Conformal Symmetry in Einstein-Cartan Gravity

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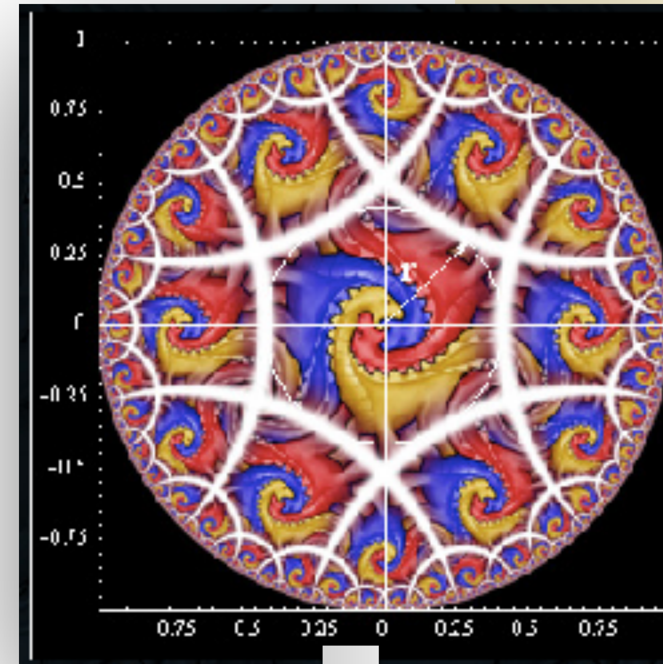
Weyl symmetry and motivation

- Renormalisable theories posses an UV fixed point of RG flow, where the theory becomes conformally invariant.

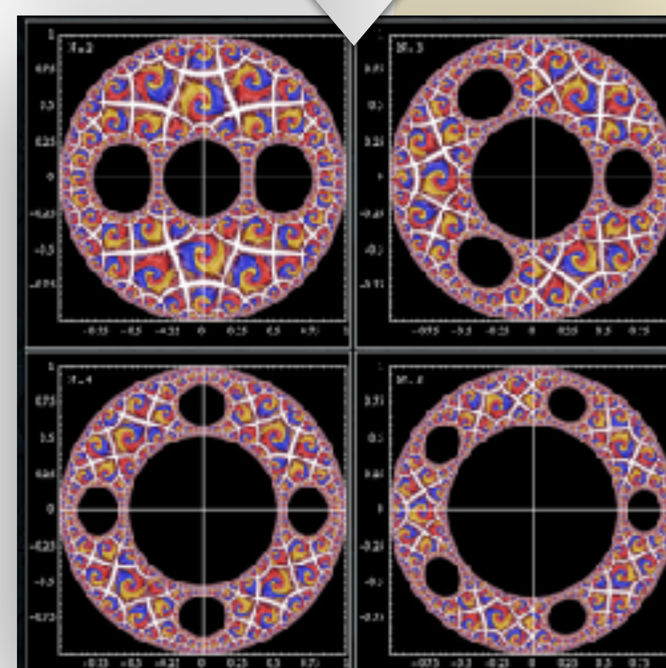
$$\beta_i = \mu \frac{\partial \lambda_i}{\partial \mu}\bigg|_{\lambda_i^*} = 0$$



- Generalisation of scale (conformal) transformation:

$$\left[\nabla_{(\mu}\xi_{\nu)} = \frac{\nabla_\lambda \xi^\lambda}{D} g_{\mu\nu}\right]$$

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$$



- Conformal symmetry, background dependent
- Weyl symmetry, in arbitrary manifolds

The conformal (trace) anomaly

- If the theory is Weyl invariant, we have,

$$\frac{\delta S}{\delta g^{\mu\nu}} \omega g^{\mu\nu} = T^{\mu}_{\mu} = 0 \implies \langle T^{\mu}_{\mu} \rangle = 0$$

