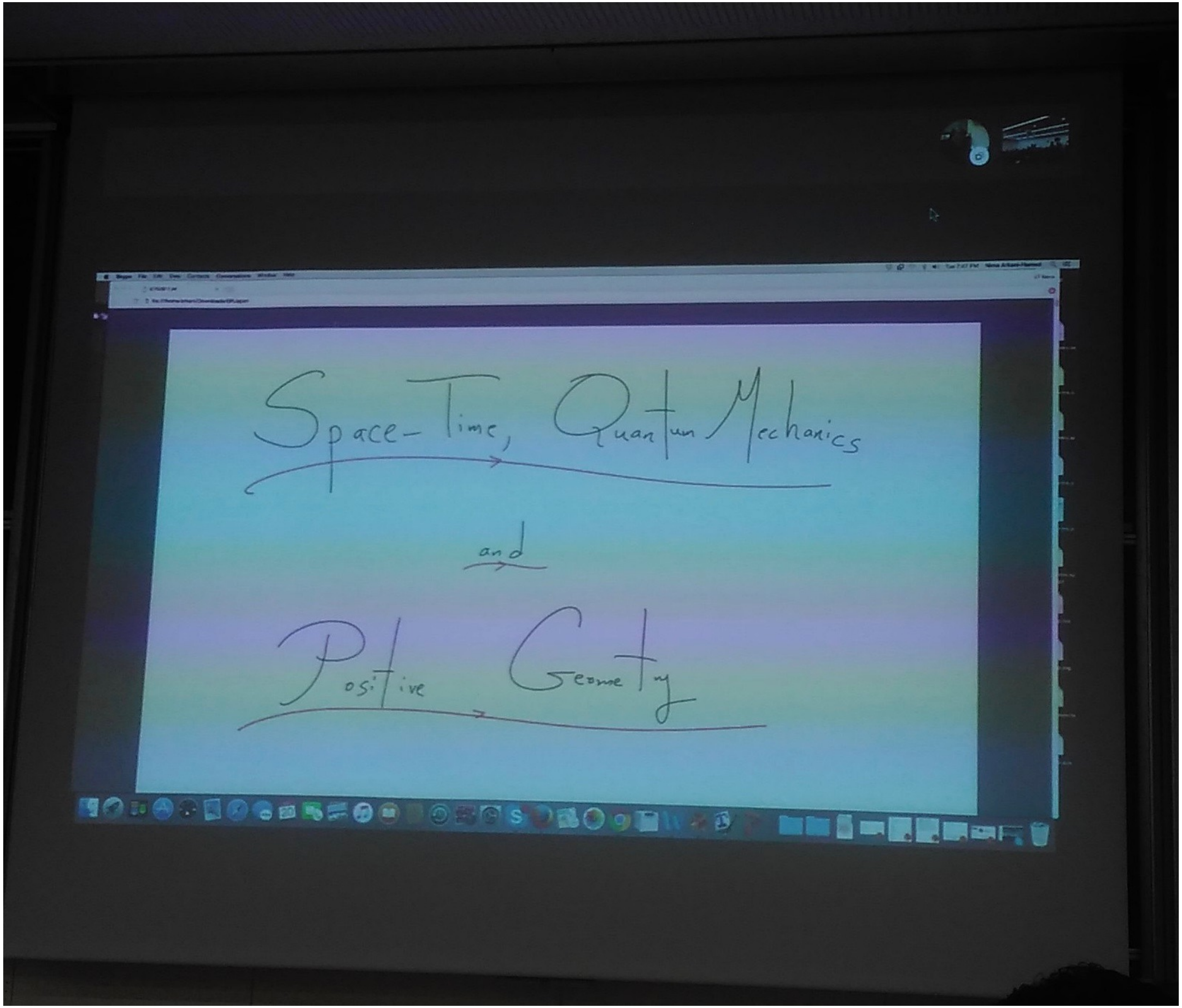
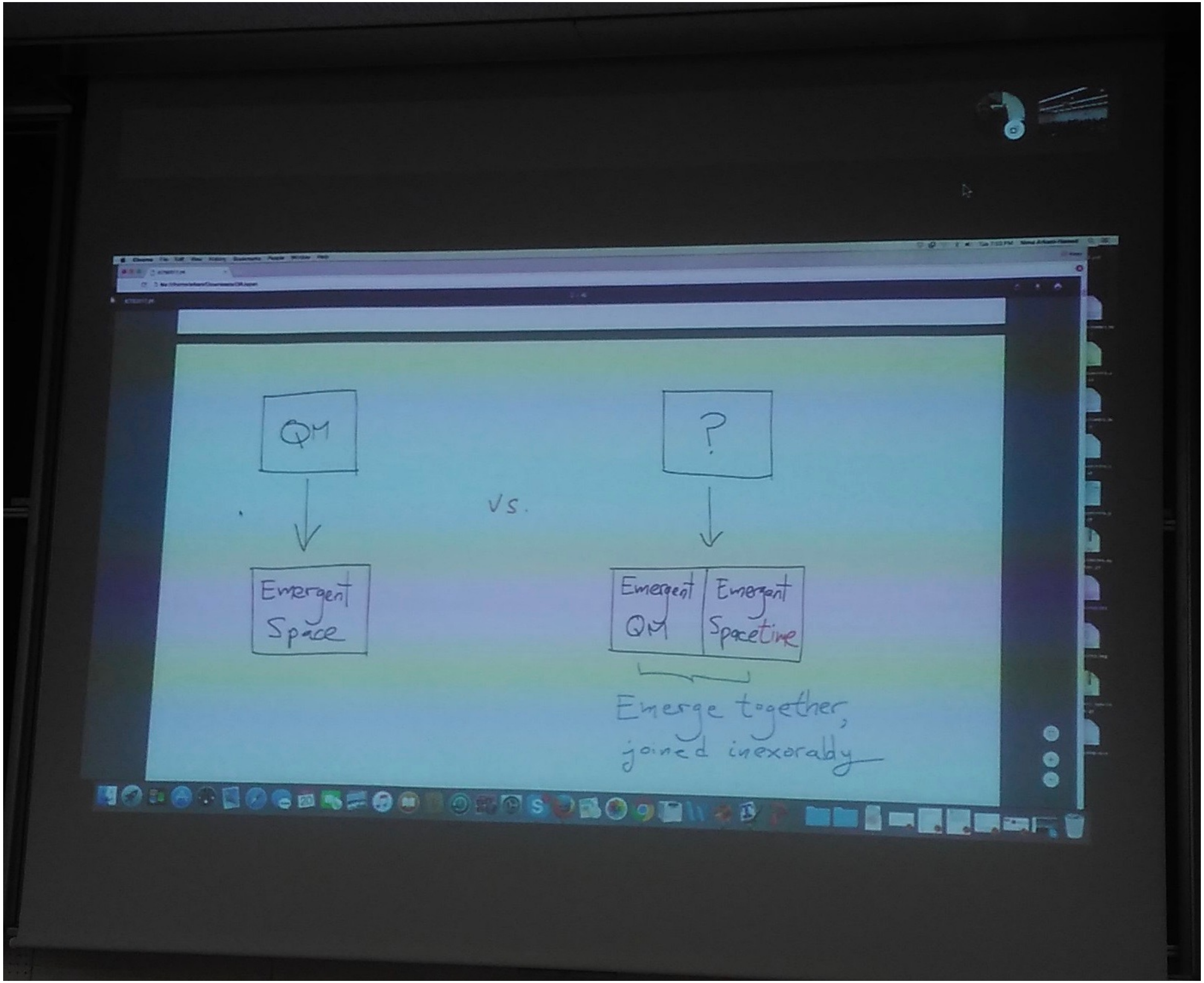
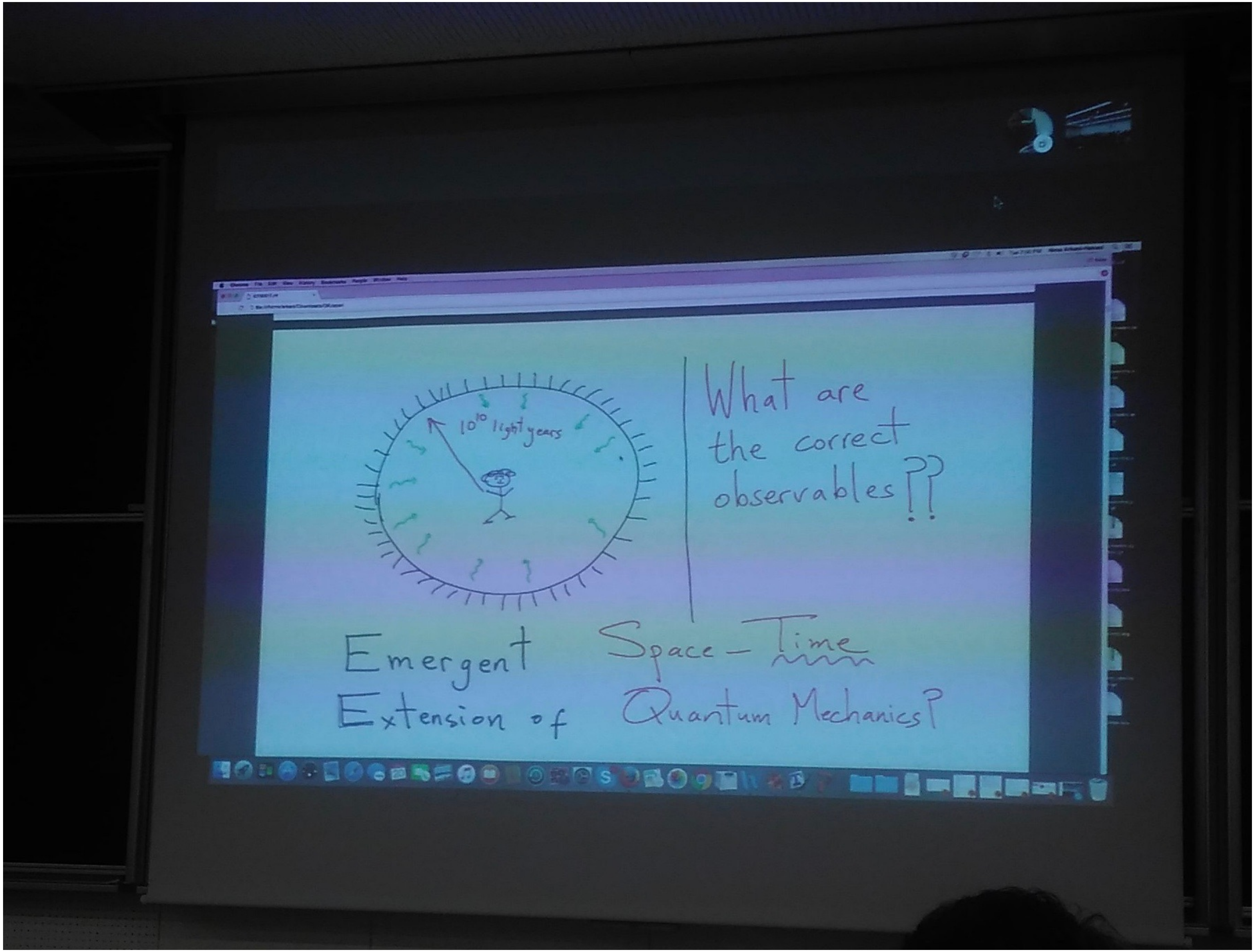
