Dynamics of Rapid Granular Fluid:
(i) Shear, Bifurcation & Gravity,
(ii) Rheology and Correlation, ...

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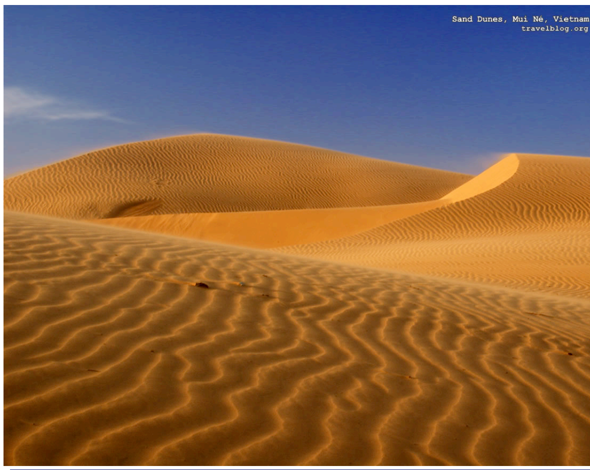
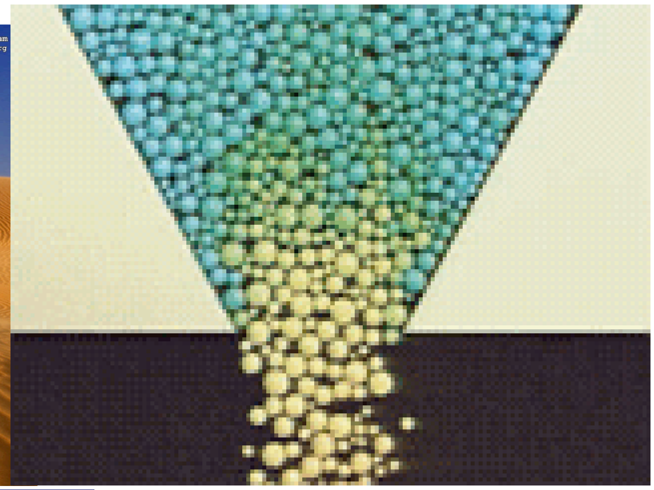
Physics of Granular Flows

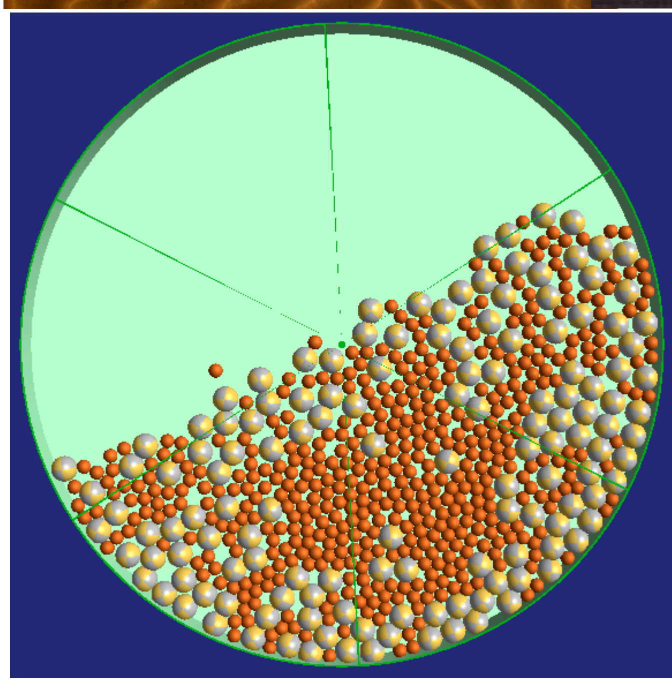
Yukawa Institute for Theoretical Physics

Kyoto, Japan, June 24 -- June 05, 2013

Granular Matter



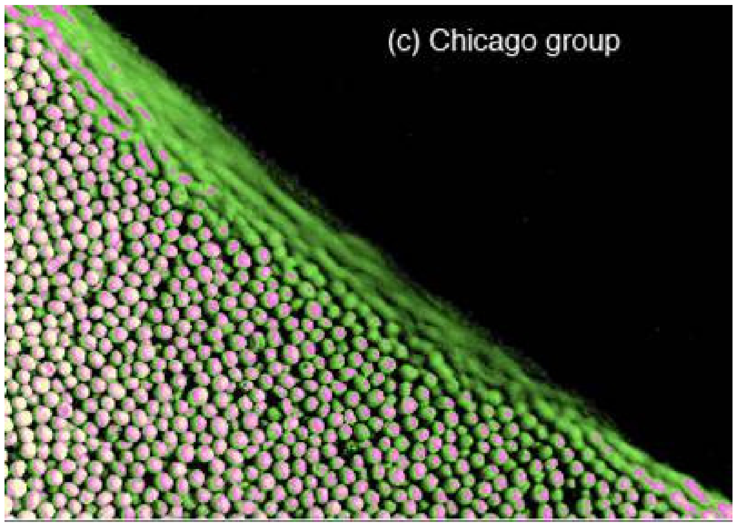


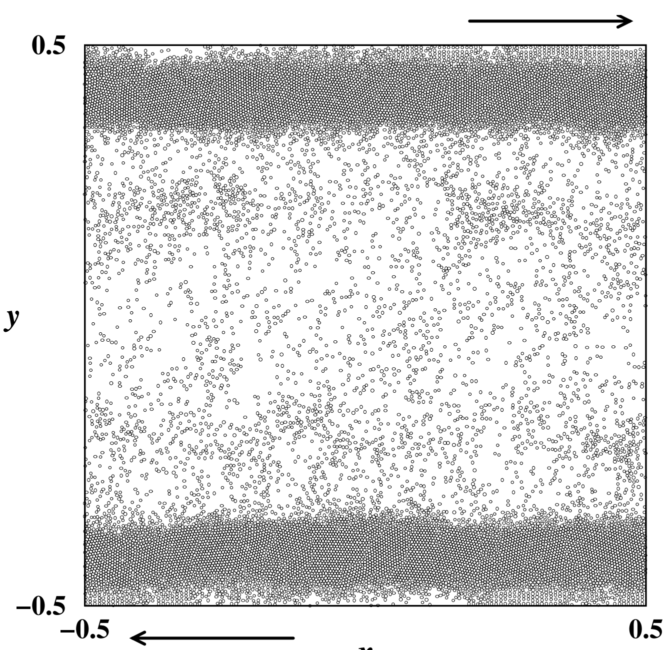


Solid, Liquid and Gas ?

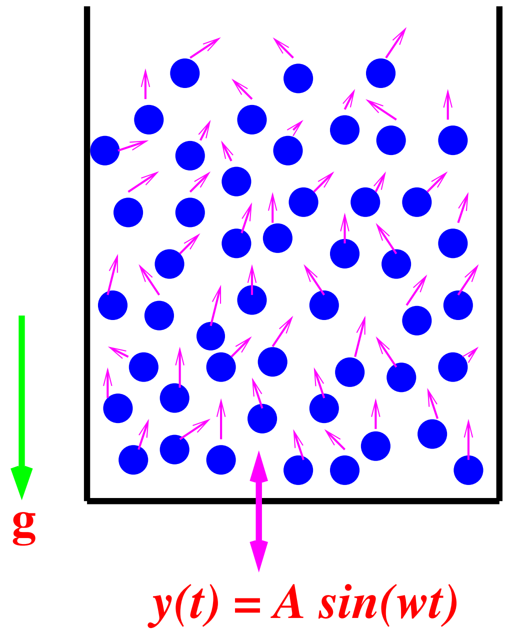


Coexistence of Different States





Granular Fluid



$\Gamma = \frac{A\omega^2}{g}$

$>> 1$

♣ **Rapid Flows (Strong Driving)**

♣ **Inelastic Hard Spheres**

♣ **Restitution Coefficient:** $0 < e < 1$

♣ **Collision Rule:** $\boldsymbol{k}\cdot\boldsymbol{c}_{12}' = -e(\boldsymbol{k}\cdot\boldsymbol{c}_{12})$

♣ **Control Parameters:**

$\nu = N\pi d_p^2/LH$, Γ, $e, \ldots$

Granular Matter: Faraday Patterns

Oscillons (Swinney & Umbanhower 1996)

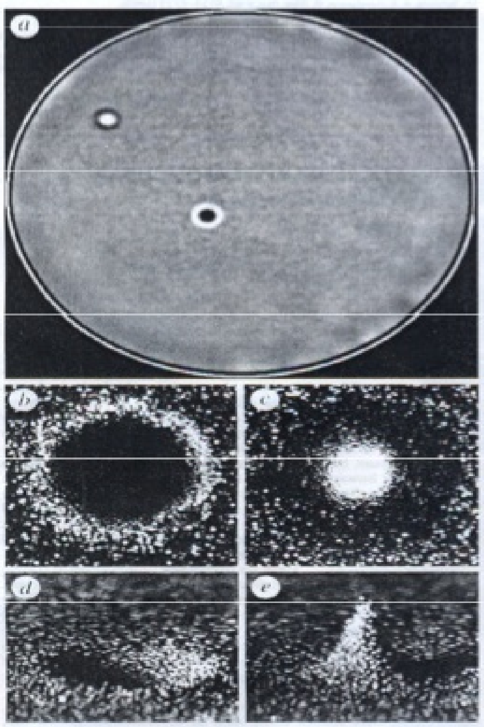


FIG. 1 *a*, Two oscillons with opposite phase. *b* and *c*, A single oscillon viewed from above, at times differing by $1/f$. *d* and *e*, Corresponding side views ($f = 26$ Hz, $\Gamma = 2.45$, layer depth of 17 particles). The individual particles (0.15–0.18-mm-diameter bronze spheres) are in close contact, as can be seen in *b*–*e*, which show 7 mm × 7 mm regions of the 127-mm-diameter container. Images are illuminated from the side, for 1/100 of a cycle, with light strobed in phase with the drive signal so that regions with large amplitude are light, and regions with small amplitude are dark. The container is evacuated to 0.1 torr, a value at which the effects of the remaining gas are negligible.

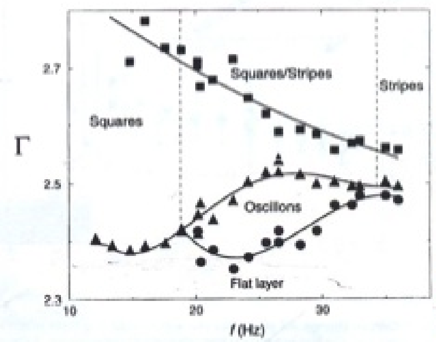
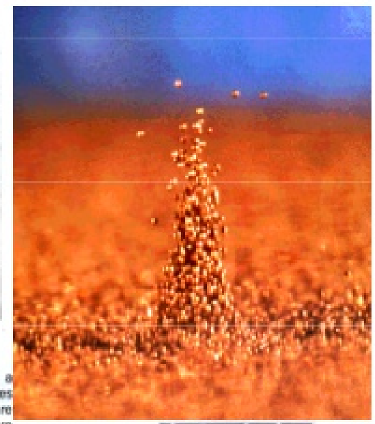
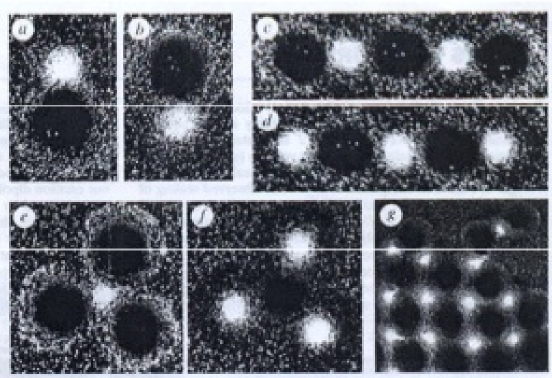
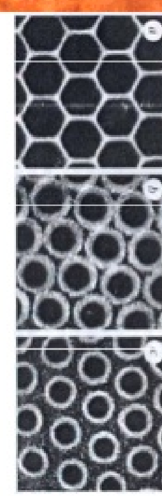


FIG. 2 Diagram showing the stability regions for different states, as a function of f and Γ, for increasing Γ (squares) and decreasing Γ (triangles and circles). The transitions from the flat layer to squares and stripes are hysteretic, but the hysteresis is much smaller for stripes. Oscillons are observed for layers greater than 13 particles deep in a range of f which increases with increasing depth. For thinner layers, the phase diagram is similar but without the oscillon region.







Patterns: Phenomenological Modelling?

Patterns in Vibrated bed can be predicted by the complex Ginzburg LE (Tsimring and Aranson 1997)

$$\partial_t \psi = \gamma \psi^* - (1 - i\omega)\psi + (1 + ib)\nabla^2 \psi - |\psi|^2 \psi - \rho\psi$$

γ normalized amplitude of forcing at 'f'
b ratio of dispersion to diffusion
ω frequency of the driving force
ρ thickness of the layer
ψ amplitude of layer oscillations

$$h \approx \exp(\lambda(k)t + ikx)$$

$$\lambda(k) = -\lambda_0 - \lambda_1 k^2$$

$$b = \frac{\mathrm{Im}(\lambda_1)}{\mathrm{Re}(\lambda_1)}$$

$$\omega = (\Omega_0 - \pi f)/\mathrm{Re}(\lambda_0)$$

$$\Omega_0 = -\mathrm{Im}(\lambda_0)$$

Layer frequency

$$f/2$$

$$h = \psi \exp(\pi i f t)$$

(Swinney etal, 1995)

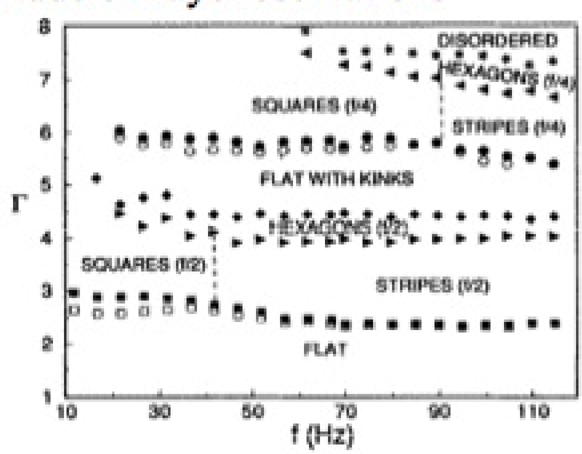


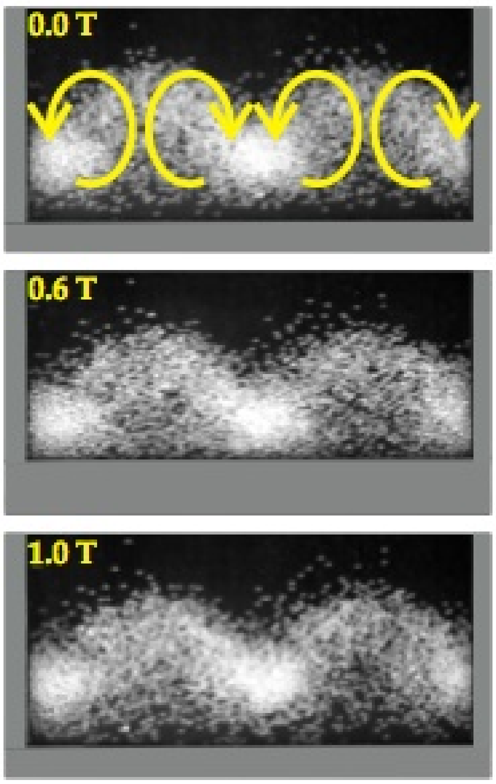
FIG. 2. Stability diagram showing transitions in a 1.2 mm deep layer. The vertical dashed lines indicate the frequencies above which only stripes appear in the square or stripe regime. Closed (open) square and circular symbols denote transitions with increasing (decreasing) Γ.

@ **Amplitude Equations for Shear Flow and Convection:**

Shukla & Alam (2009-2013)

Saitoh & Hayakawa (2011-2013)

Theoretical Issues: Patterns



Eshuis, ..., & Lohse (Phys. Fluid 2007)

Patterns from Continuum Viewpoint ??

$\Uparrow$

Derive Continuum Equations

$\Uparrow$

Granular Fluid

$\Uparrow$

Analogy with Dense Gas

Granular Hydrodynamics: Kinetic Theory

♣ **Kinetic Theory of Inelastic Dense Gases**

Hydrodynamic Fields

♣ **Mass density**

$$\rho(\mathbf{x},t) \equiv mn(\mathbf{x},t) = m\int f(\mathbf{c},\mathbf{z},t)\mathrm{d}\mathbf{c} = \rho_p\nu(\mathbf{x},t)$$

@ $\nu(\mathbf{x},t) =$ **Volume fraction of particles**

♣ **Hydrodynamic Velocity**

$$\mathbf{u}(\mathbf{x},t) = < \mathbf{c} > = \frac{1}{n}\int \mathbf{c} f(\mathbf{c},\mathbf{z},t)\mathrm{d}\mathbf{c}$$

♣ **Granular Temperature**

$$T(\mathbf{x},t) = \frac{1}{d}m < \mathbf{C}\cdot\mathbf{C} > = \frac{1}{nd}\int mC^2 f(\mathbf{c},\mathbf{z},t)\mathrm{d}\mathbf{c}$$

@ This is not thermodynamic temperature!

