EKPYROTIC SCENARIO IN STRING THEORY

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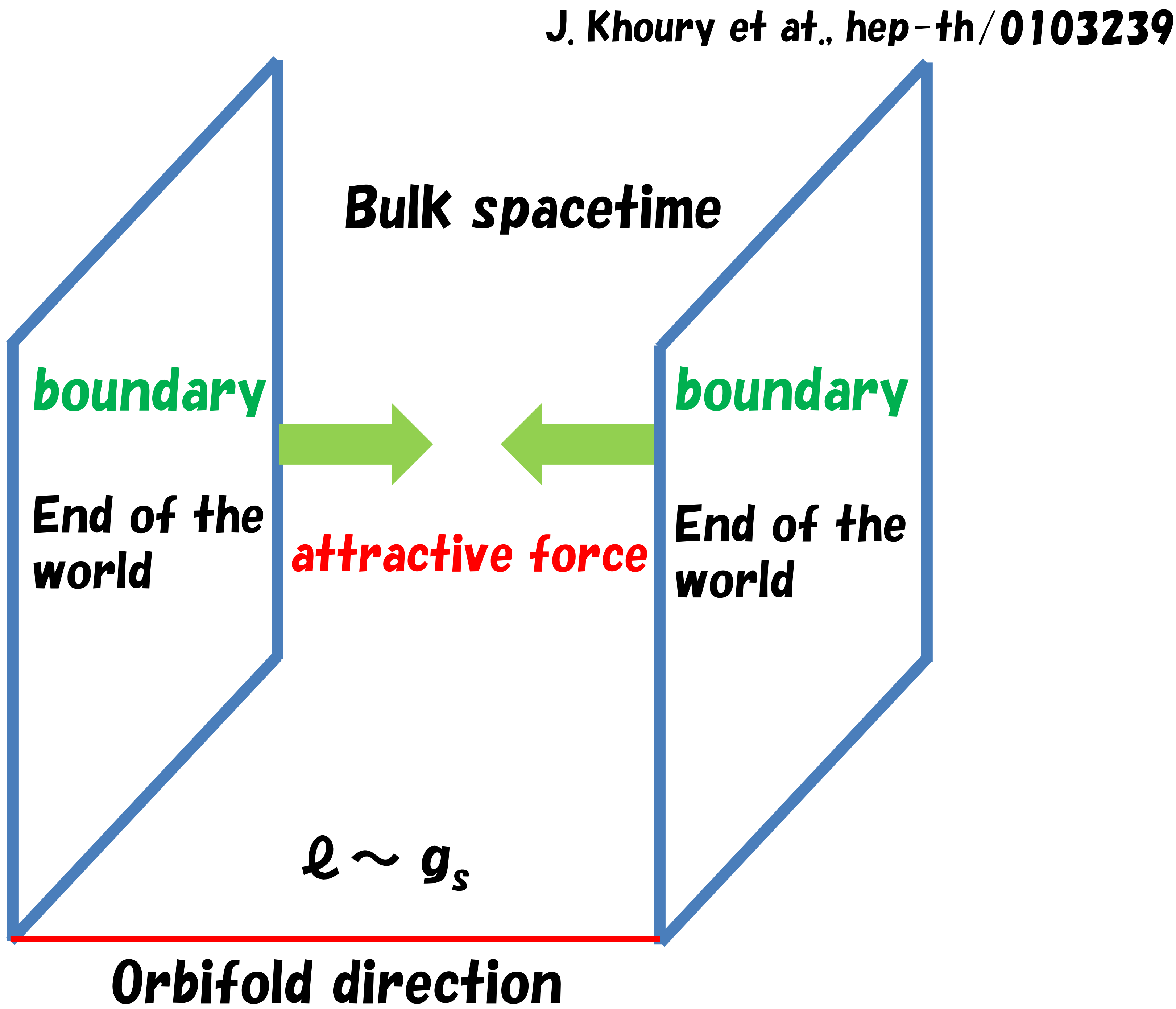
[1] Introduction

- The Ekpyrosis inspired by string theory and brane world model suggests alternative solutions to the early universe puzzles such as inflation and dark energy.
  (P. Horava, E. Witten, hep-th/9510209)
  (A. Lukas et al., hep-th/9710208)
  (J. Khoury et at., hep-th/0103239)

- Since the big bang is described as a collision of branes, there is also a new ekpyrotic phase or a cyclic universe due to another brane collision with the creation of new matter.