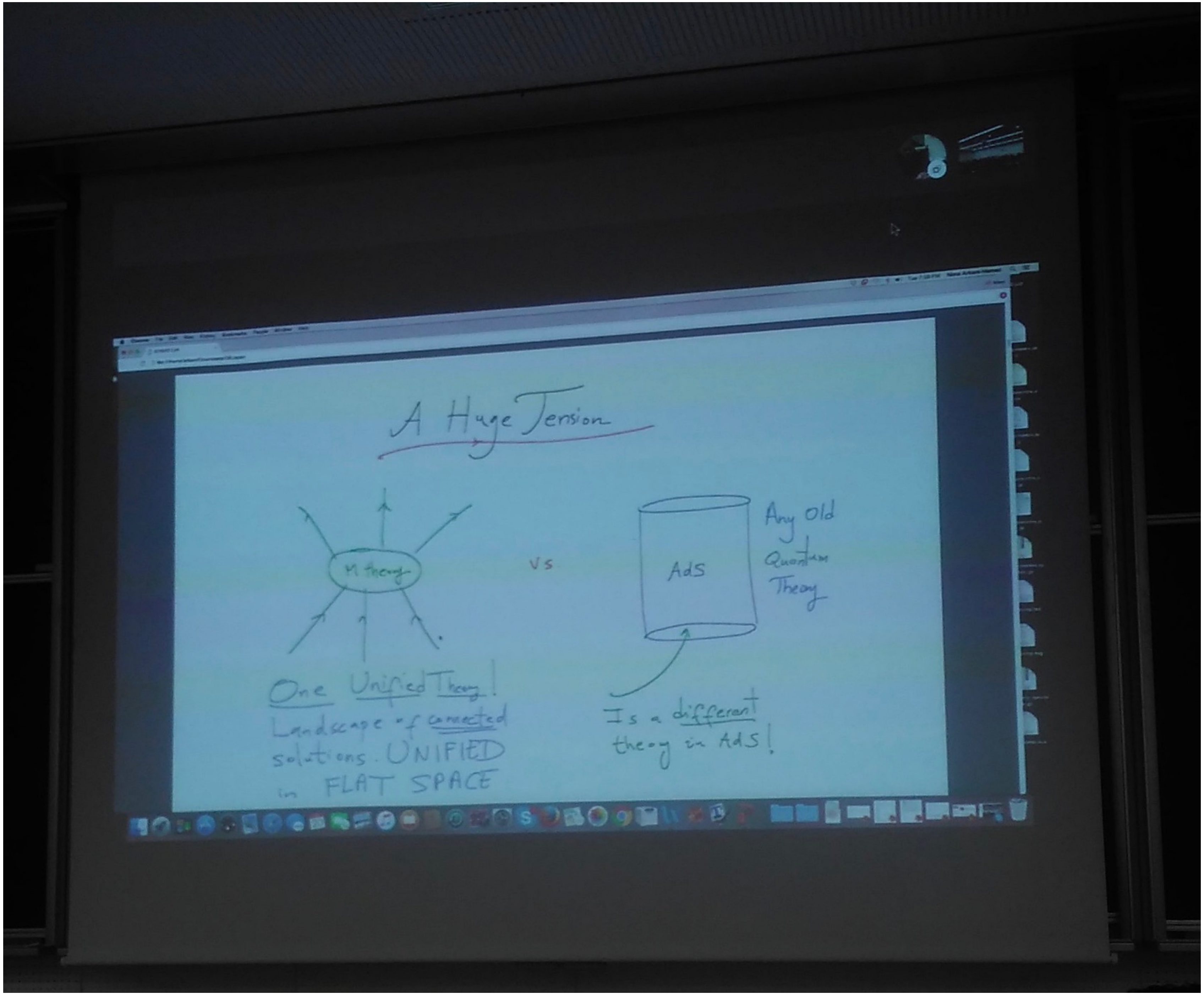
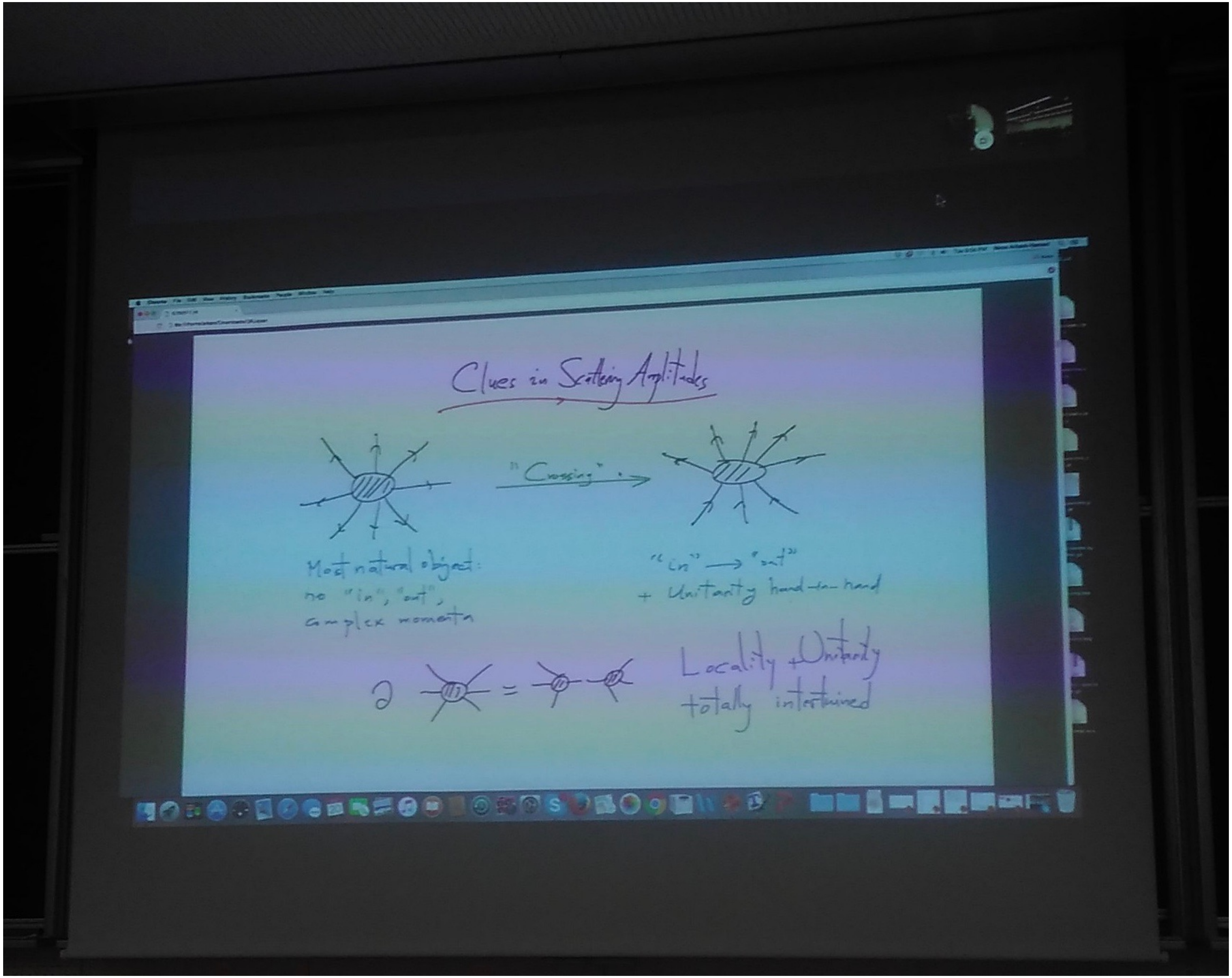
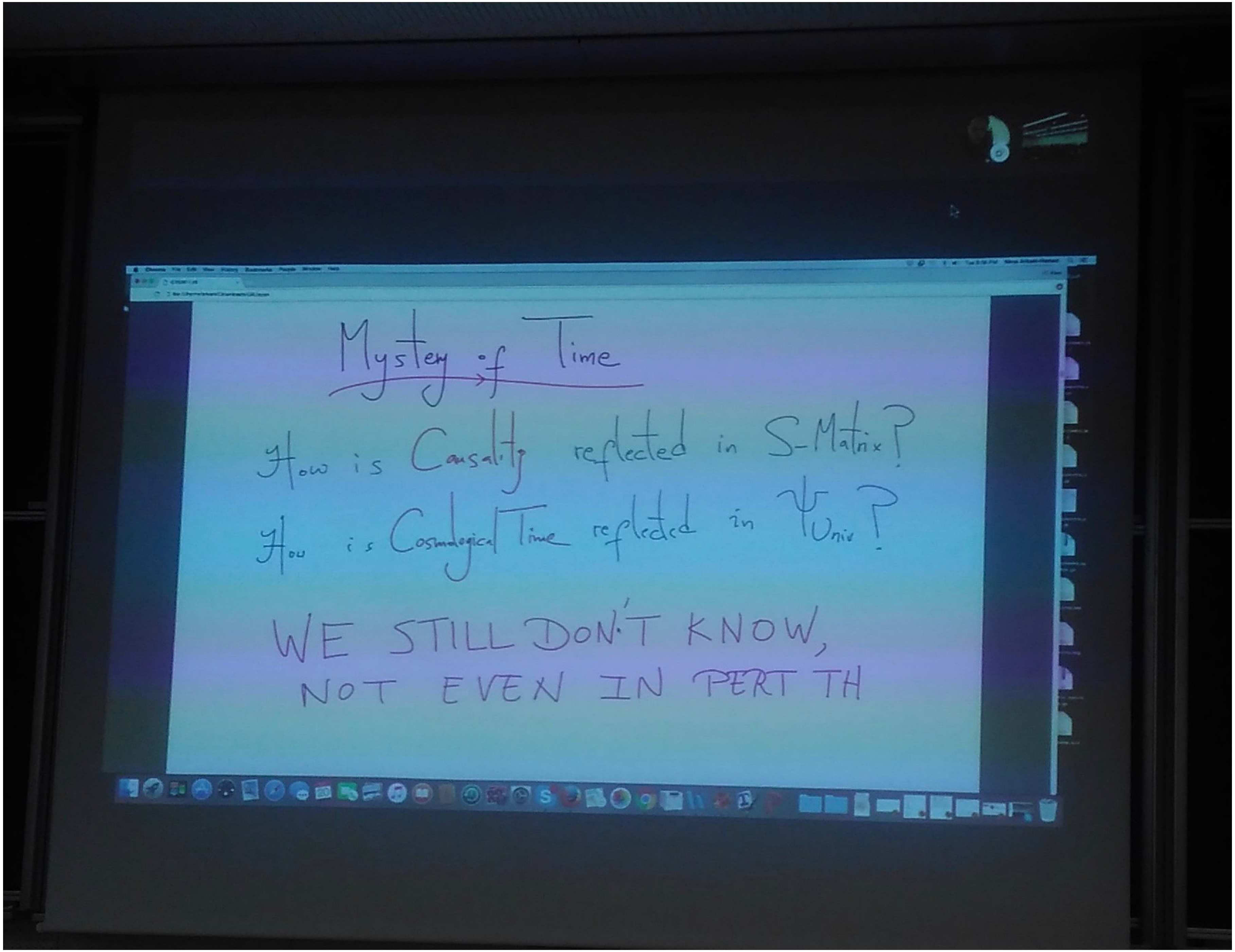

