New insights on the cosmic strings stochastic gravitational wave background

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Outline

CR & Teruaki Suyama arXiv:1709.03845

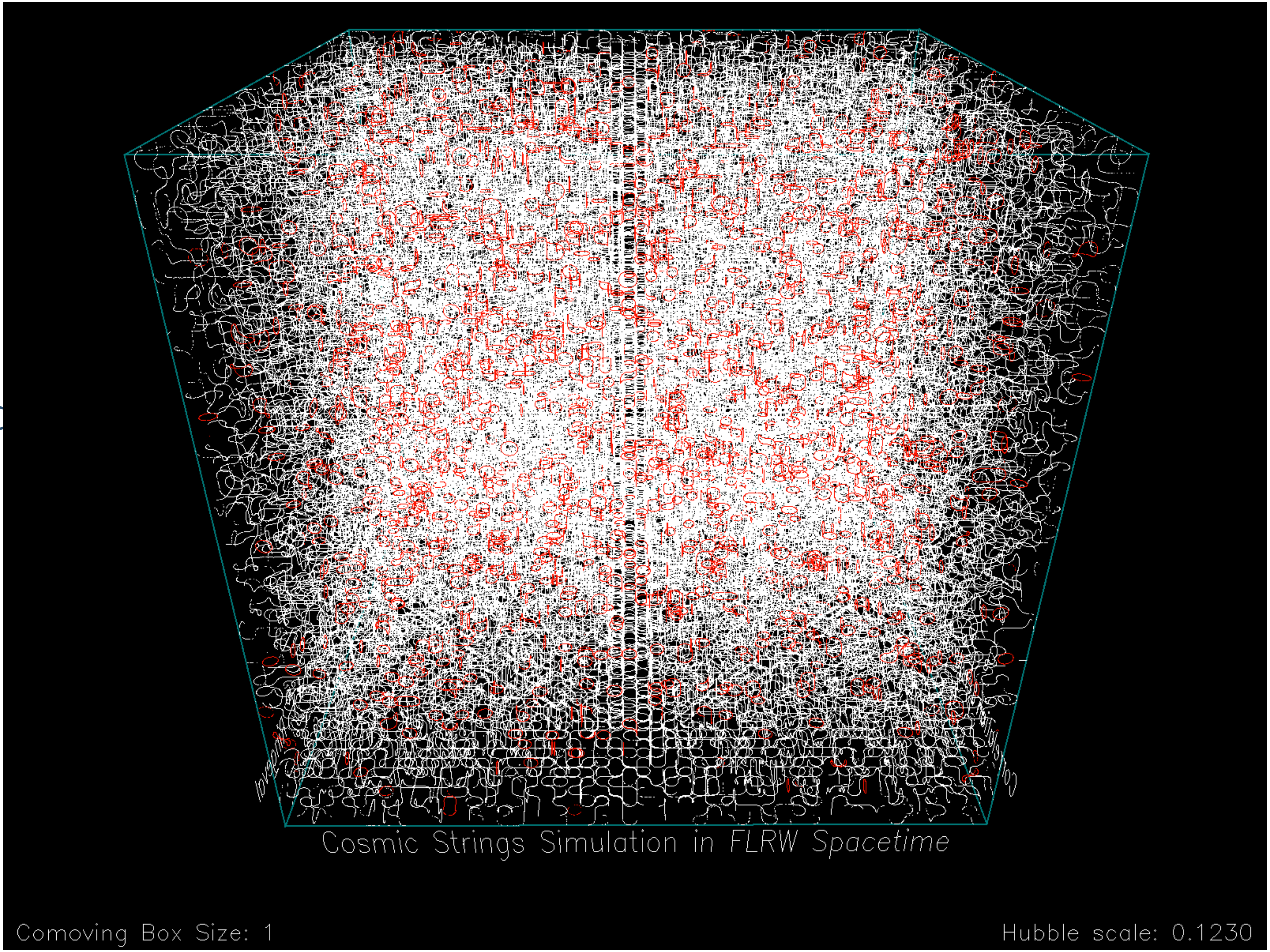
Cosmic strings

- Topological defects
  - Global strings [Davis:1985, Durrer:1998rw, Yamaguchi:1999yp]
  - Non-Abelian strings [Vilenkin:1984rt, Dvali:1993qp, Spergel:1996ai, Bucher:1998mh, McGraw:1998]
  - K- and DBI-strings [Babichev:2006cy, Babichev:2007tn, Sarangi:2007mj]
  - Current-carrying strings [Witten:1984eb, Davis:1988ip,Carter:1989dp, Peter:1992dw, Peter:1992ta]
- Line-like energy density distributions
  - Semi-local strings: energetically favoured for $m_{\mathrm{b}} > m_{\mathrm{h}}$ [Vachaspati:1991, Hindmarsh:1991jq, Achucarro:1999it]
  - Cosmic superstrings: bound states made of p F-strings and q $D1$-brane [Witten:1985fp,Copeland:2009ga, Sakellariadou:2008ie, Polchinski:2004ia, Davis:2008dj]
  - Nambu–Goto strings: Lorentz invariant two-dimensional worldsheet [Goto:1971ce,Nambu:1974]
  - Carter strings [Carter:1989xk, Carter:1992vb, Carter:1994zs, Carter:2000wv]

- Simplest: Nambu–Goto strings, one parameter: U

$$S = -U \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\gamma}, \quad \gamma_{ab} = g_{\mu\nu} X^{\mu}_{,a} X^{\nu}_{,b} \text{ (induced metric)}$$

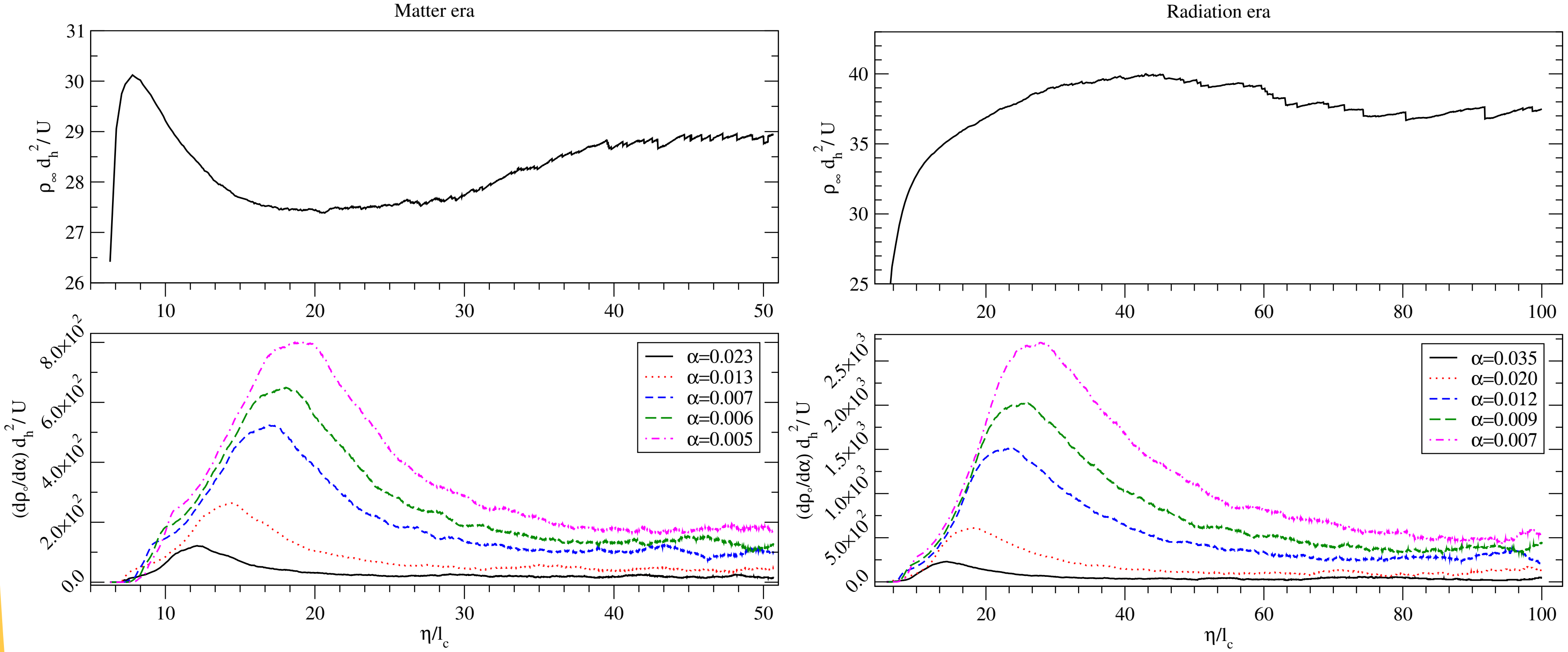
Cosmological evolution



Scaling of the energy density

- Scaling of the energy densities for loops and long strings

[Ringeval:2005kr,Blanco-Pillado:2013qja]



