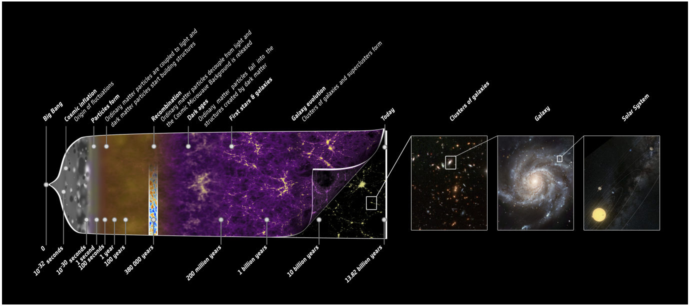
# Stabilities of Vector Fields during Inflationary Epochs

Ryo Namba

McGill University

Gravity and Cosmology 2018 @ YITP: February 16, 2018

Emami, Mukohyama, Namba & Zhang, JCAP 1703, 03, 058 (2017)
Beltran Jimenez, Heisenberg, Kase, Namba & Tsujikawa, PRD 95, 6, 063533 (2017)
Namba, Dimastrogiovanni & Peloso, JCAP 1311, 045 (2013)
Namba, Phys. Rev. D 86, 083518 (2012)




## Inflation + Hot Big Bang cosmology

- Compelling description of the evolution of the universe
- **Inflation** solves puzzles in HBB
  - ◇ Horizon problem — sky $\sim \mathcal{O}(10^4)$ causally-disconnected patches
  - ◇ Flatness problem — $|1 - \Omega_0| \lesssim \mathcal{O}(10^{-3})$
  - ◇ Unwanted relics problem — topological defects in early universe
  - ◇ Seeds of inhomogeneity — large-scale structure

# Symmetries for inflation

Soda '12

- **Homogeneity** $\Leftrightarrow$ spatial translation $\Leftrightarrow$ momentum conservation
  - ▷ $\langle \zeta(\vec{k})\,\zeta(\vec{k}')\rangle = \delta^{(3)}(\vec{k}+\vec{k}')\,k^{-3}P_\zeta(\vec{k})$
- **Spatial isotropy** $\Leftrightarrow$ rotational symmetry
  - ▷ $P_\zeta(\vec{k}) = P_\zeta(|\vec{k}|)$
- **De Sitter** $\Leftrightarrow$ invariance under $t \to t + c, x^i \to \mathrm{e}^{-H\,c} x^i$
  - ▷ $P_\zeta(|\vec{k}|) \approx$ const.

# Symmetries for inflation

Soda '12

- **Homogeneity** $\Leftrightarrow$ spatial translation $\Leftrightarrow$ momentum conservation
  - ▷ $\langle\zeta(\vec{k})\,\zeta(\vec{k}')\rangle = \delta^{(3)}(\vec{k}+\vec{k}')\,k^{-3}P_\zeta(\vec{k})$
- **Spatial isotropy** $\Leftrightarrow$ rotational symmetry
  - ▷ $P_\zeta(\vec{k}) = P_\zeta(|\vec{k}|)$
- **De Sitter** $\Leftrightarrow$ invariance under $t \to t+c,\, x^i \to e^{-Hc}x^i$
  - ▷ $P_\zeta(|\vec{k}|) \approx$ const.

We know: Soft breaking of **de Sitter**

▷ Quasi de Sitter $H \neq$ const. — inflation ends

▷ Small $|\vec{k}|$ dependence (spectral index $|n_s - 1| \ll 1$)

# Symmetries for inflation

Soda '12

- **Homogeneity** $\Leftrightarrow$ spatial translation $\Leftrightarrow$ momentum conservation
  - ▷ $\langle \zeta(\vec{k})\, \zeta(\vec{k}') \rangle = \delta^{(3)}(\vec{k} + \vec{k}')\, k^{-3} P_\zeta(\vec{k})$
- **Spatial isotropy** $\Leftrightarrow$ rotational symmetry
  - ▷ $P_\zeta(\vec{k}) = P_\zeta(|\vec{k}|)$
- **De Sitter** $\Leftrightarrow$ invariance under $t \to t + c$, $x^i \to e^{-Hc} x^i$
  - ▷ $P_\zeta(|\vec{k}|) \approx$ const.