- However, a background field analysis for arbitrary metric reveals this is not realised in a renormalised field theory:

$$\langle T^{\mu}_{\mu} \rangle = C_1 \mathcal{E}_4 + C_2 W_{\alpha\beta\gamma\delta} W^{\alpha\beta\gamma\delta} + C_3 \Box R + \sum_i \beta_i \frac{\partial \mathcal{L}}{\partial \lambda_i}$$

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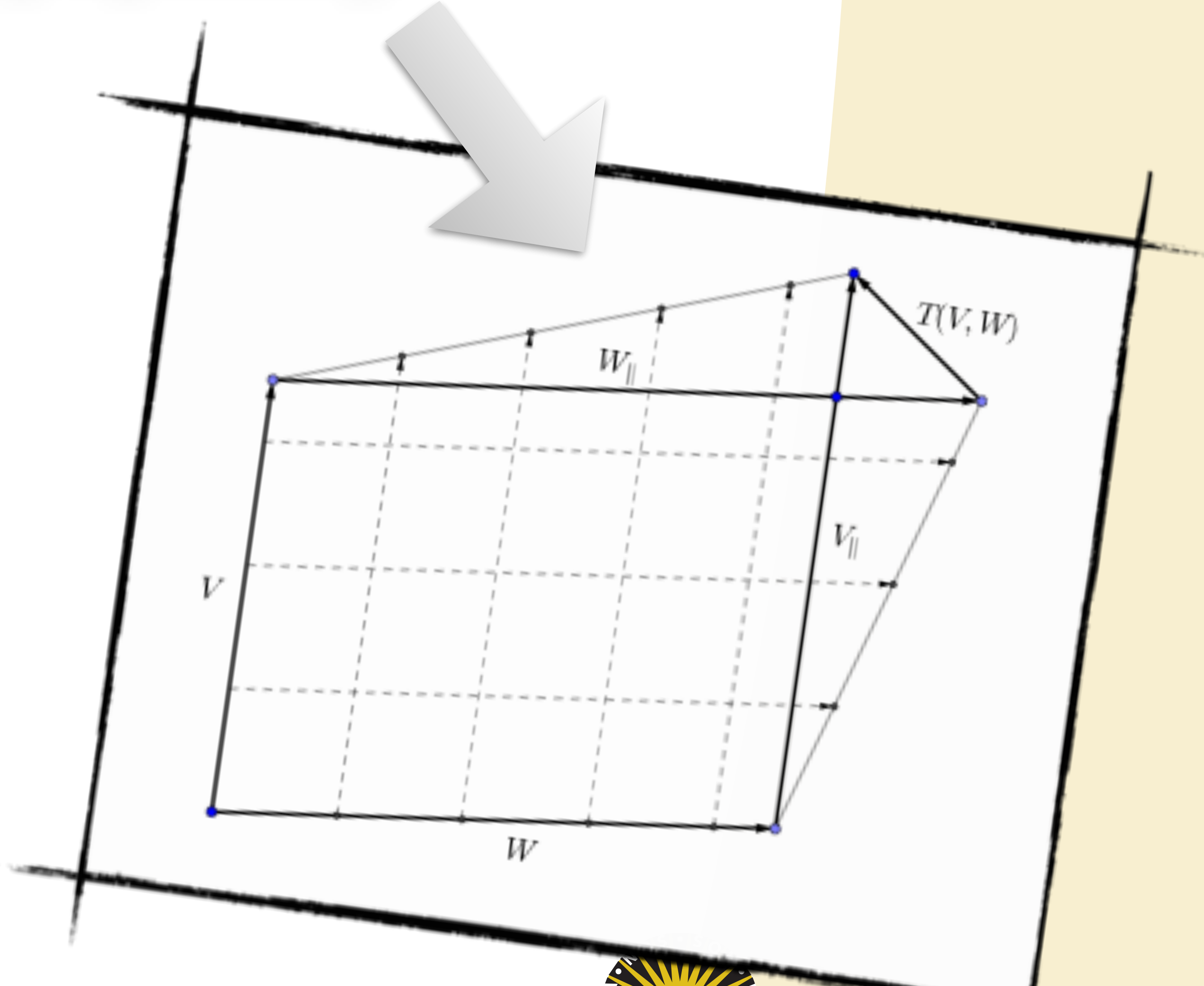
The link between torsion and Weyl symmetry