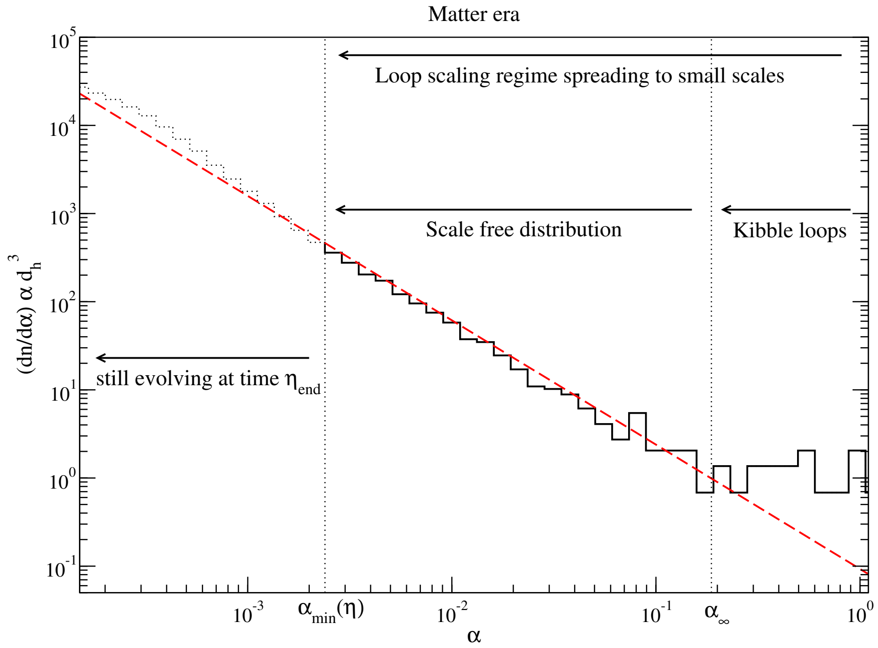
$$\rho_{\rm inf} \frac{d_{\rm h}^2}{U}\bigg|_{\rm mat} = 28.4 \pm 0.9 \qquad \rho_{\rm inf} \frac{d_{\rm h}^2}{U}\bigg|_{\rm rad} = 37.8 \pm 1.7$$

$$\mathrm{d}\rho_\circ \times \frac{d_{\rm h}^2}{U} = \mathcal{S}(\alpha) \quad \text{(time independent)}$$

Scaling of the loop distribution

- By the end of the run

Scaling parts



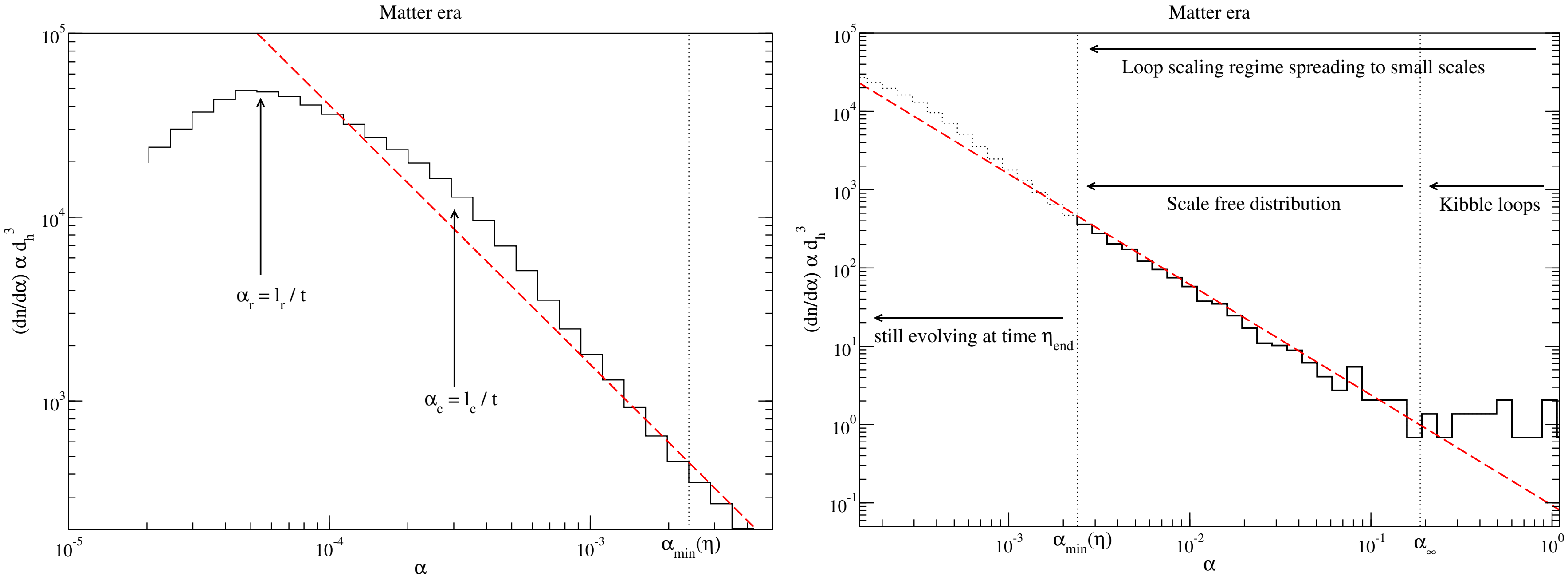
- Scaling form $\mathcal{S}(\alpha) = \dfrac{C_\circ}{\alpha^p}$ with

$$\left\{\begin{array}{ccc} p & \underset{\text{mat}}{=} & 1.41\,^{+0.08}_{-0.07} \\ C_\circ & \underset{\text{mat}}{=} & 0.09\,^{-0.03}_{+0.03} \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{ccc} p & \underset{\text{rad}}{=} & 1.60\,^{+0.21}_{-0.15} \\ C_\circ & \underset{\text{rad}}{=} & 0.21\,^{-0.12}_{+0.13} \end{array}\right.$$

Scaling of the loop distribution

- By the end of the run

Non-scaling parts — Scaling parts



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$$\begin{cases} p \underset{\text{mat}}{=} 1.41\,^{+0.08}_{-0.07} \\ C_\circ \underset{\text{mat}}{=} 0.09\,^{-0.03}_{+0.03} \end{cases} \quad \text{and} \quad \begin{cases} p \underset{\text{rad}}{=} 1.60\,^{+0.21}_{-0.15} \\ C_\circ \underset{\text{rad}}{=} 0.21\,^{-0.12}_{+0.13} \end{cases}$$

Polchinski-Rocha model

- No fragmentation, no reconnection, loops from long string only
  [Polchinski:2006ee,Dubath:2007mf,Rocha:2007ni]
  - Predicts a power law scaling function

$$\mathcal{S}(\alpha) \propto \alpha^{2\chi-2} \implies p = 2(1-\chi)$$

  - Parameter χ is related to two-point functions [Hindmarsh:2008dw]

$$\left\langle \acute{X}^A(\sigma)\acute{X}^B(\sigma')\right\rangle = \frac{1}{2}\delta^{AB}T(\sigma-\sigma') \qquad T(\sigma) \simeq \bar{t}^2 - c_1\left(\frac{\sigma}{\hat{\xi}}\right)^{2\chi}$$

- Agreement with simulations suggests that all neglected effects mostly renormalise $C_\circ$ but not χ
- $\Rightarrow$ use the PR model to understand the loop distribution down to the length scales unreachable with numerical simulations
- $+$ Boltzmann equation...

Including loop's gravitational radiation

- Boltzmann equation + PR production function
  - PR loop production function (from string shape correlations)

$$t^5\mathcal{P}(\ell,t) = c\left(\frac{\ell}{t}\right)^{2\chi-3}$$

  - In an expanding universe

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3\mathcal{P}(\ell,t)$$

  - A loop shrinks due to GW emission ($\gamma \equiv \ell/t$) [Allen:1992]

