Dark energy & Modified gravity in scalar-tensor theories

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Introduction

• So far, GR seems compatible with all observations.

• Several motivations for exploring modified gravity
  – Quantum gravity effects
  – Explain cosmological acceleration (or possibly dark matter)
  – Explore alternative gravitational theories
  – Testing gravity

• Many models of dark energy & modified gravity:
  quintessence, K-essence, f(R) gravity, massive gravity…

• Generalized framework for scalar-tensor theories, allowing for 2nd order derivatives in their Lagrangian
Traditional scalar-tensor theories

- Simplest extensions of GR: add a scalar field

\[
S = \int d^4 x \sqrt{-g} \left[ F(\phi)(^4 R - Z(\phi) \partial_\mu \phi \partial^\mu \phi - U(\phi) \right] + S_m[\psi_m; g_{\mu\nu}]
\]

- K-essence/ k-inflation: non standard kinetic term

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{M_P^2}{2} (^4 R + P(X, \phi) \right] \\
X \equiv \nabla_\mu \phi \nabla^\mu \phi
\]
Higher order scalar-tensor theories

• Traditional scalar-tensor theories: \( \mathcal{L}(\nabla \chi \phi, \phi) \)

• Generalized theories with second order derivatives

\[
\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla \chi \phi, \phi)
\]

• In general, they contain an extra degree of freedom, expected to lead to Ostrogradsky instabilities

\[
L(\ddot{q}, \dot{q}, q)
\]

• But there are exceptions...
Horndeski theories

- Combination of the Lagrangians (a.k.a. Generalized Galileons)

\[
L^H_2 = G_2(\phi, X) \\
L^H_3 = G_3(\phi, X) \Box \phi \\
L^H_4 = G_4(\phi, X) (^{(4)}R - 2G_{4X}(\phi, X)(\Box \phi^2 - \phi^{\mu\nu} \phi_{\mu\nu}) \\
L^H_5 = G_5(\phi, X) (^{(4)}G_{\mu\nu} \phi^{\mu\nu} + \frac{1}{3} G_{5X}(\phi, X)(\Box \phi^3 - 3 \Box \phi \phi_{\mu\nu} \phi^{\mu\nu} + 2 \phi_{\mu\nu} \phi^{\mu\sigma} \phi^{\nu}_{\sigma})
\]

- Second order equations of motion for the scalar field and the metric

- They contain 1 scalar DOF and 2 tensor DOF. 
  No dangerous extra DOF!
Beyond Horndeski & DHOST theories

- Extensions “beyond Horndeski”  Gleyzes, DL, Piazza & Vernizzi ’14
  \[ L^bH_4 \equiv F_4(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \]
  \[ L^bH_5 \equiv F_5(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'} \]
  leading to **third order** equations of motion.

- Earlier hint: disformal transformation of Einstein-Hilbert Zumalacarregui & Garcia-Bellido ’13

- Even if EOM are higher order, **no extra DOF** if the Lagrangian is “degenerate”.
  
  **DHOST** theories  DL & K. Noui ‘15

  (Degenerate Higher-Order Scalar-Tensor)
Higher order scalar-tensor theories

• Traditional theories: $\mathcal{L}(\nabla_\lambda \phi, \phi)$

• Generalized theories: $\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$
Degenerate Lagrangians

DL & K. Noui ‘1510

- **Scalar-tensor theories**: scalar field + metric

- Simple toy model: \( \phi(x^\lambda) \rightarrow \phi(t) \), \( g_{\mu\nu}(x^\lambda) \rightarrow q(t) \)

- Lagrangian

  \[
  L = \frac{1}{2} a \dddot{\phi}^2 + b \dddot{q} \dot{q} + \frac{1}{2} c \dddot{q}^2 + \frac{1}{2} \dddot{\phi}^2 - V(\phi, q)
  \]

- Equations of motion are higher order
  (4th order if a nonzero, 3rd order if a=0)
Degrees of freedom

• Introduce the auxiliary variable \( Q \equiv \dot{\phi} \)

\[
L = \frac{1}{2} a \dot{Q}^2 + b \dot{Q} \dot{q} + \frac{1}{2} c \dot{q}^2 + \frac{1}{2} Q^2 - V(\phi, q) - \lambda(Q - \dot{\phi})
\]

• Equations of motion

\[
\begin{align*}
    a \ddot{Q} + b \ddot{q} &= Q - \lambda \\
    b \ddot{Q} + c \ddot{q} &= -V_q
\end{align*}
\]

\[
\begin{align*}
    \dot{\phi} &= Q, \quad \dot{\lambda} = -V_\phi
\end{align*}
\]