We know: Soft breaking of **de Sitter**

▷ Qua

What if **VECTOR FIELDS** drive inflation?

▷ Small $|\vec{k}|$ dependence (spectral index $|n_s - 1| \ll 1$)

# Vector fields driving inflation

**Difficult to sustain a prolonged period of expansion**

◇ Standard gauge fields **conformally couple** to gravity (in 4D)

$$\mathcal{L}_{\rm EM}^{\rm free} = -\frac{\sqrt{-g}}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \quad \text{invariant under} \quad g_{\mu\nu} \to \boldsymbol{\Omega^2}\, g_{\mu\nu}$$

▷ **"Standard" massless vectors do not feel/drive expanding universe**

# Vector fields driving inflation

**Difficult to sustain a prolonged period of expansion**

- Standard gauge fields **conformally couple** to gravity (in 4D)

$$\mathcal{L}_{\rm EM}^{\rm free} = -\frac{\sqrt{-g}}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \quad \text{invariant under} \quad g_{\mu\nu} \to \mathbf{\Omega^2}\, g_{\mu\nu}$$

  - **“Standard” massless vectors do not feel/drive expanding universe**

**PAST ATTEMPTS: Need to break conformal invariance**

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ — Ford ’89
- Fixed norm: $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ — Ackerman, Carroll & Wise ’07
- Coupling to gravity: $\Delta\mathcal{L} = \xi\, R\, A_\mu A^\mu$ — Golovnev, Mukhanov & Vanchurin ’08

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

- Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\text{longitudinal}}^2 + \ldots \right]$$

- Turned out $\boldsymbol{m_{\text{eff}}^2 < 0}$ $\Rightarrow$ **ghost**

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

- Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\text{longitudinal}}^2 + \dots \right]$$

- Turned out $\boldsymbol{m_{\text{eff}}^2 < 0}$ $\Rightarrow$ **ghost**
  - $\Delta\mathcal{L} = V(A_\mu A^\mu)$ $\Rightarrow$ to sustain long anisotropic period

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

- ◇ Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}^2_{\text{longitudinal}} + \dots \right]$$

- ◇ Turned out $\boldsymbol{m^2_{\text{eff}} < 0}$ $\Rightarrow$ **ghost**
  - ▷ $\Delta\mathcal{L} = V(A_\mu A^\mu)$ $\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

- Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\text{longitudinal}}^2 + \dots \right]$$

- Turned out $\boldsymbol{m_{\text{eff}}^2 < 0} \;\; \Rightarrow$ **ghost**
  - $\Delta\mathcal{L} = V(A_\mu A^\mu) \;\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 
  - $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2) \;\Rightarrow$ to produce large-scale fluctuation

# Ghost Instabilities

These models suffer **ghost instabilities** — Himmetoglu, Contaldi & Peloso '08

- ◇ Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\rm longitudinal}^2 + \dots \right]$$

- ◇ Turned out $\boldsymbol{m_{\rm eff}^2 < 0}$ $\Rightarrow$ **ghost**

  - ▷ $\Delta\mathcal{L} = V(A_\mu A^\mu)$ $\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 

  - ▷ $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ $\Rightarrow$ to produce large-scale fluctuation $\Rightarrow$ 

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

- ◇ Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\text{longitudinal}}^2 + \dots \right]$$

- ◇ Turned out $\boldsymbol{m_{\text{eff}}^2 < 0}$ $\Rightarrow$ **ghost**
  - ▷ $\Delta\mathcal{L} = V(A_\mu A^\mu)$ $\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 
  - ▷ $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ $\Rightarrow$ to produce large-scale fluctuation $\Rightarrow$ 
  - ▷ $\Delta\mathcal{L} = \xi \, R \, A_\mu A^\mu$ $\Rightarrow$ to sustain long anisotropic period