J. Khoury et at., hep-th/0103239
Bulk spacetime
boundary
End of the world
boundary
End of the world
attractive force
$\ell \sim g_s$
Orbifold direction

🕶 Property of eKypirosis
(J. Khoury et at., hep-th/0103239)

eKpyrosis – the world would continuously be consumed by a great inferno only to arise again like phenix.

- According to this scenario, the universe is in a slowly contracting phase before big bang, and universe undergoes a slow expansion.

- To take place eKpyrosis the scalar field rolls down its potential and kinetic energy of the scalar increases.

- The potential during ekpyrosis is negative and steeply falling: it can be modeled by the exponential form $V(\phi)=-V_0\, e^{-c\phi}$ $(c \gg 1)$.
  (J. Khoury et at., hep-th/0103239)

$$a(t) = (-t)^p\,, \quad p = \frac{2}{c^2}$$

- There was plenty of time before the big bang for the universe to be in causal contact over large regions.

- The scalar potential obeys fast-roll condition.
  (S. Gratton, et al., astro-ph/0301395)

$$\varepsilon = \frac{V^2}{\sum_i \left(\partial_{\phi_i} V\right)^2}\,, \quad \eta = 1 - \frac{V \sum_i \partial^2_{\phi_i} V}{\sum_i \left(\partial_{\phi_i} V\right)^2}$$

- In ekpyrotic models with single scalar field, the spectrum of the curvature perturbation is blue, in disagreement with observations.

- It is necessary to consider two scalar fields at least in the 4-dimensional theory.
  $\Rightarrow$ new Ekpyrotic scenario
  (E.I. Buchbinder, et al., hep-th/0702154)

- Our work:

- We investigate whether (the new) Ekpyrotic scenario can be embedded into 10D string theory (no go theorem).

- We use that the scalar potential obtained from compactifications of type II supergravity with sources has a universal scaling with respect to the dilaton and the volume mode.

[2] Compactifications of the type II theory

Compactifications of the type II theory to 4-diemnsional spacetime on compact manifold

☆10-dimensional action

$$S = \frac{1}{2\bar{\kappa}^2} \int d^{10}x \sqrt{-g} \left[\mathrm{e}^{-2\phi} \left(R + 4g^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} \sum_p |F_p|^2 \right]$$

$$- \sum_p (T_{\mathrm{D}p} + T_{\mathrm{O}p}) \int d^{p+1}x \sqrt{-g_{p+1}}\, \mathrm{e}^{-\phi} \,,$$

F_p (p=0, 2, 4, 6, 8): R-R p-form field strengths

T_{Dp} , (T_{Op}) : Dp-brane (Op-plane) tension

★To compactify the theory to 4 dimensions, we consider the a metric ansatz of the form:

6-dimensional internal space

$$ds^2 = g_{MN}dx^M dx^N = \boxed{q_{\mu\nu}dx^\mu dx^\nu} + \boxed{\rho\, u_{ij}(\mathrm{Y})dy^i dy^j}$$

4-dimensional universe

$q_{\mu\nu}$: 4-dimensional metric

ρ : volume modulus of the compact space

4-dimensional effective action (Einstein frame)

(M. P. Hertzberg, arXiv:0711.2512)

$$S_{\mathrm{E}} = \int d^4x\sqrt{-\bar{q}}\left[\frac{1}{2\kappa^2}\bar{R} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\rho}\partial_\nu\bar{\rho} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\tau}\partial_\nu\bar{\tau} - V(\bar{\tau}\,,\bar{\rho})\right]$$

$\bar{R}$: Ricci scalar constructed from $\bar{q}_{\mu\nu}$

$$q_{\mu\nu} = \left(\frac{\bar{\kappa}}{\tau\kappa}\right)^2 \bar{q}_{\mu\nu}$$

κ^2 : 4-dimensional gravitational constant

τ : dilaton modulus

$$\bar{\rho} = \sqrt{\frac{3}{2}}\kappa^{-1}\ln\rho\,, \qquad \bar{\tau} = \sqrt{2}\kappa^{-1}\ln\tau\,, \qquad \tau = \mathrm{e}^{-\phi}\rho^{3/2}$$