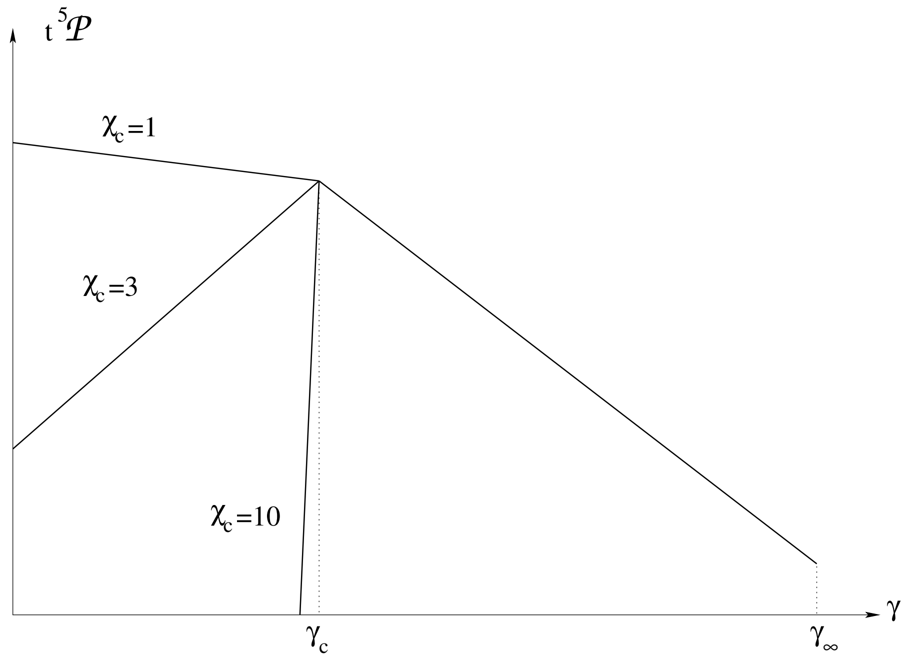
$$\frac{\mathrm{d}\ell}{\mathrm{d}t} = -\gamma_\mathrm{d} \simeq 50GU$$

- Evolution equation [Rocha:2007ni,Lorenz:2010sm]

$$\frac{\partial}{\partial t}\left(a^3\frac{\mathrm{d}n}{\mathrm{d}\ell}\right) - \gamma_\mathrm{d}\frac{\partial}{\partial \ell}\left(a^3\frac{\mathrm{d}n}{\mathrm{d}\ell}\right) = a^3\mathcal{P}(\ell,t)$$

Inclusion of gravitational backreaction

- PR model + GW emission + GW backreaction [Lorenz:2010sm]
  - GW backreaction: $\gamma_\mathrm{c} \simeq 20(GU)^{1+2\chi}$ [Polchinski:2007]
  - Postulated piecewise scaling loop production function



$$t^5\mathcal{P}\left(\gamma = \frac{\ell}{t}, t\right) \propto \gamma^{2\chi - 3}$$

$$\gamma_\mathrm{c} \ll \gamma_\mathrm{d} \ll \gamma_\infty \lesssim 1$$

- Allows us to extrapolate numerical simulations to small ℓ
- Boltzmann equation can be completely solved analytically (see arXiv.1006.0931)

Cosmological attractor

- From any initial loop distribution $\mathcal{N}_{\text{ini}}(\ell)$, one gets $\mathcal{F}(\gamma,t) \equiv \dfrac{\mathrm{d}n}{\mathrm{d}\ell}(\gamma,t)$
- Scaling attractor does not depend on $\mathcal{N}_{\text{ini}}$ nor on GW backreaction details

$$t^4\mathcal{F}(\gamma \geq \gamma_{\text{c}}, t) = \left(\frac{t}{t_{\text{ini}}}\right)^4 \left(\frac{a_{\text{ini}}}{a}\right)^3 t_{\text{ini}}^4\, \mathcal{N}_{\text{ini}}\left\{\left[\gamma + \gamma_{\text{d}}\left(1 - \frac{t_{\text{ini}}}{t}\right)\right] t\right\} + C(\gamma+\gamma_{\text{d}})^{2\chi-3} f\left(\frac{\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\right)$$
$$- C(\gamma+\gamma_{\text{d}})^{2\chi-3}\left(\frac{t}{t_{\text{ini}}}\right)^{2\chi+1}\left(\frac{a_{\text{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\frac{t_{\text{ini}}}{t}\right),$$

$$t^4\mathcal{F}(\gamma_{\mathcal{T}} \leq \gamma < \gamma_{\text{c}}, t) = \left(\frac{t}{t_{\text{ini}}}\right)^4 \left(\frac{a_{\text{ini}}}{a}\right)^3 t_{\text{ini}}^4\, \mathcal{N}_{\text{ini}}\left\{\left[\gamma + \gamma_{\text{d}}\left(1 - \frac{t_{\text{ini}}}{t}\right)\right] t\right\} + C_{\text{c}}(\gamma+\gamma_{\text{d}})^{2\chi_{\text{c}}-3} f_{\text{c}}\left(\frac{\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\right)$$
$$- C(\gamma+\gamma_{\text{d}})^{2\chi-3}\left(\frac{t}{t_{\text{ini}}}\right)^{2\chi+1}\left(\frac{a_{\text{ini}}}{a}\right)^3 f\left(\frac{\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\frac{t_{\text{ini}}}{t}\right)$$
$$+ K\left(\frac{\gamma_{\text{c}}+\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\right)^4 \left[\frac{a\left(\dfrac{\gamma+\gamma_{\text{d}}}{\gamma_{\text{c}}+\gamma_{\text{d}}}t\right)}{a(t)}\right]^3,$$

$$t^4\mathcal{F}(0 < \gamma < \gamma_{\mathcal{T}}, t) = \left(\frac{t}{t_{\text{ini}}}\right)^4 \left(\frac{a_{\text{ini}}}{a}\right)^3 t_{\text{ini}}^4\, \mathcal{N}_{\text{ini}}\left\{\left[\gamma + \gamma_{\text{d}}\left(1 - \frac{t_{\text{ini}}}{t}\right)\right] t\right\} + C_{\text{c}}(\gamma+\gamma_{\text{d}})^{2\chi_{\text{c}}-3} f_{\text{c}}\left(\frac{\gamma_{\text{d}}}{\gamma+\gamma_{\text{d}}}\right)$$

$$\gamma_{\mathcal{T}}(t) \equiv (\gamma_{\text{c}}+\gamma_{\text{d}})\frac{t_{\text{ini}}}{t} - \gamma_{\text{d}}, \qquad \mu \equiv 3\nu - 2\chi - 1$$
$$f(x) \equiv {}_2\mathrm{F}_1(3-2\chi, \mu; \mu+1; x) \qquad f_{\text{c}}(x) \equiv {}_2\mathrm{F}_1(3-2\chi_{\text{c}}, \mu_{\text{c}}; \mu_{\text{c}}+1; x)$$