@ $E_{potential} >> k_B T_{ther}$

Granular Hydrodynamics at NS-order

Balance Laws

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \boldsymbol{\nabla\cdot u}$$

$$\rho\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = \rho\mathbf{g} - \boldsymbol{\nabla}\cdot\mathbf{P}$$

$$\frac{d}{2}\rho\frac{\mathrm{D}T}{\mathrm{D}t} = -\boldsymbol{\nabla\cdot q} - \mathbf{P}:\boldsymbol{\nabla u} - \mathcal{D}$$

Rheological Model

♣ Stress Tensor

$$\mathbf{P} = \left[p(\nu,T) - \zeta(\nu,T)\boldsymbol{\nabla\cdot u}\right]\mathbf{I} - 2\mu(\nu,T)\mathbf{S}$$

$$\mathbf{S} = \tfrac{1}{2}\left(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^{T}\right) - \tfrac{1}{d}\left(\boldsymbol{\nabla\cdot u}\right)\mathbf{I}$$

♣ Heat Flux

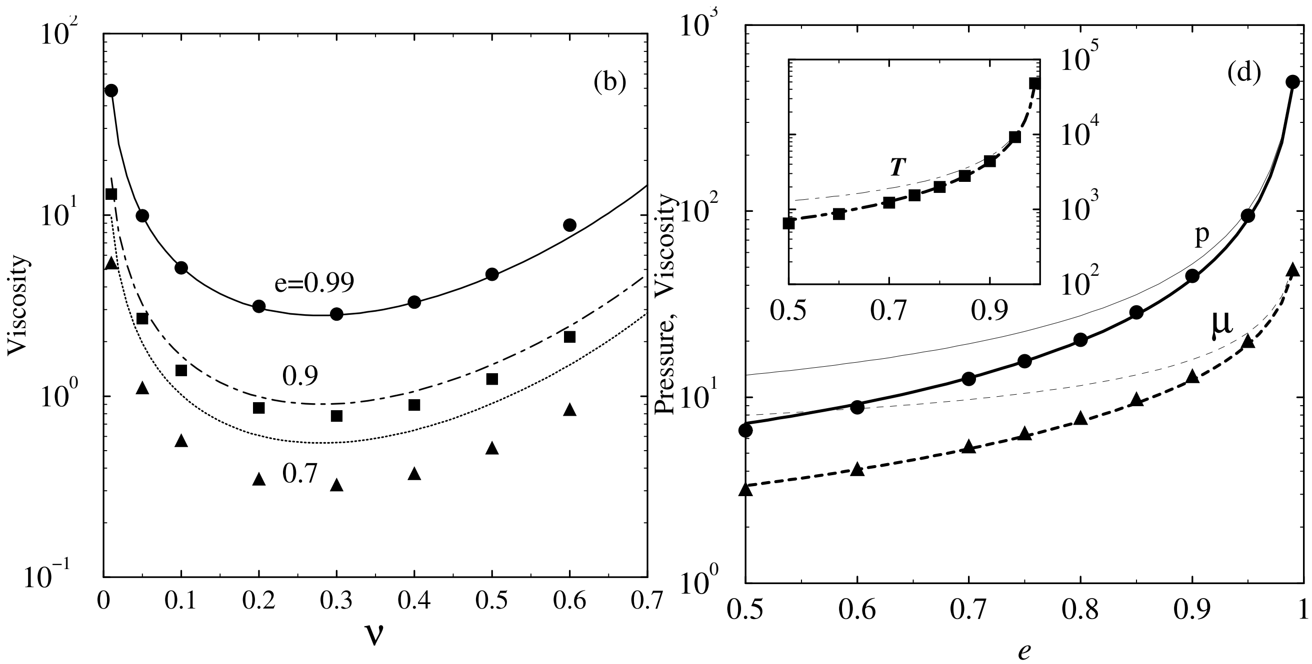
$$\mathbf{q} = -\kappa(\nu,T)\boldsymbol{\nabla}T - \kappa_h(\nu,T)\boldsymbol{\nabla}\nu$$

♣ Collisional Dissipation

$$\mathcal{D} = \frac{\rho_p}{d_p} f_5(\nu,e) T^{3/2} \sim (1-e^2)$$

♣ Lun et al. (1984, JFM); Jenkins & Richman (1985)

How good is NS-order Rheology?



- **Stress Tensor:** $\mathbf{P} = -p\mathbf{I} + \Pi$
- **Viscosity:** $\mu = \Pi_{xy}/(du/dy)$
- $\mathcal{N}_1 = (\Pi_{xx} - \Pi_{yy})/p \neq 0$, $\mathcal{N}_2 = (\Pi_{yy} - \Pi_{zz})/p \neq 0$

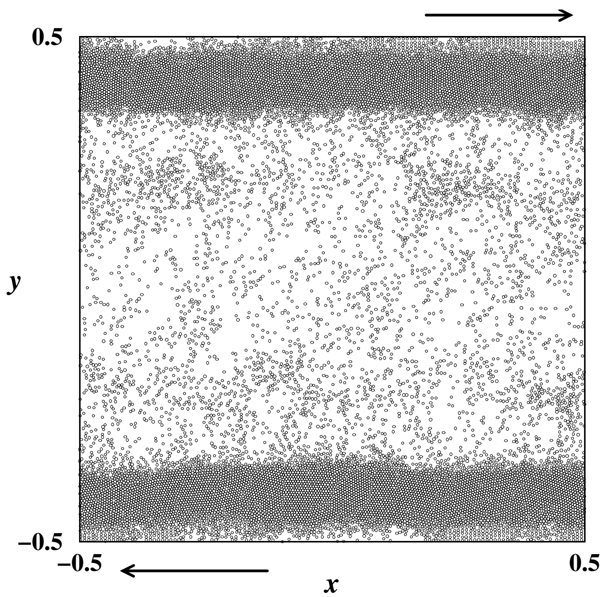
- **Disagreement due to "measurable" normal stress differences**

- **KT-models are good for nearly elastic systems ($e \sim 1$)**

  - *Jenkins & Richman (J. Fluid Mech 1988)*
  - *Alam & Luding (J. Fluid Mech, 2003)*

Can we predict patterns?

(NS-Theory, and Simulation)



$\nu = 0.3; \quad e = 0.8$

♣ **Patterns** $\Longleftarrow$ **Instability**

♣ **Route to Capture Instability**

@ **Continuum Equations**

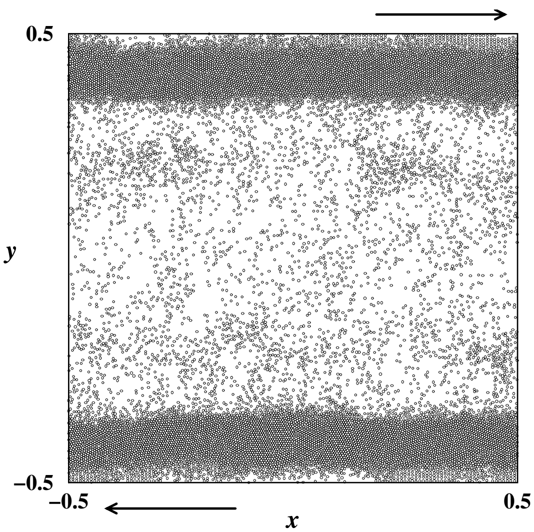
@ **Base Flow**

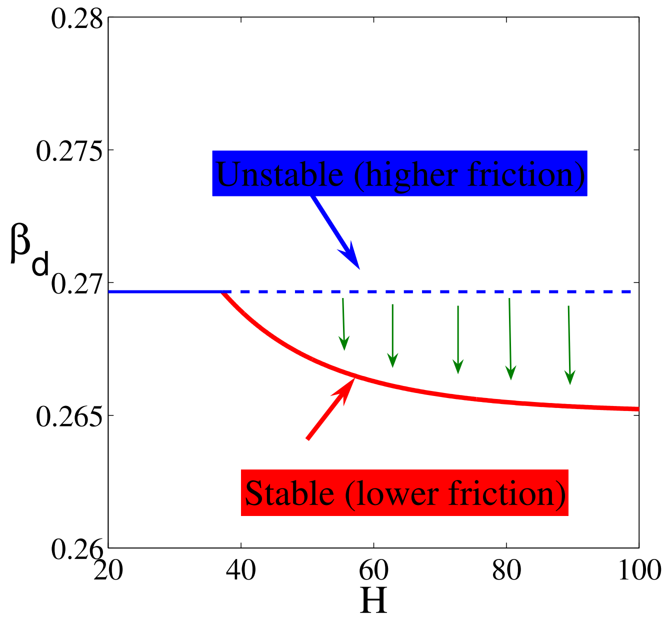
@ **Introduce Perturbations**

@ **Dynamics of Perturbations**

@ **J. Fluid Mech. (??–2013)**

Origin of Shear-banding?





♣ Dynamic Friction Coefficient

$$\beta_d = \frac{\text{shear stress}}{\text{pressure}} = \frac{\mu(\frac{du}{dy})}{p}$$

@ Disordered/Uniform-Shear State

⇑

"Higher" Dynamic Friction Coefficient

@ Ordered/Shear-banded State

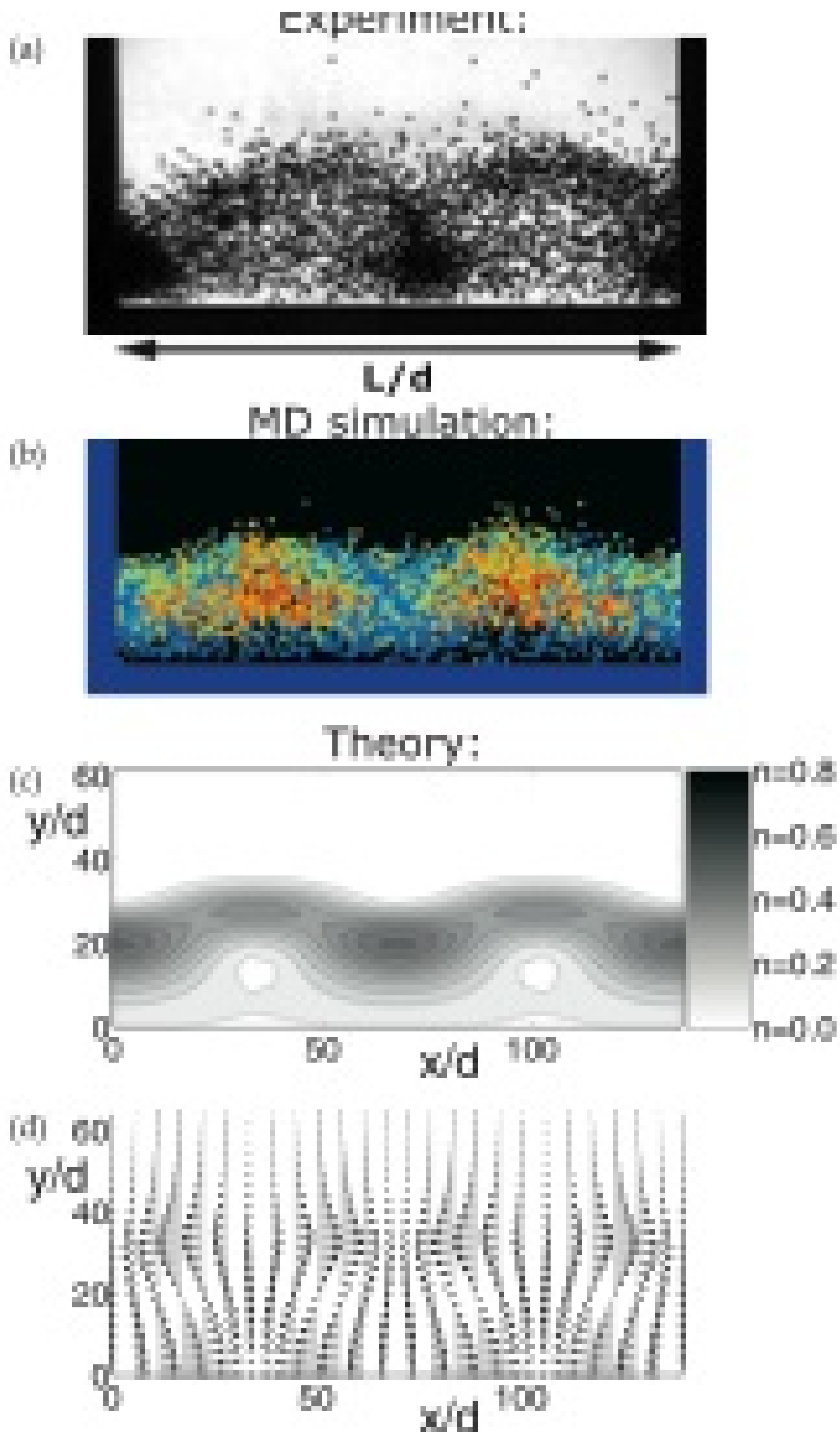
⇑

"Lower" Dynamic Friction Coefficient

@ Alam (Unpublished)

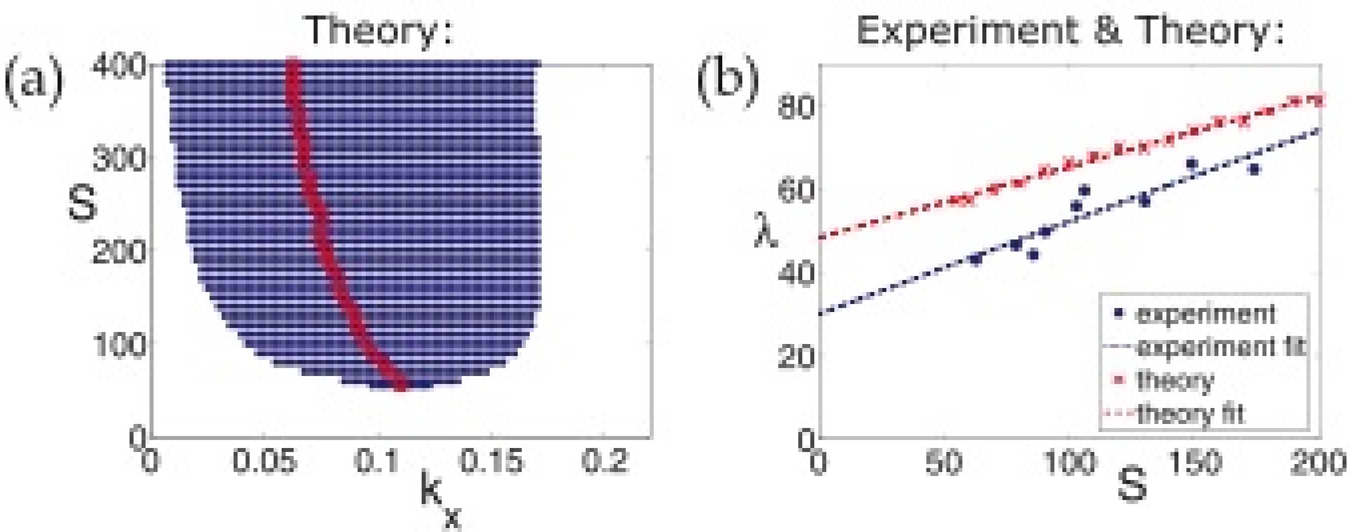
@ Alam, Shukla & Luding (J. Fluid Mech. 2008)

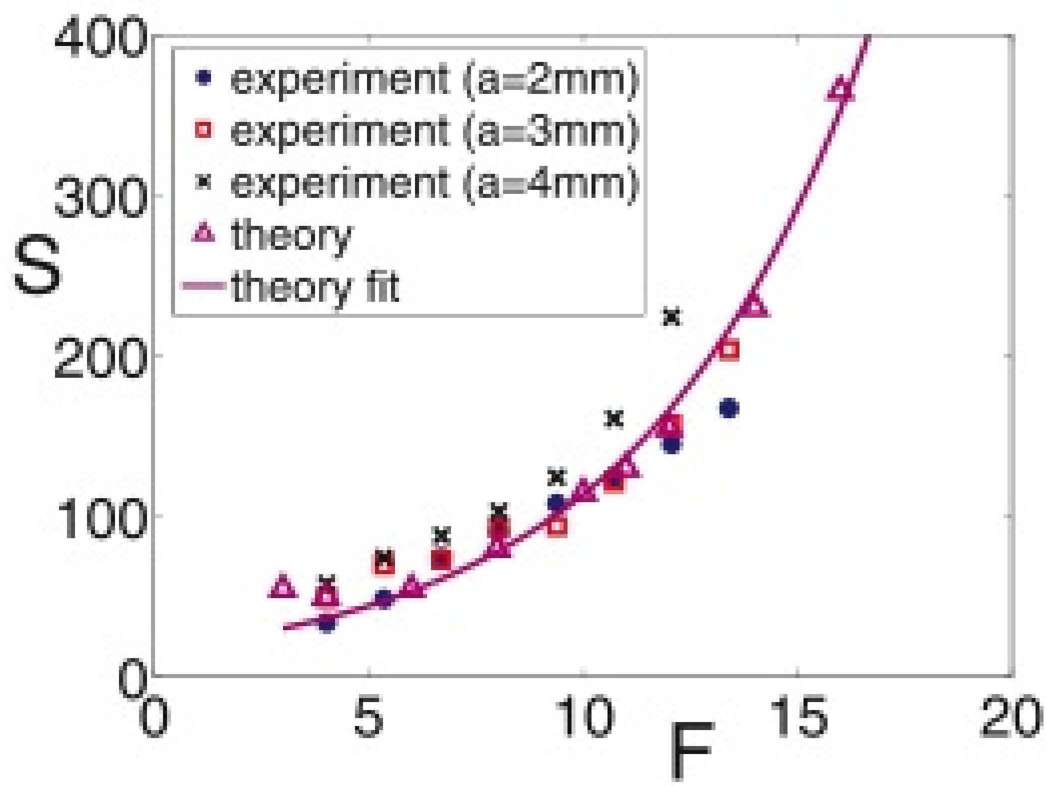
Experiment, Simulation and Theory



@ Eshuis, ..., Alam & Lohse (Gran. Matt. 2013, in press!)

