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Holographic self-tuning of the cosmological constant

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Bibliography

Ongoing work with

Francesco Nitti, Lukas Witkowski (APC, Paris 7), Christos Charmousis, Evgeny Babichev (U. d'Orsay)

C. Charmousis, E. Kiritsis, F. Nitti JHEP 1709 (2017) 031
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Emerging (Holographic) gravity and the SM

• We can envisage the physics of the SM+gravity (plus maybe other ingredients) as emerging from 4d UV complete QFTs:

Kiritsis

a) A large N/strongly coupled stable (near-CFT)

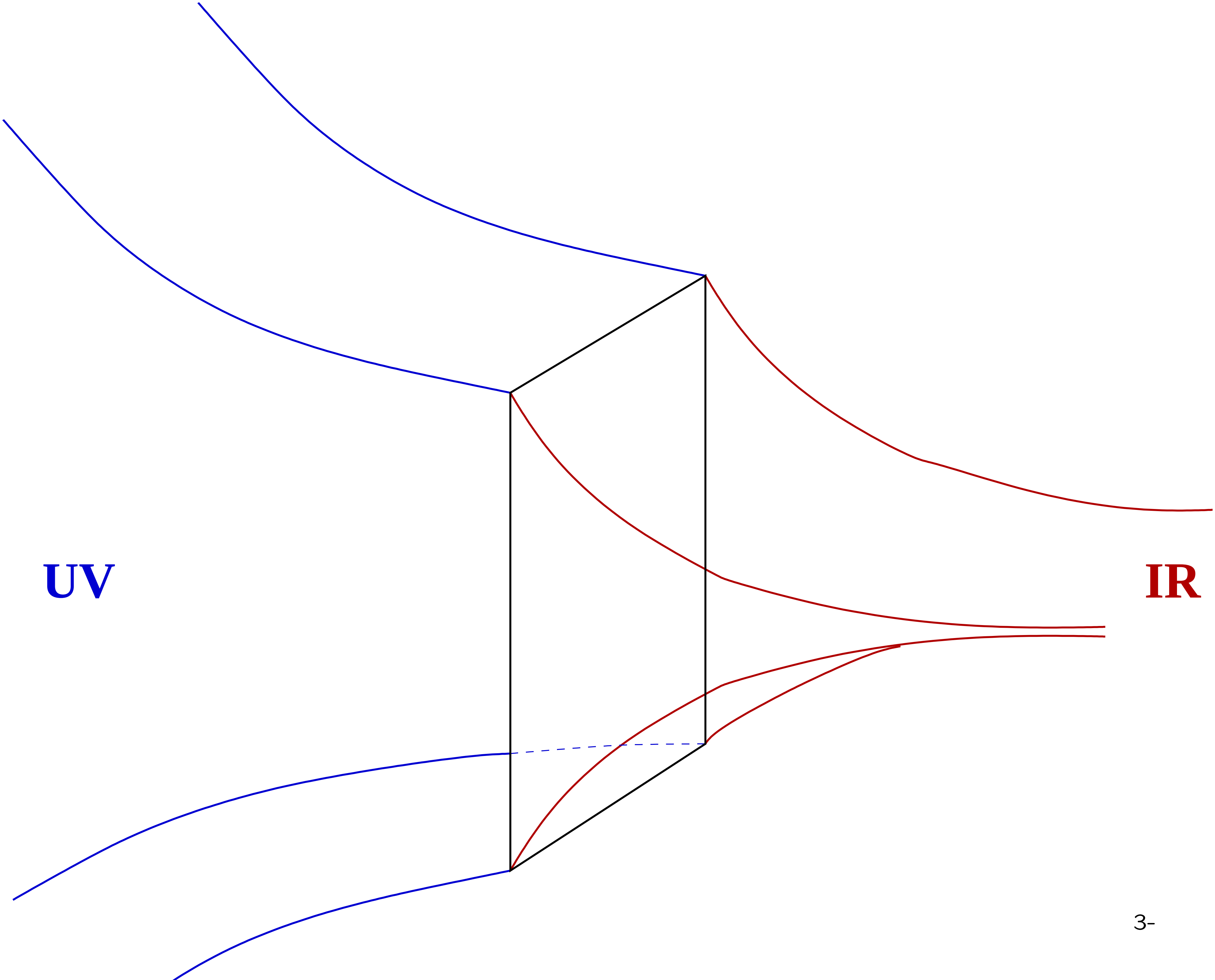
b) The Standard Model

c) A massive sector of mass Λ, (the "messengers") that couples the two theories (in a UV-complete manner).

• (a) has a holographic description in a 5d space-time.

• For $E \ll \Lambda$ we can integrate out the "messenger" sector and obtain directly the SM coupled to the bulk gravity.

• The holographic picture is that of a brane (the SM) embedded in the bulk at $r \gtrsim \frac{1}{\Lambda}$.


UV
IR

• This picture has a UV cutoff: the messenger mass Λ.

• Λ will turn out to be essentially the 4d Planck scale.

• The configuration resembles string theory orientifolds and possible SM embeddings have been classified in the past.

Anastasopoulos+Dijkstra+Kiritsis+Schellekens

• The SM couples to all operators/fields of the bulk QFT.

• Most of them they will obtain large masses of $O(\Lambda)$ due to SM quantum effects.

• The only protected fields are the metric, the universal axion $\sim Tr[F \wedge F]$ and possible vectors (aka graviphotos).

The strategy

• We consider a large-N QFT, in its dual gravitational description, in 5 space-time dimensions.

• We consider its coupling to the (4-dimensional) SM brane, embedded in the 5-dimensional bulk.

• We will assume that there is a (large) cosmological constant on the brane (due to SM quantum corrections)

• We will try to find a full solution where the brane metric is flat.

• If successful, then we will worry about many other things.

• Branes in a cutoff-AdS_5 space were used to argue that this offers a context in which brane-world scales run exponentially fast, putting the hierarchy problem in a very advantageous framework.

Randall+Sundrum

• It is in this context that the first attempts of "self tuning" of the brane cosmological constant were made.

Arkani-Hamed+Dimopoulos+Kaloper+Sundrum,Kachru+Schulz+Silverstein,

• The models used a bulk scalar to "absorb" the brane cosmological constant and provide solutions with a flat brane metric despite the non-zero brane vacuum energy.

• The attempts failed as such solutions had invariantly a bad/naked bulk singularity that rendered models incomplete.

• More sophisticated setups were advanced and more general contexts have been explored but without success: the naked bulk singularity was always there.

Csaki+Erlich+Grojean+Hollowood

Bulk equations and RG flows

• We will consider a large N, strongly coupled QFT (a CFT perturbed by a relevant scalar operator)

$$S_{bulk} = M^3 \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - V_{bulk}(\Phi) \right]$$

• We have kept, out of an infinite number of fields, the metric (dual to the stress tensor) and a single scalar (dual to some relevant scalar operator $O(x)$) in the large-N QFT.

$$S_{QFT} = S_* + \phi_0 \int d^4x \; O(x)$$

• The near-boundary region of the bulk geometry corresponds to the UV region of the QFT.

• The far interior of the bulk geometry corresponds to the IR of the QFT.
• Lorentz invariant solutions lead to the ansatz

$$ds^2 = du^2 + e^{2A(u)}(-dt^2 + d\vec{x}^2) \quad , \quad \Phi(u)$$

• The independent bulk gravitational scalar-Einstein equations can be written in first order form

$$\dot{A}(u) := -\frac{1}{6}W(\Phi) \quad , \quad \dot{\Phi}(u) = W'(\Phi)$$

in terms of the "superpotential" $W(\phi)$ that satisfies

$$V_{bulk}(\Phi) = \frac{1}{2}W'^2(\Phi) - \frac{1}{3}W^2(\Phi)$$

• This is equivalent to the EM everywhere where $\dot{\Phi} \neq 0$

• One of the integration constants ϕ_1 is hidden in the non-linear superpotential equation.

• It is fixed, by asking the gravitational solution is regular at the interior of the space-time (IR in the QFT).

• Conclusion: given a bulk action, the regular solution is characterized by the unique* superpotential function $W(\Phi)$.

• So far we described the solution that describes the ground state of the QFT without the SM brane.