Space-Time, Quantum Mechanics
and
Positive Geometry

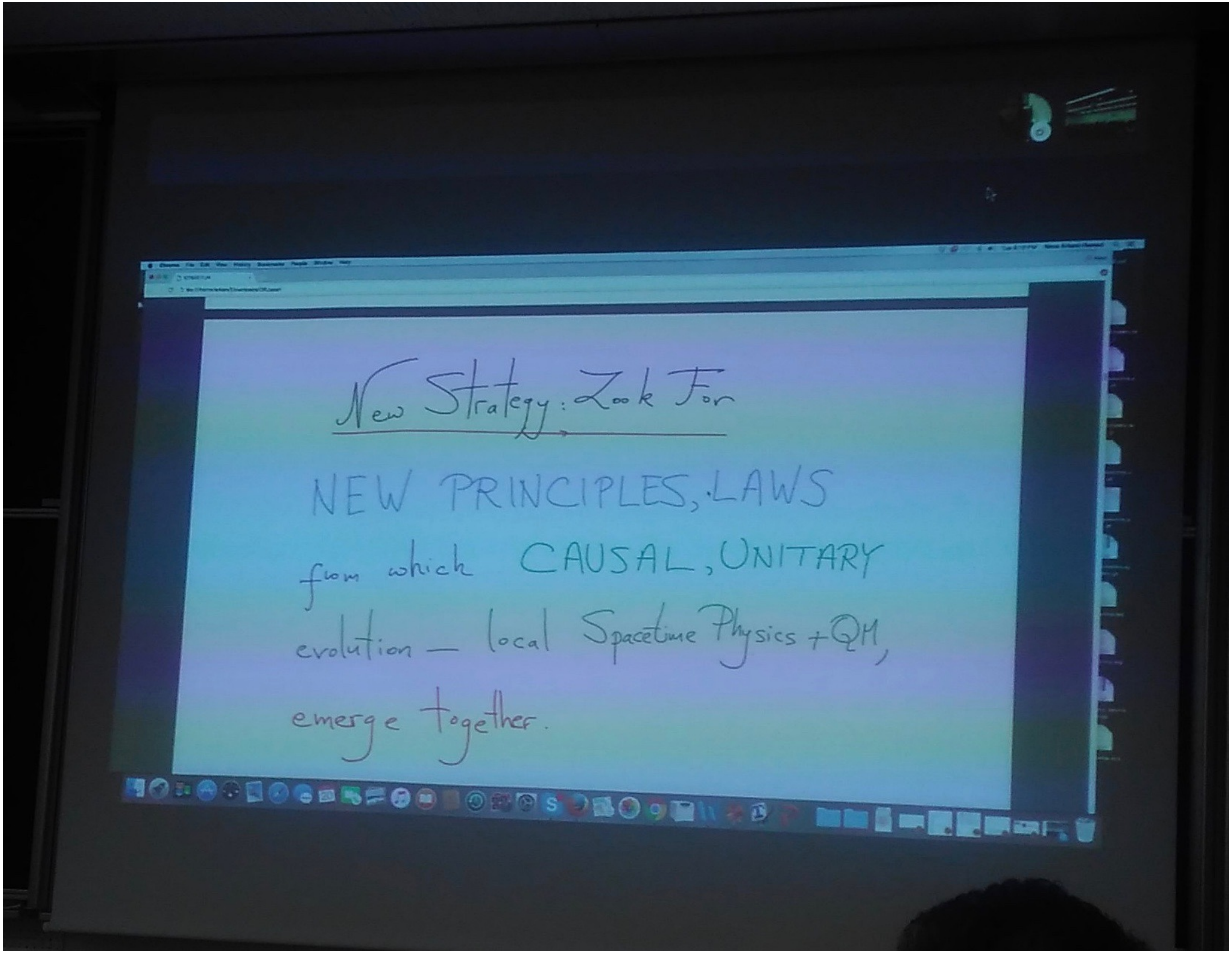

QM
Emergent Space
vs.
?
Emergent QM
Emergent Spacetime
Emerge together, joined inexorably

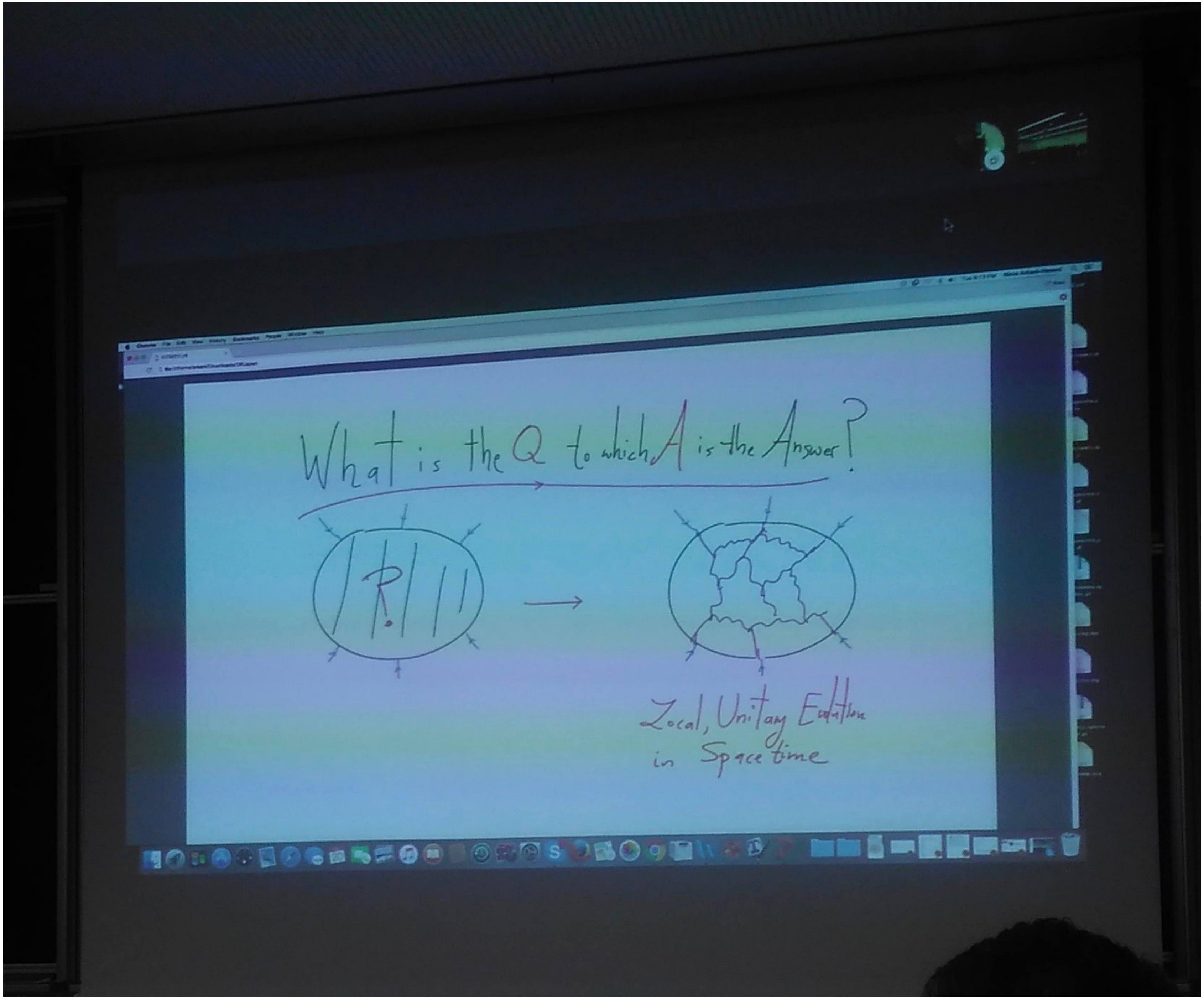

10^{10} light years
What are the correct observables??
Emergent Space-Time
Extension of Quantum Mechanics?