- Why should torsion be linked to Weyl symmetry?

$$\omega^a_b \to \omega^a_b$$

$$e^a_\mu \to e^{\theta(x)} e^a_\mu$$

$$T^a \to T^a + e^a \wedge \mathrm{d}\theta$$



- The torsion trace is naturally linked to scale transformations.
- Transforming torsion and vierbein leaves the Cartan connection invariant.

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Geometrical properties

- Riemann curvature and geodesics trajectories are frame invariant.

$$R^{\lambda}{}_{\sigma\mu\nu} \to R^{\lambda}{}_{\sigma\mu\nu}$$

$$\nabla_{\dot\gamma}\dot\gamma^{\mu} \to e^{-\theta(x)}\nabla_{\dot\gamma}\dot\gamma^{\mu}$$

Proper time reparametriz-ation

$$\mathrm{d}\tau_{g.i.} = \Phi \times \mathrm{d}\tau$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}$$

$$T_{\mu\nu} \to e^{-(D-2)\theta(x)}T_{\mu\nu}$$

$$\kappa \equiv \frac{\alpha}{\Phi^2(x)} \to e^{(D-2)\theta(x)}\frac{\alpha}{\Phi^2(x)}$$

- Trajectories of free falling bodies invariant up to a reparametrization of time.

- Absence of dimension-full parameters requires dynamical Planck Mass.

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Scale symmetry and dilatation current

- Scale invariant theory possess a Noether charge, the dilatation current

$$\Pi^\mu = -\frac{D-2}{2}\phi\partial^\mu\phi$$

- If scale invariance is exact on the state of the field, the scale current is conserved and energy tensor is traceless

$$\partial_\mu \Pi^\mu = 0$$

$$T^\mu_\mu = 0$$

- If the theory is scale invariant, the equation of motion imply

$$T^\mu_\mu = -\partial_\mu \Pi^\mu$$

$$\Longrightarrow \quad \Pi^\mu = T^\mu_\nu x^\nu \text{ if } g_{\mu\nu} = \eta_{\mu\nu}$$

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Interactions in scalar theory

- For scalars the dilatation current is:

$$\Pi^\mu = \frac{D-2}{2}\phi\partial^\mu\phi \implies \partial_\mu\Pi^\mu = T^\mu_\mu$$

- Idea: couple dilatation current to torsion trace (and complete theory by requiring symmetry).

$$\mathcal{L}_{int} = T_\mu\Pi^\mu + \left(\frac{D-2}{2}\right)^2 T_\mu T^\mu\phi^2$$

- Extension of gravitational field sources. Equation of motion imply the fundamental equation:

$$\Pi^\mu = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T_\mu}$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\nabla_\mu\Pi^\mu + T^\mu_\mu = 0$$

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Weyl symmetry in the quantum theory(formally)

- Phase space quantisation is manifestly Weyl invariant:

$$\pi = \frac{\delta S}{\delta \dot{\phi}}$$

$$[\phi, \pi] = i\hbar\delta^{(D-1)}(\vec{x} - \vec{x}')$$

- This means that the Weyl symmetry Ward identities are preserved:

$$\int \mathcal{D}\phi\mathcal{D}\pi \exp\left(iS[\phi,\pi]\right) =$$
$$= \int \mathcal{D}\phi \det{}^{\frac{1}{2}}\left(\sqrt{-g}g^{00}\delta^{(D-1)}\left(\vec{x} - \vec{x}'\right)\right) \exp\left(iS[\phi]\right)$$

$$\langle\nabla_\mu \hat{\Pi}^\mu\rangle + \langle\hat{T}^\mu_\mu\rangle = 0$$

- Source dilatation current by generating Energy momentum trace

- Identity "broken" by terms which vanish upon regularisation, e.g.

$$(D-4)\lambda\langle\hat{\phi}^4\rangle$$

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Can we then show this in a renormalised field theory?