# Ghost Instabilities

These models suffer **ghost instabilities** Himmetoglu, Contaldi & Peloso '08

◇ Models break gauge invariance $\Longrightarrow$ additional d.o.f. (longitudinal mode)

$$S = \int d^4x \, m_{eff}^2(t) \left[ \dot{A}_{\rm longitudinal}^2 + \ldots \right]$$

◇ Turned out $\boldsymbol{m_{\rm eff}^2 < 0}$ $\Rightarrow$ **ghost**

▷ $\Delta\mathcal{L} = V(A_\mu A^\mu)$ $\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 

▷ $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ $\Rightarrow$ to produce large-scale fluctuation $\Rightarrow$ 

▷ $\Delta\mathcal{L} = \xi \, R \, A_\mu A^\mu$ $\Rightarrow$ to sustain long anisotropic period $\Rightarrow$ 

[Note: if we do not require $A_\mu$ to drive inflation, $m_{\rm eff} > 0$ is certainly possible]

Dimopoulos, Karciauskas & Wagstaff '10; RN '12

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ Ford '89
- Fixed norm: $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ Ackerman, Carroll & Wise '07
- Coupling to gravity: $\Delta\mathcal{L} = \xi R A_\mu A^\mu$ Golovnev, Mukhanov & Vanchurin '08

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ — Ford '89
- Fixed norm: $\Delta\mathcal{L} = \lambda(A_\mu A^\mu - v^2)$ — Ackerman, Carroll & Wise '07
- Coupling to gravity: $\Delta\mathcal{L} = \xi\, R\, A_\mu A^\mu$ — Golovnev, Mukhanov & Vanchurin '08

- **Horndeski-Yang-Mills** $\cdots$ non-Abelian field: $\Delta\mathcal{L} = \beta R^{\mu\nu\rho\sigma}\tilde{F}^a_{\mu\nu}\tilde{F}^a_{\rho\sigma}$
  Davydov & Gal'tsov '15

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ Ford '89
- Fixed norm: $\Delta\mathcal{L} =$ , Carroll & Wise '07
- Coupling to gravity Mukhanov & Vanchurin '08

**GHOST INSTABILITIES**

- **Horndeski-Yang-M** $\beta R^{\mu\nu\rho\sigma} \tilde{F}^a_{\mu\nu} \tilde{F}^a_{\rho\sigma}$

Himmetoglu, Contaldi & Peloso '08; Beltrán Jiménez et al. '17

Davydov & Gal'tsov '15

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ Ford '89
- Fixed norm: $\Delta\mathcal{L} =$ , Carroll & Wise '07
- Coupling to gravity Mukhanov & Vanchurin '08

**GHOST INSTABILITIES**

- **Horndeski-Yang-M** $\beta R^{\mu\nu\rho\sigma}\tilde{F}^a_{\mu\nu}\tilde{F}^a_{\rho\sigma}$
  Himmetoglu, Contaldi & Peloso '08; Beltrán Jiménez et al. '17
  Davydov & Gal'tsov '15

- **Gauge-flation** $\cdots$ non-Abelian field: $\Delta\mathcal{L} = \kappa\left(F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}\right)^2$
  Maleknejad & Sheikh-Jabbari '11

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ Ford '89
- Fixed norm: $\Delta\mathcal{L} =$ , Carroll & Wise '07
- Coupling to gravity Mukhanov & Vanchurin '08

**GHOST INSTABILITIES**

- **Horndeski-Yang-M** $\beta R^{\mu\nu\rho\sigma} \tilde{F}^a_{\mu\nu} \tilde{F}^a_{\rho\sigma}$ Davydov & Gal'tsov '15