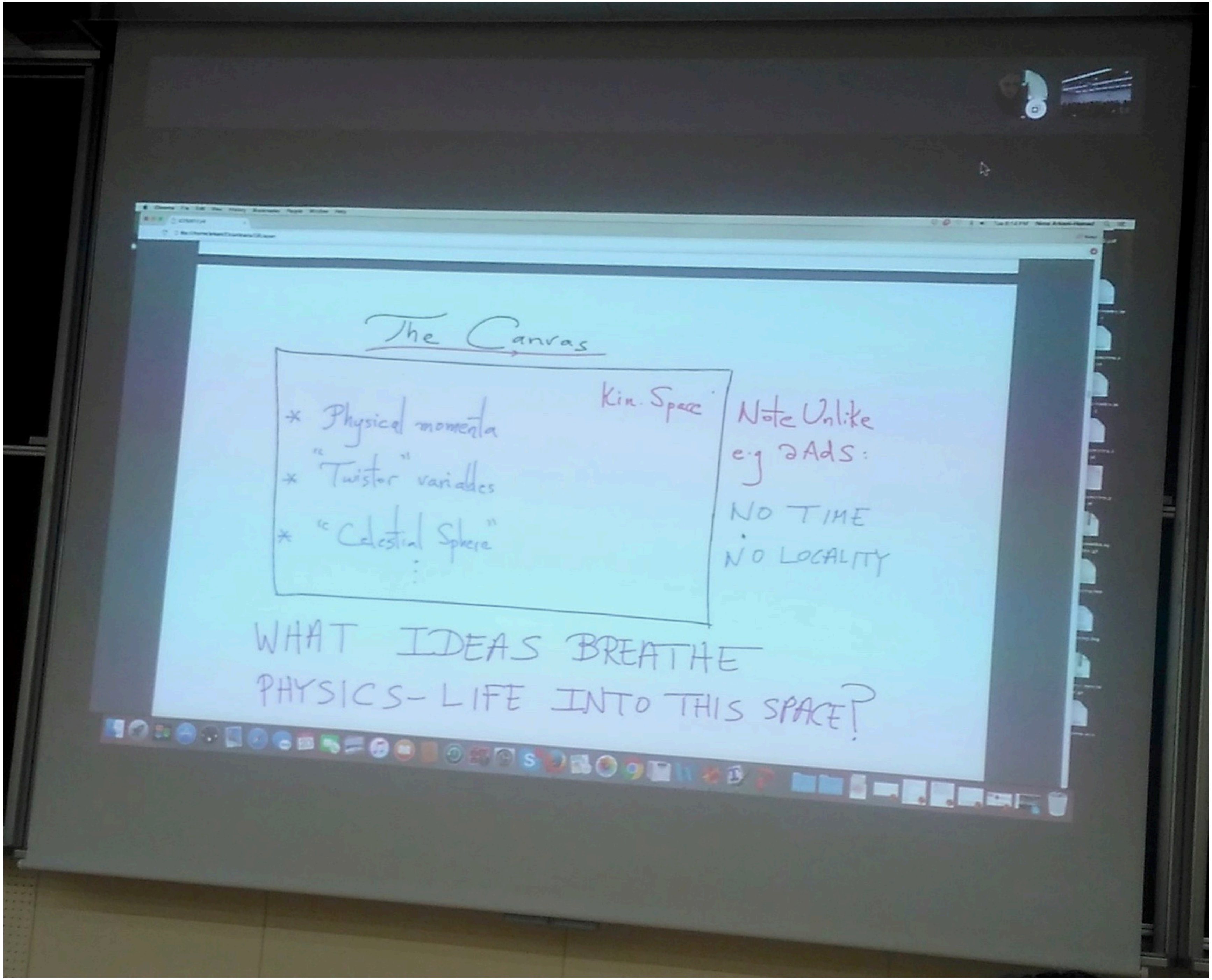

A Huge Tension
M theory
vs
AdS
Any Old Quantum Theory
One Unified Theory!
Landscape of connected solutions. UNIFIED in FLAT SPACE
Is a different theory in AdS!


Clues in Scattering Amplitudes
"Crossing"
Most natural object:
no "in", "out",
complex momenta
"in" → "out"
+ Unitarity hand-in-hand
Locality + Unitarity
totally intertwined

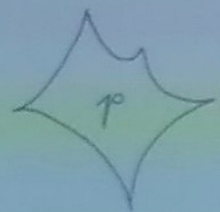

Mystery of Time
How is Causality reflected in S-Matrix?
How is Cosmological Time reflected in Ψ_{Univ}?
WE STILL DON'T KNOW,
NOT EVEN IN PERT TH


New Strategy: Look For
NEW PRINCIPLES, LAWS
from which CAUSAL, UNITARY
evolution — local Spacetime Physics + QM,
emerge together.


What is the Q to which A is the Answer?
Local, Unitary Evolution
in Spacetime


The Canvas
Kin. Space
* Physical momenta
* "Twistor" variables
* "Celestial Sphere"
⋮
Note Unlike
e.g ∂AdS:
NO TIME
NO LOCALITY
WHAT IDEAS BREATHE
PHYSICS-LIFE INTO THIS SPACE?

Positive Geometries



Region $\mathcal{P}$ w/ boundaries of all codimension

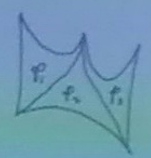
Real Geometry

Canonical Forms

$\longleftrightarrow$ $\Omega_{\mathcal{P}}$: unique form with logarithmic singularities on (+ only on) boundaries of $\mathcal{P}$ $\left\{ \text{locally } \Omega \to \frac{dx_1}{x_1} \cdots \frac{dx_D}{x_D} \right\}$

Complex Form

"Triangulation"

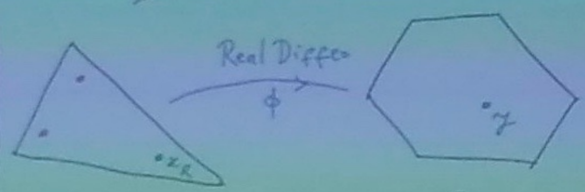


$\mathcal{P}$ tiled by $\mathcal{P}_i$

$\Downarrow$

$$\Omega_{\mathcal{P}} = \sum_i \Omega_{\mathcal{P}_i}$$

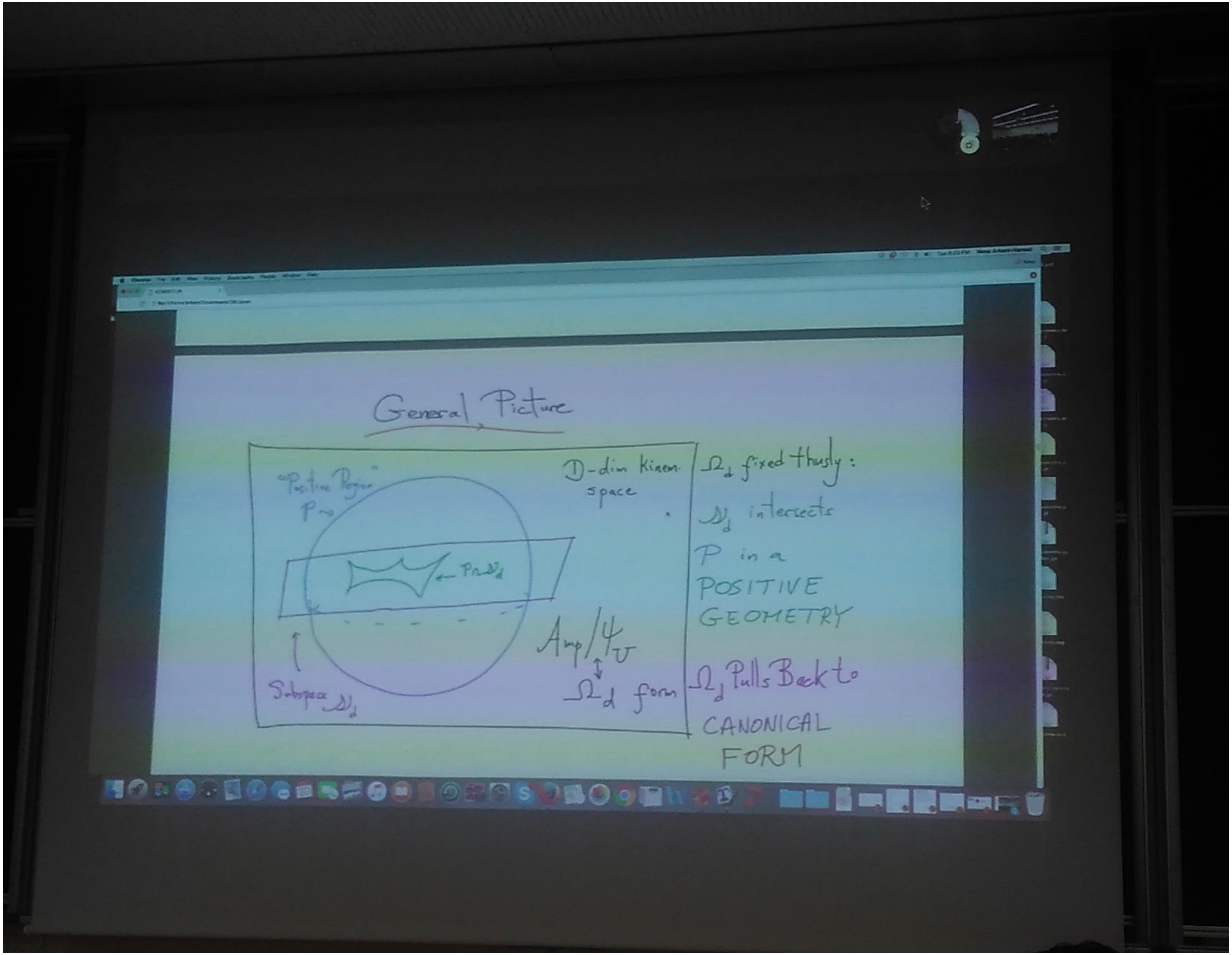
"Push-Forward"

