The Bispectrum Beyond Slow-Roll in the Unified EFT of Inflation

Passaglia & Hu, In Prep.

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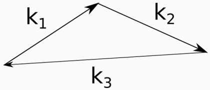
Bispectrum is a test of the *Physics* of Inflation

Size and shape of bispectrum probes **inflaton interactions**

$$\langle \hat{\mathcal{R}}_{\mathbf{k_1}} \hat{\mathcal{R}}_{\mathbf{k_2}} \hat{\mathcal{R}}_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_{\mathcal{R}}(k_1, k_2, k_3)$$



$$f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B_{\mathcal{R}}(k_1, k_2, k_3)}{P_{\mathcal{R}}(k_1) P_{\mathcal{R}}(k_2) + \text{perm.}} \quad .$$

Planck: $f_{\mathrm{NL}}^{\mathrm{squeeze.}} \sim 1 \pm 5, f_{\mathrm{NL}}^{\mathrm{equil}} \sim 0 \pm 40, f_{\mathrm{NL}}^{\mathrm{ortho.}} \sim -30 \pm 20.$

Constraints will **improve**

Slow-roll violating models produce **new discovery modes.**

(Future Bispectrum Prospects: CMB: Abazajian et al. 16, LSS: Gleyzes et al. 17, Bauldoff et al. 16)(Beyond Slow-Roll Signals, e.g., Hannestad et al. 2010)

In This Talk

Simple expressions for the bispectrum for any* single-field model even when slow-roll is violated

This enables precision tests of individual models and of the single-field paradigm

Deriving the interactions

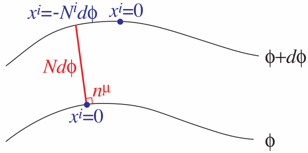
The Unified EFT of Inflation

- EFT of inflation: **Explicitly break time diffs** in action.
- **Extended** to gravitational operators **in Dark Energy context**.
- Motohashi & Hu 2017 studied these operators in **inflationary power spectra**.

We use this framework to compute bispectrum for general models.

(EFT Inflation: Creminelli et al. 06, Cheung et al. 07, Baumann & Green 11)(Operator Extensions: Gleyzes et al. 13, Gleyzes et al. 14, Kase and Tsujikawa 14, Gleyzes et al. 15)

Work directly in 3+1 Split



$$S = \int \mathrm{d}^4 x N \sqrt{h}\, L(N, K^i{}_j, R^i{}_j, t),$$

- Easily connects to **model space** and to **observables**
- **Perturb around FLRW** up to **cubic order** .
- Only $\mathcal{R}$ dynamical. Enforce **standard dispersion relation**

$$\partial^2 \mathcal{R} = \frac{1}{a Q c_s^2} \frac{d}{dt}(a^3 Q \dot{\mathcal{R}})$$

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Third-Order Action for Perturbations

After **integration by parts** and use of the **equation of motion**

$$S_3 = \int \mathrm{d}^3x\,\mathrm{d}t \Big[a^3 F_1 \mathcal{R}\dot{\mathcal{R}}^2 + a F_2 \mathcal{R} \left(\partial\mathcal{R}\right)^2 + a^3 F_3 \dot{\mathcal{R}}^3 + a^3 F_4 \dot{\mathcal{R}} \partial_a \mathcal{R} \partial_a \partial^{-2} \dot{\mathcal{R}} + a^3 F_5 \partial^2 \mathcal{R} \left(\partial\partial^{-2}\dot{\mathcal{R}}\right)^2 + \frac{F_6}{a} \dot{\mathcal{R}} \partial^2 \mathcal{R} \partial^2 \mathcal{R} + \frac{F_7}{a^3} \left(\partial_a \partial_b \mathcal{R}\right)^2 \partial^2 \mathcal{R} + \frac{F_8}{a^3} \partial^2 \mathcal{R} \partial^2 \mathcal{R} \partial^2 \mathcal{R} + \frac{F_9}{a} \partial^2 \mathcal{R} \left(\partial_a \partial_b \mathcal{R}\right) \left(\partial_a \partial_b \partial^{-2} \dot{\mathcal{R}}\right) \Big],$$

- k-Inflation, Horndeski+GLPV, EFT

Consistency Relation Problem?