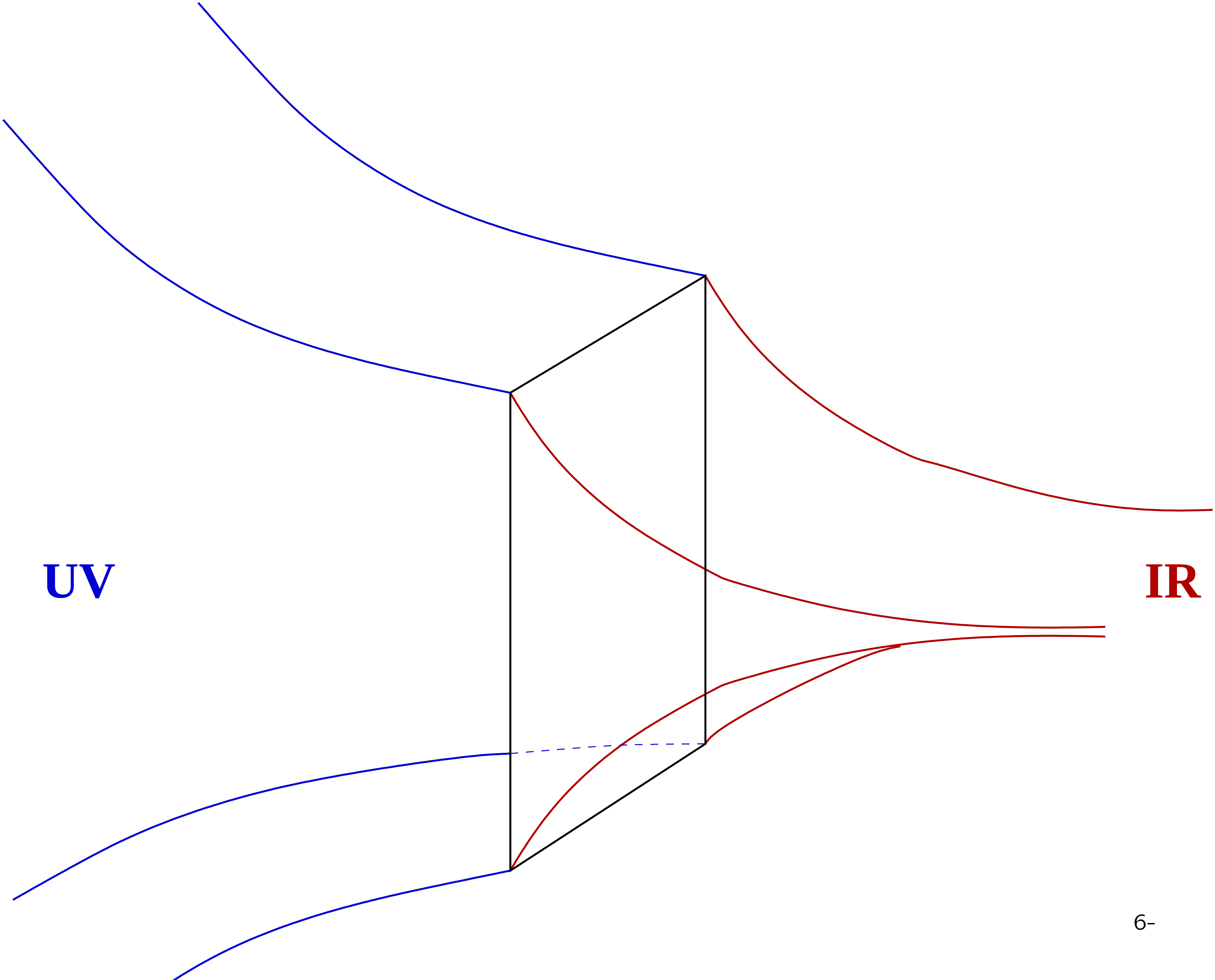
Adding the SM brane

- We add the SM brane inserted at some radial position $u = u_0$.

- The SM fields couple to the bulk fields Φ and $g_{\mu\nu}$.

$$S_{brane} = M^2 \delta(u-u_0) \int d^4x \sqrt{-\gamma} \left[W_B(\Phi) - \frac{1}{2} Z(\Phi) \gamma^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + U(\Phi) R^B + \cdots \right]$$

- The localized action on the brane is due to quantum effects of the SM fields.


UV
IR

$$S_{brane} = M^2\delta(u-u_0)\int d^4x\sqrt{-\gamma}\left[W_B(\Phi) - \frac{1}{2}Z(\Phi)\gamma^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + U(\Phi)R^B + \cdots\right]$$

- $W_B(\phi)$ is the cosmological term.

- The Israel matching conditions are:

1. Continuity of the metric and scalar field:

$$\left[g_{ab}\right]_{IR}^{UV} = 0, \qquad \left[\Phi\right]_{UV}^{IR} = 0$$

2. Discontinuity of the extrinsic curvature and normal derivative of Φ:

$$\left[K_{\mu\nu} - \gamma_{\mu\nu}K\right]_{UV}^{IR} = -\frac{1}{\sqrt{-\gamma}}\frac{\delta S_{brane}}{\delta\gamma^{\mu\nu}}, \qquad \left[n^a\partial_a\Phi\right]_{UV}^{IR} = \frac{\delta S_{brane}}{\delta\Phi},$$

- These conditions involve the first radial derivatives of A and Φ

- We have two W: W_{UV} and W_{IR}.

- They are both solutions to the superpotential equation:

$$\frac{1}{3}W^2 - \frac{1}{2}\left(\frac{dW}{d\Phi}\right)^2 = V(\Phi).$$

- A, Φ are continuous at the position of the brane.

- The jump conditions are

$$W^{IR} - W^{UV}\Big|_{\Phi_0} = W^B(\Phi_0) \quad , \quad \frac{dW^{IR}}{d\Phi} - \frac{dW^{UV}}{d\Phi}\Big|_{\Phi_0} = \frac{dW^B}{d\Phi}(\Phi_0)$$

- Assuming regularity of W_{IR}, the Israel conditions determine W_{UV} and Φ_0.

Recap

To recapitulate:

- We have shown that generically, a flat brane solution exists irrespective of the details of the "cosmological constant" function $W_B(\Phi)$

- The position of the brane in the bulk, determined via Φ_0, is fixed by the dynamics. There is typically a single such equilibrium position.

- We must analyze the stability of such an equilibrium position.

- We must analyze the nature of gravity and the equivalence principle on the brane.

- We must then analyze "cosmology" (how to get there).

Induced gravity

- The tensor mode on the brane satisfies the Laplacian equation in the bulk

$$\partial_r^2 \widehat{h}_{\mu\nu} + 3(\partial_r A)\partial_r \widehat{h}_{\mu\nu} + \partial^\rho \partial_\rho \widehat{h}_{\mu\nu} = 0$$

- $\widehat{h}_{\mu\nu}$ is continuous and satisfies the jump condition

$$\left[\widehat{h}'_{IR} - \widehat{h}'_{UV}\right]_{r_0} = -U(\phi_0) \;\; e^{-A_0} \quad\;\; \partial^\mu \partial_\mu \widehat{h}(r_0),$$

- This is the same condition as in DGP in flat space

Dvali+Gabadadze+Porrati

- The main difference is that now the bulk is curved.

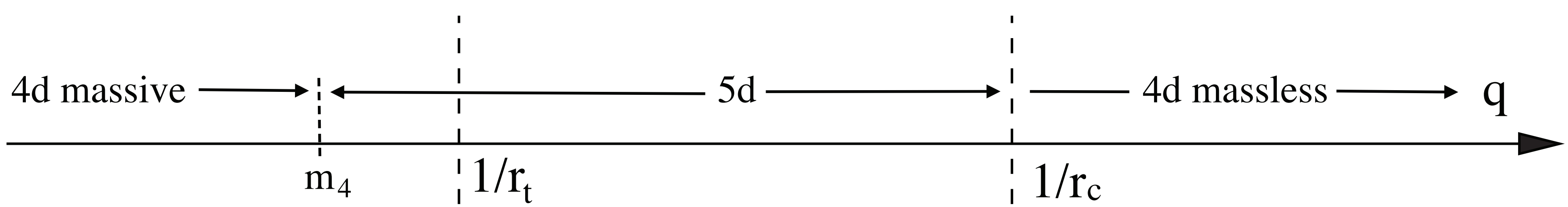
- This affects the nature of gravity on the brane:

DGP and massive gravity

• There are two relevant scales: the DGP Scale r_c and the "holographic scale r_t.

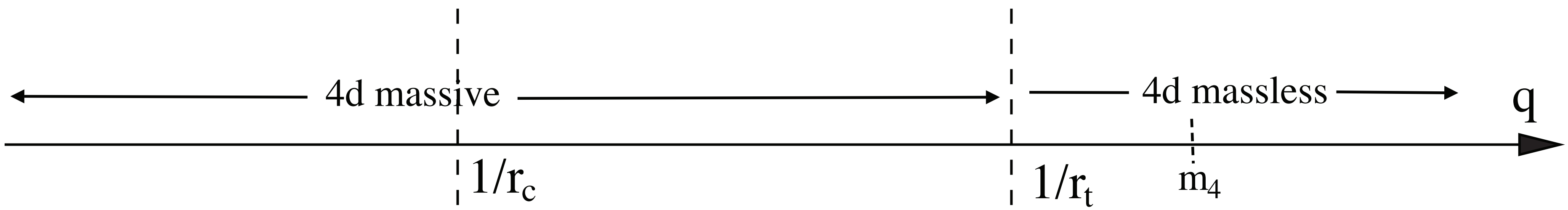
• When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:

$$\tilde{G}_4(p) \simeq \begin{cases} -\frac{1}{2M_P^2} \quad \frac{1}{p^2} & p \gg \frac{1}{r_c}, \qquad , M_P^2 = r_c M^3 \\ -\frac{1}{2M^3} \quad \frac{1}{p} & \frac{1}{r_c} \gg p \gg m_0 \\ -\frac{1}{2M_P^2} \quad \frac{1}{p^2 + m_0^2} & p \ll m_0, \qquad m_0^2 \equiv \frac{1}{2r_c d_0} \end{cases}$$



• Massive 4d gravity ($r_t < r_c$)

• In this case, at all momenta above the transition scale, $p \gg 1/r_t > 1/r_c$, we are in the 4-dimensional regime of the DGP-like propagator.