Experiment, Simulation and Theory





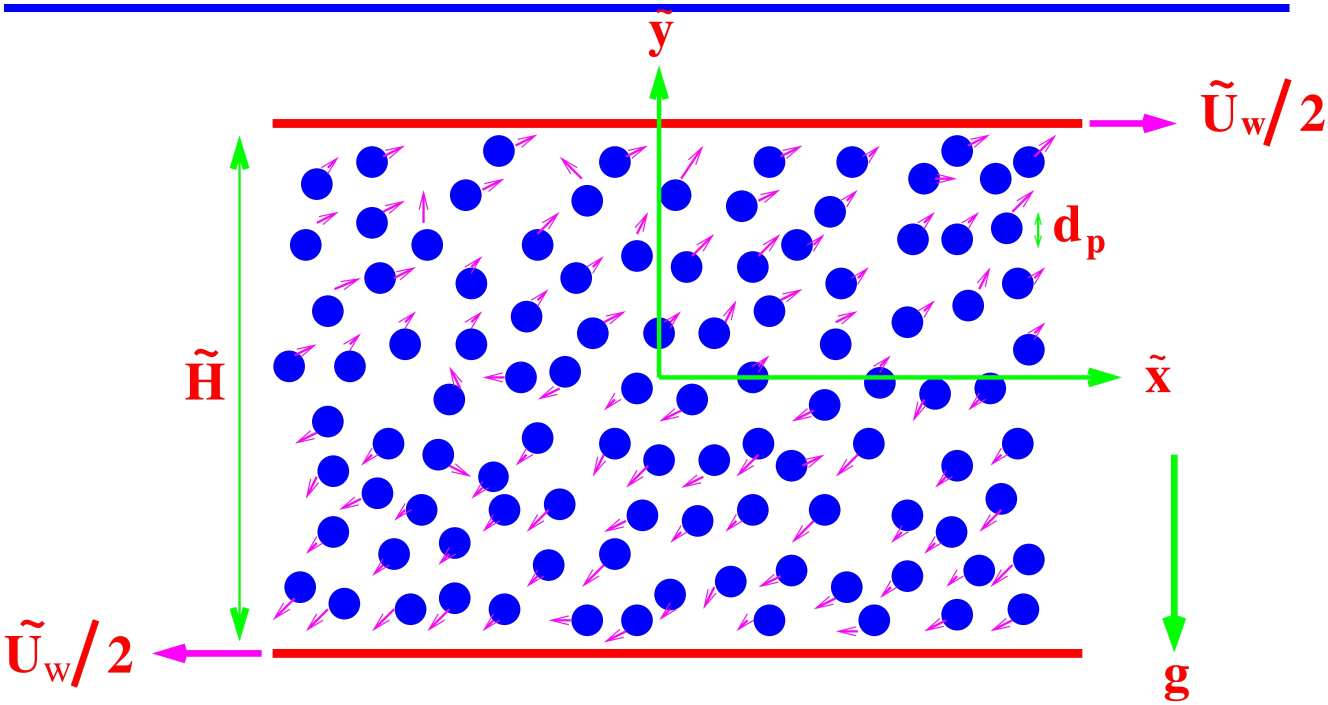
@ Eshuis, ..., Alam & Lohse (PRL 2010)

Outline of Talk

- ♣ **Introduction**
- ♣ **Patterns from Particle Simulations**
- ♣ **Patterns from Hydrodynamic Instability**
- ♣ **Ordering and Role of Gravity**
- ♣ **Universal Unfolding and Normal Form**
- ♣ **Conclusions**
- ♣ **Normal Stress Difference?**
- ♣ **Rotation and Friction: Micropolar effects?**

@ *Alam (J. Fluid Mech., 2005, vol. 523)*
Shukla & Alam (PRL 2009; JFM 2011-13)

Plane Couette Flow



♣ Control Parameters

@ **Mean volume fraction of particles:** $\nu = V_p N/V$

@ **Wall separation:** $H = \tilde{H}/d_p$

@ **Restitution coefficient:** $0 < e < 1$

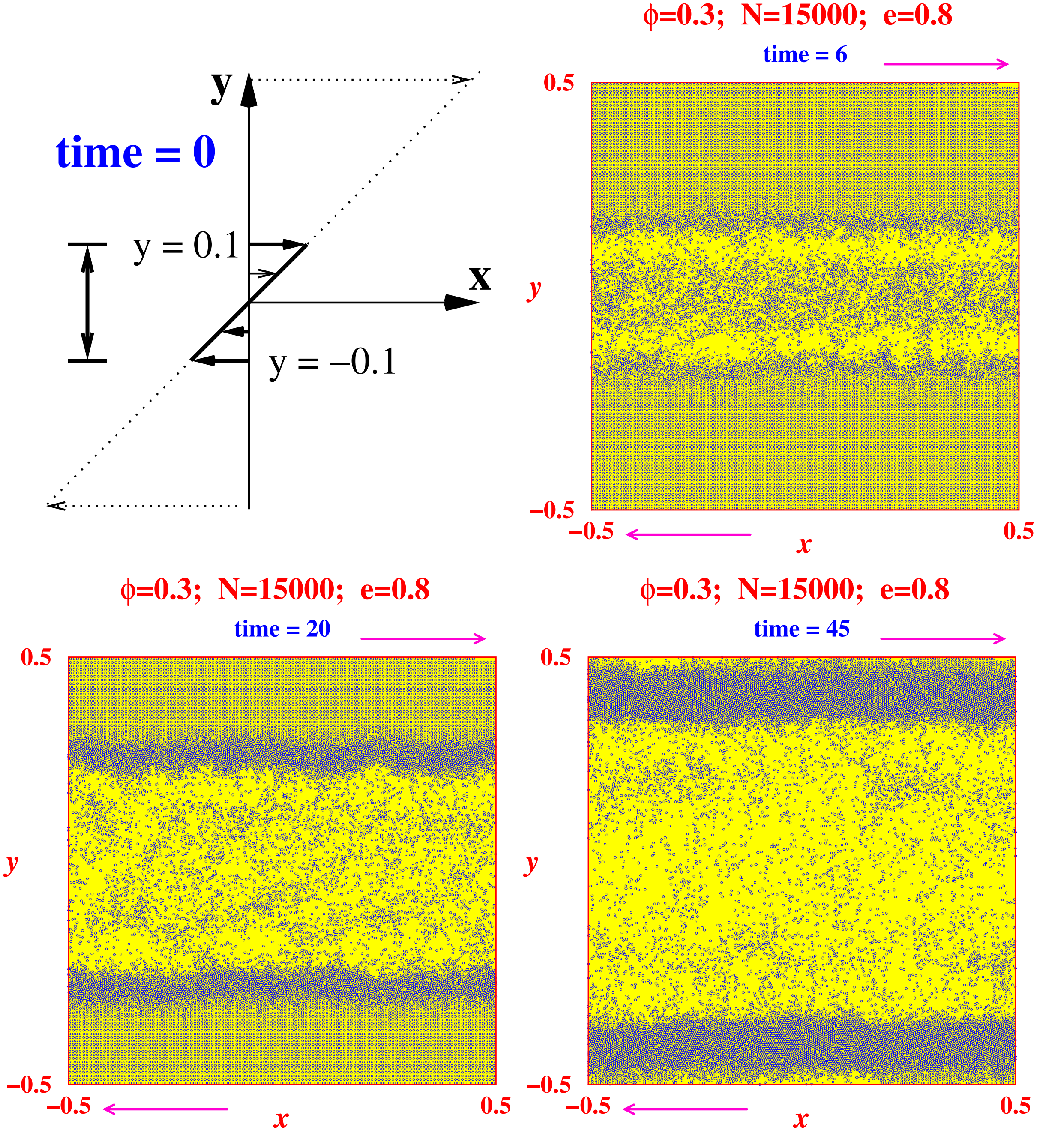
@ **Froude number:** $Fr = \tilde{U}_w/\sqrt{gd_p} = \frac{\tau_{\mathbf{g}}}{\gamma^{-1}}$

- $\tau_g = H(d_p/g)^{1/2}$
- $\gamma = \tilde{U}_w/\tilde{H}$

@ $\boxed{g = 0} \Rightarrow \boxed{Fr^{-1} = 0 \quad \textbf{or} \quad Fr = \infty} \Leftarrow \boxed{\gamma = \infty}$

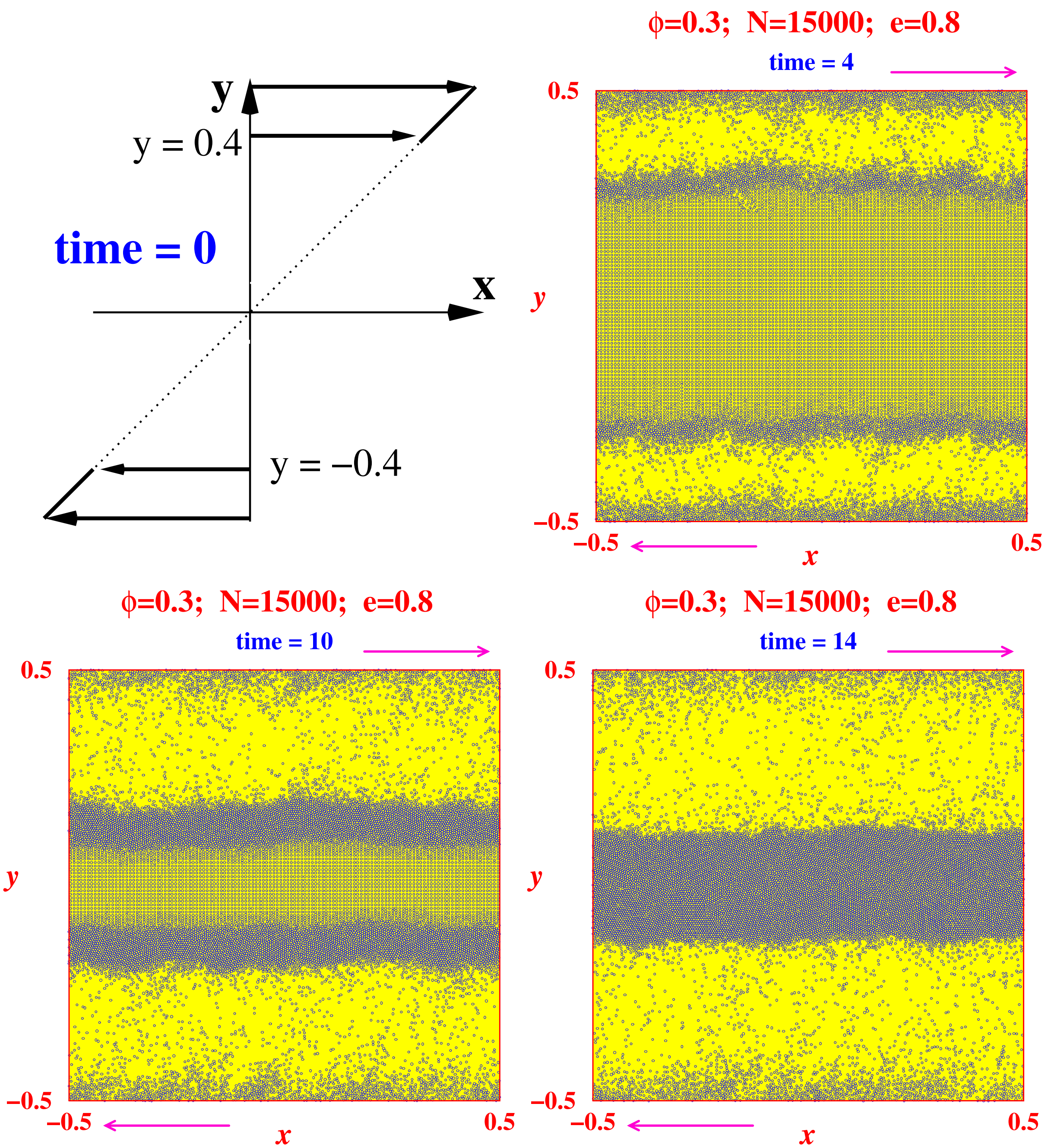
@ $\boxed{g > 0} \Rightarrow \boxed{Fr^{-1} > 0 \quad \textbf{or} \quad Fr < \infty}$

Ordering/Shearbanding Transition



$$\left.\begin{aligned} \nu(y) &= \nu(-y) \\ \mathbf{T(y)} &= \mathbf{T(\text{-}y)} \\ \mathbf{u(y)} &= \mathbf{\text{-}u(\text{-}y)} \end{aligned}\right\} \quad \textbf{as} \quad y \longrightarrow -y$$

Ordering/Shearbanding Transition



$$\nu(y) = \nu(-y); \quad T(y) = T(-y); \quad u(y) = -u(-y)$$

Density Patterns from Simulations

- ♣ **Properties of Patterns**
  - @ **Ordering transition** $\Rightarrow$ **'Plugs'**
  - @ **Driven by inelasticity**
  - @ **Smaller systems inhibit pattern formation**
  - @ **Other patterns (convective/stationary)?**

Boundary Conditions

♣ Issues

@ **No-slip boundary condition does not hold**

@ **Walls act either as a sink or source of granular energy**

♣ Boundary Characterization

@ **Roughness Parameter,** $0 \le \phi' \le 1$

@ **Restitution Coefficient for particle-wall collisions,** e_w

♣ Non-dimensional B.C.

$$\frac{\mathbf{u}_{slip}}{|\mathbf{u}_{slip}|} \cdot \mathbf{P} \cdot \mathbf{n} = H\mathbf{S}^{wall}$$

$$\mathbf{n} \cdot \mathbf{q} = H^3 \mathbf{u}_{slip} \cdot \mathbf{S}^{wall} - H\mathcal{D}^{wall}$$

@ $\mathbf{u}_{slip} = \mathbf{u}^{fw} - \mathbf{u}^{wall}$

@ $\mathbf{S}^{wall} = \dfrac{\phi' \pi \nu \chi(\nu) \sqrt{T} \mathbf{u}_{slip}}{2\sqrt{3}\nu_{max}}$

@ $\mathcal{D}^{w} = \dfrac{\sqrt{3}\pi \nu \chi(\nu) T^{3/2} (1 - {e_w}^2)}{4\nu_{max}}$

@ **Hui etal. (1984); Johnson & Jackson (1987)**

♣ **...,** *Chikkadi & Alam (PRE, 2009)*

Plane Couette Flow

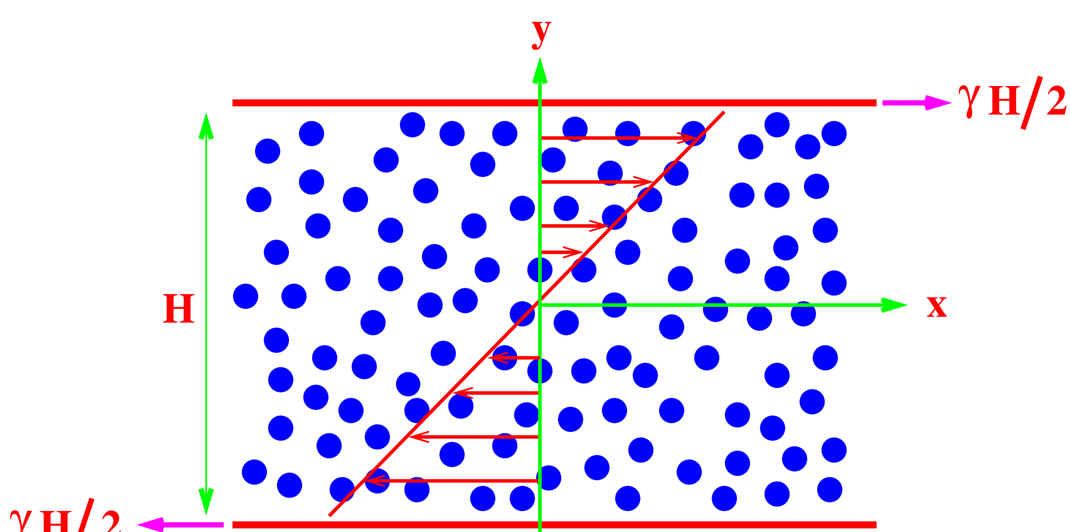
♣ Assumptions

- @ **No-slip B.C.** ($\mathbf{u} = \mathbf{u_w}$)
- @ **Zero heat-flux B.C.** ($\mathbf{n} \cdot \mathbf{q} = \mathbf{0}$)
- @ **Steady** ($\partial/\partial t = 0$), **fully developed** ($\partial/\partial x = 0$) **flow**
- @ **Gravity is absent** ($g = 0$)

$\Downarrow$

Uniform Shear Flow

- @ $\nu(y) =$ **constant**
- @ $[u(y), v(y), w(y)] = [\dot{\gamma} y, 0, 0]$
- @ $T(y) = \dot{\gamma}^2 d_p^2 \left[f_2(\nu, e) / f_5(\nu, e) \right]$



Linear Stability Analysis

♣ Linearized Equations for Perturbed Fields

$$\left.\begin{aligned} \frac{\partial \mathbf{X}}{\partial t} &= \mathcal{L}\mathbf{X} \\ \mathcal{B}_1\mathbf{X} &= 0 \\ \mathcal{B}_2\mathbf{X} &= 0 \end{aligned}\right\} \quad \textbf{where } \mathbf{X} = (\nu', u', v', w', T')^T$$