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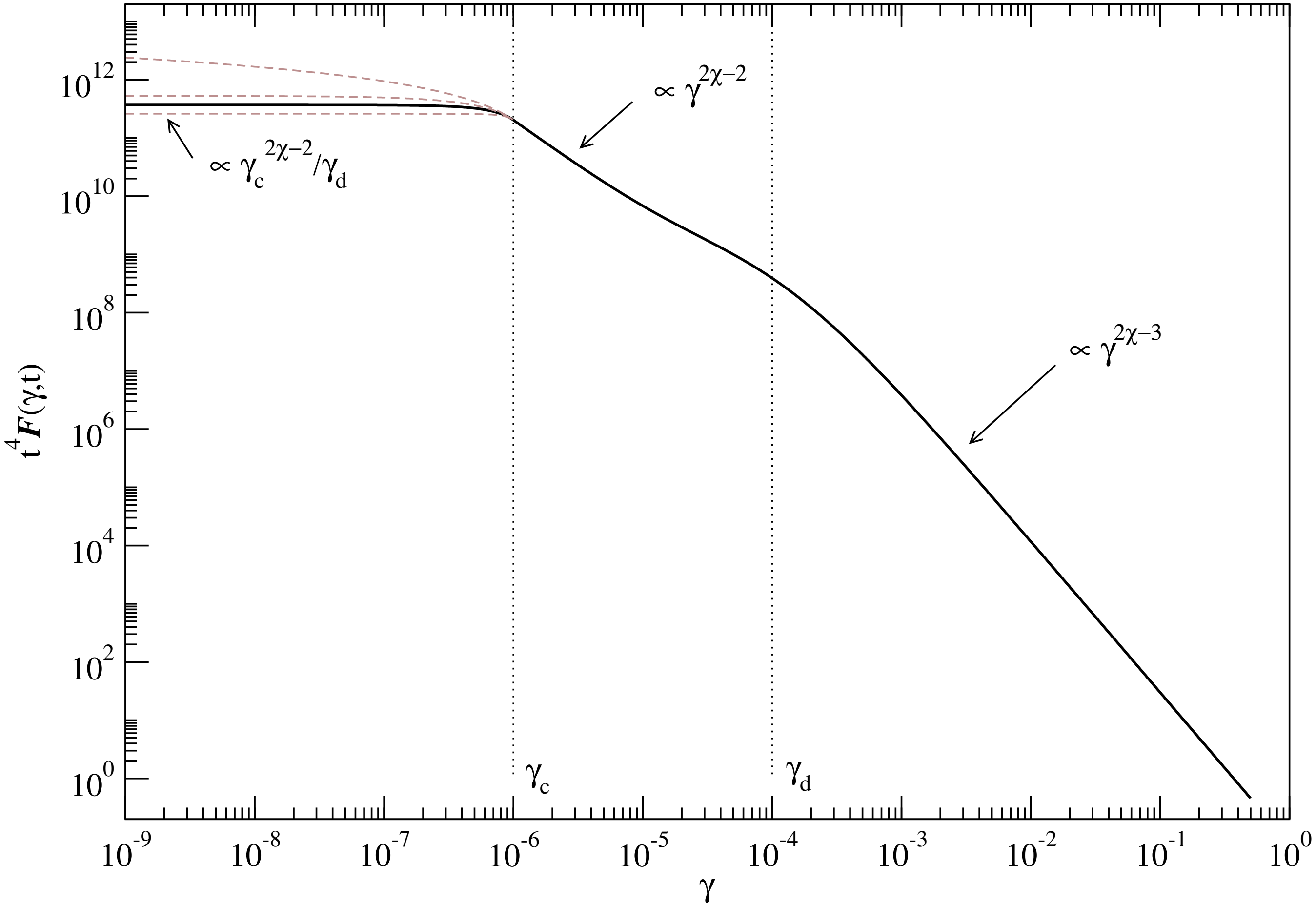
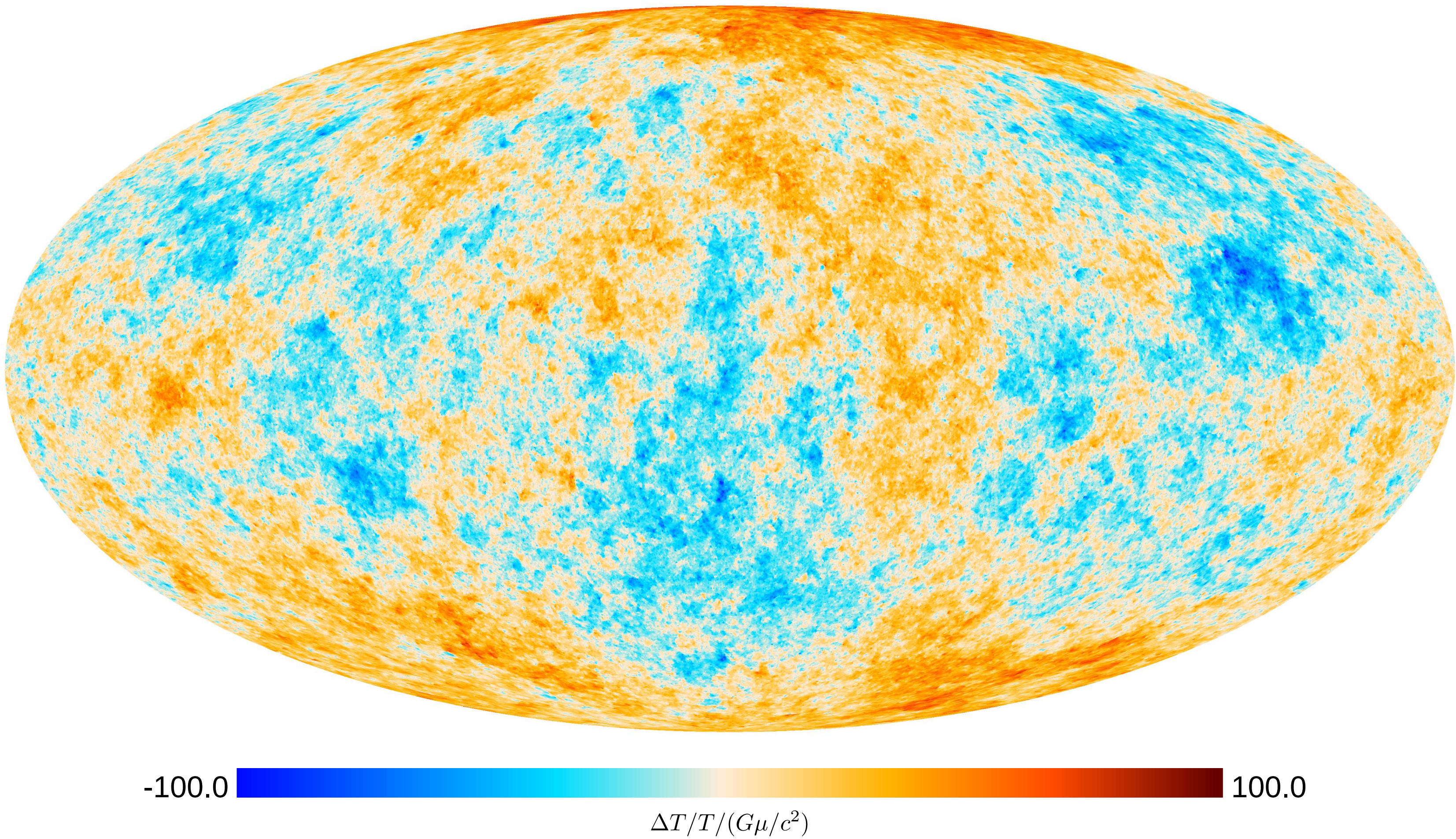


CMB constraints on long strings

- Full sky synthetic string map of 2×10^8 pixels [Ringeval&Bouchet:2012tk, Ade:2013xla]
- Planck imposes: $GU < \mathcal{O}(1) \times 10^{-7}$



Gravitational wave bursts from loops

- Leading order for a loop of $T = \ell/2$, frequency $\varpi_n = 2\pi n/T$

$$\bar{h}^{\mu\nu}_{\flat}(\varpi_n, |\boldsymbol{r}|\hat{\boldsymbol{n}}) = \frac{GU}{T}\frac{e^{i\varpi_n|\boldsymbol{r}|}}{|\boldsymbol{r}|}C^{\mu\nu}, \qquad C^{\mu\nu} \equiv I^{\mu}_{+}I^{\nu}_{-} + I^{\nu}_{+}I^{\mu}_{-}$$

$$I^{\mu}_{\epsilon} \equiv \int \mathrm{d}\sigma_\epsilon \exp\left(\frac{i\varpi_n\sigma_\epsilon}{2} - \frac{i\varpi_n\hat{\boldsymbol{n}}\cdot\boldsymbol{X}_\epsilon}{2}\right)\frac{\mathrm{d}X^{\mu}_{\epsilon}}{\mathrm{d}\sigma_\epsilon}$$

- Maximal GW emission when [Damour&Vilenkin:2001]
  - ✦ Both $I^{\mu}_{\pm}$ have saddle points: $\hat{\boldsymbol{n}} = \acute{\boldsymbol{X}}_{+} = \acute{\boldsymbol{X}}_{-} \Rightarrow$ cusp

$$\Omega_{\mathrm{beam}} = \pi\theta^2_{\mathrm{beam}} = \pi\left(\frac{8\pi}{\sqrt{3}\varpi\ell}\right)^{2/3}, \qquad C^{\mu\nu} \propto \varpi^{-4/3}$$

  - ✦ One $I^{\mu}_{\pm}$ has a saddle point and $\acute{\boldsymbol{X}}_{\mp}$ discontinuous $\Rightarrow$ kink

$$\Omega_{\mathrm{beam}} = 2\pi\theta_{\mathrm{beam}}, \qquad C^{\mu\nu} \propto \varpi^{-5/3}$$

  - ✦ Both $\acute{\boldsymbol{X}}_{\pm}$ are discontinous: two kinks collide

$$\Omega_{\mathrm{beam}} = 4\pi, \qquad C^{\mu\nu} \propto \varpi^{-2}$$

Stochastic GW spectrum

- Time-averaged GW strain power $[k = (\omega, \omega\hat{\boldsymbol{n}})]$ for one loop

$$\bar{h}_{\mathrm{c}}^2(\omega, \ell, z) = \left[\frac{GU(1+z)}{\chi(z)}\right]^2 \bar{\mathcal{C}}^2\left[k(1+z)\right]\Theta[\omega - \omega_1(\ell, z)]$$

$$\bar{\mathcal{C}}^2 \equiv C^*_{\alpha\beta}C^{\alpha\beta} - \frac{1}{2}\left|C\right|^2$$