• If the Hessian matrix is invertible, one finds 3 DOF.

\[
M \equiv \left( \frac{\partial^2 L}{\partial v^a \partial v^b} \right) = \begin{pmatrix} a & b \\ b & c \end{pmatrix}
\]

[6 initial conditions]
Degrees of freedom

• If the Hessian matrix is degenerate, i.e.
\[ ac - b^2 = 0 \]
then only 2 DOF (at most).

\[ \dot{\phi} \text{ can be absorbed in } \dot{x} \equiv \dot{q} + \frac{b}{c} \ddot{\phi} \]

• Hamiltonian analysis: primary constraint and secondary constraint
\[ p_a = \frac{\partial L}{\partial v^a}(v) \text{ cannot be inverted } \]
Generalization (classical mechanics)

Motohashi, Noui, Suyama, Yamaguchi & DL 1603

[See also Klein & Roest 1604]

- Consider a general Lagrangian

\[ L(\ddot{\phi}^\alpha, \dot{\phi}^\alpha, \phi^\alpha; \dot{q}^i, q^i) \quad \alpha = 1, \ldots, n; \ i = 1, \ldots, m \]

In general, \(2n+m\) DOF. But the \(n\) extra DOF can be eliminated by requiring:

1. **Primary conditions** (\(n\) primary constraints)

\[ L \dot{Q}^\alpha \dot{Q}_\alpha - L \dot{Q}^\alpha \dot{q}^i (L^{-1})^{ij} L \dot{q}_j \dot{Q}_\beta = 0 \]

2. **Secondary conditions** (\(n\) secondary constraints)

\[ L \dot{Q}^\alpha \dot{\phi}_\beta - L \dot{Q}^\beta \dot{\phi}_\alpha = 0 \quad \text{if} \quad m = 0 \]

- Third-order time derivatives… Motohashi, Suyama, Yamaguchi 1711
Quadratic DHOST theories

• Consider all theories of the form

\[
S[g, \phi] = \int d^4 x \sqrt{-g} \left[ f_2^{(4)} R + C_{(2)}^{\mu \nu \rho \sigma} \nabla_\mu \nabla_\nu \phi \nabla_\rho \nabla_\sigma \phi \right]
\]

where \( f_2 = f_2(X, \phi) \) and \( C_{(2)}^{\mu \nu \rho \sigma} \) depends only on \( \phi \) and \( \nabla_\mu \phi \).

• All possible contractions of \( \phi_{\mu \nu} \phi_{\rho \sigma} \)?

  e.g. \( g^{\mu \nu} g^{\rho \sigma} \phi_{\mu \nu} \phi_{\rho \sigma} = (\Box \phi)^2 \) or \( \phi^{\mu} \phi^{\nu} \phi^{\rho} \phi^{\sigma} \phi_{\mu \nu} \phi_{\rho \sigma} = (\phi^{\mu} \phi_{\mu \nu} \phi^{\nu})^2 \)

In summary:

\[
C_{(2)}^{\mu \nu \rho \sigma} \phi_{\mu \nu} \phi_{\rho \sigma} = \sum a_A(X, \phi) L_A^{(2)}
\]

\[
L_1^{(2)} = \phi_{\mu \nu} \phi^{\mu \nu} , \quad L_2^{(2)} = (\Box \phi)^2 , \quad L_3^{(2)} = (\Box \phi) \phi^{\mu} \phi_{\mu \nu} \phi^{\nu}
\]

\[
L_4^{(2)} = \phi^{\mu} \phi_{\mu \rho} \phi^{\rho \nu} \phi^{\nu} , \quad L_5^{(2)} = (\phi^{\mu} \phi_{\mu \nu} \phi^{\nu})^2
\]
Quadratic DHOST theories

- Lagrangians of the form
  \[ L = f_2(X, \phi) (4)R + \sum_{A=1}^{5} a_A(X, \phi) L_A^{(2)} \]
  which depend on 6 arbitrary functions.

- **Degeneracy** yields **three conditions** on the 6 functions.