♣ Fourier decomposition

@ $\mathbf{X}(x, y, z, t) = \hat{\mathbf{X}}(y) e^{i(k_x x + k_z z) + \omega t}$

♣ Temporal stability

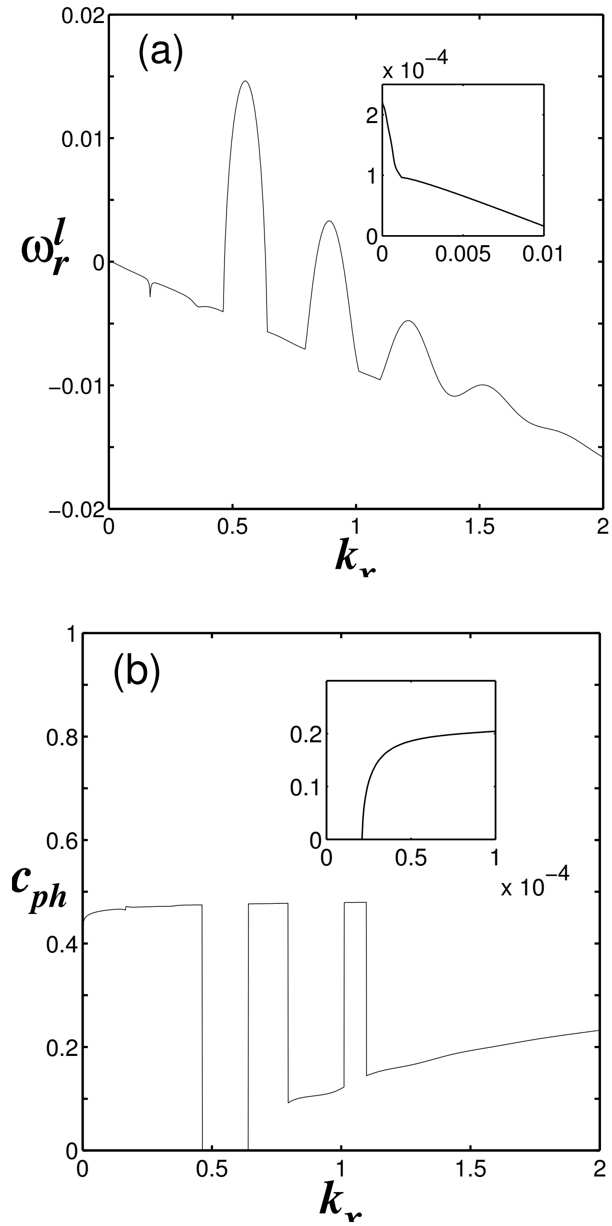
@ $\omega = \omega_r + i\omega_i \Rightarrow \begin{cases} \omega_r < 0 & (\textbf{Stable}) \\ \omega_r = 0 & (\textbf{Neutral}) \\ \omega_r > 0 & (\textbf{Unstable}) \end{cases}$

@ **Least-Stable Mode:** $\omega_r^l = \max \omega_r$

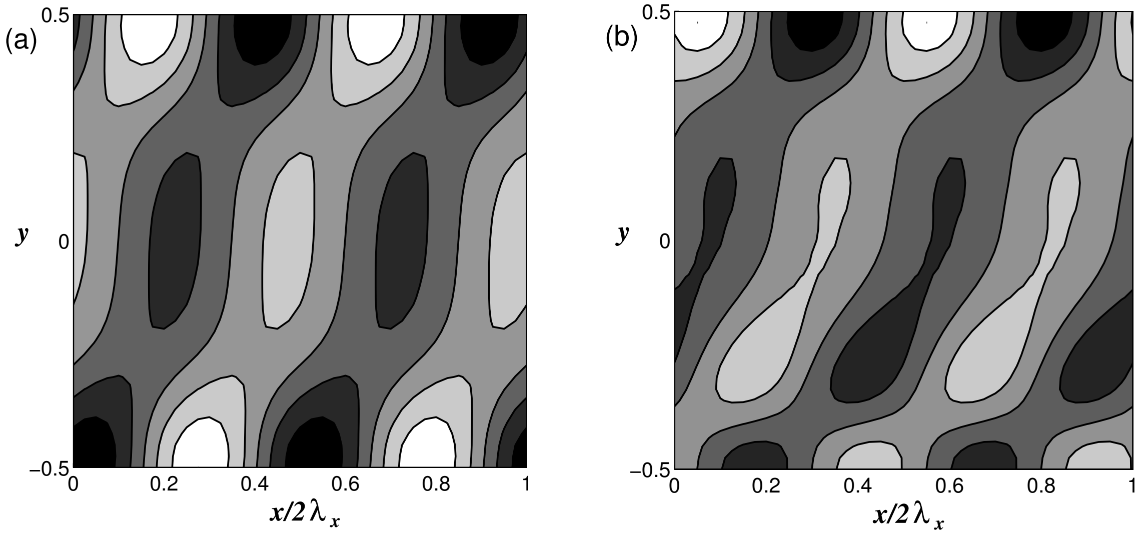
@ **Phase Velocity:** $c_{ph} = \dfrac{\omega_i}{\sqrt{k_x^2 + k_z^2}}$

Types of Instabilities

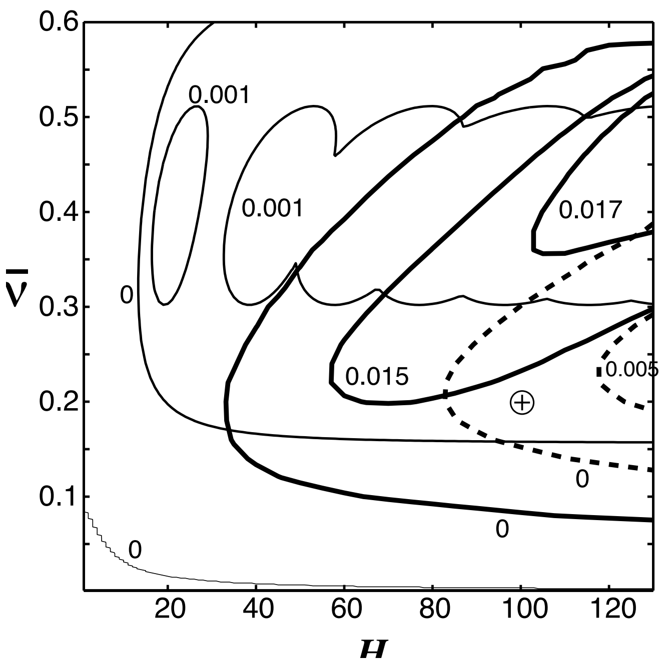
$\overline{\nu} = 0.2$; $H = 100$; $e = 0.8$



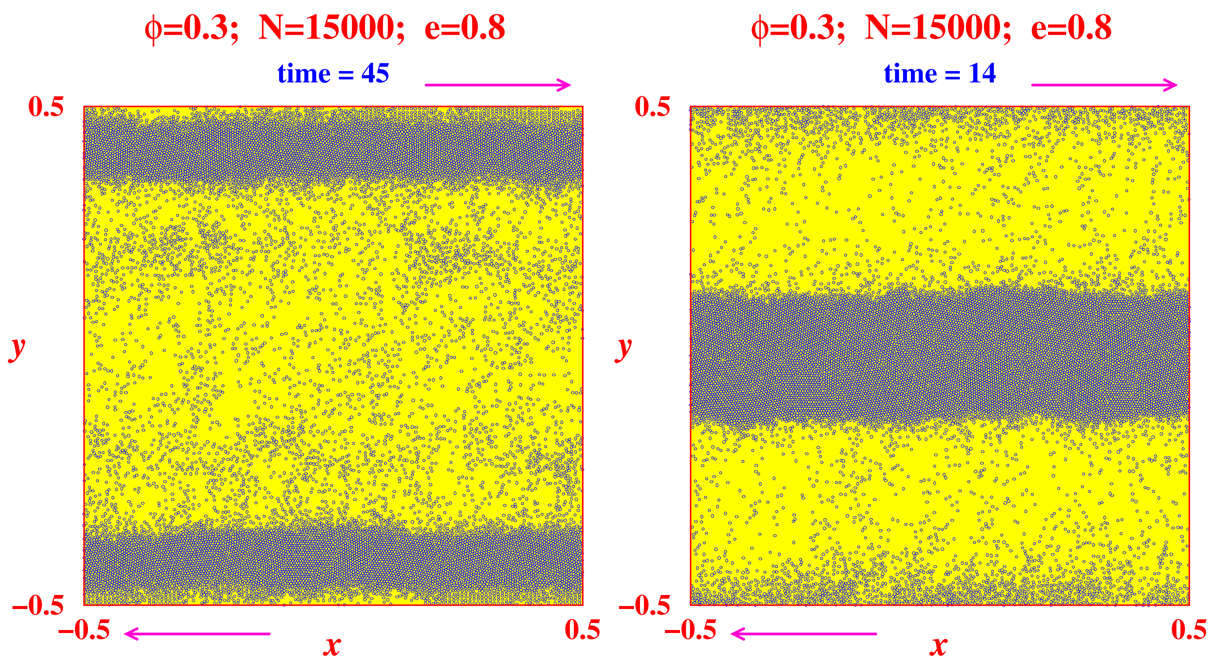
Density Patterns



(a) S-Wave (b) T-Wave



Ordering/Shearbanding Transition



$$\nu(y) = \nu(-y); \quad T(y) = T(-y); \quad u(y) = -u(-y)$$

$\Downarrow$

"Streamwise-independent" Flow?

$\Downarrow$

"Shear-banding" Instability!

$$(k_x = 0)$$

Shear-banding Instability ($k_x = 0$)

♣ Perturbed Fields

$$[\nu, T](y) = [\nu_1, T_1] \cos n\pi(y \pm 1/2)$$
$$(u, v](y) = [u_1, v_1] \sin n\pi(y \pm 1/2)$$

@ Mode Number: $n = 1, 2, 3, \ldots$

$\Downarrow$

Shape of Eigenfunctions

♣ Symmetries

$$\left.\begin{aligned} [\nu, T](y) &= [\nu, T](-y) \\ (u, v](y) &= -[u, v](-y) \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} [\nu, T](y) &= -[\nu, T](-y) \\ (u, v](y) &= [u, v](-y) \end{aligned}\right\} \qquad (2)$$

(1) $\Rightarrow$ **Symmetry-Preserving Modes:** $n = 2, 4, \ldots$

(2) $\Rightarrow$ **Symmetry-Breaking Modes:** $n = 1, 3, \ldots$

Shear-banding Instability ($k_x = 0$)

♣ Stationary ($\omega_i = 0$) Instability

$\Downarrow$ Real Eigenvalue

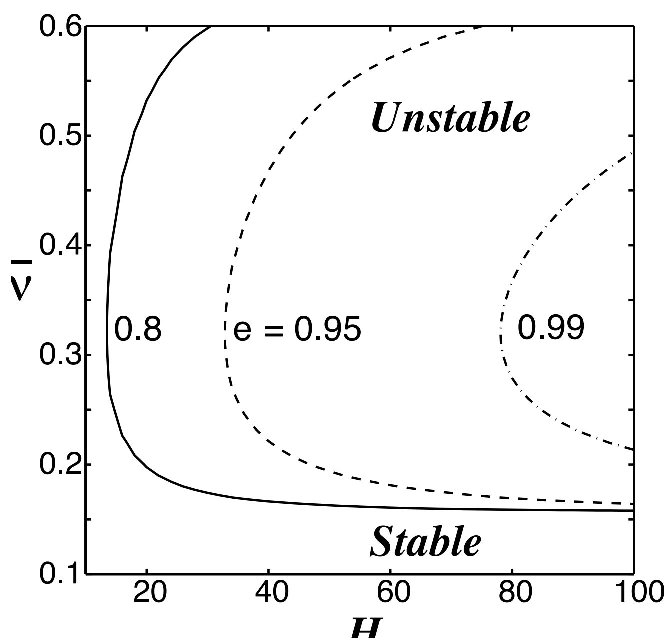
Pitchfork Bifurcation

♣ Bifurcation Loci:

$$H_n^c = n\pi\psi(\nu, e) \sim (1 - e^2)^{-1/2}$$

$\Downarrow$ $n = 1, 2, 3 \ldots$

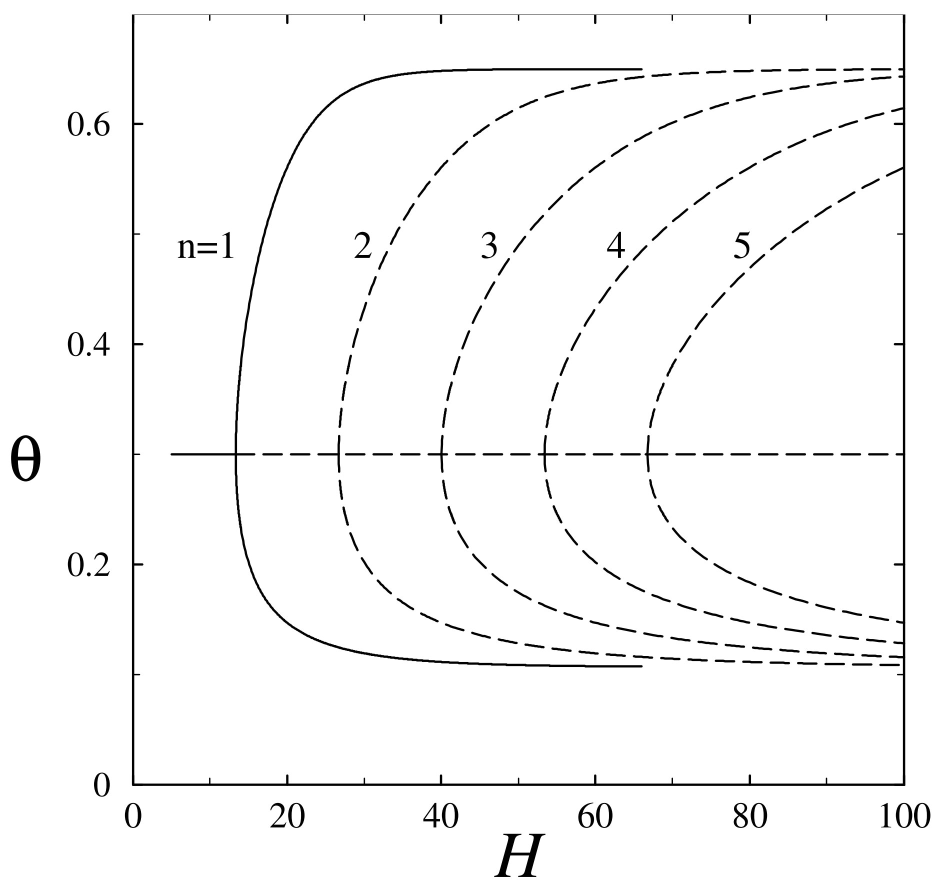
@ Infinite-hierarchy of BPs: $H_1^c, H_2^c, \ldots$



Instability and Bifurcation

♣ **Supercritical Bifurcation** $(\overline{\nu} = 0.3, \quad e = 0.8)$

@ $\Phi \equiv \nu(1/2) =$ **Density at Top Wall**



♣ **Stationary Instability**

$\Downarrow$

♣ **Pitchfork Bifurcations:** $\frac{d\Phi}{dt} = \alpha_3 \Phi^3 + (H - H_n^c)\Phi$

$\Downarrow$

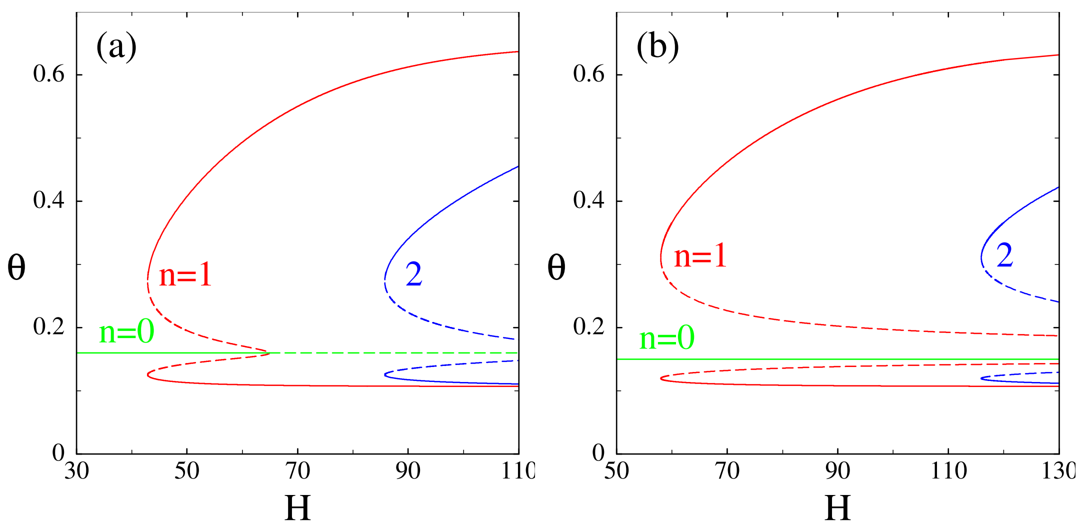
♣ **Infinite-hierarchy of Solutions**

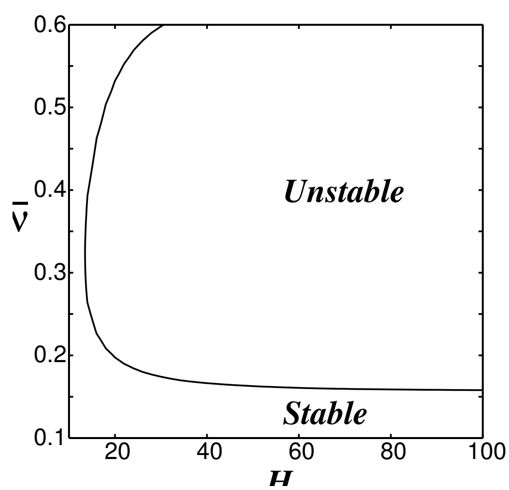
$(n = 1, 2, 3, \ldots)$

Instability and Bifurcation

♣ Subcritical Bifurcations

$\overline{\nu} = 0.16$ $\overline{\nu} = 0.15$





♣ Supercritical Bifurcation (large $\overline{\nu}$)