• Below the transition, $p \ll 1/r_t$, we have again a massive-graviton propagator.

• The behavior is four-dimensional at all scales, and it interpolates between massless and massive four-dimensional gravity.

EK+Tetradis+Tomaras

Small perturbations summary

- The ratio of the graviton mass to the Planck scale can be made arbitrarily small naturally (taking $N \gg 1$)

- The lighter scalar mode is also healthy under mild assumptions.

- The breaking of the equivalence principle and the Vainshtein mechanism is under current investigation.

Conclusions and Outlook

• A large-N QFT coupled holographically to the SM offers the possibility of tuning the SM vacuum energy.

• The graviton fluctuations have DGP behavior while the graviton is massive at large enough distances.

• There are however many extra constraints that need to be analyzed in detail:

• Constraints from the healthy behavior of scalar modes. Constraints from the equivalence principle and the Vainshtein mechanism

• The cosmological evolution must be elucidated.

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THANK YOU

Linear perturbations around a flat brane

- We investigate the dynamics of bulk fluctuations equations.

$$ds^2 = a^2(r)\left[(1+2\phi)dr^2 + 2A_\mu dx^\mu dr + (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu\right], \Phi(x) = \Phi_0(r) + \chi$$

where the fields $\phi, A_\mu, h_{\mu\nu}, \chi$ depend on (r, x_μ) and are small perturbations.

- We further decompose the bulk modes into tensor, vector and scalar perturbations as usual:

$$A_\mu = \partial_\mu \mathcal{W} + A^T_\mu, \quad h_{\mu\nu} = 2\eta_{\mu\nu}\psi + \partial_\mu\partial_\nu E + 2\partial_{(\mu}V^T_{\nu)} + \widehat{h}_{\mu\nu}$$

with

$$\partial^\mu A^T_\mu = \partial_\mu V^T_\mu = \partial^\mu \widehat{h}_{\mu\nu} = \widehat{h}^\mu_\mu = 0$$

• Before we insert a brane in the bulk, it is known that there are two non-trivial (propagating) fluctuations: $\widehat{h}_{\mu\nu}$ and a scalar mode ζ.

• The physical bulk scalar can be identified with the gauge-invariant combination:

$$\zeta = \psi - \frac{A'}{\Phi'}\chi.$$

• In the presence of the brane there is also the embedding mode $X^A(\sigma^\alpha)$ where $X^A = (r, x^\mu)$ and σ^α are world-volume coordinates.

• We choose the gauge $\sigma^\alpha = x^\mu \delta^\alpha_\mu$, so the embedding is completely specified by the radial profile $r(x^\mu)$.

• We consider a small deviation from the equilibrium position r_0:

$$r(x^\mu) = r_0 + \rho(x^\mu)$$

• The brane scalar mode ρ represents brane bending.

Induced gravity

- We proceed to solve the fluctuation equations:

- The tensor mode satisfies the Laplacian equation in the bulk

$$\partial_r^2 \hat{h}_{\mu\nu} + 3(\partial_r A)\partial_r \hat{h}_{\mu\nu} + \partial^\rho \partial_\rho \hat{h}_{\mu\nu} = 0$$

- $\hat{h}_{\mu\nu}$ is continuous and satisfies the jump condition

$$\left[\hat{h}'_{IR} - \hat{h}'_{UV}\right]_{r_0} = -U(\phi_0) \quad e^{-A_0} \qquad \partial^\mu \partial_\mu \hat{h}(r_0),$$

- This is the same condition as in DGP in flat space

Dvali+Gabadadze+Porrati

- The main difference is that now the bulk is curved.

The gravitational interaction on the brane

• The field equations together with the matching conditions can be obtained by extremizing

$$S[h] = M^3 \int d^4x dr \sqrt{-g} g^{ab} \partial_a \hat{h} \partial_b \hat{h} + M^3 \int_{r=r_0} d^4x \sqrt{\gamma}\; U^B(\phi) \gamma^{\mu\nu} \partial_\mu \hat{h} \partial_\nu \hat{h},$$

where $g_{ab} = e^{A(r)} \eta_{ab}$ and $\gamma_{\mu\nu} = e^{A_0}\, \eta_{\mu\nu}$ are the unperturbed bulk metric and induced metric on the brane, respectively.

• We introduce brane-localized matter sources,

$$S_m = \int d^d x \sqrt{\gamma} \;\; \mathcal{L}_m(\gamma_{\mu\nu}, \psi_i)$$

where ψ_i denotes collectively the matter fields.

• The interaction of brane stress tensor $T_{\mu\nu}$ can be written in terms of the propagator G satisfying:

$$\left[\partial_r\left(e^{3A(r)}\partial_r\right)+\left[e^{3A(r)}+U_0e^{2A_0}\delta(r-r_0)\right]\partial_\mu\partial^\mu\right]G(r,x;r',x')=$$
$$=\delta(r-r_0)\delta^{(4)}(x-x')$$

and is given by

$$S_{int}=-\frac{e^{4A_0}}{2M^3}\int d^4xd^4x'\ G(r_0,x;r_0,x')\left(T_{\mu\nu}(x)T^{\mu\nu}(x')-\frac{1}{3}T_\mu{}^\mu(x)T_\nu{}^\nu(x')\right)$$

• Notice that the combination above is appropriate for a massive graviton exchange.

• The metric on the brane is $\gamma_{\mu\nu}=e^{2A_0}\eta_{\mu\nu}$.

• The brane-to-brane propagator in momentum space ($G(r_0,x;r_0,x')\to G(p)$) is given by:

$$G(p,r_0)=-\frac{1}{M^3}\quad\frac{D(p,r_0)}{1+[U_0\,D(p,r_0)]p^2}$$

where $D(p,r)$ solves the equation:

$$\left[e^{-3A(r)}\ \partial_r\ e^{3A(r)}\partial_r-p^2\right]D(p,r)=-\delta(r-r_0)\ .$$

• This is roughly the DGP structure.

• When

$$U_0 D(p,r_0) \quad p^2 \gg 1 \quad , \quad G(p) \simeq - \frac{1}{M^3 U_0} \; \frac{1}{p^2}$$

the propagator is 4-dimensional

$$M_P^2 = U_0 M^3 \sim \; \Lambda^2$$

• The detailed behavior of the propagator is determined by the function $D(p,r)$ evaluated at the position of the brane r_0.

• It is determined by the Laplacian in the UV and IR part of the geometry, with continuity and unit jump at the brane.

The bulk propagator

- At large Euclidean p^2, we can approximate the bulk equations as in flat space,

$$D(p,r_0) \simeq \frac{1}{2p}, \qquad pr_0 \gg 1$$

- At small momenta the bulk propagator has always an expansion in powers of p^2 and we can solve perturbatively in p^2.

- If the geometry gives a gapped spectrum (confining holographic theory), the expansion is analytic in p^2

- If the bulk QFT is gapless, then after p^4 non-analyticities appear.

- We find that as $p \to 0$

$$D(p,r) = d_0 + d_2\, p^2 + d_4\, p^4 + \cdots$$

The coefficients d_i can be explicitly computed from the bulk unperturbed solution. For example

$$d_0 = e^{3A_0} \int_0^{r_0} dr' e^{-3A_{UV}(r')} \quad > \quad 0$$

The characteristic scales

• There are the following characteristic distance scales that play a role, besides r_0.

• The *transition scale* r_t around which $D(r_0, p)$ changes from small to large momentum asymptotics:

$$D(r_0, p) \simeq \begin{cases} \dfrac{1}{2p} & p \gg \frac{1}{r_t}, \\ d_0 + O(p^2) & p \ll \frac{1}{r_t} \end{cases}$$

• The transition scale r_t depends on r_0 and the bulk QFT dynamics.

• The *crossover scale,* or DGP scale, r_c:

$$r_c \equiv \frac{U_0}{2};$$

This scale determines the crossover between 5-dimensional and 4-dimensional behavior, and enters the 4D Planck scale and the graviton mass.

• The *gap scale* d_0

$$d_0 \equiv D(r_0, 0) = e^{3A_0} \int_0^{r_0} dr' e^{-3A_{UV}(r')},$$

which governs the propagator at the largest distances (in particular it sets the graviton mass as we will see).

• In generic cases, $d_0 \lesssim r_0$

• In confining bulk backgrounds we have instead

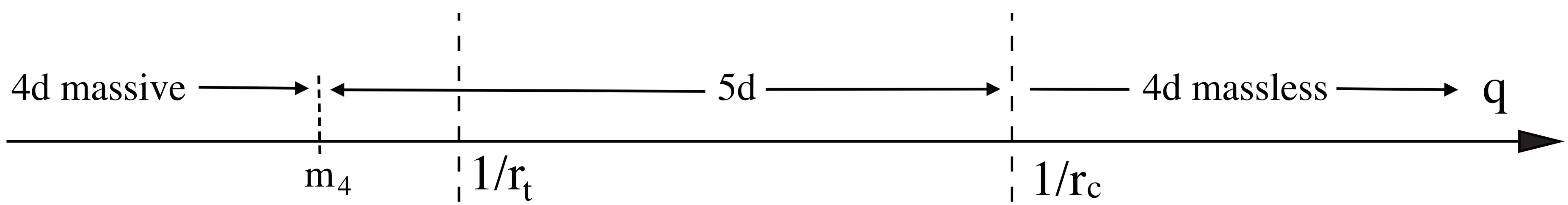
$$d_0 \simeq \frac{1}{6\Lambda_{QCD}^2 \; r_0}$$

• In the far IR, $\Lambda r_0 \gg 1$ and d_0 can be made arbitrarily small.