- Classification: 7 subclasses (4 with \( f_2 \neq 0 \), 3 with \( f_2 = 0 \))

  [See also Crisostomi et al ‘1602; Ben Achour, DL & Noui ’1602; de Rham & Matas ‘1604]

- This includes, in particular, \( L^H_4 \) and \( L^{bH}_4 \)
  \[ f_2 = G_4, \quad a_1 = -a_2 = 2G_{4X} + XF_4, \quad a_3 = -a_4 = 2F_4 \]
Cubic DHOST theories

[Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato ’1608]

• Action of the form

\[ S[g, \phi] = \int d^4 x \sqrt{-g} \left[ f_3 G^{\mu\nu} \phi_{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \phi_{\mu\nu} \phi_{\rho\sigma} \phi_{\alpha\beta} \right] \]

depends on eleven functions:

\[ C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \phi_{\mu\nu} \phi_{\rho\sigma} \phi_{\alpha\beta} = \sum_{i=1}^{10} b_i(X, \phi) L_i^{(3)} \]

• This includes the Lagrangians \( L_5^H \) and \( L_5^{bH} \).

• 9 degenerate subclasses: 2 with \( f_3 \neq 0 \), 7 with \( f_3 = 0 \)

• 25 combinations of quadratic and cubic theories (out of 7x9) are degenerate.
Disformal transformations

• Transformations of the metric
  \[ g_{\mu \nu} \rightarrow \tilde{g}_{\mu \nu} = C(X, \phi) g_{\mu \nu} + D(X, \phi) \partial_\mu \phi \partial_\nu \phi \]

• Starting from an action \( \tilde{S} [\phi, \tilde{g}_{\mu \nu}] \), one can define the new action
  \[ S[\phi, g_{\mu \nu}] \equiv \tilde{S} [\phi, \tilde{g}_{\mu \nu} = C g_{\mu \nu} + D \phi_\mu \phi_\nu] \]

• Disformal transformation of quadratic DHOST theories
  \[ \tilde{S} = \int d^4 x \sqrt{-\tilde{g}} \left[ \tilde{f}_2^{(4)} \tilde{R} + \sum_I \tilde{a}_I \tilde{L}_I^{(2)} \right] \]

The structure of DHOST theories is preserved and all seven subclasses are stable.

[Bekenstein ’93]

[Ben Achour, DL & Noui ’1602]
Disformal transformations

- Stability under
  \[ g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \partial_{\mu} \phi \partial_{\nu} \phi \]

- When **matter** is included (with minimal coupling), two disformally related theories are **physically inequivalent**!
Cosmology:
Effective description of Dark Energy & Modified Gravity
Parametrized Effective Description

Observational constraints

Theories
Effective description of Dark Energy

[ See e.g review: Gleyzes, DL & Vernizzi 1411.3712 ]

• Restriction: **single scalar field** models

• The scalar field defines a **preferred slicing**
  Constant time hypersurfaces = uniform field hypersurfaces

\[
\phi = \phi_1 \\
\phi = \phi_2 \\
\phi = \phi_3
\]

• All perturbations embodied by the metric only
Uniform scalar field slicing

• 3+1 decomposition based on this preferred slicing

• Basic ingredients
  – Unit vector normal to the hypersurfaces

\[ n^\mu = -\frac{\nabla^\mu \phi}{\sqrt{-\left(\nabla \phi\right)^2}} \]

  – Projection on the hypersurfaces:

\[ h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \]
ADM formulation

- ADM decomposition of spacetime

\[ ds^2 = -N^2 dt^2 + h_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right) \]

Extrinsic curvature:

\[ K_{ij} = \frac{1}{2N} \left( \dot{h}_{ij} - D_i N_j - D_j N_i \right) \]

Intrinsic curvature:

\[ R_{ij} \]

- Generic Lagrangians of the form

\[ S_g = \int d^4x \ N \sqrt{h} \ L(N, K_{ij}, R_{ij}; t) \]
Homogeneous background & linear perturbations

- **Background**
  \[ ds^2 = -\tilde{N}^2(t) \, dt^2 + a^2(t) \, \delta_{ij} \, dx^i \, dx^j \]
  \[ \bar{L}(a, \dot{a}, \tilde{N}) \equiv L \left[ K^i_j = \frac{\dot{a}}{Na} \, \delta^i_j, R^i_j = 0, N = \tilde{N}(t) \right] \]

- **Perturbations:**
  \[ \delta N \equiv N - \tilde{N}, \quad \delta K^i_j \equiv K^i_j - H \delta^i_j, \quad \delta R^i_j \equiv R^i_j \]

- **Expanding the Lagrangian** \( L(q_A) \) with \( q_A \equiv \{N, K^i_j, R^i_j\} \)
  yields \( L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A} \delta q^A + \frac{1}{2} \frac{\partial^2 L}{\partial q_A \partial q_B} \delta q_A \delta q_B + \ldots \)

- The **quadratic** action describes the **dynamics of linear perturbations**
Horndeski & beyond Horndeski