$\Downarrow$

♣ Subcritical Bifurcation (moderate $\overline{\nu}$)

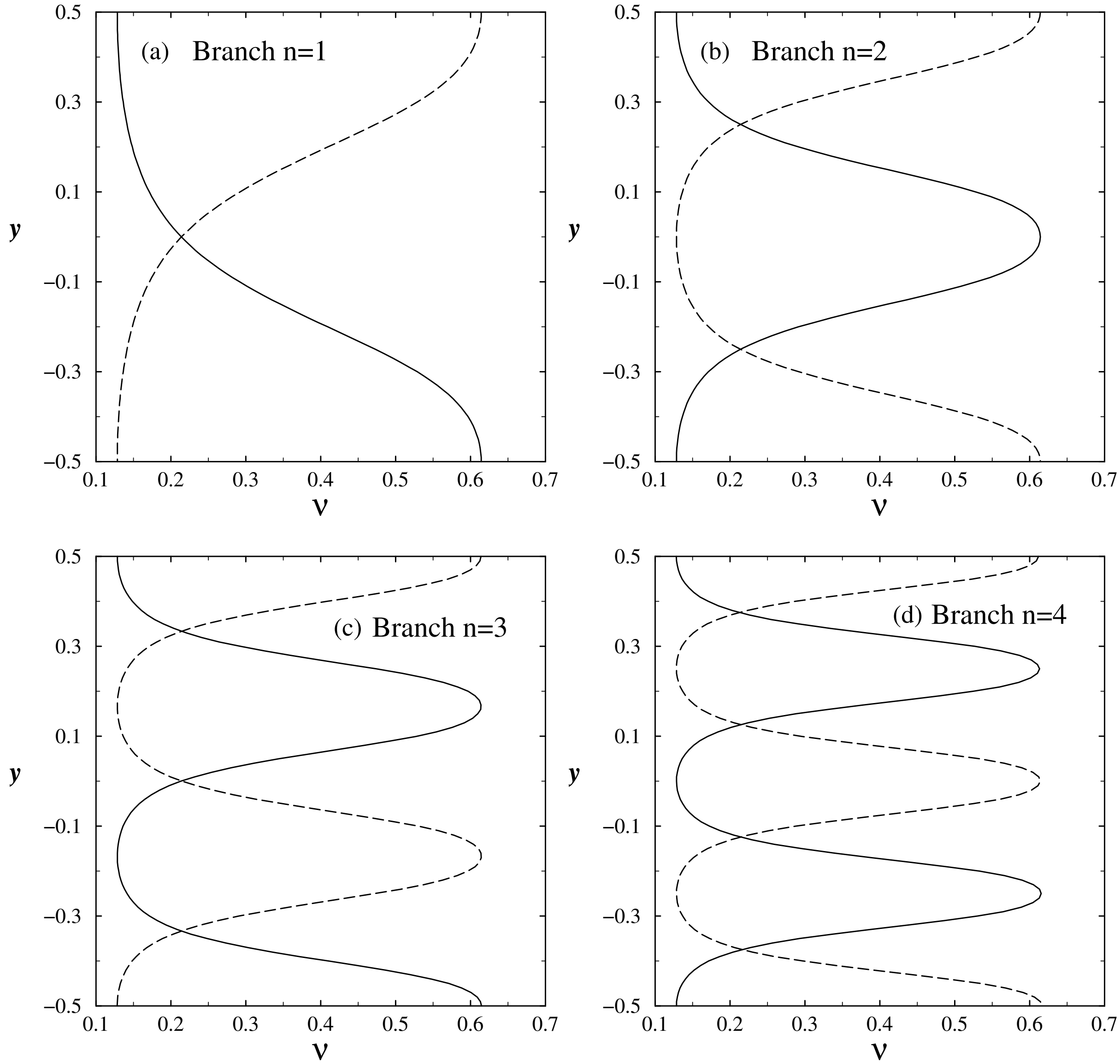
$\Downarrow$

♣ Bifurcation from Infinity (low $\overline{\nu}$)

(Rosenbluth & Davis, 1979, SIAM)

Instability and Bifurcation

♣ Density Profiles



♣ **Segregation in transverse direction**

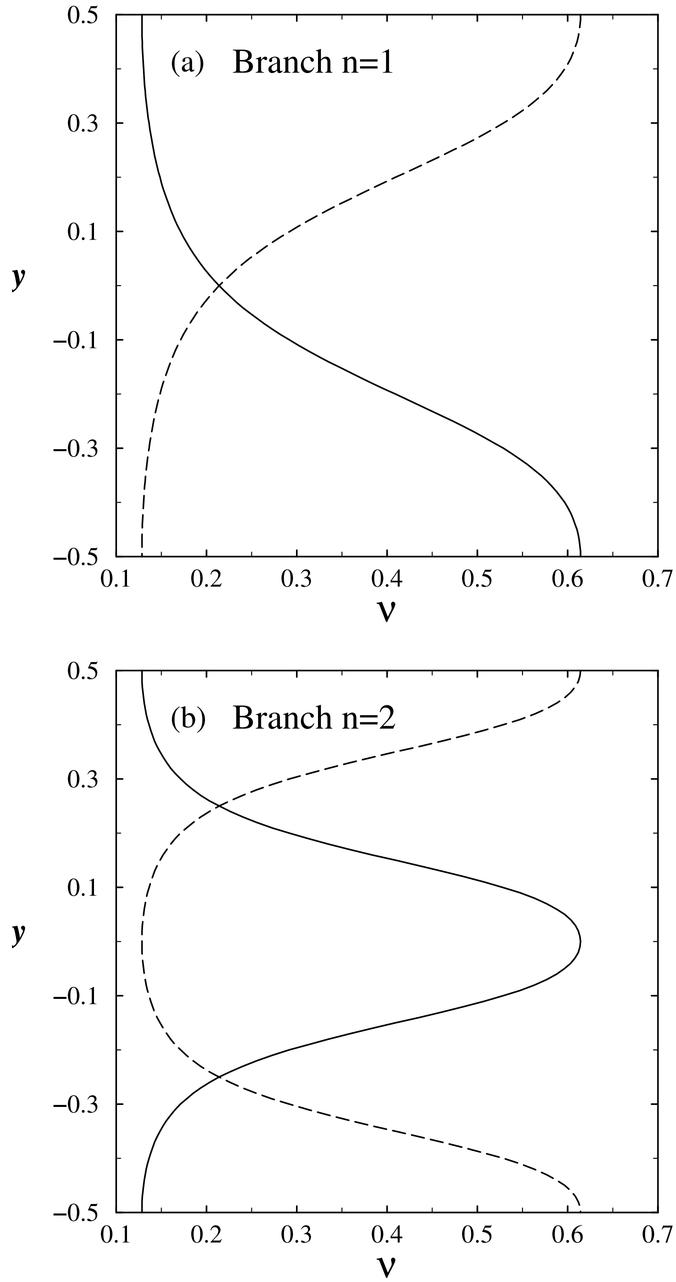
⇓

♣ **Shear-banding Patterns**

♣ **Expt. of Glasser etal. (2006): Central Plug?**

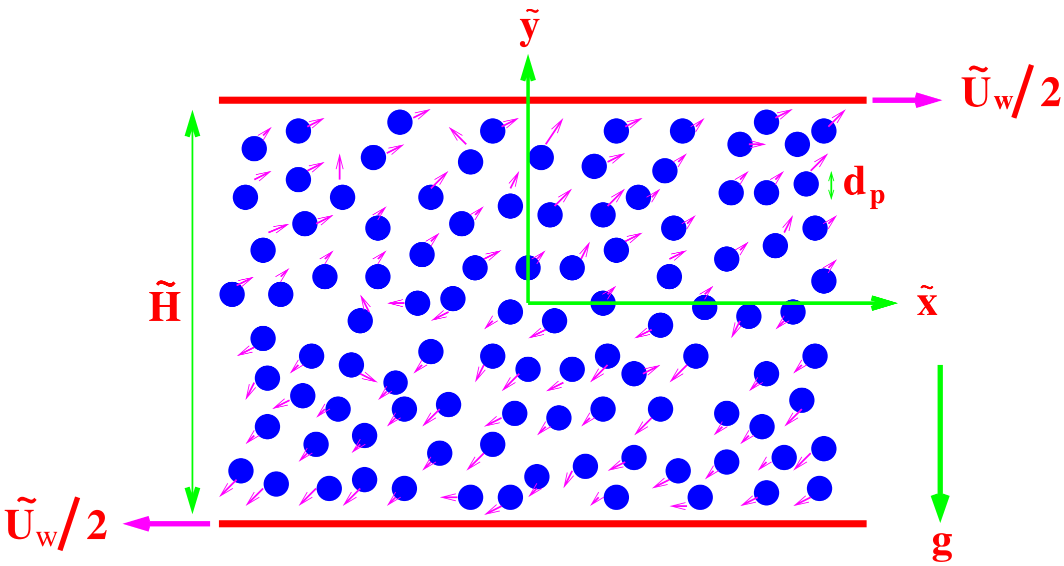
Instability and Ordering Transition

Shearbanding Instability⟺Ordering Transition



- ♣ **Shear-rate, $\gamma(y)$, is "non-uniform" across y**
- ♣ **γ "small" in dense zone and "large" in dilute zone**

Plane Couette Flow (with gravity)



♣ Control Parameters

@ **Mean volume fraction of particles:** $\nu = V_p N/V$

@ **Wall separation:** $H = \tilde{H}/d_p$

@ **Restitution coefficient:** $0 < e < 1$

@ **Froude number:** $Fr = \tilde{U}_w/\sqrt{g d_p} = \dfrac{\tau_g}{\gamma^{-1}}$

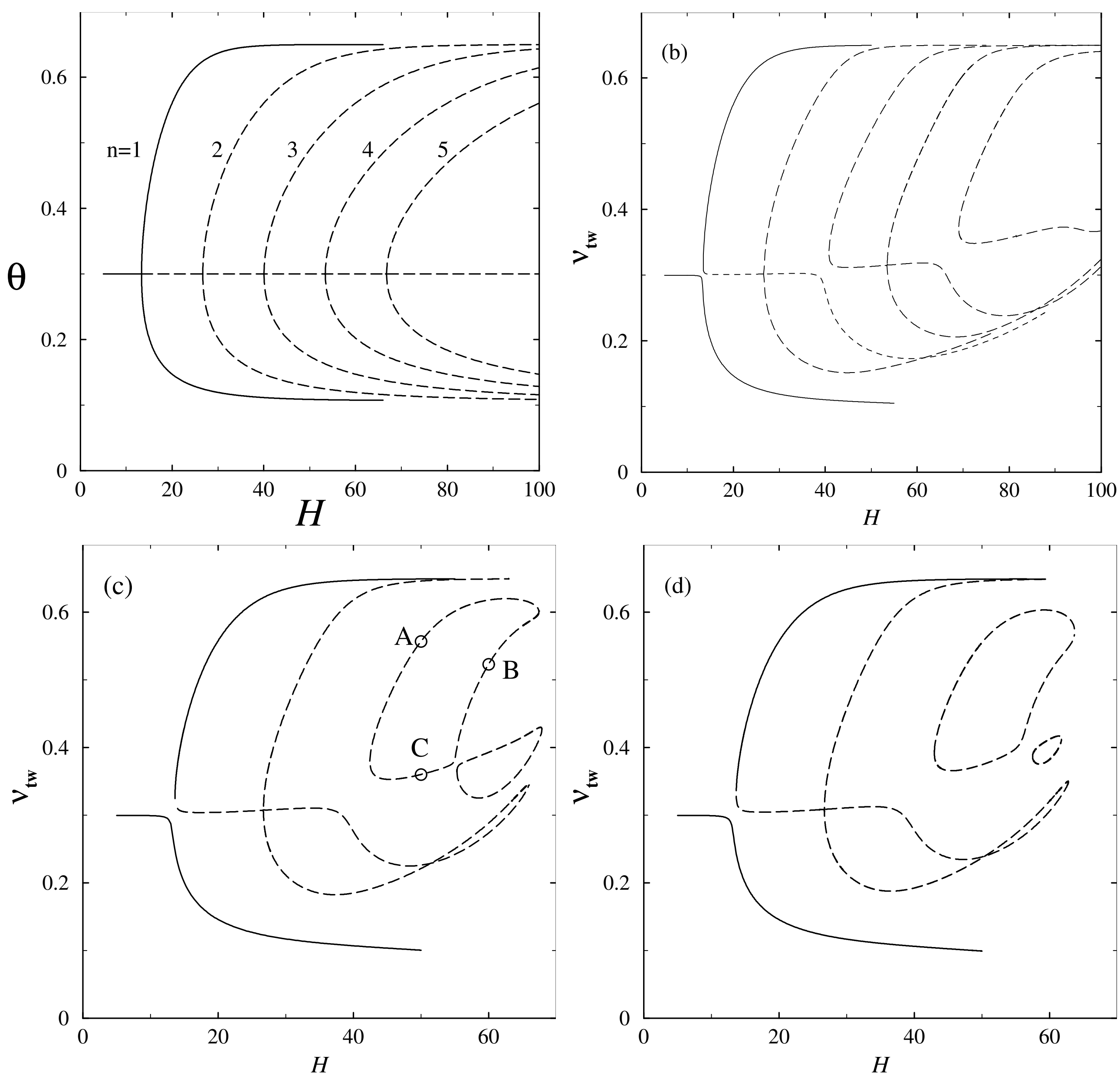
- $\tau_g = H(d_p/g)^{1/2}$
- $\gamma = \tilde{U}_w/\tilde{H}$

@ $\boxed{g = 0} \Rightarrow \boxed{Fr^{-1} = 0 \quad \text{or} \quad Fr = \infty} \Leftarrow \boxed{\gamma = \infty}$

@ $\boxed{g > 0} \Rightarrow \boxed{Fr^{-1} > 0 \quad \text{or} \quad Fr < \infty}$

Bifurcation with Gravity

♣ Birth and Death of “Isola”