DGP and massive gravity

• When $r_t > r_c$ we have three regimes for the gravitational interaction on the brane:

$$\tilde{G}_4(p) \simeq \begin{cases} -\frac{1}{2M_P^2} \quad \frac{1}{p^2} & p \gg \frac{1}{r_c}, \qquad , M_P^2 = r_c M^3 \\ -\frac{1}{2M^3} \quad \frac{1}{p} & \frac{1}{r_c} \gg p \gg m_0 \\ -\frac{1}{2M_P^2} \quad \frac{1}{p^2+m_0^2} & p \ll m_0, \qquad m_0^2 \equiv \frac{1}{2 r_c d_0} \end{cases}$$



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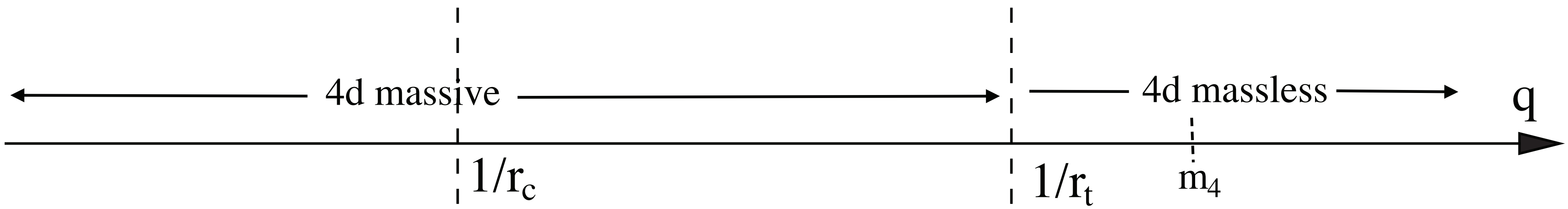

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Kiritsis+Tetradis+Tomaras

More on scales

• Scales depend on the bulk dynamics=the nature of the RG flow.

• They depend on "SM" data (the brane potential and the cutoff scale Λ).

• They can depend on boundary conditions = the UV coupling constant of the bulk QFT.

• Φ_0 at the position of the brane is fixed by the Israel conditions and is independent of boundary conditions.

• The two important parameters for 4d gravity do not depend on b.c.

$$\frac{m_0}{M_P} \sim \left(\frac{M}{\Lambda}\right)^2 \frac{1}{N^{\frac{2}{3}}} \quad , \quad m_0 \; M_P = \left(\frac{M^3}{\bar{d}}\right)^{\frac{1}{2}}$$

• $\bar{d}$ is the "rescaled" value of the bulk propagator at $p = 0$ at the position of the brane (so that it is independent of boundary conditions). It depends only on the bulk action.

• The choice of a small ratio $\frac{m_0}{M_P} \sim 10^{-60}$ is (technically) natural from the QFT point of view.

• There is important numerology to be analyzed for typical classes of holographic theories.

Scalar Perturbations

• The next step is to study the scalar perturbations. They are of interest, as they might destroy the equivalence principle.

• The equations for the scalar perturbations can be derived and they are complicated.

• Unlike previous analysis of similar systems they cannot be factorized to a relatively simple system as the graviton.

• There are two scalar modes on the brane:

• In one gauge, the brane bedding mode can be “eliminated” but the scalar perturbation is discontinuous on the brane.

• In another gauge the perturbation is continuous but the brane bending mode is present.

The effective quadratic interactions for the scalar modes are of the form

$$S_4 = -\frac{\mathcal{N}}{2}\int d^4x\sqrt{\gamma}((\partial\phi)^2 + m^2\phi^2)$$

• We need both $\mathcal{N} > 0$ and $m^2 > 0$.

• In general the two scalar modes couple to two charges:

(a) the "scalar charge" and

(b) the trace of the brane stress tensor.

• The mode that couples to the scalar charge has a "heavy" mass of the order of the cutoff/Planck Scale.

• The mode that couples to the trace of the stress-tensor has a mass that is O(1) in cutoff units (like the graviton mass).

• All the stability conditions for the scalars depend on more details of the brane induced functions $W_B(\Phi)$, $U_B(\Phi)$, $Z_B(\Phi)$.

• They can be investigated further from the known parameter dependence of the vacuum energy in the SM.

Kounnas+Pavel+Zwirner, Dimopoulos+Giudince+Tetradis

• There is a vDVZ discontinuity that (as usual) cannot be cancelled at the linearized order if the theory is positive.

• It should be cancelled by the Vainshtein mechanism. To derive the relevant constraints on parameters, we must study the non-linear interactions of the scalar-graviton modes.

Connecting the Hierarchy Problem

• We can include the Higgs scalar in the effective potential on the brane:

$$S_{Higgs} = M_p^2 \int d^dx \sqrt{-\gamma} \left[-X(\Phi)|H|^2 - S(\Phi)|H|^4 + T(\Phi)R|H|^2 + \cdots \right]$$

• We must also add the equations of motion for the Higgs:

$$(X(\Phi) + 2S(\Phi)|H|^2)\ H = 0$$

• We expect that the bulk scalar field Φ will start far from the equilibrium position Φ_0 and will roll towards it.

• If $X(\Phi) > 0$ far from equilibrium and $X(\Phi) < 0$ near equilibrium, then EW symmetry breaking will be correlated with the cosmological constant self-tuning mechanism.

• This contains the "radiative breaking" idea as a component.

• Whether it works depends on the structure of the function $X(\Phi)$ that can be computed from SM physics.

Introduction

• The cosmological constant problem is arguably the most important shortcoming today of our understanding of the physical world.

• It signifies the violent clash between gravity and quantum field theory, (probably more so than the black hole information paradox problem).

• In four-dimensional Einstein gravity a non-zero vacuum energy entails irrevocably the acceleration of the univers:

$$G_{\mu\nu} = \frac{1}{2}\Lambda\ g_{\mu\nu}$$

• One can fine-tune the cosmological constant (this sometimes comes under the "anthropic" context).

Schellekens, Bousso+Polchinski

• It turns out that this is today compatible with cosmological data but soon it will be tested verified or excluded.

Bellazzini+Csaki+Serra+Terming

• The reason is that the cosmological constant is scale-dependent and changes with the energy scale.

reviews: Weinberg, Rubakov, Hebecker+Wetterich,Burgess

• For several decades efforts amounted to proving that, by symmetry, the cosmological constant should vanish.

• The advent of inflation made this approach less and less credible.

• The “detection” of the acceleration of the universe at the end of the 20th century has put an end in such approaches.

• Several other approaches have been tried over the years. Some still stand in principle:

♠ The Bousso-Polchinski anthropic "solution".

♠ "Sequestering mechanisms" for the vacuum energy.

Gabadadze+Yu,Kaloper+Padilla+Stafanyszyn+Zahariade

♠ "Degravitation" ideas.

Arkani-Hamed+Dimopoulos+Dvali+Gabadadze, Dvali+Hofmann+Khoury

♠ "Brane-world" related ideas.

Rubakov+Shaposhnikov, Akama,.....

All must pass a very stringent "filter": Weinberg argument.

The higher-dimensional arena

• It was argued by several authors that the existence of higher (than four) dimensions offers the possibility to alleviate the cosmological constant problem.

• The rough idea is that the SM-induced vacuum energy, instead of curving the 4-d world/brane, could be absorbed by bulk fields.

• For this idea to be effective, the mechanism must be quasi-generic: "any or most" cosmological constants must "relax", absorbed by the bulk dynamics.

• Any such mechanism must be intertwined tightly with cosmology as we have good reasons to believe that a large cosmological constant played an important role in the early universe, with observable consequences today.

Brane worlds and early attempts

• String Theory D-branes offer a concrete, calculable realization of a brane universe.

Polchinski

• Branes in a cutoff-AdS_5 space were used to argue that this offers a context in which brane-world scales run exponentially fast, putting the hierarchy problem in a very advantageous framework.

Randall+Sundrum

• It is in this context that the first attempts of "self tuning" of the brane cosmological constant were made.

Arkani-Hamed+Dimopoulos+Kaloper+Sundrum,Kachru+Schulz+Silverstein,

• The models used a bulk scalar to "absorb" the brane cosmological constant and provide solutions with a flat brane metric despite the non-zero brane vacuum energy.

• The attempts failed as such solutions had invariantly a bad/naked bulk singularity that rendered models incomplete.

• More sophisticated setups were advanced and more general contexts have been explored but without success: the naked bulk singularity was always there.

Csaki+Erlich+Grojean+Hollowood,

• The Randall-Sundrum Z_2 orbifold boundary conditions were relaxed to consider even more general setups, but this did not improve the situation.

Padilla

• The RS setup and its siblings is related via holographic ideas to cutoff-CFTs and this provides independent intuition on the physics.

Maldacena,Witten,Hawking+Hertog+Reall, Arkani-Hamed+Porrati+Randall

• In view of our current understanding of holography, these failures were to be expected.