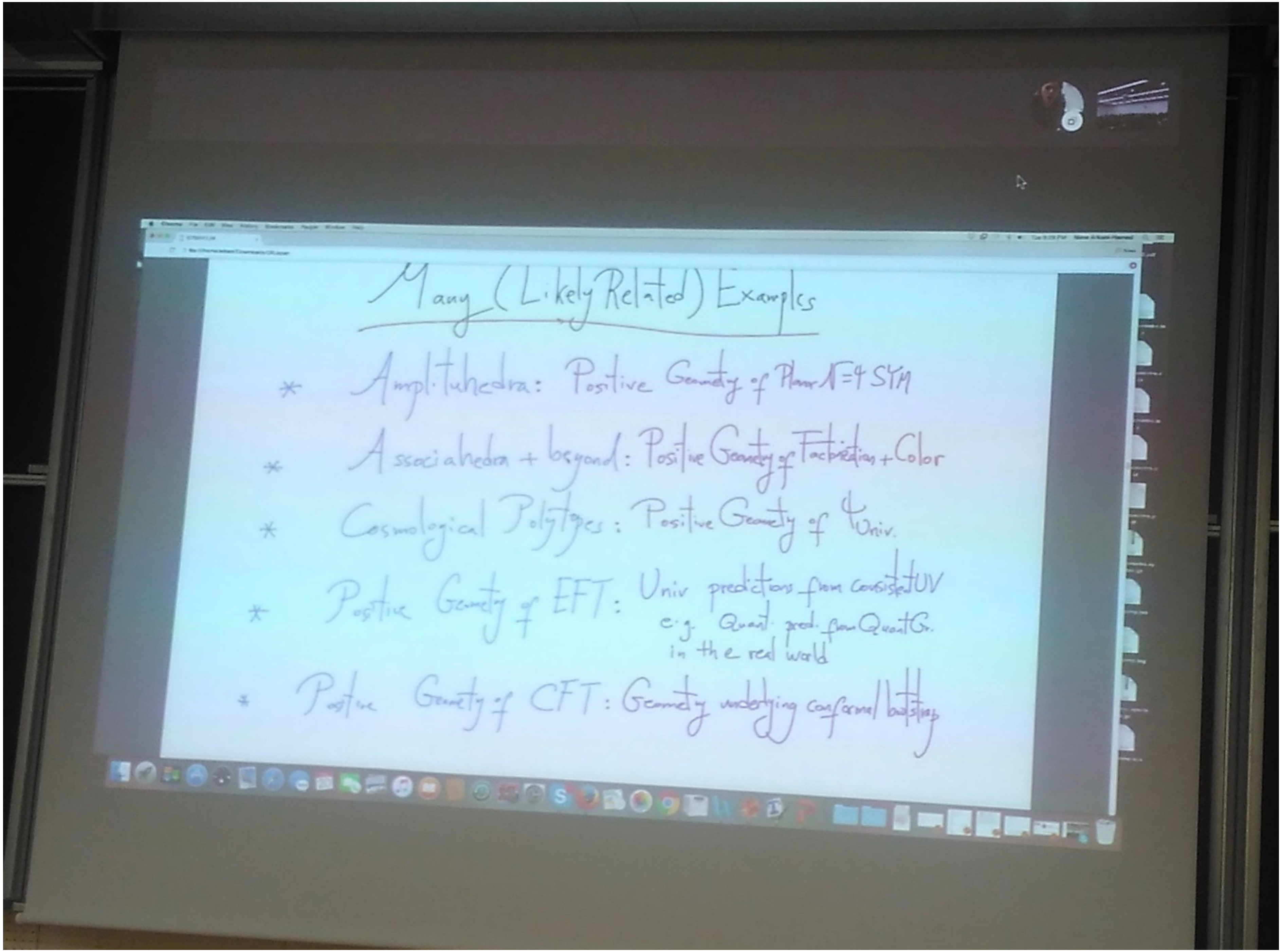


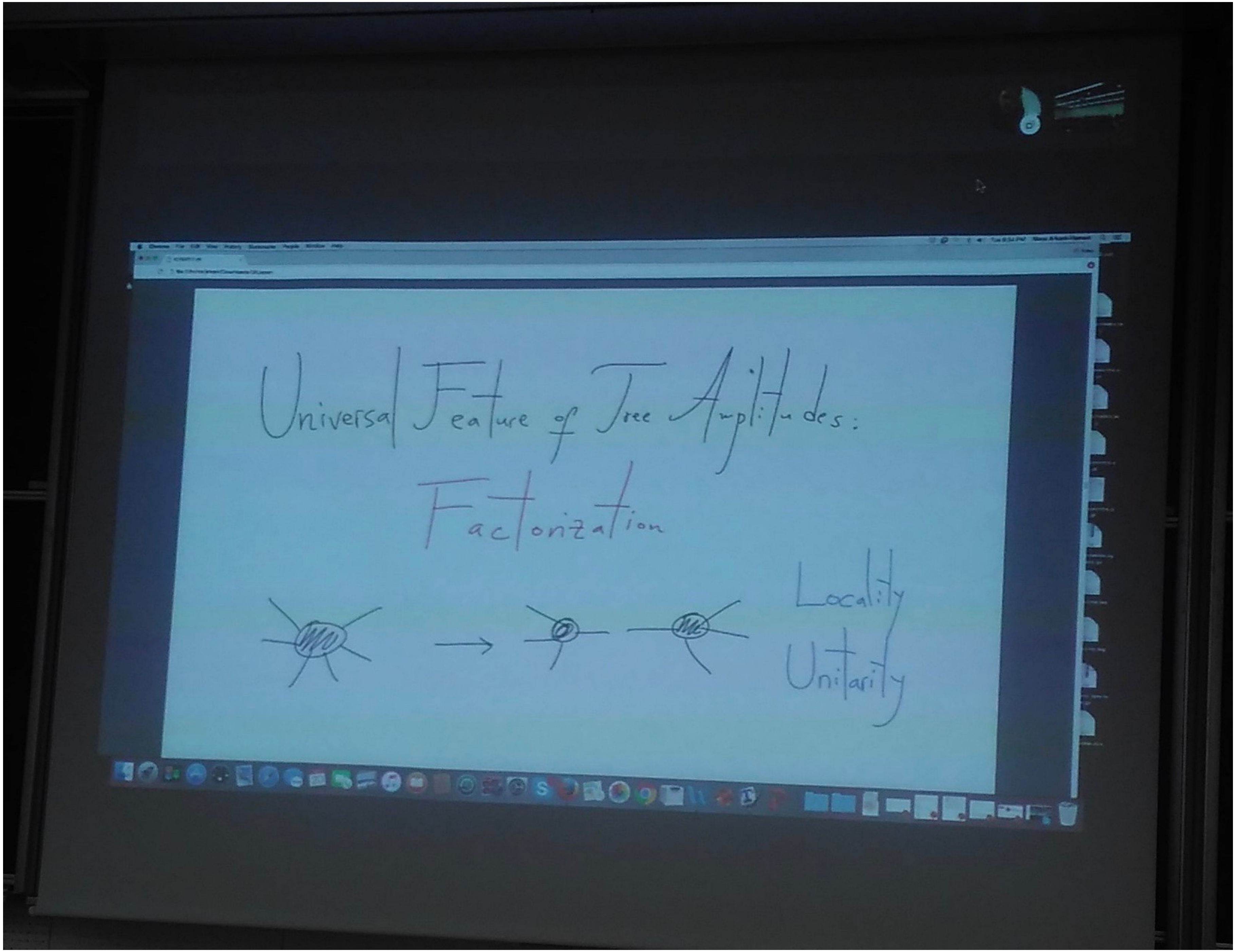
$\mathcal{P}_1$ $\mathcal{P}_2$

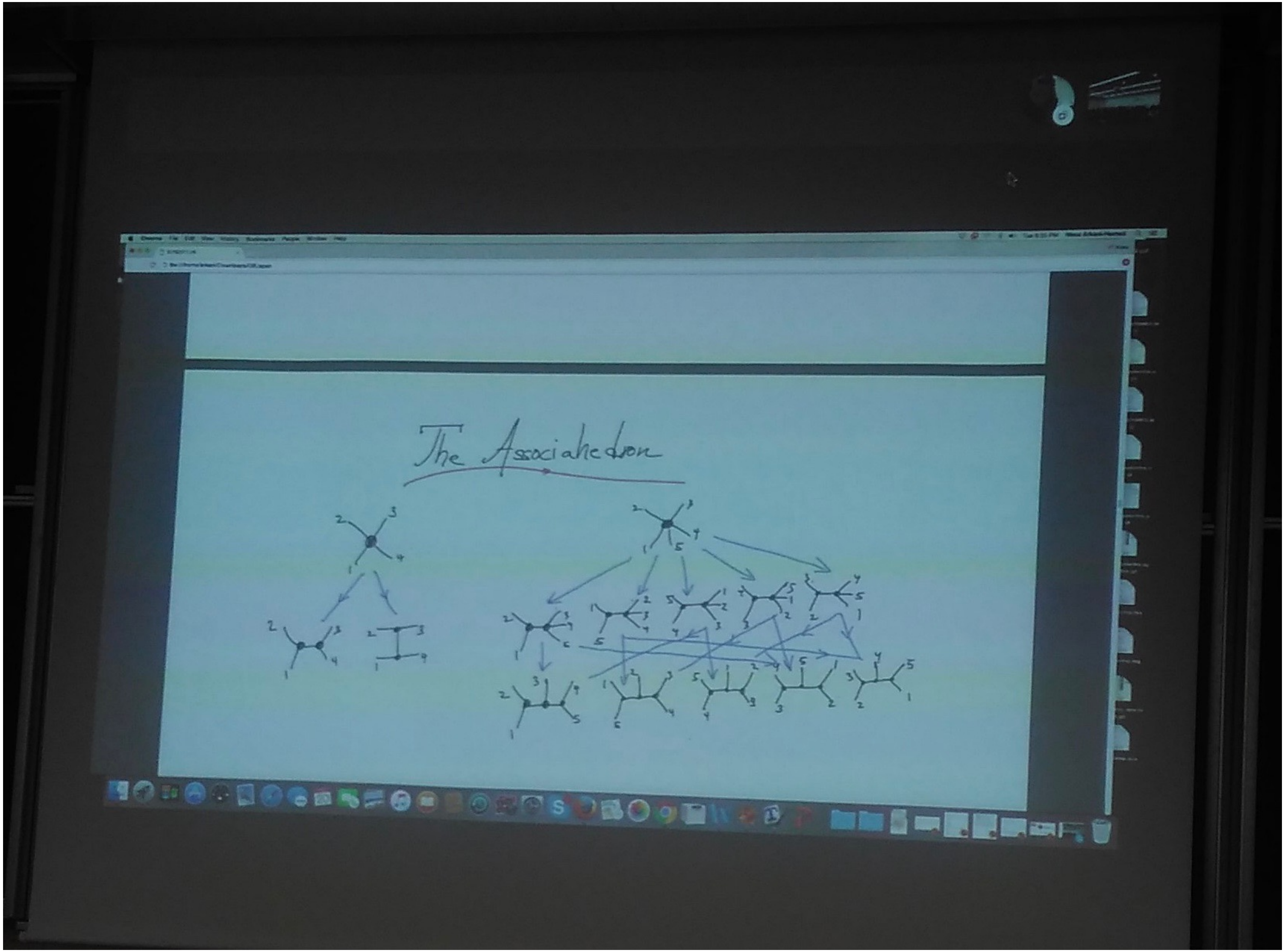
$$\Omega_{\mathcal{P}_2}[y] = \sum_{\phi^{-1}(y)} \Omega_{\mathcal{P}_1}[\phi^{-1}(y)]$$