- Quadratic action

\[ S^{(2)} = \int dx^3 dt a^3 \frac{M^2}{2} \left[ \delta K^i_j \delta K^j_i - \delta K^2 + \alpha_K H^2 \delta N^2 + 4 \alpha_B H \delta K \delta N \right. \]

\[ + \left. (1 + \alpha_T) \delta_2 \left( \frac{\sqrt{h}}{a^3} R \right) + (1 + \alpha_H) R \delta N \right] \]

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<th>( \alpha_K )</th>
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Gleyzes, DL, Piazza & Vernizzi '13, [notation: Bellini & Sawicki '14]
Scalar degree of freedom

- Scalar perturbations: \( \delta N \), \( N_i \equiv \partial_i \psi \), \( h_{ij} = a^2(t) e^{2\zeta} \delta_{ij} \)

- Quadratic action for the **physical degree of freedom**: 

\[
S^{(2)} = \frac{1}{2} \int dx^3 dt \ a^3 \left[ K_t \dot{\zeta}^2 + K_s \frac{(\partial_i \zeta)^2}{a^2} \right]
\]

\[
K_t \equiv \frac{\alpha_K + 6\alpha_B^2}{(1 + \alpha_B)^2}, \quad K_s \equiv 2M^2 \left\{ 1 + \alpha_T - \frac{1 + \alpha_H}{1 + \alpha_B} \left( 1 + \alpha_M - \frac{\dot{H}}{H^2} \right) - \frac{1}{H} \frac{d}{dt} \left( \frac{1 + \alpha_H}{1 + \alpha_B} \right) \right\}
\]

- Stability (neither ghost nor gradient instability)

\[
K_t > 0 \quad c_s^2 \equiv -\frac{K_s}{K_t} > 0
\]
Tensor degrees of freedom

- Quadratic action for the tensor modes:

\[ S^{(2)}_\gamma = \frac{1}{2} \int dt \, d^3x \, a^3 \left[ \frac{M^2}{4} \dot{\gamma}_{ij}^2 - \frac{M^2}{4} (1 + \alpha_T) \frac{(\partial_k \gamma_{ij})^2}{a^2} \right] \]

- Stability:

\[ M^2 > 0 \quad \text{and} \quad c_T^2 \equiv 1 + \alpha_T > 0 \]
Extension to DHOST theories

DL, Mancarella, Noui & Vernizzi ’1703

• Quadratic action in terms of 9 functions of time

\[ S_{\text{quad}} = \int d^3x \, dt \, a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left( 1 + \frac{2}{3} \alpha_L \right) \delta K^2 + (1 + \alpha_T) \left( R \frac{\delta \sqrt{h}}{a^3} + \delta_2 R \right) \\
+ H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H) R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\} \]

9-3=6 independent coefficients

• Degeneracy conditions: 2 categories

\[ C_I: \alpha_L = 0, \beta_2 = -6\beta_1^2, \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1 (1 + \alpha_T)] \]

\[ C_{II}: \beta_1 = -(1 + \alpha_L) \frac{1 + \alpha_H}{1 + \alpha_T}, \beta_2 = -6(1 + \alpha_L) \left( \frac{1 + \alpha_H}{1 + \alpha_T} \right)^2, \beta_3 = 2 \left( \frac{1 + \alpha_H}{1 + \alpha_T} \right)^2 \]

\[ C_{II} \]: gradient instability either in the scalar or the tensor sector
Scalar-tensor theories

Type I

DHOST I

Beyond Horndeski

Horndeski

Type II

(D Gradient instability !)
Disformal transformations

- Disformal transformations: \( \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \partial_\mu \phi \partial_\nu \phi \)

![Diagram showing disformal transformations and types I and II](image_url)
Disformal transformations

- Disformal transformations: \( \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \partial_\mu \phi \partial_\nu \phi \)

- Mimetic gravity & extensions (non-invertible transf) are DHOST theories of type II (some of type I too) and all **unstable**.