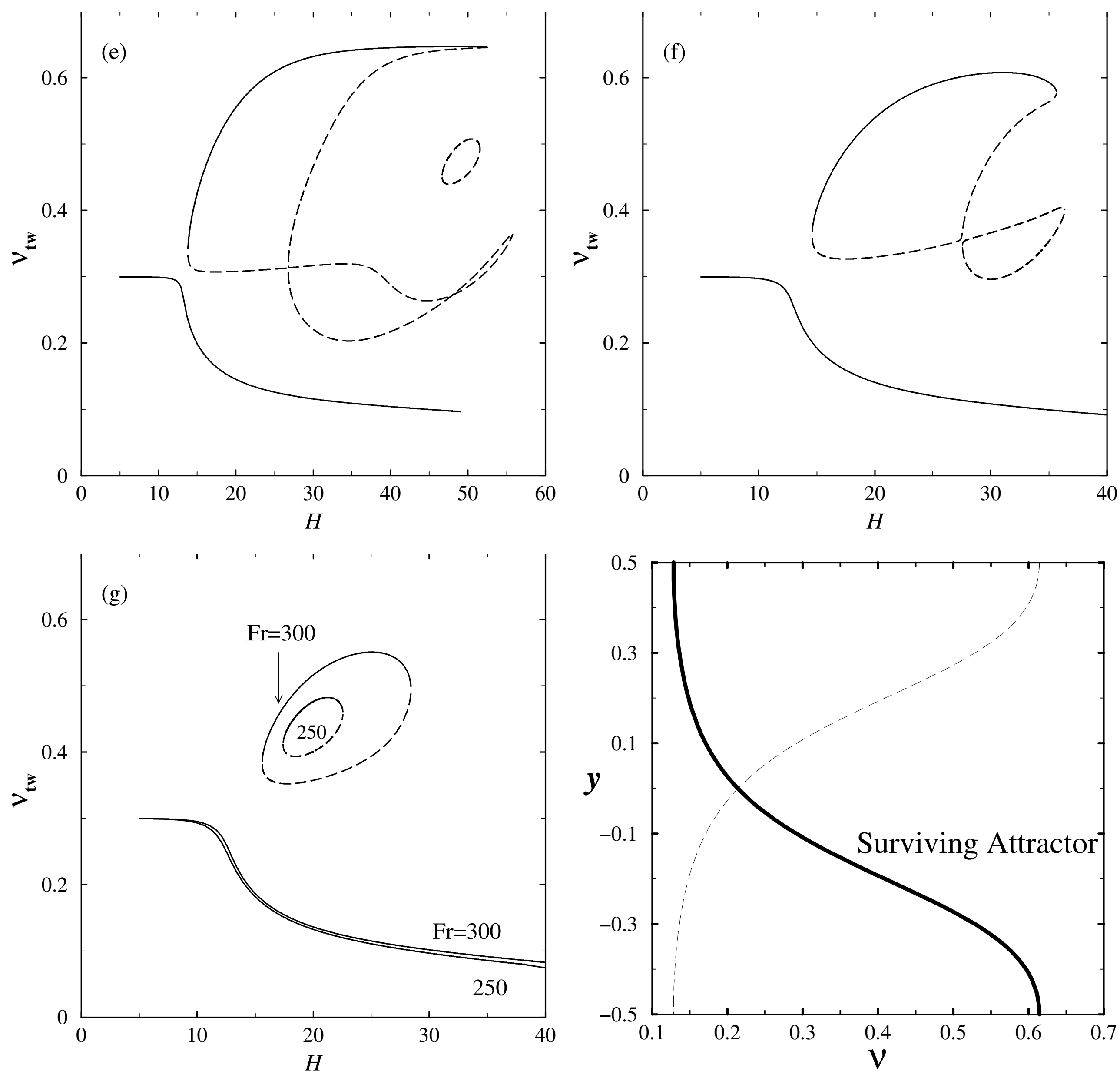


♣ $\overline{\nu} = 0.3$; $e = 0.8$

- **(a)** $Fr = \infty$; **(b)** 2000, **(c)** 1000, **(d)** 922

Bifurcation with Gravity

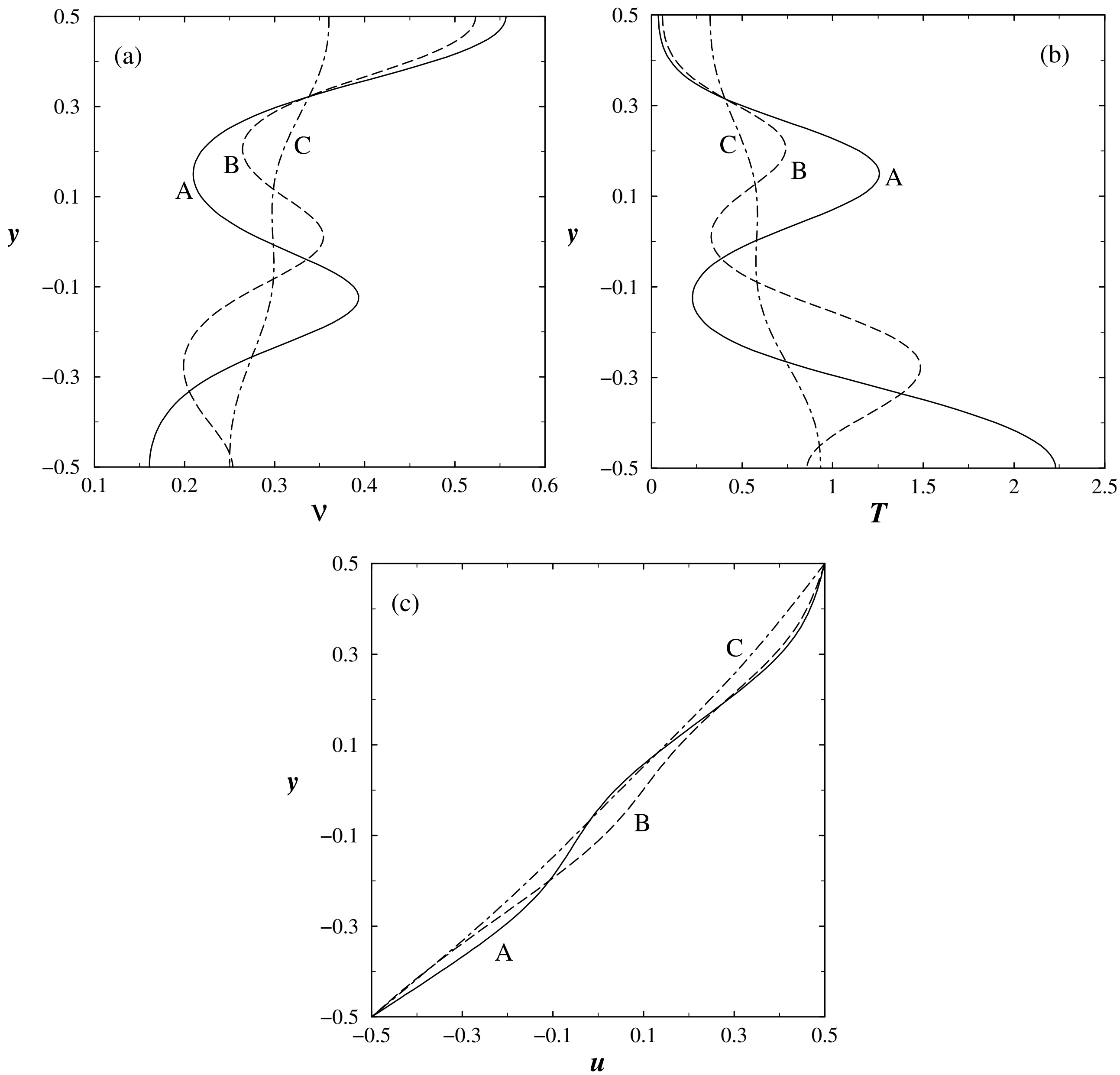
♣ From "Isola" to "Surviving Attractor" (SA)



- **(e)** $Fr = 750$; **(f)** 400; (g) $300,\ 250$

Bifurcation with Gravity

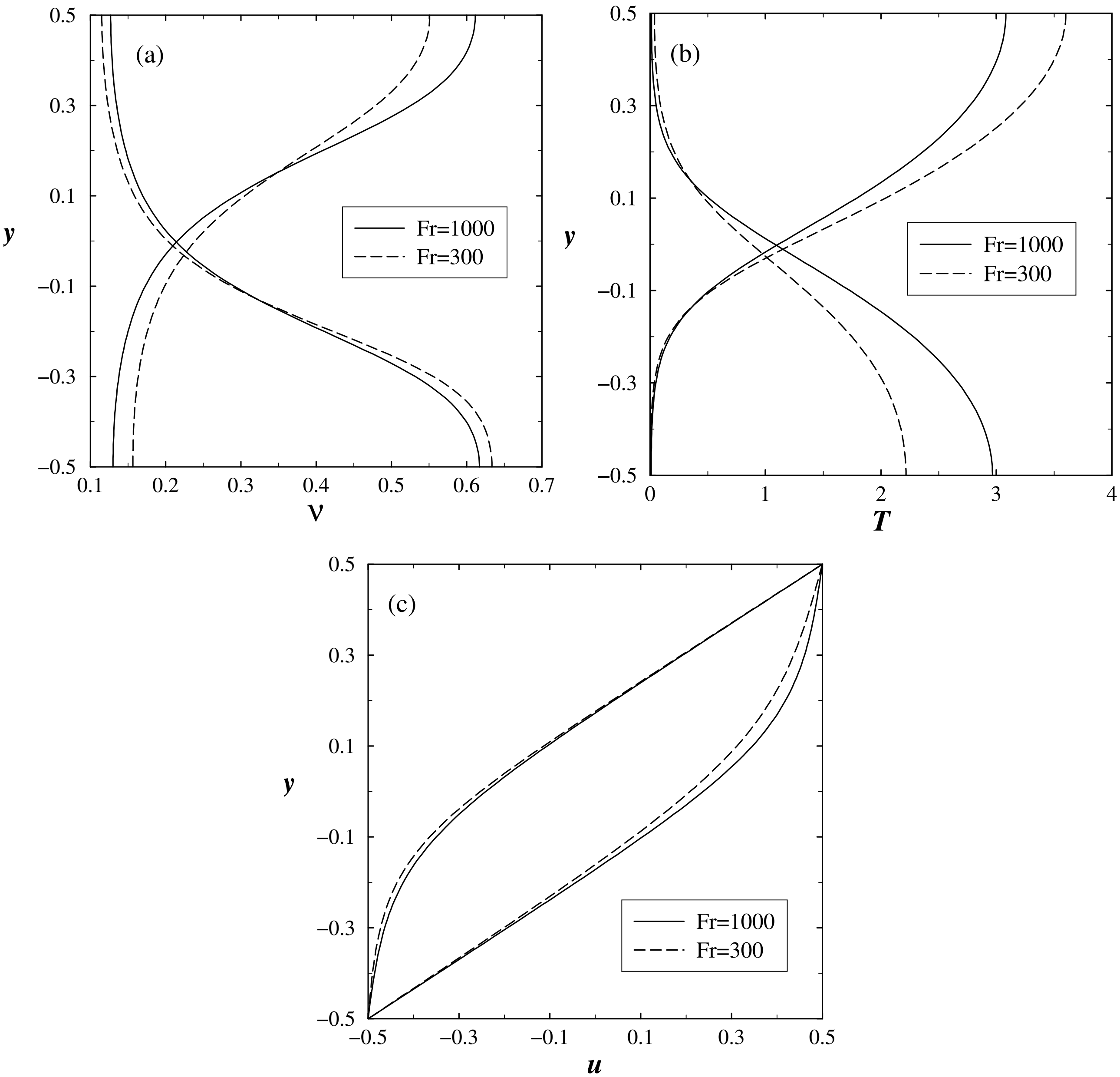
♣ Solution Profiles for an Isola:



- $\overline{\nu} = 0.3$; $e = 0.8$; $Fr = 1000$

Bifurcation with Gravity

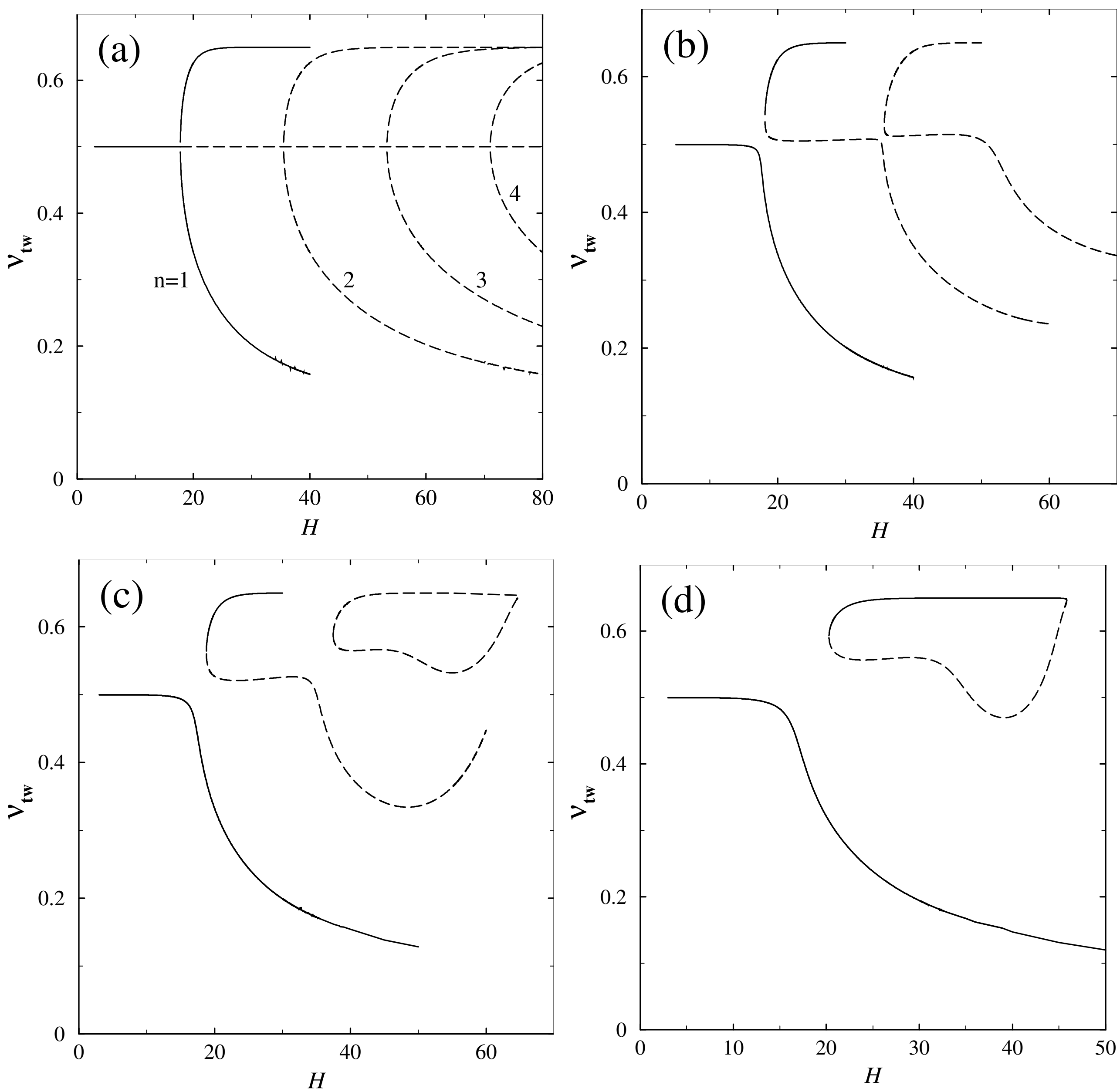
♣ Asymmetric Solutions:



- $\overline{\nu} = 0.3$; $e = 0.8$; $H = 25$

Bifurcation with Gravity

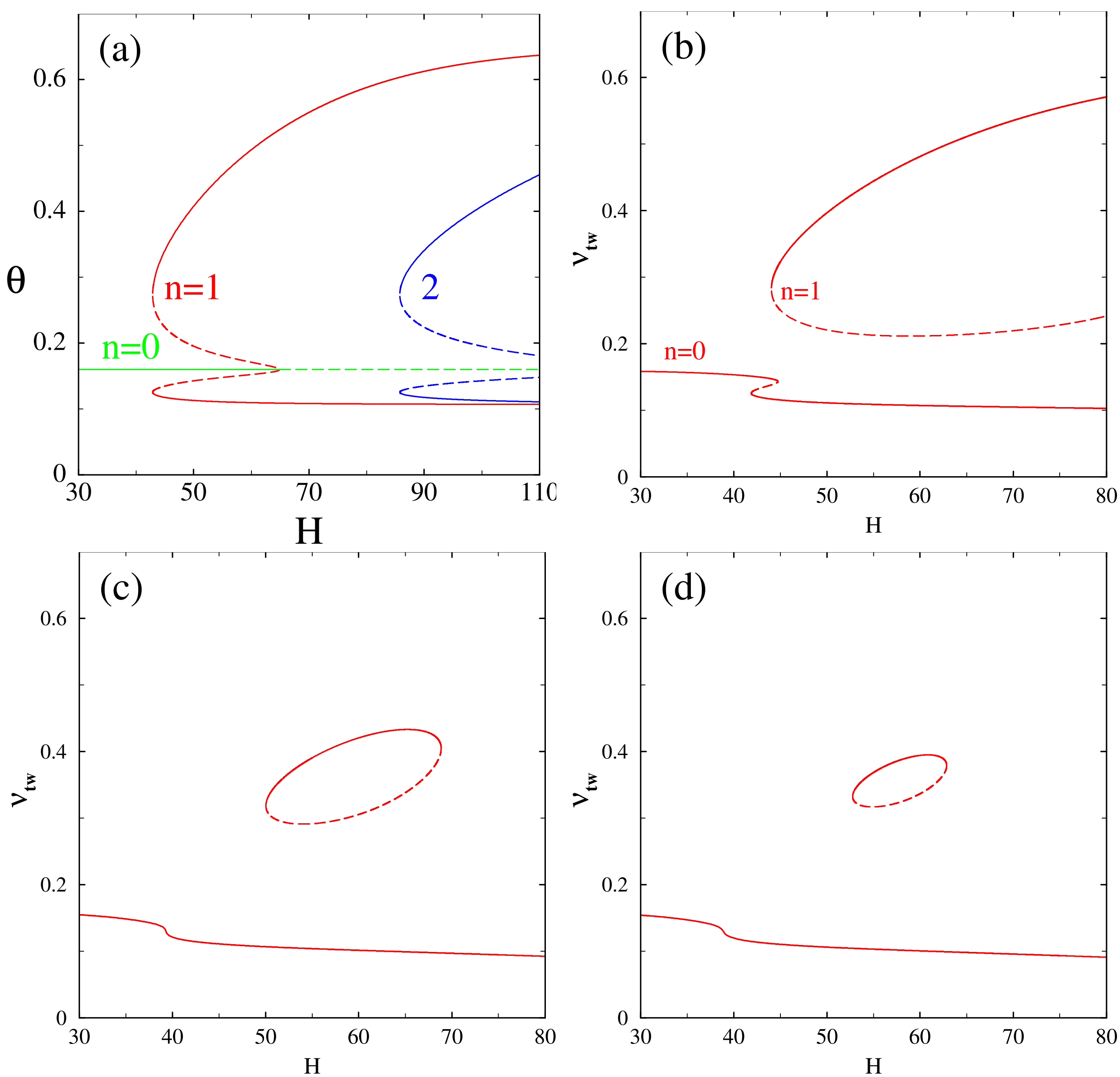
♣Dense Flows ($\overline{\nu} = 0.5$)