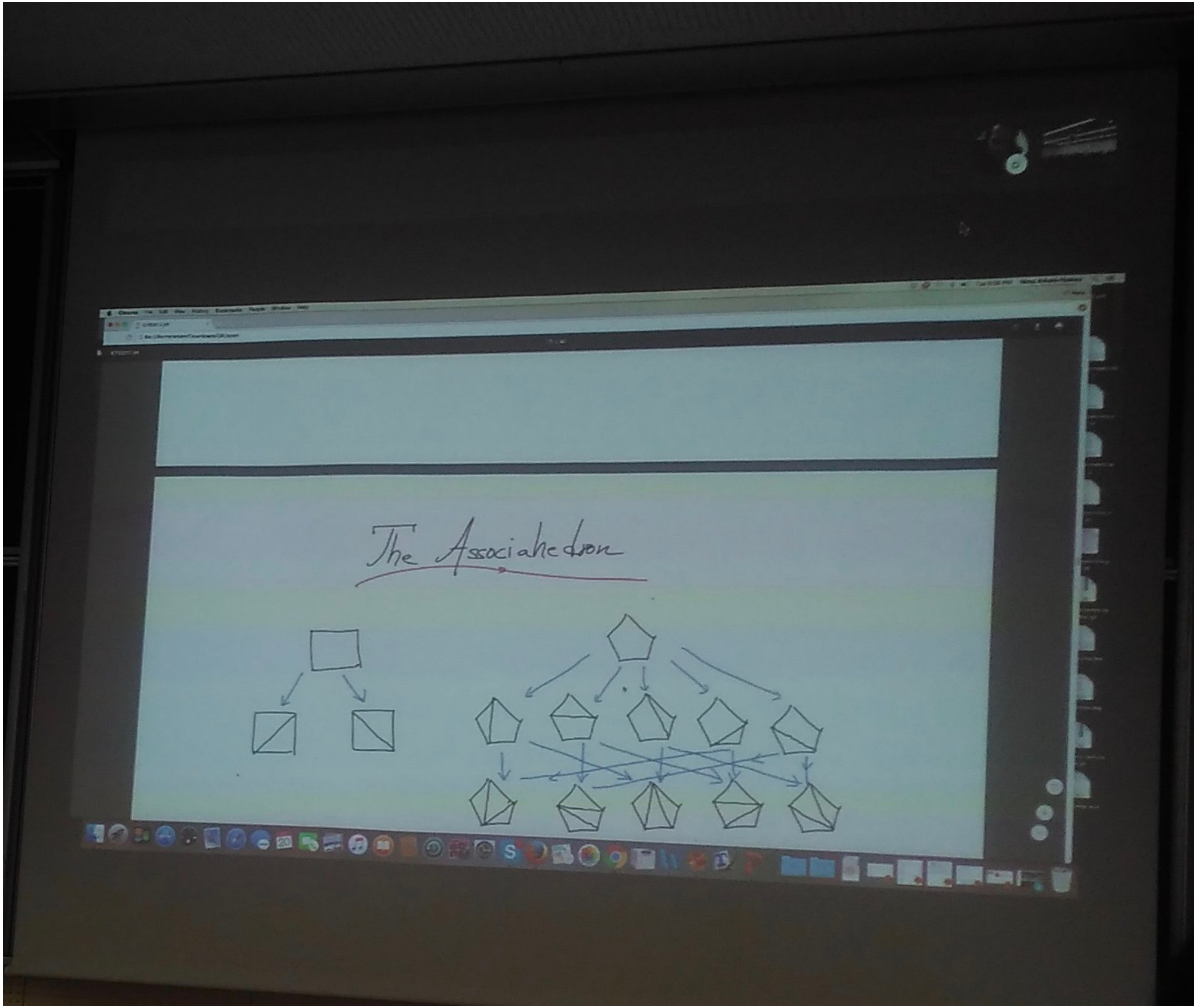
Magic: single real solution $x_R \in \mathcal{P}_1$ iff $y_R \in \mathcal{P}_2$


General Picture
"Positive Region"
P
D-dim Kinem. space
Subspace $\mathcal{S}_d$
Amp/Ψ_U
Ω_d form
Ω_d fixed thusly:
$\mathcal{S}_d$ intersects
P in a
POSITIVE
GEOMETRY
Ω_d Pulls Back to
CANONICAL
FORM

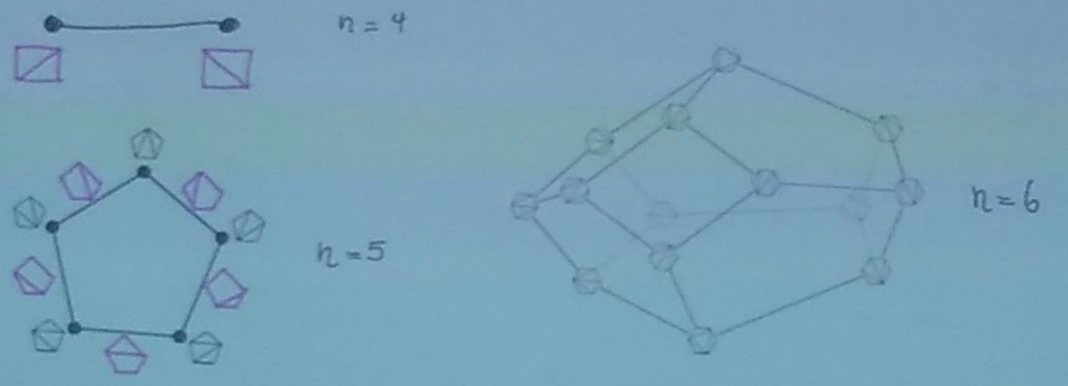

Many (Likely Related) Examples
* Amplituhedra: Positive Geometry of Planar N=4 SYM
* Associahedra + beyond: Positive Geometry of Factorization + Color
* Cosmological Polytopes: Positive Geometry of ψ Univ.
* Positive Geometry of EFT: Univ predictions from consistent UV
e.g. 'Quant. pred. from Quant Gr.
in the real world'
* Positive Geometry of CFT: Geometry underlying conformal bootstrap


Universal Feature of Tree Amplitudes:
Factorization
Locality
Unitarity


The Associahedron


The Associahedron

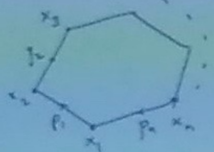
Quite beautifully, this network of inclusion relationships is that of a convex polytope in $(n-3)$ dimensions:



Mandelstam Kinematics

* n particles ; $s_{ab} = p_a \cdot p_b$; $\sum_{b \neq a} s_{ab} = 0$; $\binom{n}{2} - n = \frac{n(n-3)}{2}$ dim space.