• Our goal: provide a 2.0 version of the self-tuning mechanism that is in line with the dictums of holography.

Old Self-Tuning

- W_{UV} and W_{IR} are determined from the superpotential equation up to one integration constant, C_{UV}, C_{IR}.

- For a generic brane potential $W^B(\Phi)$, the two matching equations

$$W^{IR} - W^{UV}\Big|_{\Phi_0} = W^B(\Phi_0) \quad , \quad \frac{dW^{IR}}{d\Phi} - \frac{dW^{UV}}{d\Phi}\Big|_{\Phi_0} = \frac{dW^B}{d\phi}(\Phi_0)$$

will fix C^{UV}, C^{IR} for *any* generic value of Φ_0.

- The fixed value of C_{IR} typically leads to a bad IR singularity.

- Moreover Φ_0 is a modulus and generates a massless mode (the radion).

Self-Tuning 2.0

• The IR constant C^{IR} should be fixed by demanding that the IR singularity is absent.

• Typically there is only one such solution to the superpotential equation (or a discreet set).

• According to holography rules, the solution W^{IR} should be fixed before we impose the matching conditions.

• Once W^{IR} is fixed by regularity, the Israel conditions will determine:

♠ The integration constant C^{UV} in the UV superpotential

♠ *The brane position* in field space, Φ_0.

• This is a desirable outcome as there would be no massless radion mode.

• It can be checked that generically such an equilibrium position exists.

The bulk propagator

• At large Euclidean p^2, we can approximate the bulk equations as in flat space,

$$\partial_r^2 \Psi^{(p)}(r) = p^2 \Psi^{(p)}(r)$$

except for small r, where the effective Schrödinger potential is $\sim 1/r^2$ and cannot be neglected.

• The solution satisfying appropriate boundary conditions (vanishing in the IR and for $r \to 0$) and jump condition is

$$\Psi_{IR}^{(p)} = \frac{\sinh pr_0}{p} e^{-pr}, \qquad \Psi_{UV}^{(p)} = \frac{e^{-pr_0}}{p} \sinh pr, \qquad p \equiv \sqrt{p^2}$$

• For large p, it is like in flat 5d space

$$D(p, r_0) = \frac{\sinh pr_0}{p} e^{-pr_0} \simeq \frac{1}{2p}, \qquad pr_0 \gg 1$$

• At small momenta the bulk propagator has always an expansion in powers of p^2 and we can solve perturbatively in p^2.

• If the geometry is gapped, the expansion is analytic in p^2

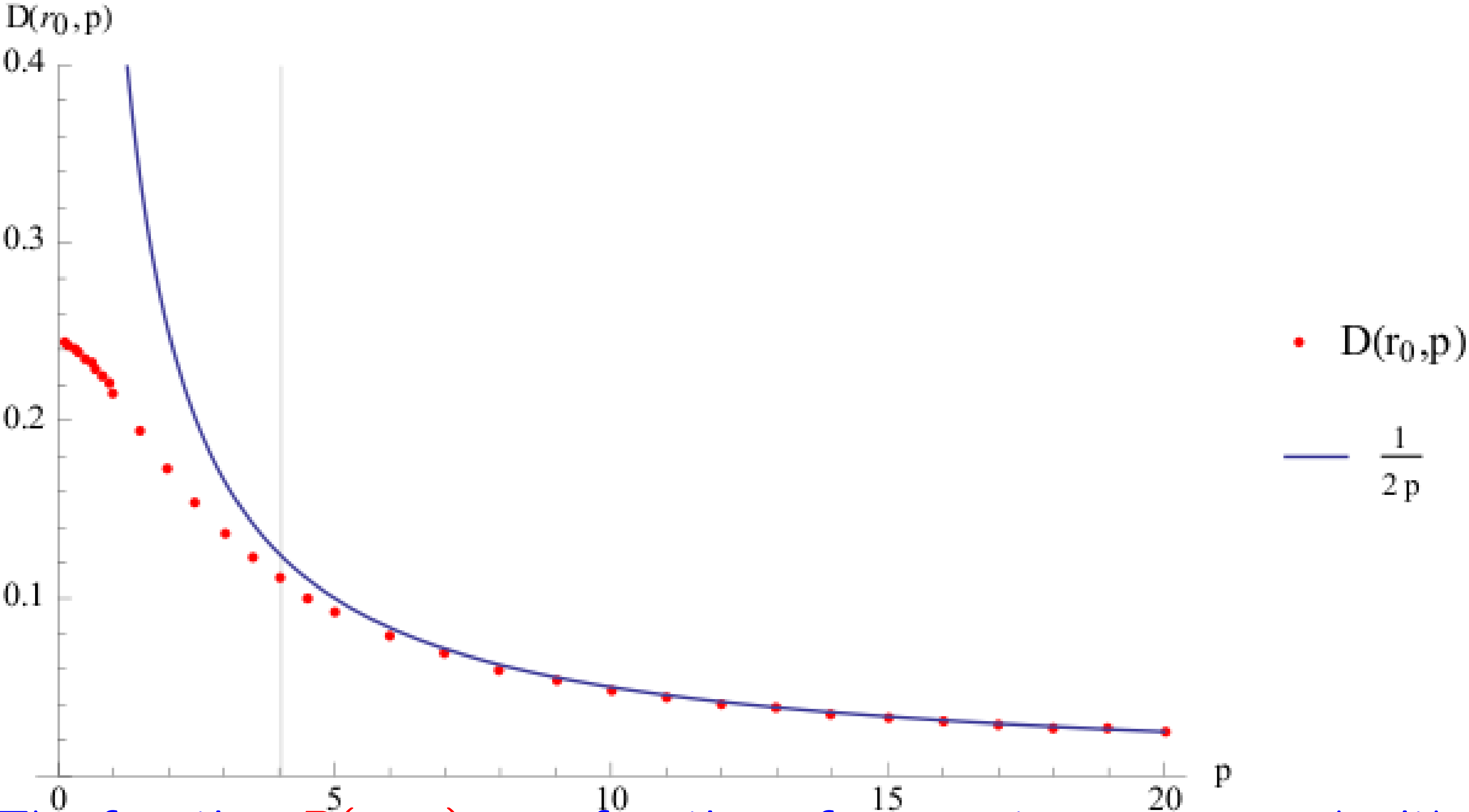
• If the geometry is gapless, then after some power of p non-analyticities appear.

• We find that as $p \to 0$

$$D(p,r) = d_0 + d_2\, p^2 + d_4\, p^4 + \cdots$$

The coefficients d_i can be explicitly computed from the bulk unperturbed solution. For example

$$d_0 = e^{3A_0} \int_0^{r_0} dr' e^{-3A_{UV}(r')}$$



The function $D(r_0, p)$ as a function of momentum, compared with $1/2p$.
The transition scale $1/r_t$ (solid line) is about 4 (in UV-AdS units)

Scalar Perturbations

- The perturbations are

$$ds^2 = a^2(r)\left[(1+2\phi)dr^2 + 2A_\mu dx^\mu dr + (\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu\right], \qquad \varphi = \bar{\varphi}(r) + \chi$$

and the scalar ones are

$$\phi, \quad \chi, \quad A_\mu = \partial_\mu B, \quad h_{\mu\nu} = 2\psi\eta_{\mu\nu} + 2\partial_\mu\partial_\nu E,$$

plus the brane-bending mode $\rho(x)$ defined as

$$r(x^\mu) = r_0 + \rho(x^\mu)$$

- Unlike the tensor modes, these fields are not gauge-invariant. Under an infinitesimal diff transformation $(\delta r, \delta x^\mu) = (\xi^5, g^{\mu\nu}\partial_\nu \xi)$ they transform as

$$\delta\psi = -\frac{a'}{a}\xi^5 \quad , \quad \delta\phi = -(\xi^5)' - \frac{a'}{a}\xi^5 \quad , \quad \delta B = -\xi' - \xi^5$$

$$\delta E = -\xi \quad , \quad \delta\chi = -\bar{\varphi}'\xi^5 \quad , \quad \delta\rho = \xi^5(r_0, x).$$

- We partly fix the gauge by choosing $B = 0$.

• We are still free to do radial gauge-transformations and r-independent space-time diffeomorphisms and keep this gauge choice.

• The matching conditions become

$$\Big[a^2(r_0+\rho)\,(2\psi\eta_{\mu\nu}+2\partial_\mu\partial_\nu E)\Big]_{IR}^{UV}=0, \qquad \Big[\bar\varphi(r_0+\rho)+\chi\Big]_{UV}^{IR}=0$$

$$\Big[\hat\psi\Big]_{IR}^{UV}=0, \quad \Big[\hat\chi\Big]_{IR}^{UV}=0, \qquad \Big[E\Big]_{UV}^{IR}=0$$

where we have defined the new *bulk* perturbations:

$$\hat\psi(r,x)=\psi+A'(r)\rho(x), \quad \hat\chi(r,x)=\chi+\bar\varphi'(r)\rho(x) \quad , \quad A'=a'/a$$

The gauge-invariant scalar perturbation has the same expression in terms of these new continues variables:

$$\zeta=\psi-\frac{A'}{\bar\varphi'}\chi=\hat\psi-\frac{A'}{\bar\varphi'}\hat\chi.$$

In general however $\zeta(r,x)$ is not continuous across the brane, since the background quantity $A'/\bar{\varphi}'$ jumps:

$$\Big[\zeta\Big]_{IR}^{UV} = \left[\frac{A'}{\bar{\varphi}'}\right]_{IR}^{UV} \widehat{\chi}(r_0)$$

Notice that this equation is gauge-invariant since, under a gauge transformation:

$$\delta\widehat{\chi}(r,x) = -\bar{\varphi}'(r)\left[\xi^5(r,x) - \xi^5(r_0,x)\right],$$

thus $\widehat{\chi}(r_0)$ on the right hand side of equation (**??**) is invariant.