- Integrating over all loops

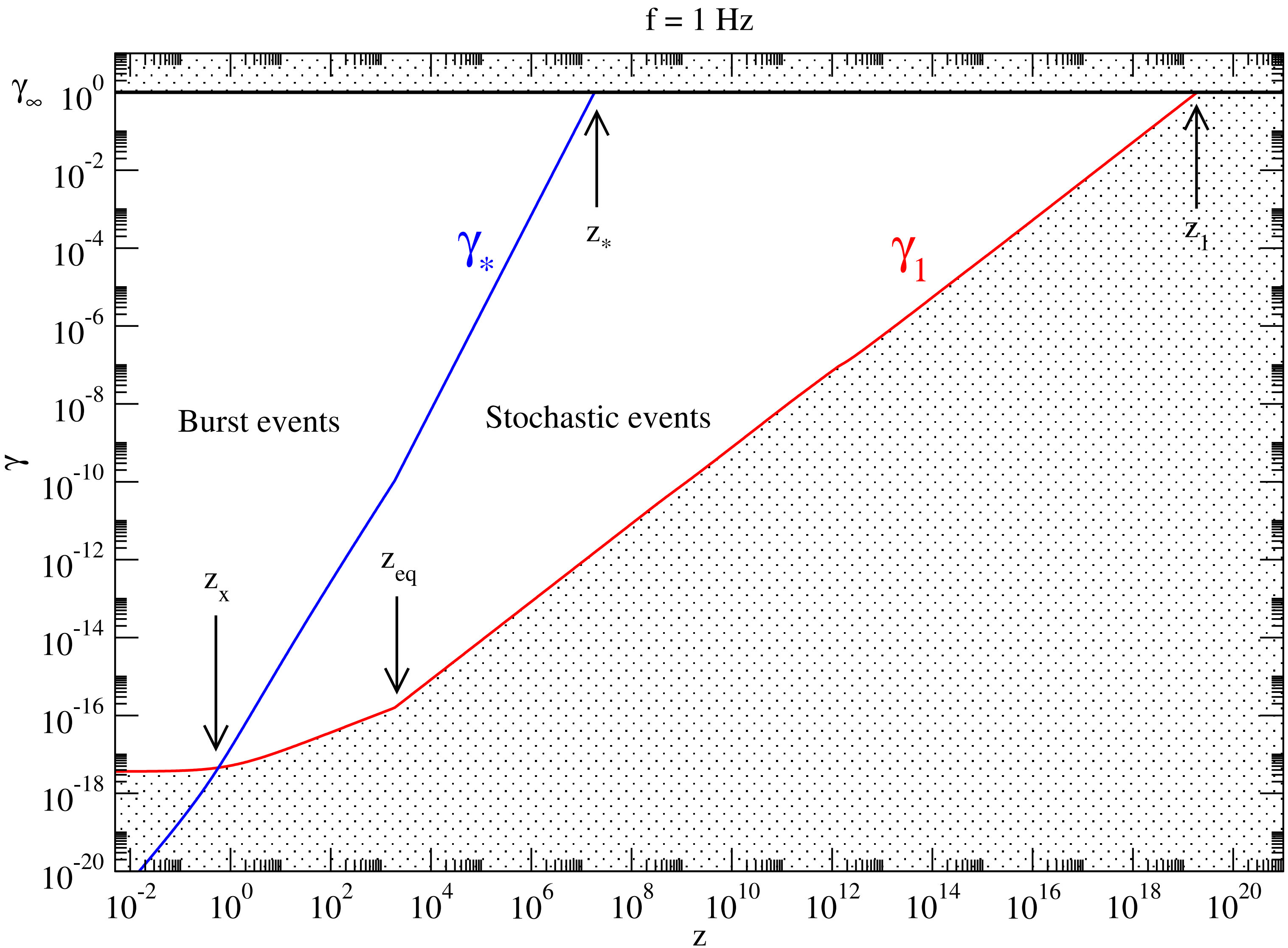
$$\Omega_{\mathrm{sgw}}(\omega) = \frac{(GU)^2\omega^3}{6\pi H_0^2}\iint \mathrm{d}z\mathrm{d}\ell \frac{\mathrm{d}V}{\mathrm{d}z}\frac{\mathcal{F}(\ell, z)}{\ell(1+z)}\Delta_{\mathrm{beam}}(\omega, \ell, z)$$
$$\times \frac{(1+z)^2}{\chi^2(z)}\bar{\mathcal{C}}^2(\omega, \ell, z)\Theta[\omega - \omega_1(\ell, z)]\,\Theta\left[\bar{h}_\star(\omega) - \bar{h}_{\mathrm{c}}(\omega, \ell, z)\right]$$

where $\bar{h}_\star(\omega)$ is solution of:

$$\iint \mathrm{d}z\mathrm{d}\ell \frac{2}{(1+z)\ell}\frac{\mathrm{d}V}{\mathrm{d}z}\mathcal{F}(\ell, z)\Delta_{\mathrm{beam}}(\omega, \ell, z)\Theta[\omega - \omega_1(\ell, z)]$$
$$\times\,\Theta\left[\bar{h}_{\mathrm{c}}(\omega, \ell, z) - \bar{h}_\star(\omega)\right] = \frac{\omega}{2\pi}$$