- **(a)** $Fr = \infty$; **(b)** 1000; **(c)** 500; **(d)** 300

Bifurcation with Gravity

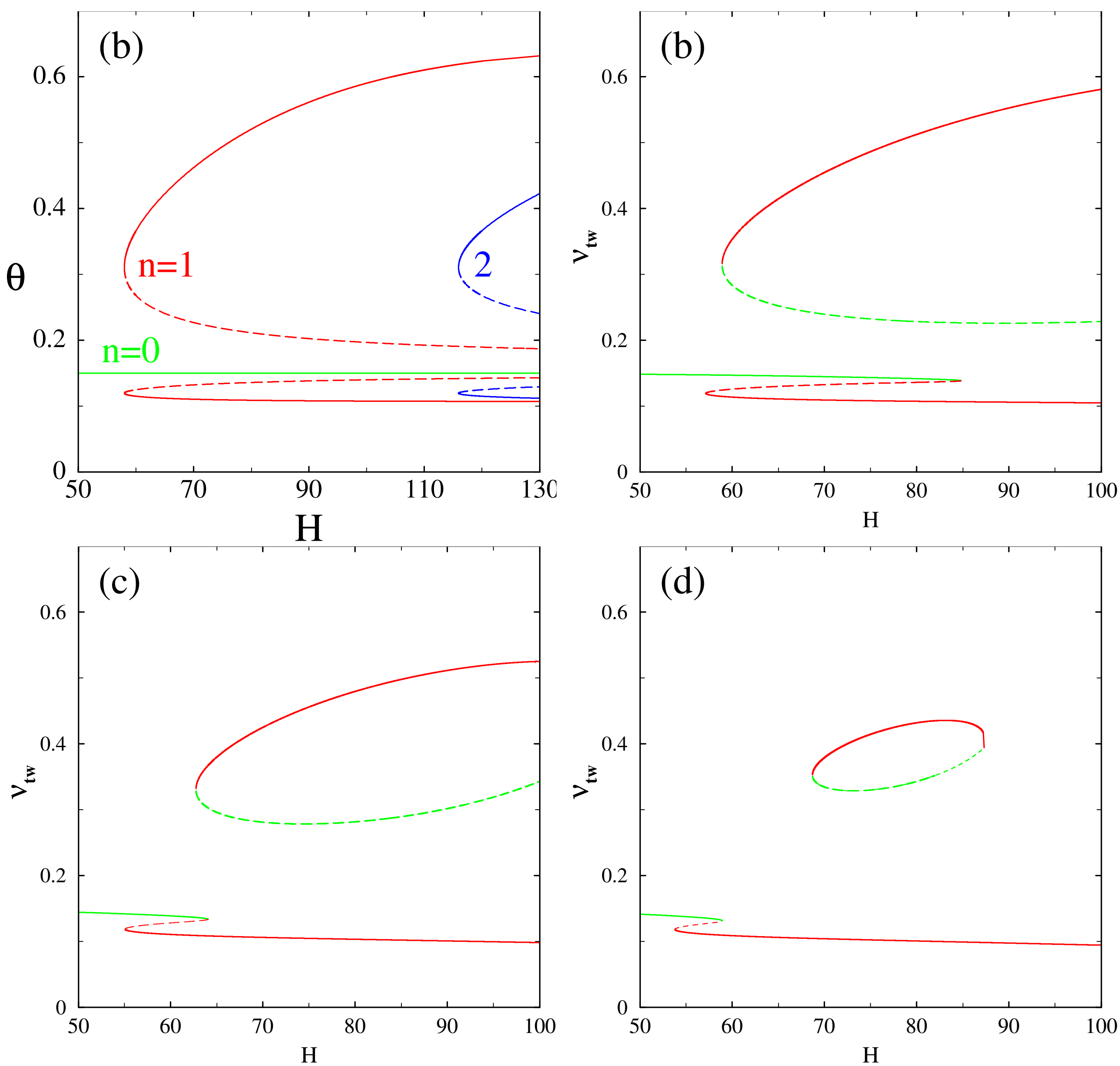
♣Subcritical Bifurcation ($\overline{\nu} = 0.16$)



- **(a)** $Fr = \infty$; **(b)** 5000; **(c)** 2500; **(d)** 2350

Bifurcation with Gravity

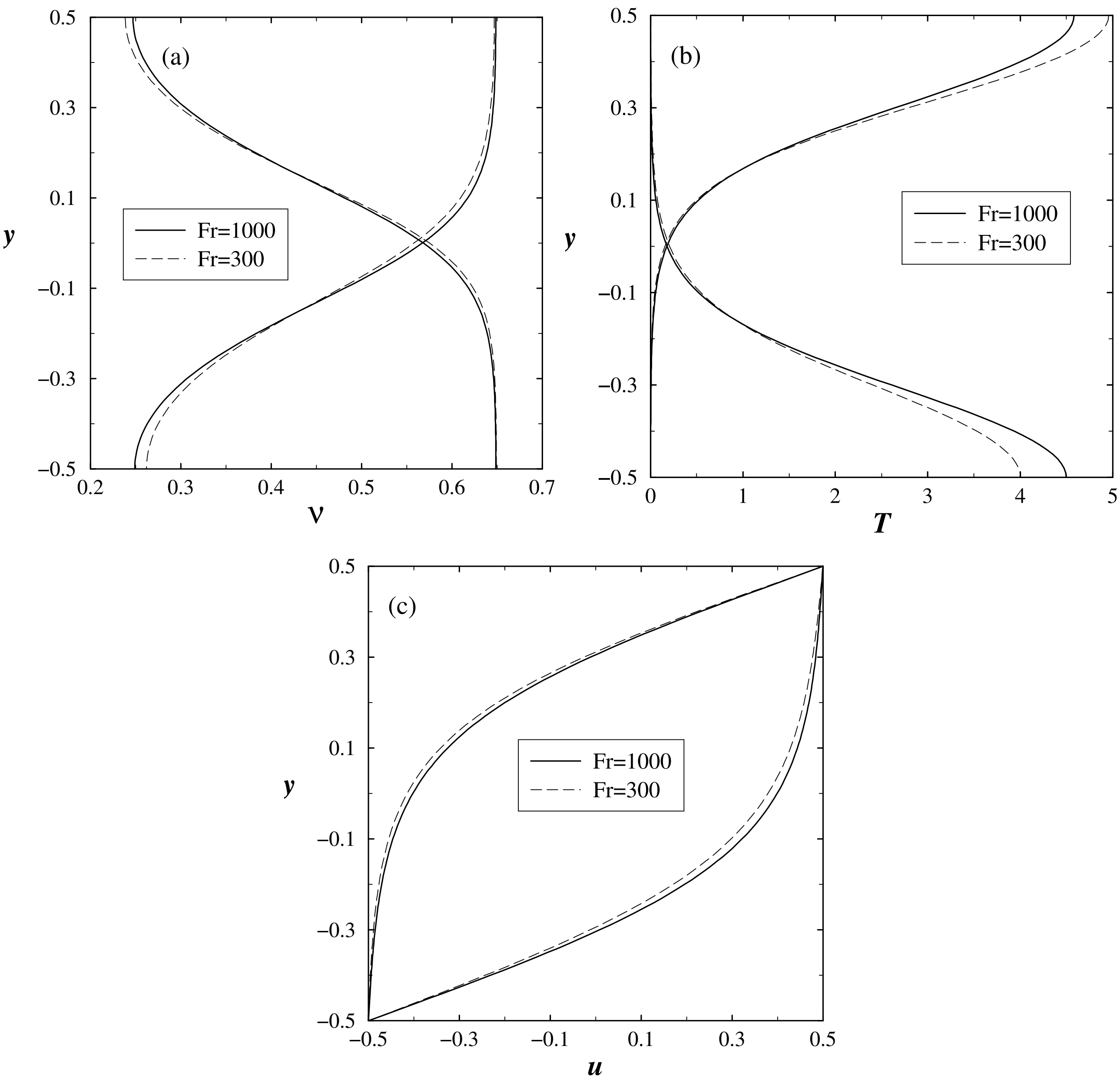
♣Bifurcation from Infinity ($\bar{\nu} = 0.15$)



- **(a)** $Fr = \infty$; **(b)** 10000; (c) 5000; (d) 4000

Bifurcation with Gravity

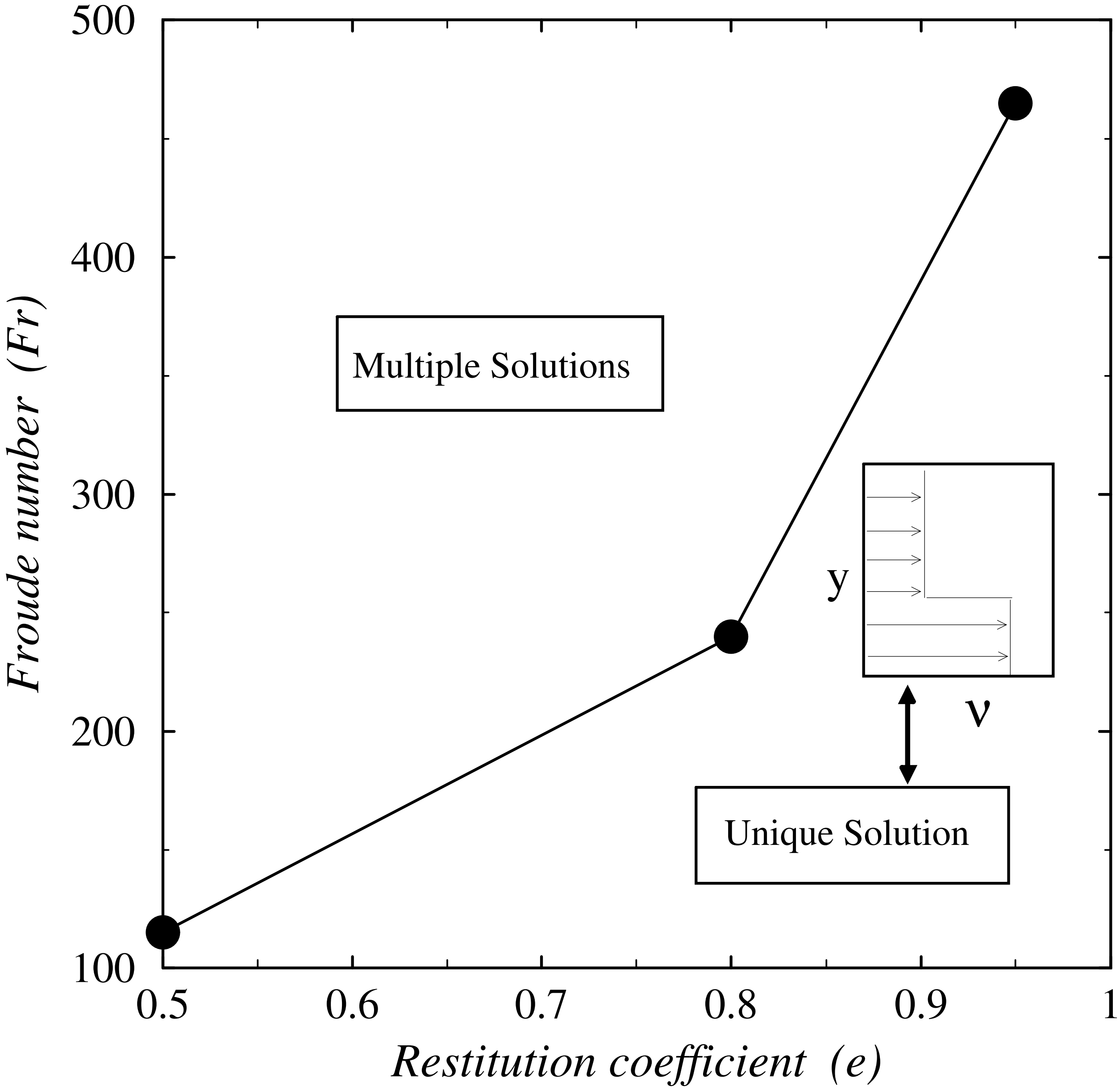
♣ Asymmetric Solutions:



- $\overline{\nu} = 0.5$; $e = 0.8$; $H = 25$

Bifurcation with Gravity

♣ **Phase Diagram:** $(\bar{\nu} = 0.3)$



@ **Froude number:** $Fr = \tilde{U}_w/\sqrt{gd_p} = \dfrac{\tau_{\mathbf{g}}}{\gamma^{-1}}$

- $\tau_g = H(d_p/g)^{1/2}$
- $\gamma = \tilde{U}_w/\tilde{H}$

“Universal Unfolding” of Pitchfork

♣ **Normal-form Equation:**

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \Phi^3 - \mathcal{B}\Phi + \alpha + \beta\Phi^2 = F(\Phi, \mathcal{B}; \alpha, \beta)$$

where

@ **Order parameter:** $\Phi \equiv \nu_{tw}$ **or** $(\nu_{tw} - \overline{\nu})$

@ **Bifurcation parameter:** $\mathcal{B} \equiv \mathcal{B}(H, e, \overline{\nu})$

@ **Imperfections:**

$$\alpha \equiv \alpha(Fr^{-1}, \mathcal{B}, \overline{\nu}) \quad \textbf{and} \quad \beta \equiv \beta(Fr^{-1}, \mathcal{B}, \overline{\nu})$$

with the following property

- $\alpha(0, H, \overline{\nu}) = 0 = \beta(0, H, \overline{\nu})$

@ **Ideal Pitchfork Bifurcation:**

- $F(\Phi, \mathcal{B}; 0, 0) \equiv f(\Phi, \mathcal{B})$.

$\Downarrow$

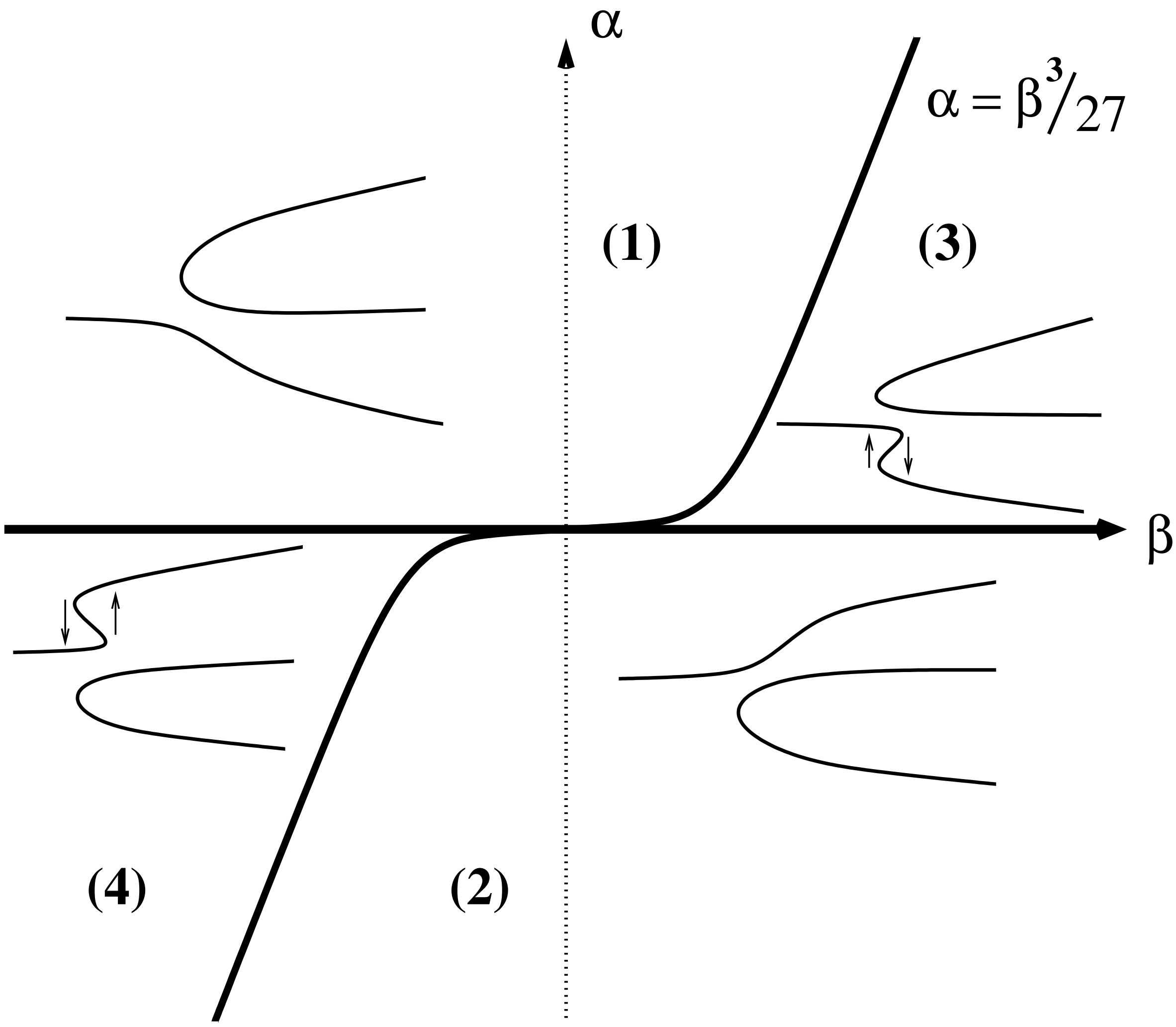
Think of $F(\Phi, \mathcal{B}; \alpha, \beta)$ **as a perturbation of** $f(\Phi, \mathcal{B})$

$\Downarrow$

$F(\Phi, \mathcal{B}; \alpha, \beta)$ **is called** **“Universal Unfolding”** **of** $f(\Phi, \mathcal{B})$

Universal Unfolding of Pitchfork

(Golubitsky & Schaeffer 1985)



♣ **Granular Plane Couette Flow shows <u>all</u> possible forms of Imperfect Pitchfork Bifurcations**

$\Downarrow$

gPCF admits Universal Unfolding of PB

(Alam, JFM, 2005, vol. 523)

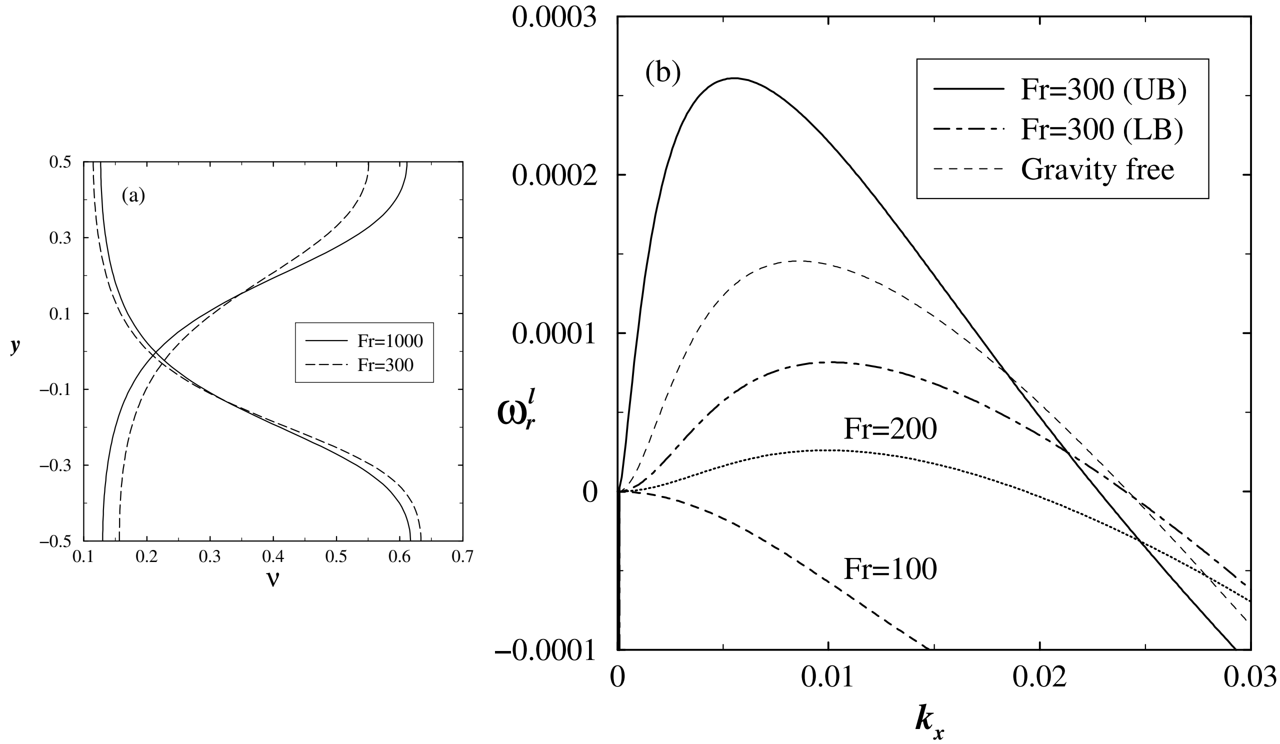
$\Uparrow$**?**

Derive Normal-Form for gPCF (Shukla & Alam 2013)?