It is convenient to fix the remaining gauge freedom by imposing:

$$\chi(r,x) = 0.$$

To do this, one needs different diffeomorphisms on the left and on the right of the brane, since $\bar{\varphi}'$ differs on both sides. The continuity for $\widehat{\chi}$ then becomes the condition:

$$\rho_{UV}(x)\bar{\varphi}'_{UV}(r_0) = \rho_{IR}(x)\bar{\varphi}'_{IR}(r_0)$$

i.e. the brane profile looks different from the left and from the right. This is not a problem, since equation (**??**) tells us how to connect the two sides given the background scalar field profile.

In the $\chi = 0$ gauge we have:

$$\zeta = \psi = \widehat{\psi} - A'\rho, \qquad \widehat{\chi}(r_0) = \bar{\varphi}'(r_0)\rho.$$

This makes it simple to solve for ϕ using the bulk constraint equation (in particular, the $r\mu$-component of the perturbed Einstein equation, for the details see the Appendix:

$$\phi = \frac{a}{a'}\psi' = \frac{a}{a'}\widehat{\psi}' + \left(\frac{a'}{a} - \frac{a''}{a}\right)\rho$$

where it is understood that this relation holds both on the UV and IR sides.

In the gauge $\chi = B = 0$, the second matching conditions to linear order in perturbations, read

$$\Big[(1-d)a'(r_0)\left(2\widehat{\psi}\,\eta_{\mu\nu} + 2\partial_\mu\partial_\nu E\right) + \frac{1}{2}a(r_0)(\bar{\varphi}')^2\rho\,\eta_{\mu\nu} +$$

$$(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^\sigma\partial_\sigma)\left(E' - \rho\right)\Big]_{UV}^{IR} = \frac{a^2(r_0)}{2}W_B(\Phi_0)\left(2\eta_{\mu\nu}\widehat{\psi} + 2\partial_\mu\partial_\nu E\right)_{r_0} +$$

$$\frac{a^2(r_0)}{2}\frac{dW_B}{d\varphi}\Big|_{\Phi_0}\bar{\varphi}'(r_0)\rho - (d-2)U_B(\Phi_0)\left(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^\sigma\partial_\sigma\right)\widehat{\psi}\ ,$$

$$\left[\frac{\bar\varphi'}{a'}\hat\psi' + \left(\frac{(\bar\varphi')^2}{6a'} - \frac{\bar\varphi''}{a\bar\varphi'}\right)\bar\varphi'\rho\right]_{UV}^{IR} =$$

$$= -\frac{d^2W_B}{d\Phi^2}\Big|_{\Phi_0}\bar\varphi'\rho + \frac{Z_B(\Phi_0)}{a^2}\bar\varphi'\partial^\sigma\partial_\sigma\rho - \frac{2(d-1)}{a^2}\frac{dU_B}{d\Phi}\Big|_{\Phi_0}\partial^\sigma\partial_\sigma\hat\psi$$

Using the background matching conditions in conformal coordinates,

$$\frac{a'}{a^2} = -\frac{1}{2(d-1)}W, \qquad \bar\varphi' = a\frac{dW}{d\Phi},$$

one can see that the first two terms on each side cancel each other, and we are left with an equation that fixes the matching condition for $E'(r,x)$:
B

$$\left[E' - \rho\right]_{UV}^{IR} = -2\frac{U_B(\Phi_0)}{a(r_0)}\hat\psi(r_0).$$

$$\left[\hat\psi\right]_{UV}^{IR} = 0\,; \qquad \left[\bar\varphi'\rho\right]_{UV}^{IR} = 0\,;$$

$$\left[\frac{\bar\varphi' a}{a'}\hat\psi'\right]_{UV}^{IR} = \left[\left(\frac{Z_B(\Phi_0)}{a}\partial^\mu\partial_\mu - \mathcal{M}_b^2\right)\bar\varphi'\rho - \frac{6}{a}\frac{dU_B}{d\Phi}(\Phi_0)\partial^\mu\partial_\mu\hat\psi\right]_{r_0}$$

where we have defined the brane mass:

$$\mathcal{M}_b^2 \equiv a(r_0)\frac{d^2W_b}{d\Phi^2}\bigg|_{\Phi_0} + \left[\left(\frac{(\bar{\varphi}')^2}{6}\frac{a}{a'} - \frac{\bar{\varphi}''}{\bar{\varphi}'}\right)\right]_{UV}^{IR}.$$

Using the background Einstein's equations this can also be written as:

$$\mathcal{M}_b^2 = \left[\frac{a'}{a} - \frac{a''}{a'}\right]_{UV}^{IR} + a\left(\frac{d^2W_B}{d\Phi^2} - \left[\frac{d^2W}{d\Phi^2}\right]_{UV}^{IR}\right),$$

We can eliminate E

$$\Box E' = -\frac{a}{a'}\left[\Box\psi + \frac{a}{a'}\left(2\frac{a'^2}{a^2} - \frac{a''}{a}\right)\psi'\right].$$

Notice that the combination multiplying ψ' can be written as $(a/a')(\bar{\varphi}')^2/6$.

The bulk equation for ζ ($\equiv \psi$ in this gauge) on both sides of the brane is:

$$\psi'' + \left(3\frac{a'}{a} + 2\frac{z'}{z}\right)\psi' + \partial^\mu\partial_\mu\psi = 0,$$

where $z = \bar{\varphi}' a/a'$.

To summarize, we arrive at the following equations and matching conditions, either in terms of ψ:

$$\psi'' + \left(3\frac{a'}{a} + 2\frac{z'}{z}\right)\psi' + \partial^\mu\partial_\mu\psi = 0,$$

$$\Big[\psi\Big]_{UV}^{IR} = -\left[\frac{a'}{a\bar{\varphi}'}\right]_{UV}^{IR}\bar{\varphi}'\rho, \qquad \Big[\bar{\varphi}'\rho\Big]_{UV}^{IR} = 0\,;$$

$$\left[\frac{a^2}{a'^2}\frac{\bar{\varphi}'^2}{6}\psi'\right]_{UV}^{IR} = \left(\frac{2U_B(\Phi_0)}{a} - \left[\frac{a}{a'}\right]_{UV}^{IR}\right)\Box\left(\psi + \frac{a'}{a}\rho\right)\;;$$

$$\left[\frac{a\bar{\varphi}'}{a'}\psi'\right]_{UV}^{IR} = -6\frac{dU_B}{d\Phi}(\Phi_0)\Box\left(\psi + \frac{a'}{a}\rho\right) + \left(\frac{Z_B(\Phi_0)}{a}\Box - \tilde{\mathcal{M}_b}^2\right)\bar{\varphi}'\rho\,;$$

$$\Box \equiv \partial^\mu\partial_\mu, \qquad z \equiv \frac{a\bar{\varphi}'}{a'}, \qquad \tilde{\mathcal{M}_b}^2 = a\left(\frac{d^2W_B}{d\Phi^2} - \left[\frac{d^2W}{d\Phi^2}\right]_{UV}^{IR}\right)\,.$$

• in terms of $\widehat{\psi}$:

$$\widehat{\psi}'' + \left(3\frac{a'}{a} + 2\frac{z'}{z}\right)\widehat{\psi}' + \partial^\mu\partial_\mu\widehat{\psi} = \mathcal{S},$$

$$\left[\widehat{\psi}\right]_{UV}^{IR} = 0, \qquad \left[\bar{\varphi}'\rho\right]_{UV}^{IR} = 0\,;$$

$$\left[\frac{a^2}{a'^2}\frac{\bar{\varphi}'^2}{6}\widehat{\psi}'\right]_{UV}^{IR} = -\left[\frac{\bar{\varphi}'}{6}\left(\frac{a''a}{a'^2} - 1\right)\right]_{UV}^{IR}\bar{\varphi}'\rho + \left(\frac{2U_B(\Phi_0)}{a} - \left[\frac{a}{a'}\right]_{UV}^{IR}\right)\Box\widehat{\psi}\,;$$

$$\left[\frac{a\bar{\varphi}'}{a'}\widehat{\psi}'\right]_{UV}^{IR} = -6\frac{dU_B}{d\Phi}(\Phi_0)\Box\widehat{\psi} + \left(\frac{Z_B(\Phi_0)}{a}\Box - \mathcal{M}_b^2\right)\bar{\varphi}'\rho\,;$$

$$\Box \equiv \partial^\mu\partial_\mu, \qquad z \equiv \frac{a\bar{\varphi}'}{a'}, \qquad \mathcal{M}_b^2 = \tilde{\mathcal{M}_b}^2 + \left[\frac{a'}{a} - \frac{a''}{a'}\right]_{UV}^{IR},$$

$$\mathcal{S} \equiv A'''\rho + 3(A' + 2z'/z)A''\rho + A'\Box\rho.$$

remarks:

- In both formulations there are 6 parameters in the system: 4 in the bulk (2 integration constants in the UV, 2 in the IR) and 2 brane parameters (ρ on each side). From these 6 we can subtract one: a rescaling of the solution, which is not a true parameter since the system is homogeneous in (ρ, ψ). There is a total of 4 matching conditions, plus 2 normalizability conditions if the IR is confining, or only one if it is not. Thus, in the confining case, we should find a quantization condition for the mass spectrum, whereas in the non-confining case the spectrum is continuous and the solution unique given the energy. The goal will be to show that such solutions exist only for positive values of m^2, defined as the eigenvalue of $\Box$. To see this, one must go to the Schrodinger formulation.