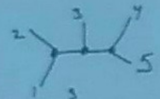
* Order



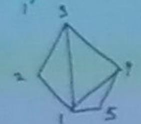
$p_a^\mu = x_{a+1}^\mu - x_a^\mu$

$X_{ij} = (x_i - x_j)^2 = (p_i + \dots + p_{j-1})^2$

*



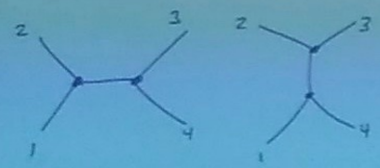
$\frac{1}{s_{12}\, s_{123}}$



$\frac{1}{X_{13} X_{14}}$

diagonals = $\frac{n(n-3)}{2}$

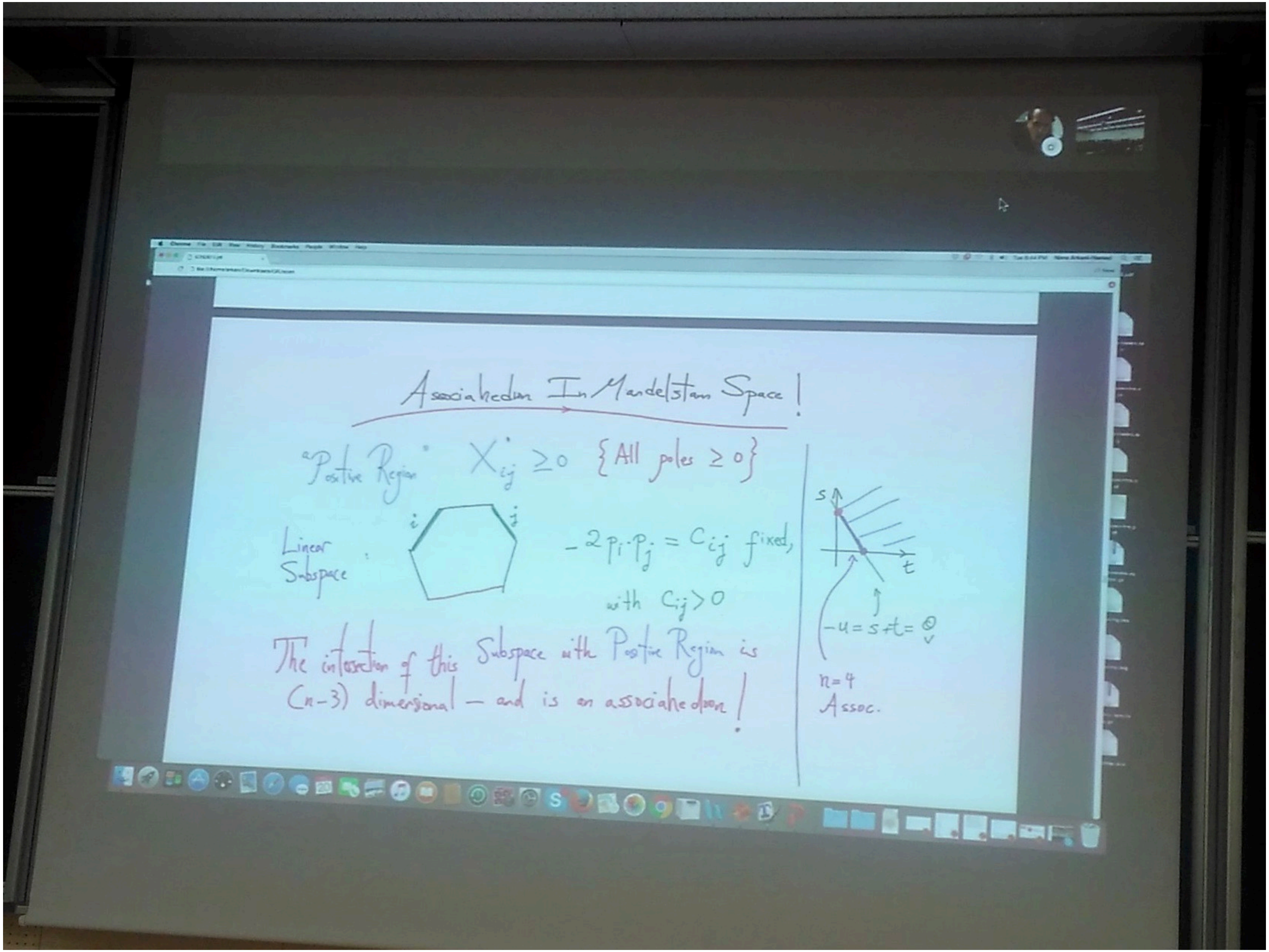
X_{ij} a basis for kinematic space



$$\frac{1}{s} + \frac{1}{t}$$

$$\frac{ds}{s} - \frac{dt}{t} \quad \left(= d\log \frac{s}{t} \right)$$

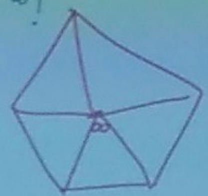
Projectivity


Associahedron In Mandelstam Space!
"Positive Region" $X_{ij} \geq 0$ {All poles ≥ 0}
Linear Subspace:
i
j
$-2p_i \cdot p_j = c_{ij}$ fixed,
with $c_{ij} > 0$
The intersection of this Subspace with Positive Region is (n−3) dimensional — and is an associahedron!
s
t
$-u = s + t = 0$
n=4
Assoc.

* There is a unique $(n-3)$ form on the big space, that pulls back to canonical form of associahedron on subspace, + gives amplitude!
* Projective invariance = Hidden symmetry of ϕ^3_{BA}
* Feynman Diagrams: one triangulation, breaks this symmetry term-by-term

Spurious Poles @ ∞!

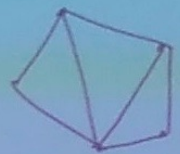
$$\frac{1}{s_{12}s_{123}}+\frac{1}{s_{23}s_{234}}+\frac{1}{s_{34}s_{345}}+\frac{1}{s_{45}s_{451}}+\frac{1}{s_{51}s_{512}}$$



Dual Assoc. Feynman Diag.

$$\frac{s_{12}+s_{234}}{s_{12}s_{34}s_{234}}+\frac{s_{12}+s_{234}}{s_{12}s_{234}s_{23}}+\frac{s_{12}-s_{123}+s_{23}}{s_{12}s_{23}s_{123}}$$