Himmetoglu, Contaldi & Peloso '08; Beltrán Jiménez et al. '17

- **Gauge-flati** 2 Sheikh-Jabbari '11

**OBSERVATIONALLY EXCLUDED**

***$n_s$* vs. *$r$***

RN, Dimastrogiovanni & Peloso '13

- Potential term: $\Delta\mathcal{L} = V(A_\mu A^\mu)$ Ford '89
- Fixed norm: $\Delta\mathcal{L} =$ Carroll & Wise '07
- Coupling to gravity Mukhanov & Vanchurin '08

**GHOST INSTABILITIES**

- **Horndeski-Yang-M** $\beta R^{\mu\nu\rho\sigma}\tilde{F}^a_{\mu\nu}\tilde{F}^a_{\rho\sigma}$
Himmetoglu, Contaldi & Peloso '08; Beltrán Jiménez et al. '17
Davydov & Gal'tsov '15

- **Gauge-flati** $^2$

Sheikh-Jabbari '11

**OBSERVATIONALLY EXCLUDED**
***$n_s$* vs. *$r$***
RN, Dimastrogiovanni & Peloso '13

**How can we get stable, vector-driven inflationary models?**

# Generalized Proca Theory

**Broken gauge invariance allows many terms**

- ◇ General model is useful to identify stability conditions

# Generalized Proca Theory

**Broken gauge invariance allows many terms**

- ◇ General model is useful to identify stability conditions

## Generalized Proca Theory

$$\mathcal{L}_{\text{GP}} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$$

Heisenberg '14; Allys, Peter & Rodriguez '15; Jiménez & Heisenberg '16

- No Ostrogradsky (higher-derivative) ghosts
  - ◇ Cosmol. solution with $H =$ const. available $\Longrightarrow$ Late-time applications

    De Felice et al. '16

# Generalized Proca Theory

**Broken gauge invariance allows many terms**

- ◇ General model is useful to identify stability conditions

## Generalized Proca Theory

$$\mathcal{L}_{\rm GP} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6$$

Heisenberg '14; Allys, Peter & Rodriguez '15; Jiménez & Heisenberg '16

- No Ostrogradsky (higher-derivative) ghosts
  - ◇ Cosmol. solution with $H =$ const. available $\Longrightarrow$ Late-time applications

    De Felice et al. '16
- Suitable deformations
  - ◇ Isotropy $\Leftrightarrow$ 3 vector fields $A_\mu^{(1)}$, $A_\mu^{(2)}$, $A_\mu^{(3)}$ with $O(3)$ $\Rightarrow$ $\mathcal{L}_3 = \mathcal{L}_5 = 0$
  - ◇ Neglect $\mathcal{L}_6$ for simplicity

$$\boxed{\mathcal{L} = G_2(A^2, F^2, F\tilde{F}, AAFF) + G_4(A^2)R + G_{4,X}\left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A^\nu \nabla_\nu A^\mu\right]}$$

## Inflationary background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \; , \qquad \langle A_i^{(a)} \rangle = a(t)\, A_{\rm BG}(t)\, \delta_i^a \; , \quad \langle A_0^{(a)} \rangle = 0$$

Quasi de Sitter solution

$$H = \frac{\dot{a}}{a} \approx \text{const.} \; , \qquad A_{\rm BG} \approx \text{const.}$$

- A local attractor under **isotropic ansatz** $\Longrightarrow$ $\boxed{1}$ condition
- A local attractor against **anisotropic expansion** $\Longrightarrow$ $\boxed{1}$ condition

## Inflationary background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \ , \qquad \langle A_i^{(a)} \rangle = a(t)\, A_{\rm BG}(t)\, \delta_i^a \ , \quad \langle A_0^{(a)} \rangle = 0$$

Quasi de Sitter solution

$$H = \frac{\dot{a}}{a} \approx \text{const.} \ , \qquad A_{\rm BG} \approx \text{const.}$$

- A local attractor under **isotropic ansatz** $\Longrightarrow$ $\boxed{1}$ condition
- A local attractor against **anisotropic expansion** $\Longrightarrow$ $\boxed{1}$ condition