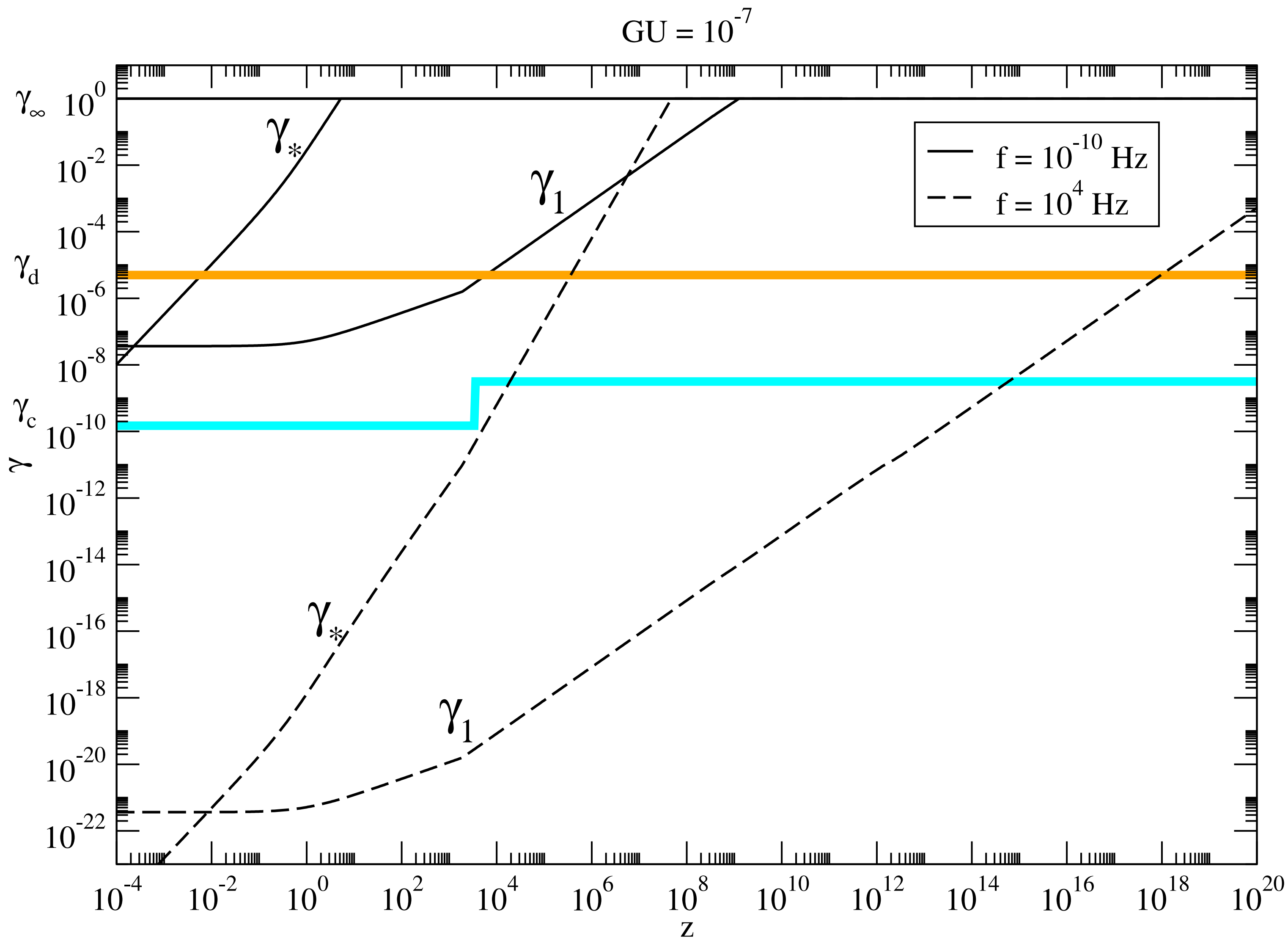
Loop visibility domains

- Separation between stochastic and non-stochastic GW from a kink

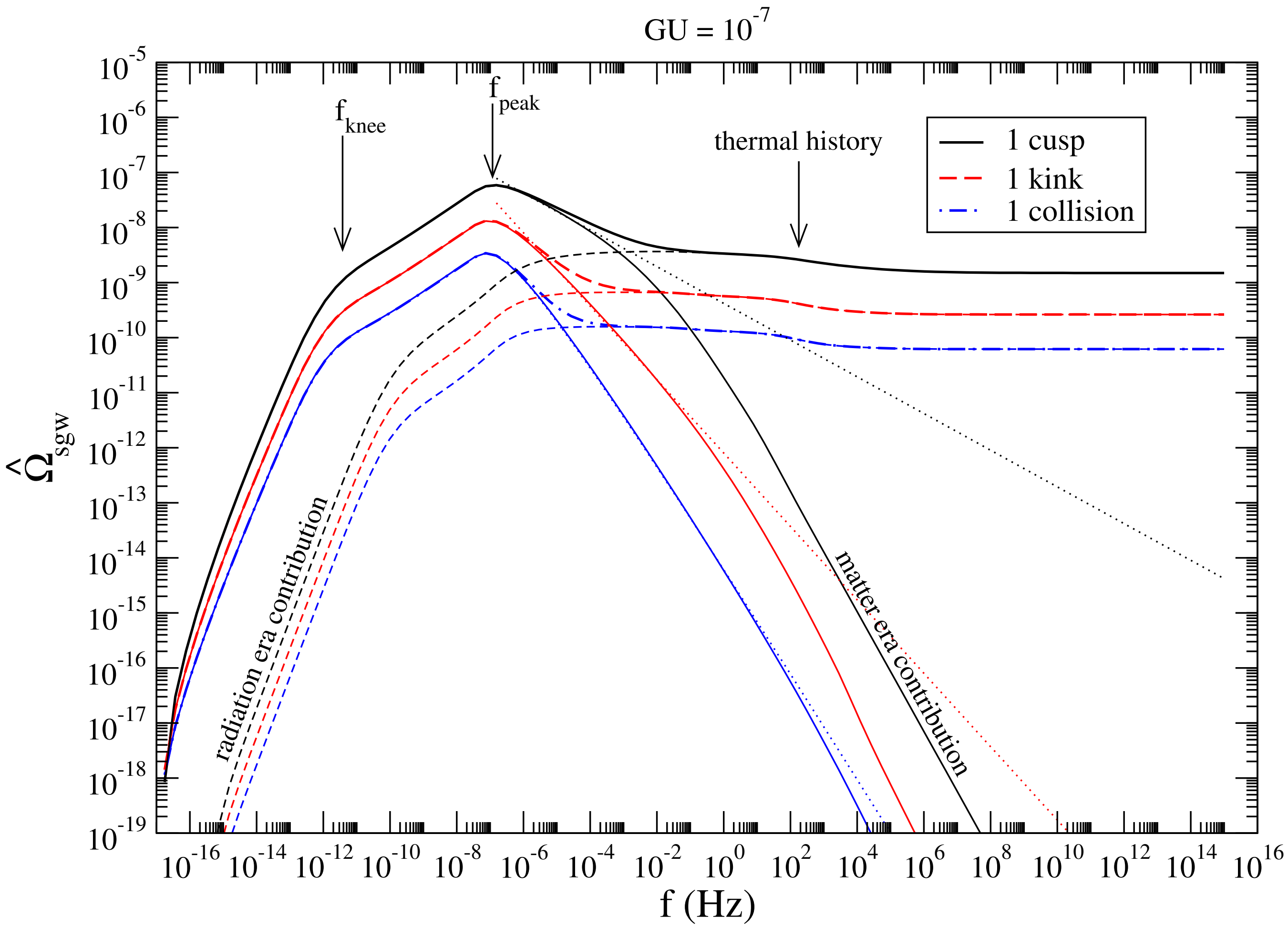


Loop visibility domains

- Separation between stochastic and non-stochastic GW from a cusp



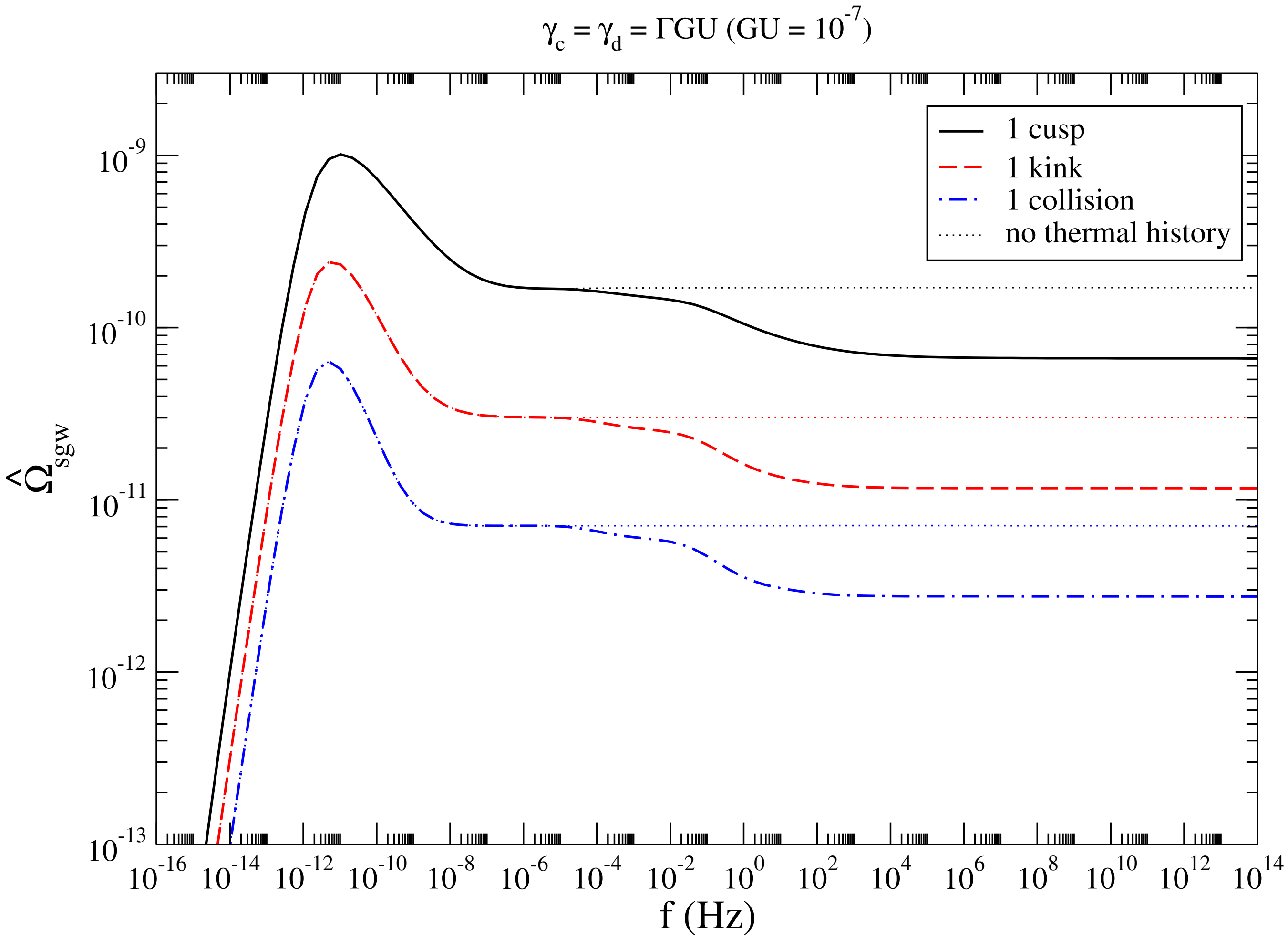
Result



- Analytical approximation: $\omega_{\text{peak}} \propto (GU)^{-(1+2\chi)}$ and $\omega_{\text{knee}} \propto (GU)^{-1}$

$$\hat{\Omega}_{\text{sgw}}\Big|_{\text{c}} \propto \omega^{-\frac{1}{3}}, \qquad \hat{\Omega}_{\text{sgw}}\Big|_{\text{k}} \propto \omega^{-\frac{2}{3}} \qquad \hat{\Omega}_{\text{sgw}}\Big|_{\text{kk}} \propto \omega^{-1} \ln \omega$$