- Notice that something interesting happens when the *second* derivative of the brane potential matches the discontinuity in the second derivative of the bulk superpotential: in that case the brane mass term for ρ vanishes. For a generic brane potential of course this is not the case, but it happens for example in fine-tuned models when the brane position is not fixed by the zeroth-order matching conditions, for example when the brane potential is chosen to be equal to the bulk superpotential, and a Z_2 symmetry is imposed. This is the generalization of the RS fine-tuning in the presence of a bulk scalar. The fact that the mass term vanishes in this case must be related to the presence of zero-modes (whether they are normalizable or not is a different story).

To put the matching conditions in a more useful form, it is convenient to eliminate $\rho_{L,R}$ altogether :

$$\left[\frac{a'}{a}\rho\right] = -[\psi], \qquad [\bar{\varphi}'\rho] = 0$$

These can be solved to express the continuous quantities $\hat{\psi}(0)$ and $\bar{\varphi}'\rho$ in terms of $\psi_{L,R}$ only:

$$\hat{\psi}(0) = \frac{[z\,\psi]}{[z]}, \qquad \bar{\varphi}'\rho = -\frac{[\psi]}{[1/z]}, \qquad z = \frac{a\bar{\varphi}'}{a'}$$

Using these results, we obtain a relation between the left and right functions and their derivatives:

$$\left[z\psi'\right] = -6\frac{dU_B}{d\Phi}\Box\frac{[z\,\psi]}{[z]} - \frac{1}{a}\left(Z_B\Box - a^2\tilde{M}^2\right)\frac{[\psi]}{[z^{-1}]}$$

$$\left[z^2\psi'\right] = 6\left(2\frac{U_B}{a} - \left[\frac{a}{a'}\right]\right)\Box\frac{[z\,\psi]}{[z]}$$

Since the left hand side is in general non-degenerate, these equations can be solved to give ψ'_L and ψ'_R as linear combinations of ψ_L and ψ_R,

$$\begin{pmatrix} \psi'_L(0) \\ \psi'_R(0) \end{pmatrix} = \Gamma \begin{pmatrix} \psi_L(0) \\ \psi_R(0) \end{pmatrix}$$

with a suitable matrix Γ.

The conditions that the scalars are not ghosts are

$$\tau_0 \equiv 6\frac{W_B}{W_{UV}W_{IR}}\Big|_{\Phi_0} - U_B(\Phi_0) > 0 \quad , \quad Z_0\tau_0 > 6\left(\frac{dU_B}{d\Phi}\right)^2\Big|_{\Phi_0} \tag{1}$$

- Asking also for no tachyons we obtain

$$\frac{d^2W_B}{d\Phi^2}\Big|_{\Phi_0} - \left[\frac{d^2W}{d\Phi^2}\right]_{UV}^{IR} > 0$$

A simple numerical example

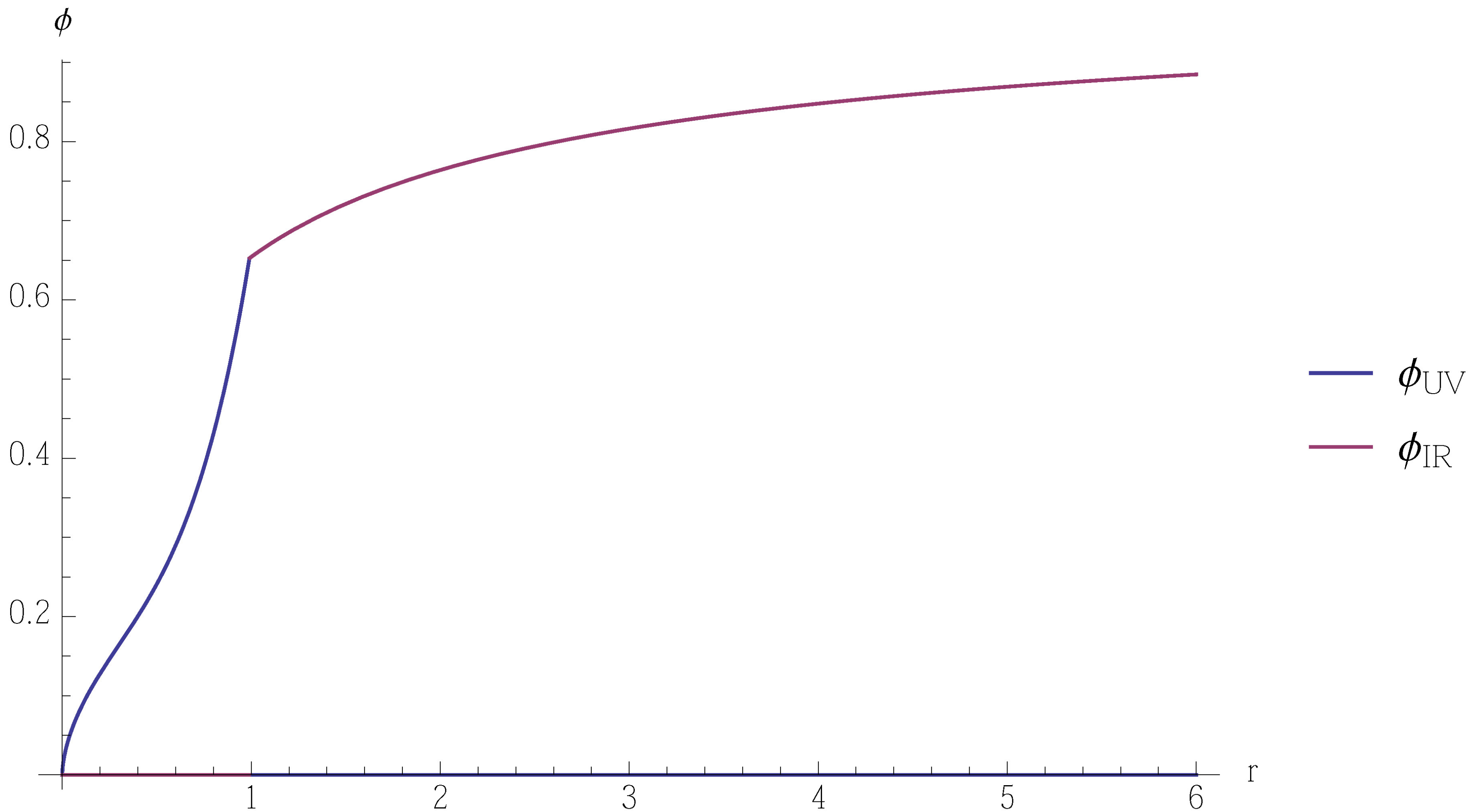
$$V(\phi) = -12 + \frac{1}{2}\left(\phi^2 - 1\right)^2 - \frac{1}{2},$$

- The flow is from $\phi = 0$ (UV Fixed point) to $\phi = 1$ (IR fixed point).

