -0.011}$$

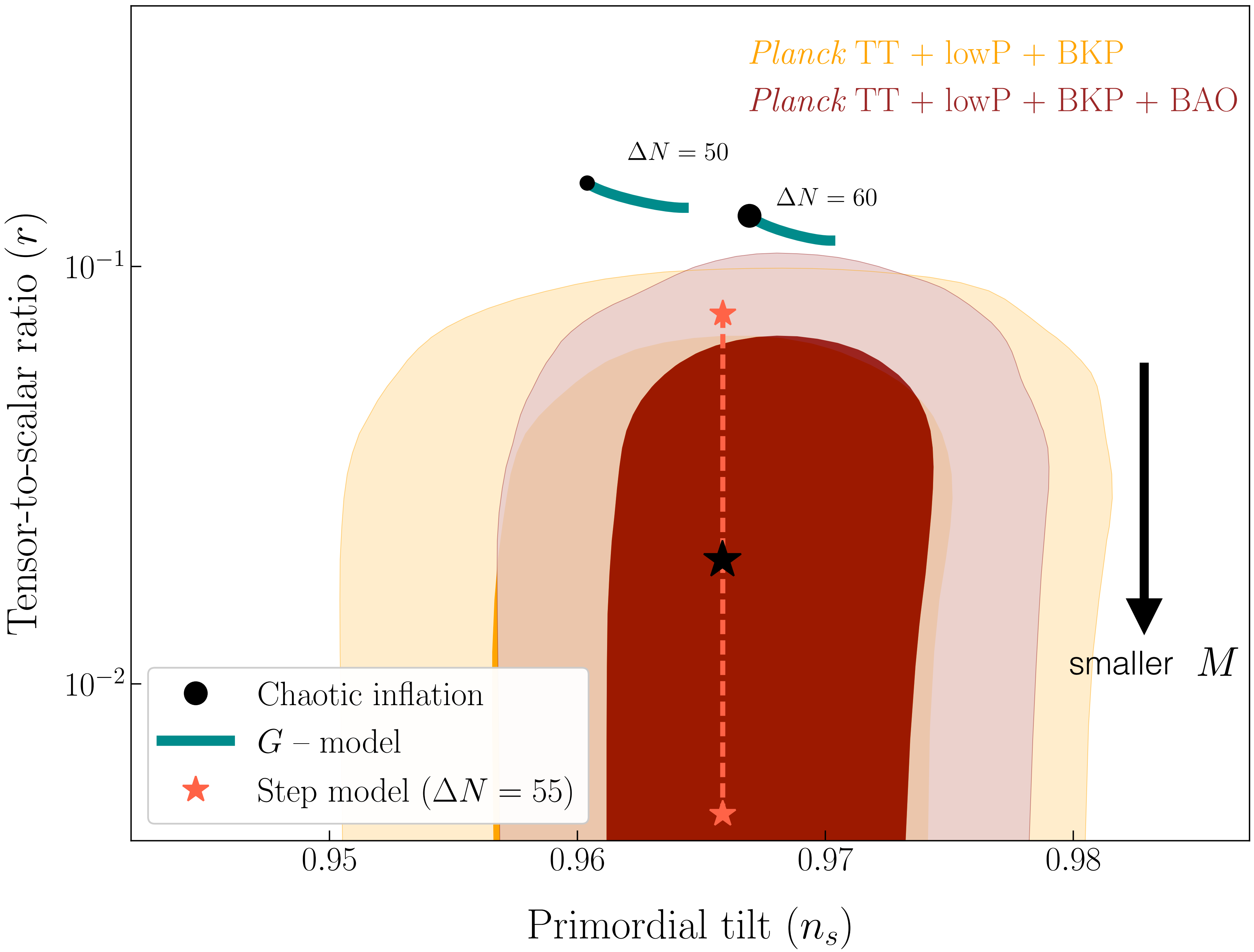
- n_s and α_s fixed.
- Find a set of values for d and ϕ_r .
- This places lower and upper bounds on r .