- Callan et al. showed that it is possible, in any generic renormalisable field theory, to construct a energy momentum tensor whose trace satisfies,

$$\Theta^{\mu}_{\mu} = \sum_i \Lambda_i \frac{\partial \mathcal{L}}{\partial \Lambda_i}$$

All dimension full couplings in the theory

$$\Theta^{\mu}_{\mu} = T^{\mu}_{\mu} + \nabla_{\mu} \Pi^{\mu}$$

$$\Theta_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-1}\left(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\nabla^2\right)\phi^2$$

- This work shows that our Ward identity is in fact satisfied in the full quantum theory, at least in the flat space limit.

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So what about local anomaly?

- Local anomaly action:

$$S_{\text{eff}} = \lim_{D\to 4} \int \mathrm{d}^D x \frac{\sqrt{-g}}{D-4} \left(\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\mu\nu\lambda\sigma}\bar{R}^{\lambda\sigma\mu\nu}\right)$$

$$\langle \hat{T}^{\mu}_{\mu} \rangle = C \left(\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta}\right) \neq 0$$

→ Local anomaly

- Including torsion trace this is compensated, and does not violate the fundamental Ward identity.

$$T^{\mu}_{\mu} + \nabla_{\mu}\Pi^{\mu} = 0$$

- This is because the Gauss Bonnet integral is a boundary term, and gets absorbed in the divergence of the dilatation current.

$$\bar{R}^2 - 4\bar{R}_{\mu\nu}\bar{R}^{\nu\mu} + \bar{R}_{\alpha\beta\gamma\delta}\bar{R}^{\gamma\delta\alpha\beta} = \nabla_{\mu}\mathcal{V}^{\mu}$$

$$\Longrightarrow \Pi^{\mu} \to \Pi^{\mu} + \mathcal{V}^{\mu}$$

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(Some) physical discussion

- Breaking of the Ward identity for chiral transformations:

$$\nabla_\mu J_5^\mu = \nabla_\mu \langle \bar{\psi}\gamma^5\gamma^\mu\psi\rangle \neq 0$$

- Means that the number of fermions is not conserved anymore.

$$\int_\Sigma \mathrm{d}\vec{x}\, J_5^0(\vec{x}) = N_F - N_{\bar{F}} \implies \frac{\mathrm{d}}{\mathrm{d}t}(N_F - N_{\bar{F}}) \neq 0$$

$$\int_\Sigma \mathrm{d}\vec{x}\, \Pi^0(\vec{x}) = \int_\Sigma \mathrm{d}\vec{x}\, \langle\{\pi(\vec{x}),\, \phi(\vec{x})\}\rangle = \frac{1}{2}\int \mathrm{d}\vec{p}\langle(ie^{2i\omega t}a^\dagger_{\vec{p}}a^\dagger_{-\vec{p}} - ie^{-2i\omega t}a_{\vec{p}}a_{-\vec{p}})\rangle$$

- Measures somehow the mixing of the state. If anomaly gets generated this is not conserved anymore. What exactly this means we still do not know…

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Summary

- We constructed a theory of gravity and torsion which is locally Weyl invariant.
- Formal arguments led us to propose that the trace anomaly is actually just a manifestation of sourcing the dilatations current, but does not actually break the local symmetry, just the global (scale symmetry) part.
- This solves the local anomaly, and predicts that only explicit violations of the Weyl symmetry result in violations of its Ward identities.
- We think this might indicate that the torsion has a role to play in UV completion of gravity, if such a theory can be described by a curved spaces CFT.

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Thanks for attention

Questions?