$$S_3^{\text{squeeze}} = \int \mathrm{d}^3x\, \mathrm{d}t \left[a^3 F_1 \mathcal{R}\dot{\mathcal{R}}^2 + a F_2 \mathcal{R} \left(\partial\mathcal{R}\right)^2 \right]$$

- $F_1, F_2 \supset$ **EFT coefficients not in power spectrum.**
- Trick (extended from Creminelli et al. 2011, Adshead et al. 2013):

$$\frac{d}{dt}\left(\frac{F\mathcal{R}\mathcal{H}_2}{H}\right) = \text{Terms that do not contribute to squeeze}$$
$$+ \text{Consistency Relation Terms}$$
$$+ \text{Terms that cancel}$$

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Cubic Action

$$
\begin{aligned}
S_3 \Rightarrow \int \mathrm{d}^3x\,\mathrm{d}t \Bigg[& a^3 Q \frac{d}{dt}\left(\epsilon_H + \frac{3}{2}\sigma + \frac{q}{2}\right)\mathcal{R}^2\dot{\mathcal{R}} && \text{SR suppressed} \\
& - \frac{d}{dt}\left[\frac{a^3 Q}{2}\color{red}{\mathbf{(1 - n_s)}\,|_{\mathbf{SR}}}\mathcal{R}^2\dot{\mathcal{R}}\right] && \text{gives consistency} \\
& + (\sigma + \epsilon_H)\mathcal{R}(\mathcal{H}_2 + 2\mathcal{L}_2) && \text{SR suppressed} \\
& + (1 - F)\frac{\dot{\mathcal{R}}\mathcal{L}_2}{H} && \text{no squeeze} \\
& + (F_3 \text{ through } F_9 \text{ terms}) && \text{no squeeze}
\end{aligned}
$$

- **Manifestly preserves consistency relation** in slow-roll.

Bispectrum Beyond Slow-Roll

In-In Formalism

$$\langle\hat{\mathcal{R}}_{\mathbf{k_1}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_2}}(t_*)\hat{\mathcal{R}}_{\mathbf{k_3}}(t_*)\rangle =$$
$$\mathrm{Re}\left[-i\left\langle\hat{\mathcal{R}}^I_{\mathbf{k_1}}(t_*)\hat{\mathcal{R}}^I_{\mathbf{k_2}}(t_*)\hat{\mathcal{R}}^I_{\mathbf{k_3}}(t_*)\int_{-\infty(1+i\epsilon)}^{t_*}\mathrm{d}t H^I(t)\right\rangle\right]$$

- $H^I \simeq -\int \mathrm{d}^3x\mathcal{L}^I_3$ at this order
- $\mathcal{R}^I$ satisfy quadratic-order equation of motion.

(Condensed Matter: Schwinger 61, Keldysh 64)(Cosmology: Jordan 86, Calzetta+Hu 87)(Inflation: Maldacena 02, Weinberg 05, Lim et al. 08, Senatore+Zaldarriaga 09, ++++)

Generalized Slow-Roll

No general analytic solution to the equation of motion

$$\partial^2\mathcal{R} = \frac{1}{aQc_s^2}\frac{d}{dt}(a^3Q\dot{\mathcal{R}})$$

GSR is an iterative solution for the modefunctions

$$y(x) = y_0(x) - \int_x^\infty \frac{\mathrm{d}w}{w^2} g(\ln\tilde{s}) y(w)\,\mathrm{Im}\left[y_0^*(w)y_0(x)\right],$$

$$y_0(x) = \left(1+\frac{i}{x}\right)e^{ix}.$$

Where $y \propto \mathcal{R}$, and orders are suppressed by $g = (f'' - 3f')/f^2$, where $\dfrac{1}{f^2} \sim \Delta^2$.

We compute to first-order in GSR.