$\boxed{n_s = 9.66}$

$\boxed{\alpha_s = -0.011}$

- A smaller α_s would shift the line upwards because the step gets wider.
- A larger α_s would be in tension with measurements.



$$n_s = 9.66$$

$$\alpha_s = -0.011$$

$$r \gtrsim 0.005$$

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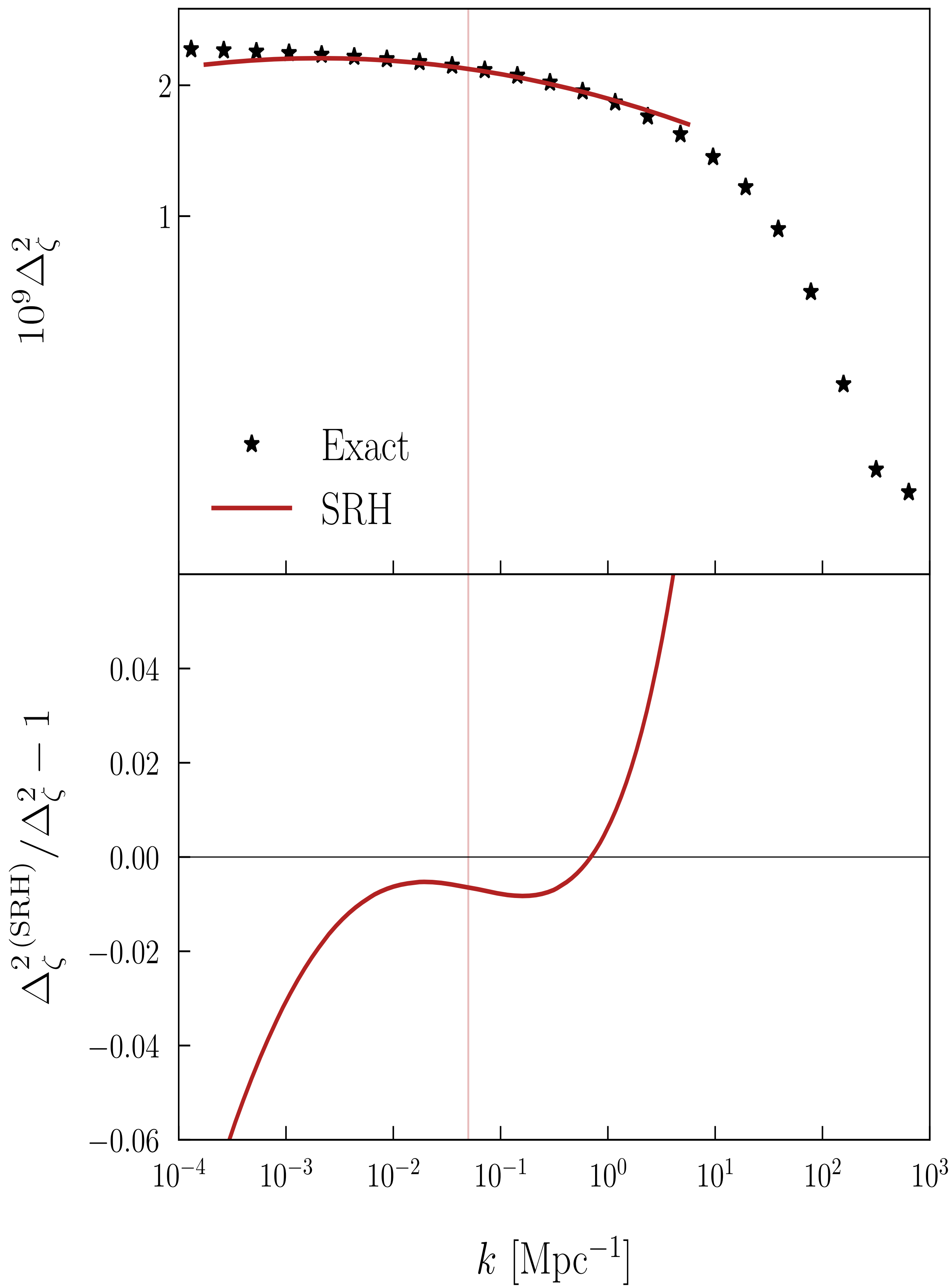
- A larger α_s would be in tension with measurements.



$$\Delta_\zeta^{2\,(\mathrm{SRH})}(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1+\frac{1}{2}\alpha_s\ln(k/k_*)}$$

Using OSR parameters

Deviations of less than 1%

Summary

- ★ Inflation in the Horndeski framework is viable and can *cure* some popular models.

- ★ The G-step model allows us to compute observables during a G-inflation period and to end inflation as canonical.

- ‣ Generalized slow-roll and Optimized slow-roll techniques are efficient tools for this type of models to compute the power spectra an also the bispectrum (see Sam's talk on Friday at 16:50 hrs.)