☆ moduli potential

$$V(\bar{\tau}\,,\bar{\rho}) = V_{\mathrm{Y}} + V_{\mathrm{H}} + V_p + V_{\mathrm{DO}}$$

$$V_{\mathrm{Y}}(\bar{\tau}\,,\bar{\rho}) = -A_{\mathrm{Y}}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right] R(\mathrm{Y})\,,$$

$$\boxed{\begin{aligned} V_{\mathrm{H}}(\bar{\tau}\,,\bar{\rho}) &= A_{\mathrm{H}}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \sqrt{6}\bar{\rho}\right)\right],\\ V_p(\bar{\tau}\,,\bar{\rho}) &= \sum_p A_p\,(\phi_i)\exp\left[-\kappa\left\{2\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}(p-3)\bar{\rho}\right\}\right], \end{aligned}}$$

$$V_{\mathrm{DO}}(\bar{\tau}\,,\bar{\rho}) = \sum_p \left[\boxed{A_{\mathrm{D}p}\,(\phi_i)} - A_{\mathrm{O}p}\,(\phi_i)\right]\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}(6-p)\bar{\rho}\right\}\right]$$

$$\times \int d^{p-3}x\sqrt{g_{p-3}}$$

positive

A_Y, A_H, A_p, A_{Dp}, A_{0p} : coefficients

If the potential form for the ekpyrotic scenario gives the negative and steep, the "fast-roll" parameters for the ekpyrosis are given by

$$\varepsilon_{\mathrm{f}} \equiv \kappa^2 \frac{V^2}{\left(\partial_{\bar{\tau}} V\right)^2 + \left(\partial_{\bar{\rho}} V\right)^2} \ll 1 \,,$$

$$|\eta_{\mathrm{f}}| \equiv \left| 1 - \frac{V\left(\partial_{\bar{\tau}}^2 V + \partial_{\bar{\rho}}^2 V\right)}{\left(\partial_{\bar{\tau}} V\right)^2 + \left(\partial_{\bar{\rho}} V\right)^2} \right| \ll 1$$

[3] The scenario with vanishing flux

☆ Statement :

Ekpyrotic scenario is prohibited in string theory with O-plane source and zero fluxes.

☆ moduli potential with vanishing flux

$$
\begin{aligned}
V(\bar{\tau}\,,\bar{\rho}) &= V_{\mathrm{Y}} + V_{\mathrm{O}p} \\
&= -A_{\mathrm{Y}}\left(\phi_i\right)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right] R(\mathrm{Y}) \\
&\quad - \sum_p A_{\mathrm{O}p}\left(\phi_i\right)\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}(6-p)\bar{\rho}\right\}\right]\int d^{p-3}x\sqrt{g_{p-3}}
\end{aligned}
$$

For the case of $R(Y)=0$, $A_Y=1$, $A_H=A_p=A_{Dp}=0$, and $A_{0p} \int d^{p-3}x (g_{p-3})^{1/2}=1$, ($p=4, 6, 8$), in the moduli potential, the parameters ε_f obeys

$$\varepsilon_{\mathrm{f}} = \kappa^2 \frac{V^2}{\left(\partial_{\bar{\tau}} V\right)^2 + \left(\partial_{\bar{\rho}} V\right)^2} > \frac{6}{31}$$

The result gives the contradiction with the fast-roll condition for ekpyrosis.

- ❑ Our results:

❀ We find strong constraints ruling out ekpyrosis from analyzing the fast-roll conditions.

☞ We conclude that a compactification in IIA string theory tend to provide potentials that are not too steep and negative (ekpyrosis).

[4] Summary and comments

(1) We studied the No-Go theorem of the ekpyrosis for string theory with vanishing flux.

(2) The 4-dimensional effective potential of two scalar fields can be constructed by postulating suitable emergent gravity, orientifold planes in terms of the compactification with smooth manifold.

(3) Since the fast-roll parameter is not small during the ekpyrotic phase, the explicit nature of the dynamics has made it impossible to realize the ekpyrotic scenario.