## Perturbations – # of dynamical D.o.F.:

- Vector fields $\delta A_\mu^{(a)}$: $3 \times 3 = 9 = 3$ "scalar" $+ 4$ "vector" $+ 2$ "tensor"
- Metric $\delta g_{\mu\nu}$: Same as GR $= 2$ tensor (after gauge fixing)
- Total: **3 scalar** $+$ **4 vector** $+$ **4 tensor**

## No ghost + Gradient stability

Positive kinetic term + Real & positive $c_s^2$

- **Tensor modes**: $2 + 3 = \boxed{5}$ conditions
- **Vector modes**: $2 + 3 = \boxed{5}$ conditions
- **Scalar modes**: $3 + 5 = \boxed{8}$ conditions

$\Longrightarrow$ $\boxed{16}$ conditions
($\exists$ redundancy)

One interesting observation:

▷ $A_\mu^{(a)} A_\nu^{(a)} F^{(b)\,\mu\rho} F^{(b)\,\nu}{}_{\rho}$ or $A_\mu^{(a)} A_\nu^{(b)} F^{(a)\,\mu\rho} F^{(b)\,\nu}{}_{\rho}$ is strictly necessary

## No ghost + Gradient stability

Positive kinetic term + Real & positive $c_s^2$

- **Tensor modes**: $2+3=\boxed{5}$ conditions
- **Vector modes**: $2+3=\boxed{5}$ conditions
- **Scalar modes**: $3+5=\boxed{8}$ conditions

$\Longrightarrow$ $\boxed{16}$ conditions
($\exists$ redundancy)

One interesting observation:

▷ $A_\mu^{(a)} A_\nu^{(a)} F^{(b)\,\mu\rho} F^{(b)\,\nu}{}_\rho$ or $A_\mu^{(a)} A_\nu^{(b)} F^{(a)\,\mu\rho} F^{(b)\,\nu}{}_\rho$ is strictly necessary

**AN EXAMPLE** — can satisfy all $\boxed{18}$ conditions

$$\mathcal{L} = F(A^2, F^2) + c_1\, A_\mu^{(a)} A_\nu^{(a)} F^{(b)\,\mu\rho} F^{(b)\,\nu}{}_\rho + c_2\, A_\mu^{(a)} A_\nu^{(b)} F^{(a)\,\mu\rho} F^{(b)\,\nu}{}_\rho$$
$$+ \left(c_3 - \frac{c_4}{2} A^2\right) R + c_4 \left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A^\nu \nabla_\nu A^\mu\right]$$

with $c_2 > 0$ and $|c_4| \gg c_2 A_{\rm BG}^2$.

Emami, Mukohyama, RN & Zhang '17

# Concluding remarks

**Q.: Can we get stable, only vector-driven inflationary models?**

# Concluding remarks

**Q.: Can we get stable, only vector-driven inflationary models?**

**A.: Yes, but dedicated terms necessary**

# Concluding remarks

**Q.: Can we get stable, only vector-driven inflationary models?**

**A.: Yes, but dedicated terms necessary**

Existing vector-driven inflation models suffered **ghost instabilities**

- ▷ Violation of gauge invariance $+$ slow-roll conditions forced $m_{\text{eff}}^2 < 0$

Analysis on a general setup – **Generalized Proca Theory**

- ▷ Identified 18 conditions for stable inflationary attractor solutions
- ▷ Provided an example with a viable parameter space to satisfy all conditions

Other possibilities

- ▷ Non-Abelian fields have specific self-interaction $\epsilon^{abc}\partial_\mu A_\nu^a A^{b,\mu} A^{c,\nu}$
- ▷ Massive/Higgsed Gauge-flation proposed

Nieto & Rodríguez '16; Adshead & Sfakianakis '17

**Further considerations – Phenomenology**

- Parity violation in tensor and vector sectors
- Existence of extra tensor modes $\sim$ gravitational-wave physics?