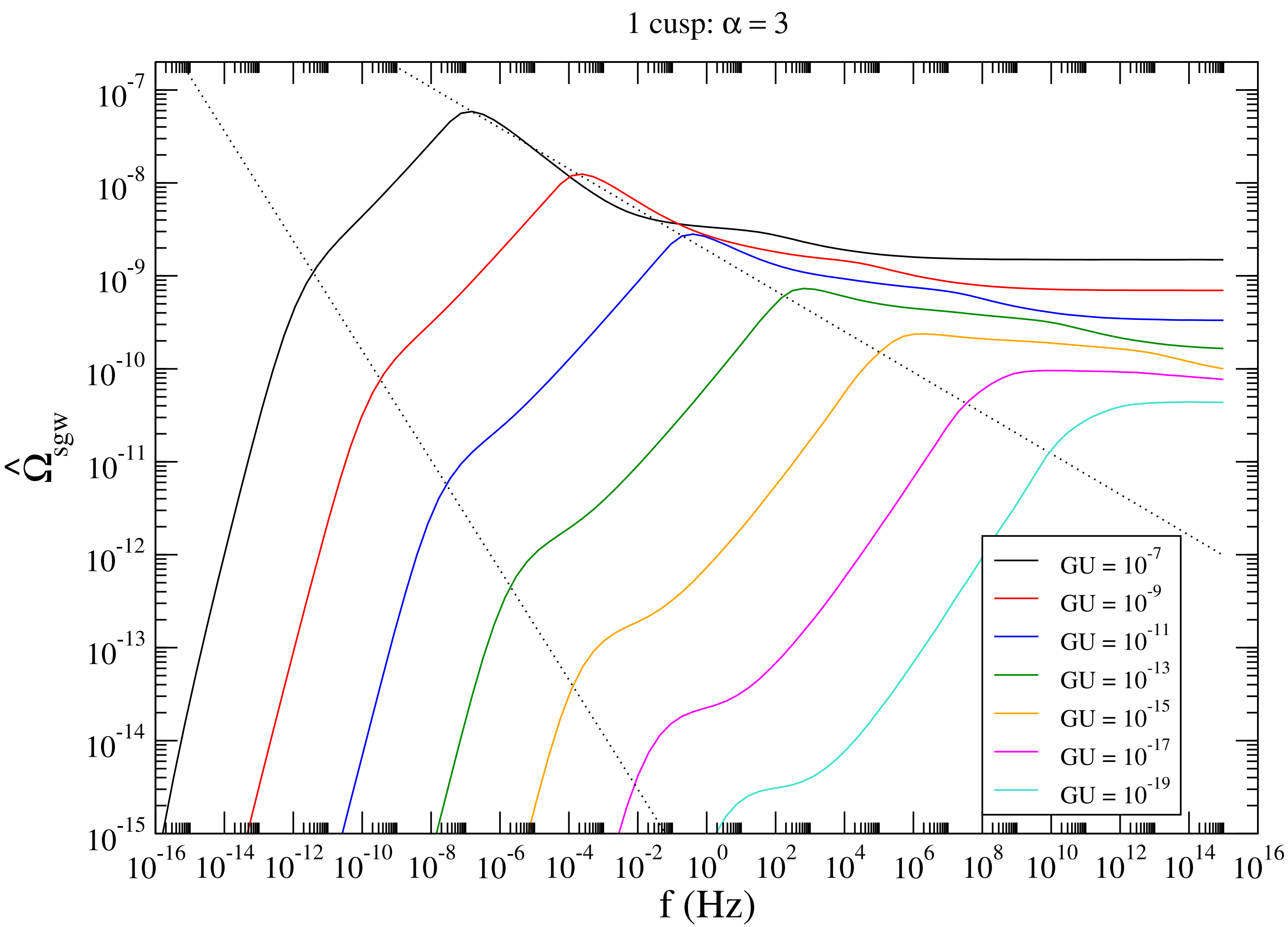
Result



- Previous works assumed that GW backreaction $=$ GW emission $\Rightarrow$ peak at knee location