DL, Mancarella, Noui & Vernizzi ’1802 [see also Takahashi & Kobayashi ’1708]
DHOST theories after GW170817
DHOST theories after GW170817

• Constraint on the speed of gravitational waves:

\[ \alpha_T < 10^{-15} \]

• Assuming \( \alpha_T = 0 \) holds exactly, this implies

1. Quadratic terms: \( a_1 = 0 \)

\[ L_{\text{ADM}} = (f - X a_1) K_{ij} K^{ij} - f^{(3)}R \]

2. No cubic term (for type I theories)

• Remain quadratic DHOST theories of type I with \( a_1 = 0 \)
DHOST theories with $c_g = c$

- Taking into account the degeneracy conditions,

$$a_1 = a_2 = 0,$$

$$a_4 = \frac{1}{8f_2} \left[ 48f_2^2X - 8(f_2 - Xf_{2X})a_3 - X^2a_3^2 \right],$$

$$a_5 = \frac{1}{2f_2} (4f_{2X} + Xa_3) a_3$$

2 free functions

- Total Lagrangian

$$L_{\text{DHOST}}^{c_g = 1} = f_2(X, \phi) (^{(4)}R + P(X, \phi) + Q(X, \phi) \Box \phi$$

$$+ a_3(X, \phi) \phi^\mu \phi^\nu \phi_{\mu\nu} \Box \phi + a_4(X, \phi) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu}$$

$$+ a_5(X, \phi) (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2$$

4 free functions of $X$ and $\phi$ (as in Horndeski without $c_g = 1$ !)
Horndeski and Beyond Horndeski with $c_g = c$

- Remaining Beyond Horndeski theories

\[ a_1 = 2G_4X + XF_4 = 0 \quad \implies \quad F_4 = -\frac{2}{X}G_4X \]

\[
S[g, \phi] = \int d^4x \sqrt{-g} \left\{ f(\phi, X) R - \frac{4}{X} f_X \left[ (\Box \phi)\phi^\mu \phi_{\mu\nu} \phi^{\nu} - \phi^\mu \phi_{\mu\nu} \phi^{\nu\rho} \phi_{\rho} \right] \right\}
\]

- Remaining Horndeski theories

\[ G_2(X, \phi), \quad G_3(X, \phi), \quad G_4(\phi) \]
Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui ’1711 [see also Crisostomi & Koyama ’1711 and Dima & Vernizzi ’1712]

• Quasi-static approximation on scales $r \ll H^{-1}$

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j$$

$$\phi = \phi_c(t) + \chi(r)$$

• Equations of motion for $\chi$, $\Phi$ and $\Psi$
  – Scalar equation
  – Metric equations

• Matter source: spherical body with density $\rho(r)$
Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui ’1711 [see also Crisostomi & Koyama ‘1711 and Dima & Vernizzi ‘1712]

- Gravitational laws

$$\frac{d\Phi}{dr} = \frac{G_N M(r)}{r^2} + \Xi_1 G_N M''(r),$$
$$\frac{d\Psi}{dr} = \frac{G_N M(r)}{r^2} + \Xi_2 \frac{G_N M'(r)}{r} + \Xi_3 G_N M''(r)$$

with $$(8\pi G_N)^{-1} \equiv 2f \left( 1 + \Xi_0 \right)$$

where the coefficients $\Xi_I$ are given in terms of $f, f_X, a_3$ and $\dot{\phi}_c$

- Breaking of the Vainshtein screening inside matter!

already noticed for Beyond Horndeski (GLPV) Kobayashi, Watanabe & Yamauchi ’14
Gravitation in DHOST with $c_g = c$

- The four coefficients $\Xi_I$ depend on only 2 parameters
  \[ \Xi_0 = -\alpha_H - 3\beta_1, \quad \Xi_1 = -\frac{(\alpha_H + \beta_1)^2}{2(\alpha_H + 2\beta_1)}, \]
  \[ \Xi_2 = \alpha_H, \quad \Xi_3 = -\frac{\beta_1(\alpha_H + \beta_1)}{2(\alpha_H + 2\beta_1)}. \]

- Constraints on the coefficients
  \[ \Xi_0 = \frac{G_{gw}}{G_N} - 1 \quad \text{Hulse-Taylor binary pulsar: } |\Xi_0| < 10^{-2} \]
  \[ \text{Beltran Jimenez, Piazza & Velten 1507} \]
  \[ -\frac{1}{12} < \Xi_1 \lesssim 0.2 \quad \text{Stars: } [-] \]
  \[ \text{[Saito, Yamauchi, Mizuno, Gleyzes & DL '15]} \]
  \[ \text{[Sakstein 15]} \]
  \[ \text{Gravitational lensing for the other coefficients…} \]
Conclusions

• **DHOST theories** provide a very general framework to describe scalar-tensor theories with higher derivatives.

  Systematic classification of "degenerate" theories that contain a single scalar DOF. They include and extend Horndeski and "beyond Horndeski" theories as particular cases.

• **Drastic reduction of viable models after GW170817.**

• These theories of modified gravity can be tested & constrained via cosmology (future LSS observations) and astrophysics.