Stewart 02, Choe et al. 04, Kadota et al. 05, Dvorkin & Hu 09

GSR Bispectrum Results

Each operator i gives j (~ 5) **shape-independent** integrals

$$I_{ij}(K) = S_{ij}(\ln s_*)W_{ij}(Ks_*) + \int_{s_*}^{\infty} \frac{ds}{s} S'_{ij}(\ln s) W_{ij}(Ks).$$

- S_{ij} are **sources** $\propto F_i$
- W_{ij} are **windows**, e.g. $\cos(x)$

Combine with a few **shape-dependent** terms and get

$$B_{\mathcal{R}}(k_1,k_2,k_3) = \frac{(2\pi)^4}{4} \frac{\Delta_{\mathcal{R}}(k_1)\Delta_{\mathcal{R}}(k_2)\Delta_{\mathcal{R}}(k_3)}{k_1^2 k_2^2 k_3^2}$$
$$\times \left\{ \sum_{ij} T_{ij} I_{ij}(K) + \sum_{i=2}^{9} [T_{iB} I_{iB}(2k_3) + \text{perm.}] \right\}.$$

- T_{ij} are k-weights for triangle shapes, e.g. $\dfrac{k_1 k_2 k_3}{(k_1+k_2+k_3)^3}$

Consistency Relation Revisited: Beyond Slow-Roll

Beyond SR, no new interactions contribute to squeeze

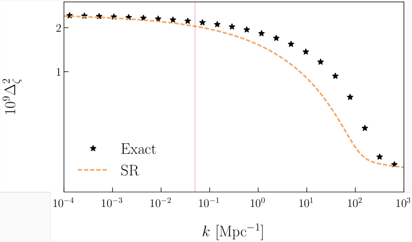
Analytically show GSR consistency relation enforced when:

1. the expansion at first-order is valid (i.e. $g \gg g^2$)
2. No large power spectrum evolution between k_S and k_L freeze-out .

We return to these conditions shortly

Example: transient G-inflation

Slow-Roll Violation in transient G-inflation



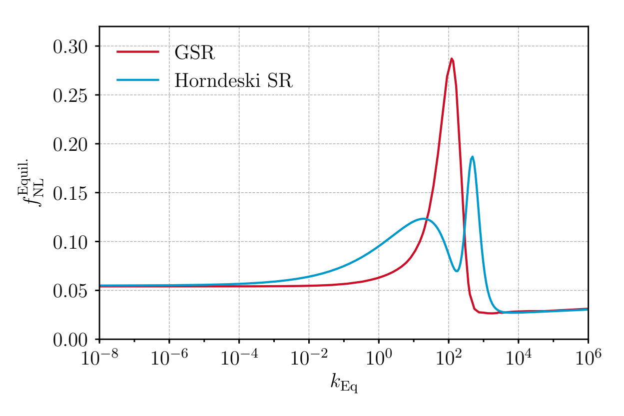
- Transition from Horndeski G-Inflation (Kobayashi et al. 11, Ohashi & Tsujikawa 12) to Chaotic Inflation.

$$\mathcal{L} \supset f_3(\phi)\frac{X}{2}\Box\phi$$

- $f_3(\phi)$ tanh step.

SR Violation: good test of our approach.

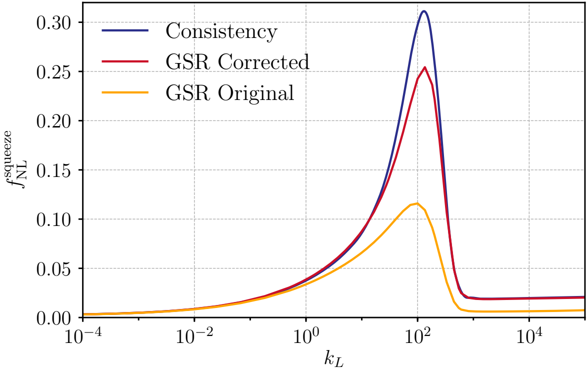
Equilateral bispectrum



GSR properly handles the transition.

Passaglia & Hu, In Prep. SR Formula from De Felice & Tsujikawa 2013

Squeeze-Limit Consistency Relation



- Correction: Modefunction evolution between freezeout epochs. (see Miranda et al. 2015 for ways to avoid)
- Residual error: Next-order GSR needed, $g^2 \sim g$.

Conclusions

Take-Home Messages

Our computation of **bispectrum beyond slow-roll** enables precision model tests.

Expressions are **easy to use**: a few 1-D integrals.

Consistency relation explicitly holds beyond slow-roll!