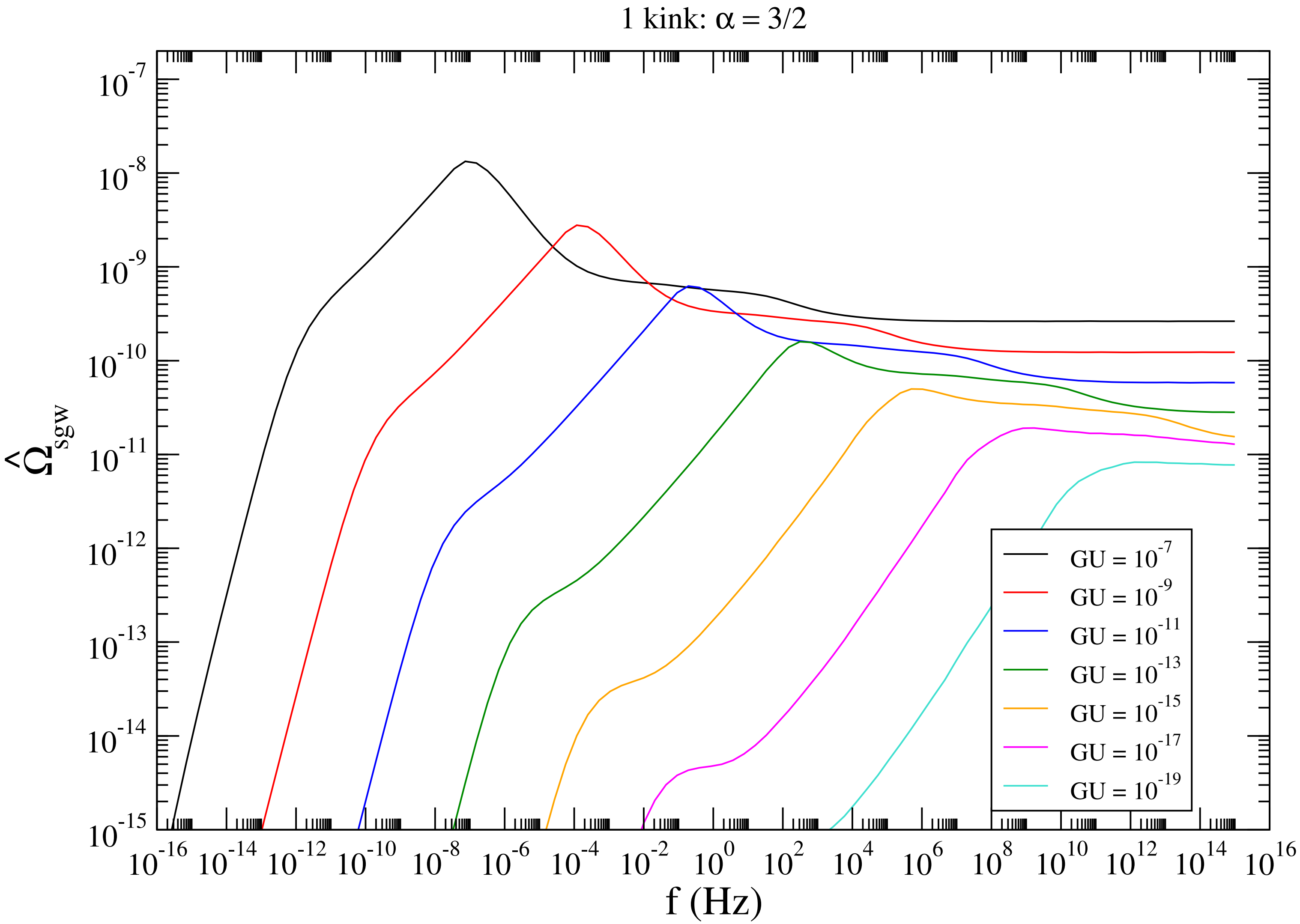
String tension dependency

- One cusp per oscillation



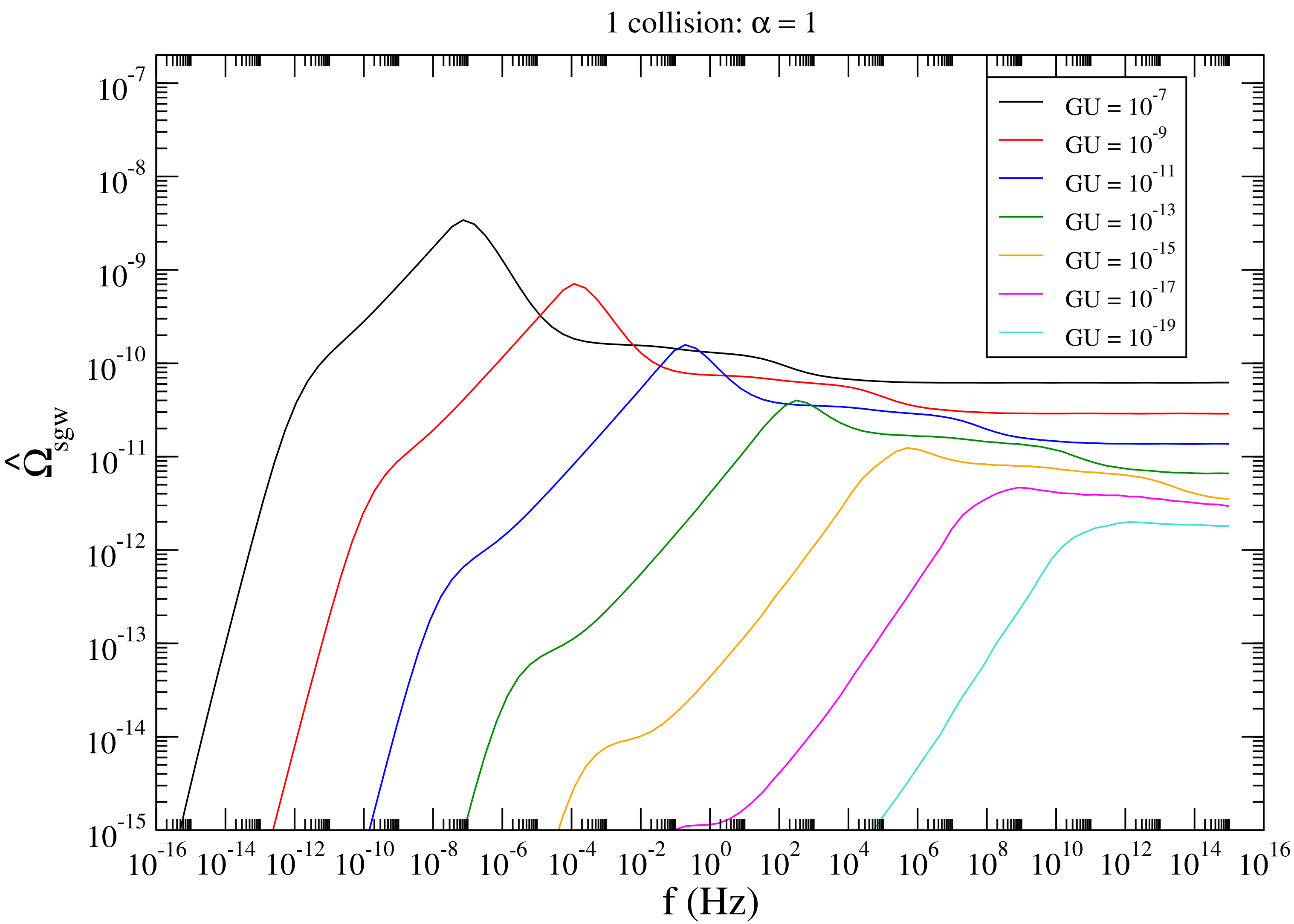
String tension dependency

- One kink per oscillation



String tension dependency

- One collision per oscillation



Microstructure effects

- Number of kinks and cusps is not known but:
  - Loop formation mechanism $\Rightarrow N_{\mathrm{kk}} = N_{\mathrm{k}}^2/4$
  - Total radiated GW power $< \Gamma G U^2$
  - For $\Gamma = 50$ this yields: $N_{\mathrm{c}} \leq 11 \quad N_{\mathrm{k}} \leq 133$
- Three prototypical models
  - Model `2C`: $N_{\mathrm{c}} = 2$, no kinks (and no collisions)
  - Model `LNK`: Only kinks with $N_{\mathrm{k}} < 20$
  - Model `HNK`: Only kinks but $20 \leq N_{\mathrm{k}} \leq 133$

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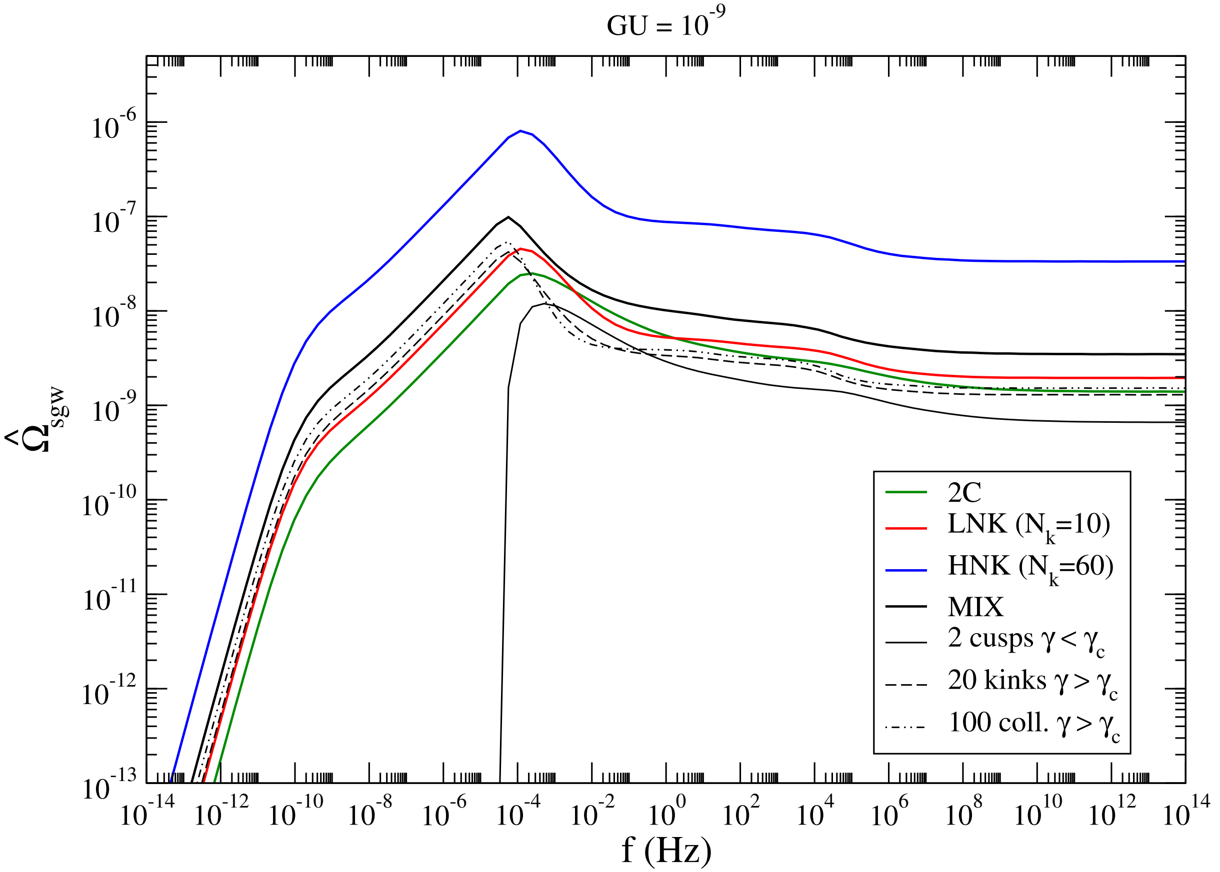
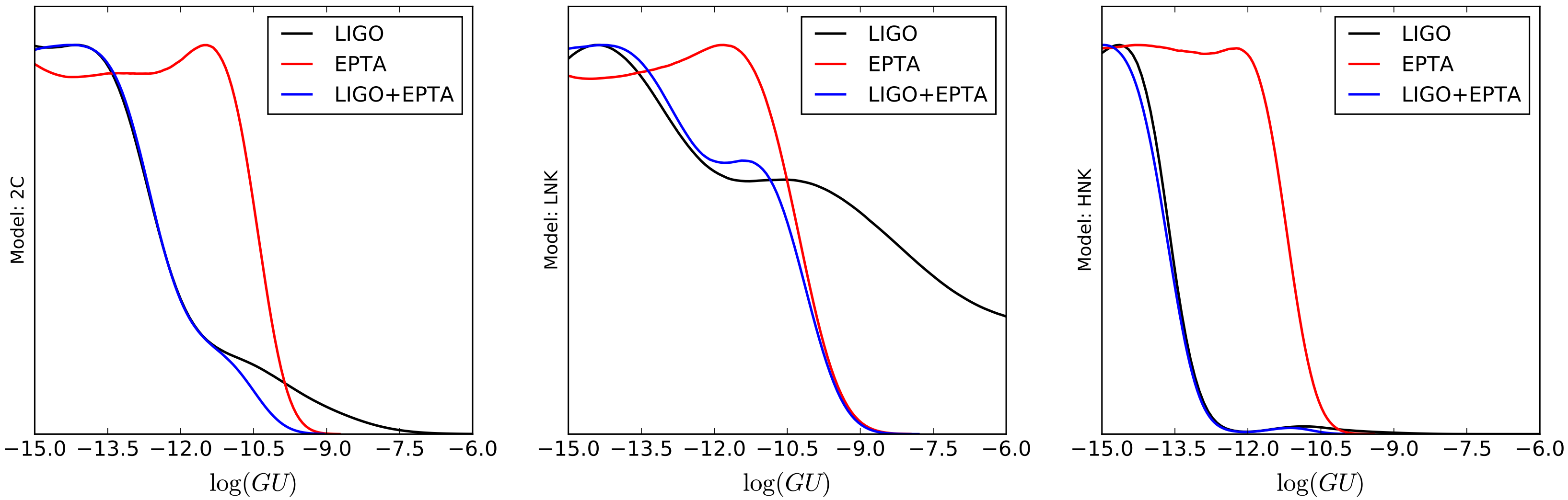


Observational constraints

- From both PTA and LIGO/VIRGO stochastic bounds
- Bayesian analysis marginalized over N_{k}



- Two-sigma upper bounds for GU

Model	LIGO	EPTA	LIGO + EPTA
2C	$GU \leq 1.1 \times 10^{-10}$	$GU \leq 3.4 \times 10^{-11}$	$GU \leq 1.0 \times 10^{-11}$
LNK	$-$	$GU \leq 6.8 \times 10^{-11}$	$GU \leq 7.2 \times 10^{-11}$
HNK	$GU \leq 8.8 \times 10^{-14}$	$GU \leq 6.4 \times 10^{-12}$	$GU \leq 6.7 \times 10^{-14}$
MIX	$GU \leq 1.4 \times 10^{-8}$	$GU \leq 1.1 \times 10^{-11}$	$GU \leq 5.9 \times 10^{-12}$