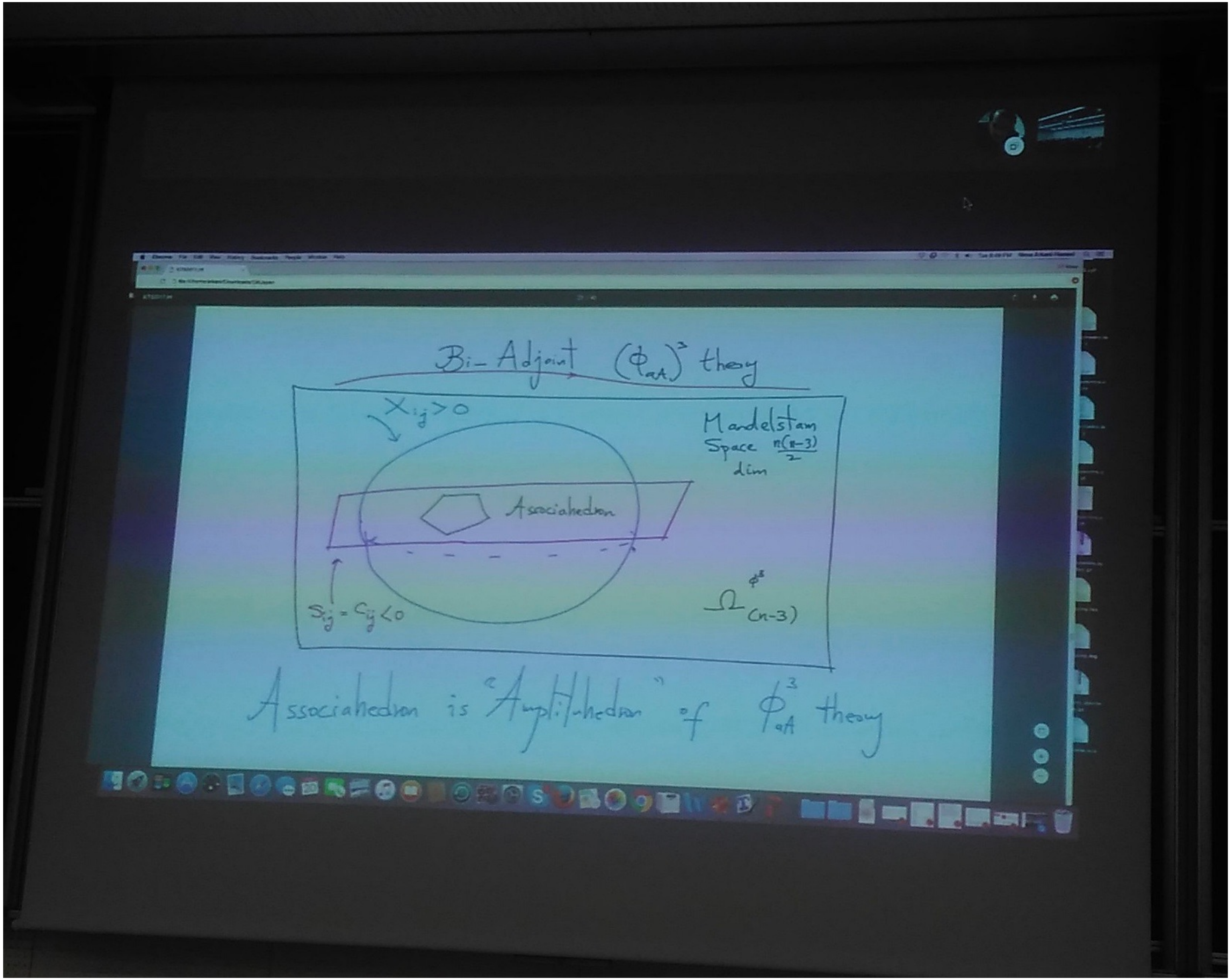
No sp poles @ ∞

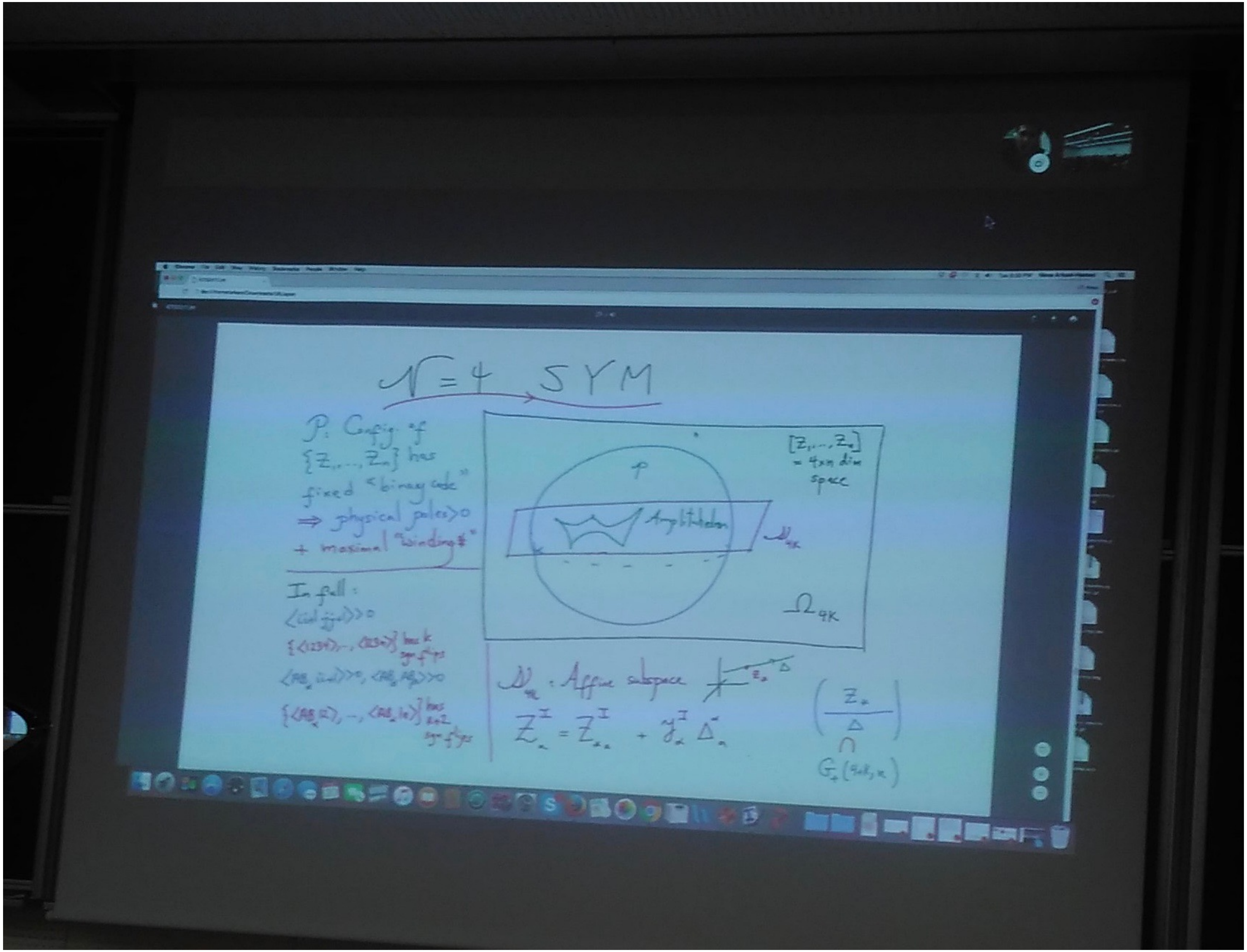


Dual Assoc. proj. manifest

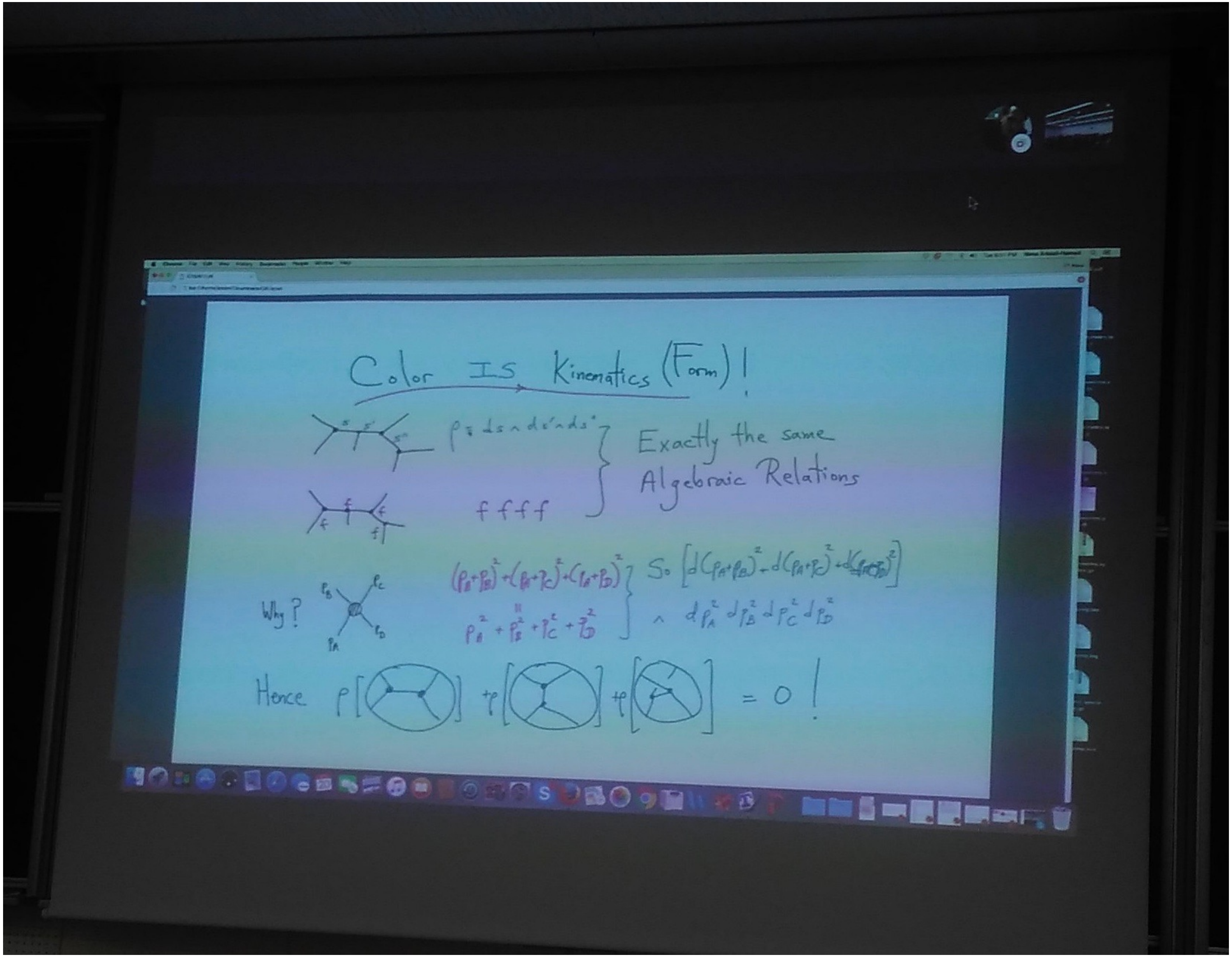
$$\frac{(X_{13}+X_{25})(X_{14}+X_{35})d^2X}{X_{13}X_{25}(X_{14}X_{25}-X_{13}X_{35})}$$
$$\frac{(X_{13}+X_{25})^2(X_{24}-X_{25}+X_{35})d^2X}{X_{25}(-X_{14}X_{25}-X_{13}X_{24}+X_{13}X_{25})(X_{14}X_{25}-X_{13}X_{35})}$$
$$\frac{(X_{13}-X_{14}+X_{24})(-X_{24}+X_{14}+X_{25})d^2X}{X_{14}X_{25}(-X_{14}X_{25}-X_{13}X_{24}+X_{13}X_{25})}$$

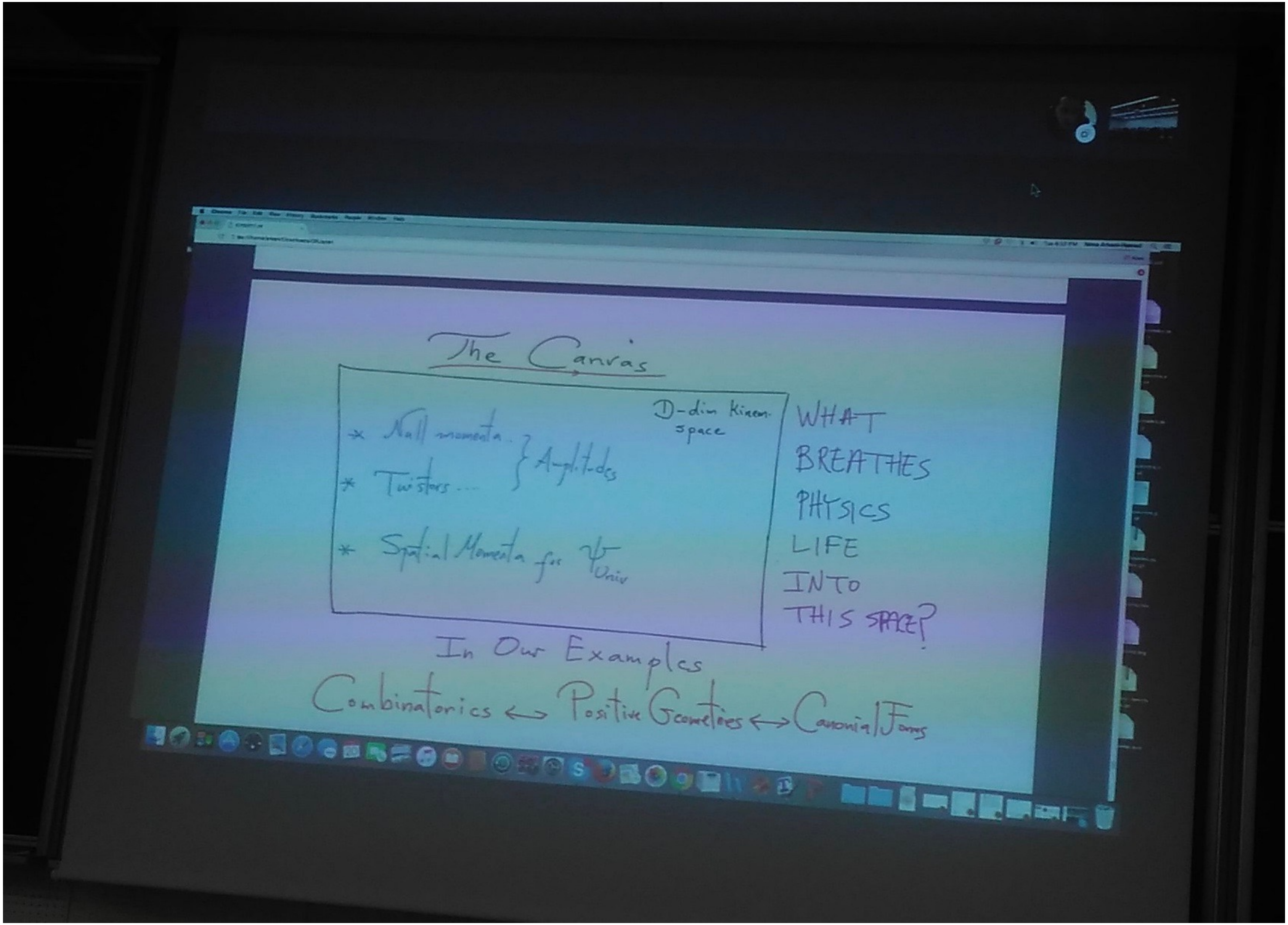
Assoc. proj + "soft" limit manifest


Bi-Adjoint (φaA)³ theory
Xij > 0
Mandelstam Space n(n-3)/2 dim
Associahedron
sij = cij < 0
Ω φ³ (n-3)
Associahedron is "Amplituhedron" of φ³aA theory



N = 4 SYM
P: Config. of {Z1, ..., Zn} has fixed "binary code"
⇒ physical poles > 0
+ maximal "winding #"
In full:
[Z1, ..., Zn] = 4xn dim space
P
Amplituhedon
Ω4K
Affine subspace
G+(4+k, n)


Color IS Kinematics (Form)!
Exactly the same
Algebraic Relations
f f f f
Why?
Hence
= 0 !


The Canvas
D-dim Kinem. space
* Null momenta...
* Twistors...
Amplitudes
* Spatial Momenta for Ψ_{Univ}
WHAT BREATHES PHYSICS LIFE INTO THIS SPACE?
In Our Examples
Combinatorics ↔ Positive Geometries ↔ Canonical Forms