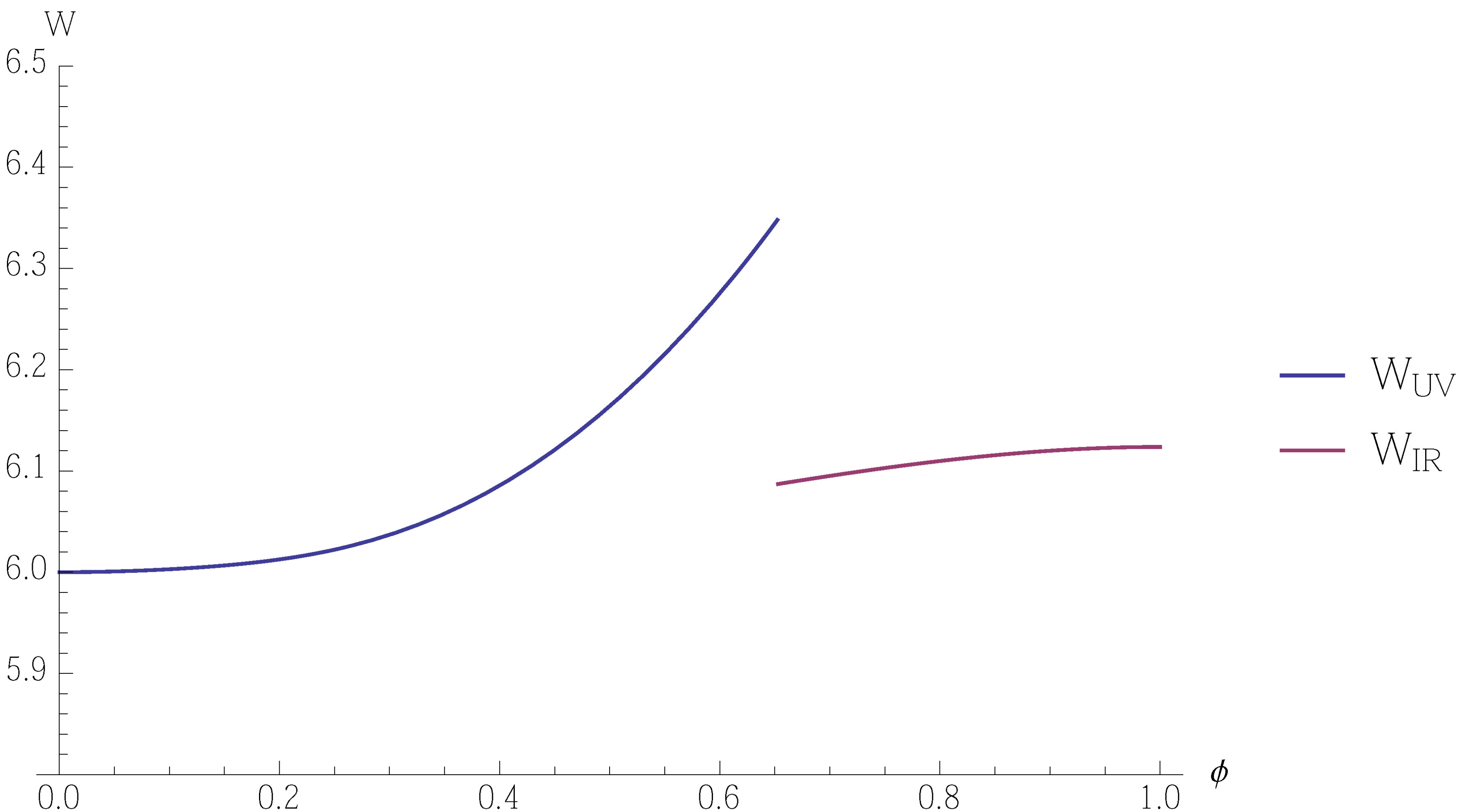
$$W_b(\phi) = \omega \ \exp[\gamma\phi].$$

$$\omega = -0.01, \ \gamma = 5 \qquad \Rightarrow \qquad \phi_0 = 0.65.$$

- This gives, in conformal coordinates, $r_0 = 0.99$.



ϕ
0.8
0.6
0.4
0.2
1
2
3
4
5
6
r
ϕ_{UV}
ϕ_{IR}


W
6.5
6.4
6.3
6.2
6.1
6.0
5.9
0.0
0.2
0.4
0.6
0.8
1.0
ϕ
W_{UV}
W_{IR}

RG

• $W(\phi)$ is the non-derivative part of the Schwinger source functional of the dual QFT =on-shell bulk action.

de Boer+Verlinde[2]

$$S_{on-shell} = \int d^dx\sqrt{\gamma}\; W(\phi) + \cdots \Big|_{u\to u_{UV}}$$

• The renormalized action is given by

$$S_{renorm} = \int d^dx\sqrt{\gamma}\; (W(\phi) - W_{ct}(\phi)) + \cdots \Big|_{u\to u_{UV}} =$$

$$= constant \int d^dx\; e^{dA(u_0)-\frac{1}{2(d-1)}\int_{\phi_{UV}}^{\phi_0} d\tilde{\phi}\frac{W'}{W}} + \cdots$$

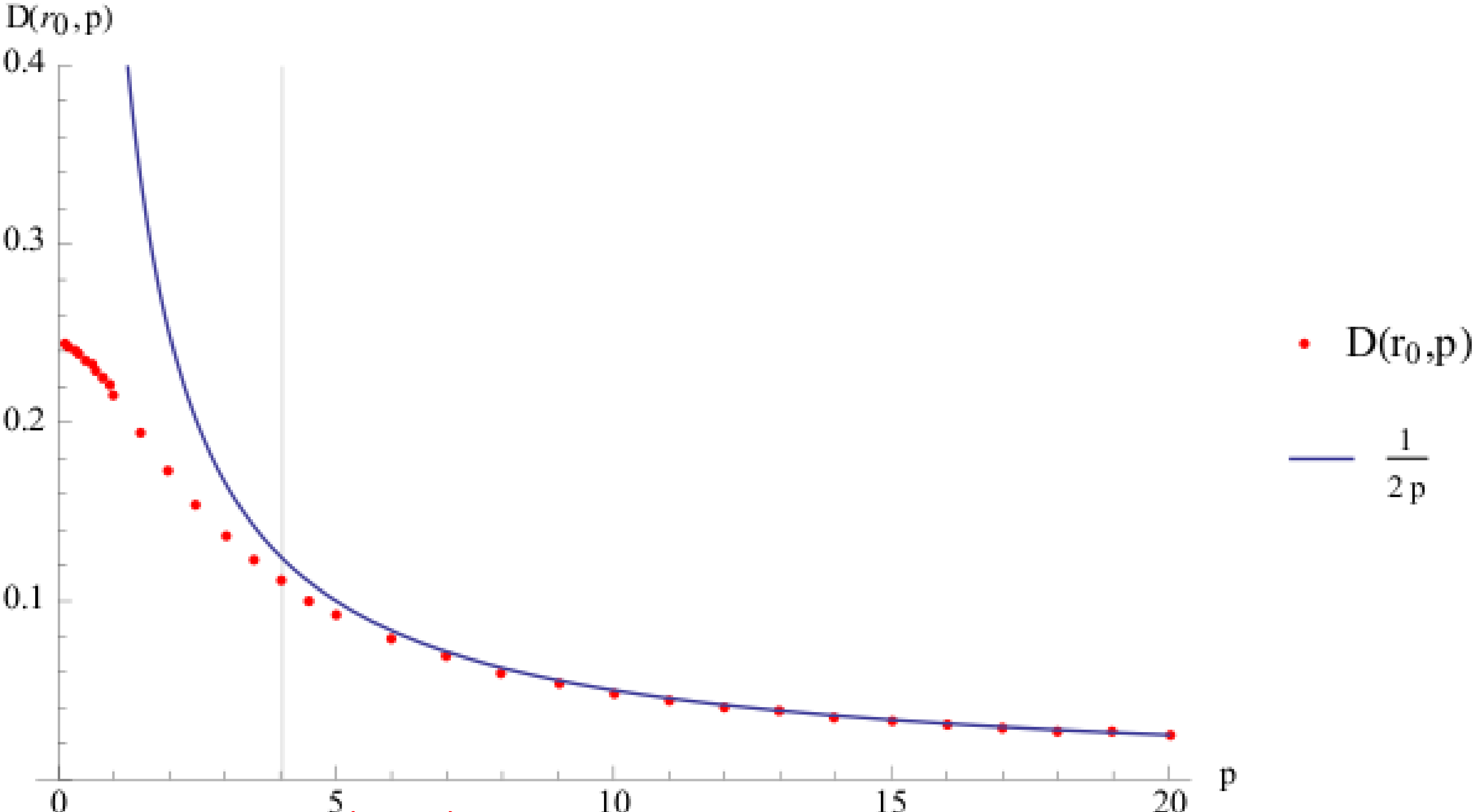
• The statement that $\frac{dS_{renorm}}{du_0} = 0$ is equivalent to the RG invariance of the renormalized Schwinger functional.

• It is also equivalent to the RG equation for ϕ.

• We can show that

$$T_\mu{}^\mu = \beta(\phi)\ \langle O \rangle$$

• The Legendre transform of S_{renorm} is the (quantum) effective potential for the vev of the QFT operator O.



The function $D(r_0, p)$ as a function of momentum, compared with $1/2p$. The transition scale $1/r_t$ (solid line) is about 4 (in UV-AdS units)

Detour: The local RG

- The holographic RG can be generalized straightforwardly to the local RG

$$\dot\phi = W' - f'\ R + \frac{1}{2}\left(\frac{W}{W'}f'\right)'(\partial\phi)^2 + \left(\frac{W}{W'}f'\right)\Box\phi + \cdots$$

$$\dot\gamma_{\mu\nu} = -\frac{W}{d-1}\gamma_{\mu\nu} - \frac{1}{d-1}\left(f\ R + \frac{W}{2W'}f'(\partial\phi)^2\right)\gamma_{\mu\nu} +$$

$$+2f\ R_{\mu\nu} + \left(\frac{W}{W'}f' - 2f''\right)\partial_\mu\phi\partial_\nu\phi - 2f'\nabla_\mu\nabla_\nu\phi + \cdots$$

Kiritsis+Li+Nitti

- $f(\phi)$, $W(\phi)$ are solutions of

$$-\frac{d}{4(d-1)}W^2 + \frac{1}{2}W'^2 = V \quad , \quad W'\ f' - \frac{d-2}{2(d-1)}W\ f = 1$$

- Like in 2d σ-models we may use it to define "geometric" RG flows.

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Emerged Holographic gravity and the SM 4 minutes
- The strategy 6 minutes
- 1rts order equations and RG Flows 10 minutes
- Adding the SM Brane 13 minutes
- Recap 14 minutes
- Conclusions and Outlook 15 minutes