Stability of Shear-banded Solutions

♣ **Growth Rate vs.** k_x

• $\overline{\nu} = 0.3$; $e = 0.8$; $H = 25$



@ **Froude number:** $Fr = \tilde{U}_w/\sqrt{g d_p} = \frac{\tau_g}{\gamma^{-1}}$

• $\tau_g = H(d_p/g)^{1/2}$ • $\gamma = \tilde{U}_w/\tilde{H}$

♣ **Floating plugs unstable to longwaves** $(k_x \sim 0)$**!**

♣ **SA (with bottom plug) is stable for all** k_x.

Conclusions

♣ **MD simulations of plane Couette flow**

@ **Shear-banding/Ordering Transition**

@ **Smaller systems inhibit pattern formation**

♣ **Newtonian Hydrodynamic Model**

@ **Uniform Shear** ($g = 0$) $+$ **Perturbations**

$\Downarrow$ **Shearbanding Instability**($k_x = 0$)

Pitchfork Bifurcation and Multiple Solutions

$\Downarrow$

Nonlinear Shearbanding Solutions

@ **Role of Gravity** ($g \neq 0$)

$\Downarrow$

Imperfection, Hysteresis, Isola, ...

$\Downarrow$

Universal Unfolding

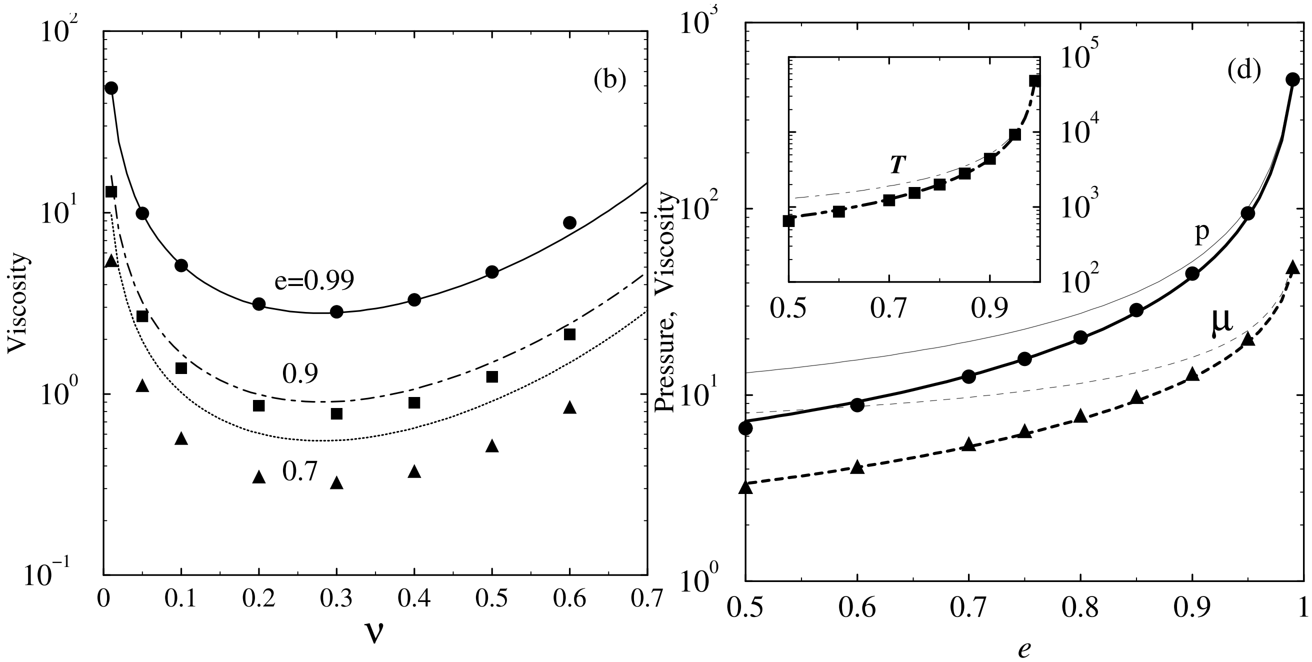
@ **Floating Plugs under Microgravity**

@ **Isolas disappear at small** Fr

@ **Surviving Attractor (SA) with a bottom-plug**

@ **Agreement with earth-bound experiments**

Return to NS-order Rheology..



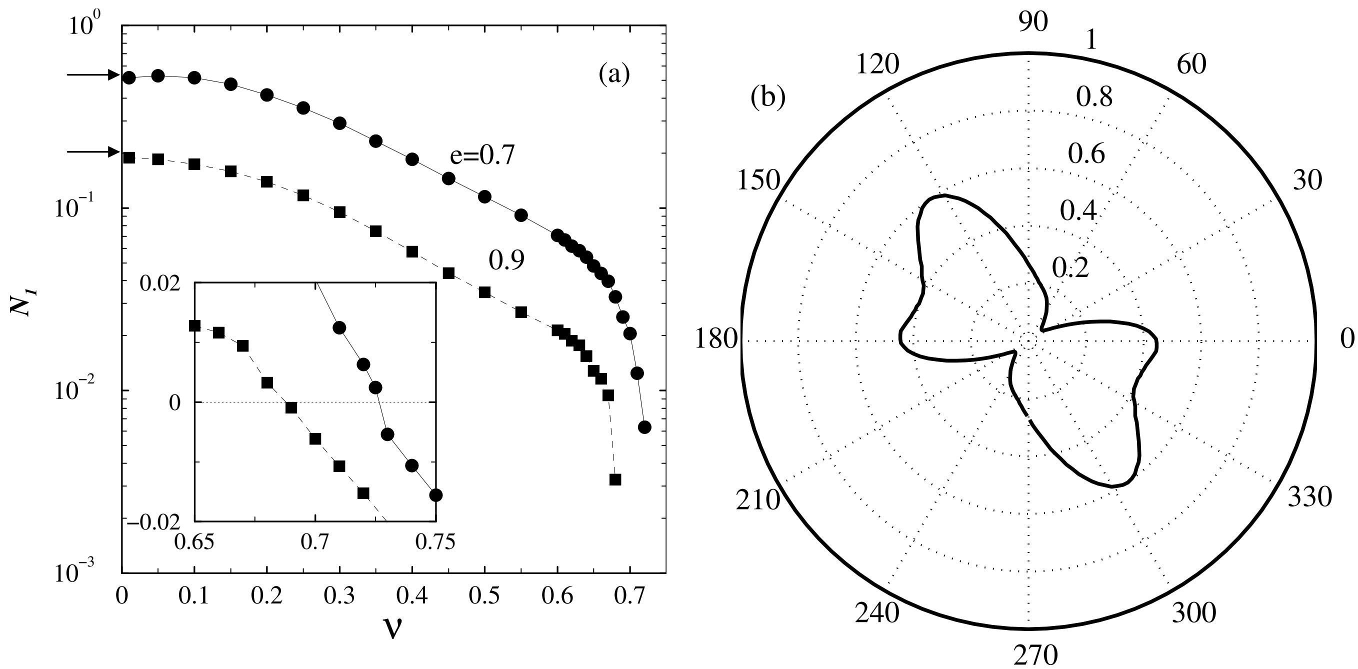
- **Stress Tensor:** $\mathbf{P} = -p\mathbf{I} + \Pi$
- **Viscosity:** $\mu = \Pi_{xy}/(du/dy)$
- $\mathcal{N}_1 = (\Pi_{xx} - \Pi_{yy})/p \neq 0$**,** $\mathcal{N}_2 = (\Pi_{yy} - \Pi_{zz})/p \neq 0$

- **Disagreement due to "measurable" normal stress differences**

- **KT-models are good for nearly elastic systems** ($e \sim 1$)

  - *Jenkins & Richman (J. Fluid Mech 1988)*
  - *Alam & Luding (J. Fluid Mech, 2003)*

Non-Newtonian Rheology



- **Large Normal Stresses**
- **Sign-reversals of $\mathcal{N}_1$ (AL2003) and $\mathcal{N}_2$ (AL 2005)**
- **Connection of $\mathcal{N}_1$ with 'microstructure' (right plot)**
- **Fabric tensor matters in dense limit!**
- **KT Models violate objectivity principle?**

@ Jeffrey's Model (2 parameters): would it hold?

$$\mathbf{\Pi} + \tau_1(\nu, e)\frac{\mathcal{D}\mathbf{\Pi}}{\mathcal{D}t} = -2\mu\left(\mathbf{S} + \frac{\lambda}{2\mu}(\nabla\cdot\mathbf{u})\,\mathbf{1} + \tau_2(\nu, e)\frac{\mathcal{D}\mathbf{S}}{\mathcal{D}t}\right)$$

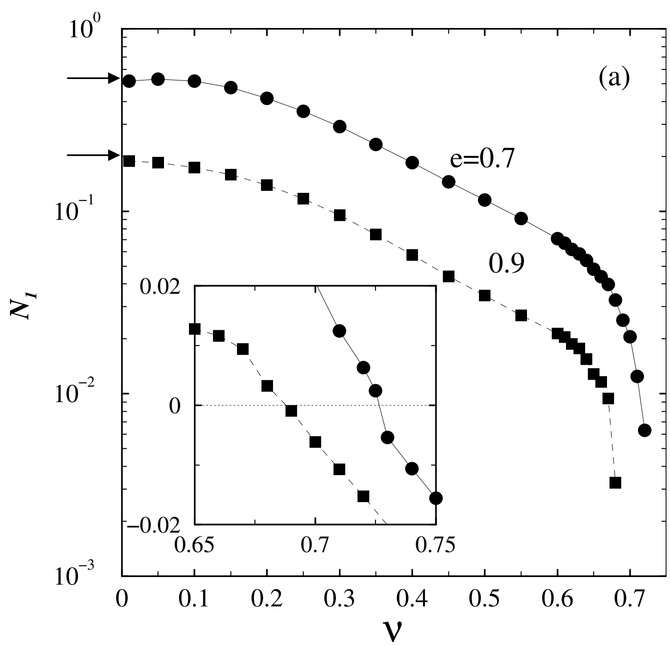
- *Alam & Luding (Physics of Fluids, 2003, 2005)*
- *Alam & Luding (Powders and Grains, 2005)*

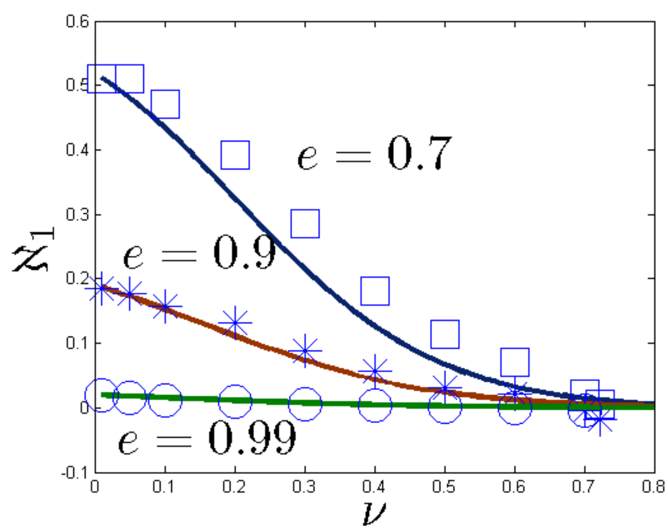
- *Weinhart, Hartkamp, ..., Luding (PoF, JCP, 2013 ...): 4-parameter model!*

Non-Newtonian Rheology

(Grad's 13-Moment Theory, Jenkins & Richman 1988)

Can we predict 'NSD' quantitatively?





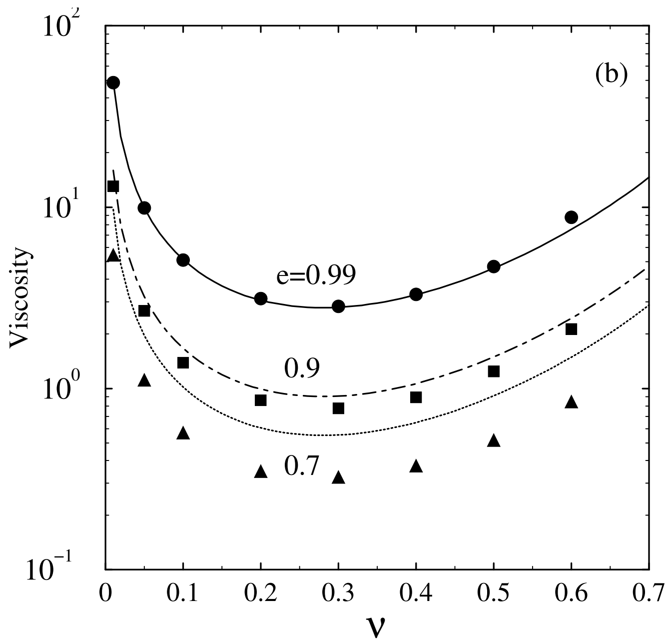
- **Yes upto freezing density, via 'Grad-level' higher-order theory!**
- **'Sign-reversal' of 1st 'NSD' beyond freezing remains illusive!**
- **Above issue is connected with 'Fabric' tensor (beyond KT!)**

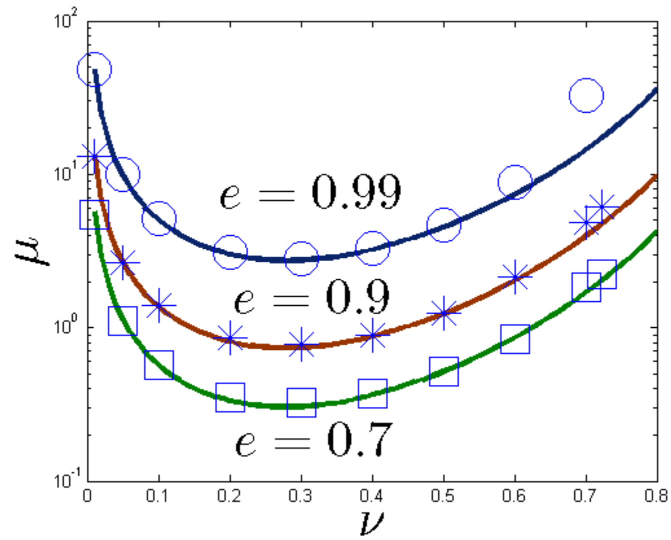
- *Jenkins & Richman (J. Fluid Mech 1988)*

Non-Newtonian Rheology

(Grad's 13-Moment Theory, Jenkins & Richman 1988)

Quantitative prediction of other transport coefficients?





- **Excellent agreement for μ, p and T, even at $e = 0.5$!**
- **Saha & Alam (2013, preprint)**

- **How simple is 14-moment theory?**

Non-Newtonian Rheology

(14-Moment Theory for Granular Fluid)

Balances of mass (1 eqn.) and momentum (3 equations) + Granular Energy ⇒ NS-order Hydrodynamics

Balance of second moment (6 equations):

$$\rho \dot{K}_{\alpha\beta} = -2Q_{\gamma\alpha\beta,\gamma} - P_{\mu\beta}u_{\alpha,\mu} - P_{\mu\alpha}u_{\beta,\mu} + \aleph_{\alpha\beta} \tag{3}$$

where K $\equiv <$ **CC** $>$ **and** $Q_\alpha = \frac{1}{2}\rho M_{\alpha\beta\beta} + \frac{1}{2}\Theta_{\alpha\beta\beta}$.

Balance of 'contracted' third moment (3 equations):

$$\rho \dot{M}_{\alpha\beta\beta} + 2Q_{n\alpha\beta\beta,n} - 3M_{(\alpha\beta}P_{\beta)n,n} + 6Q_{n(\alpha\beta}u_{\beta),n} = \aleph_{\alpha\beta\beta} \tag{4}$$

+ Balance equation for fully contracted fourth moment (Required to obtain 'correct' form of NS-order heat-flux via 'Maxwell-Iteration'! Marqueres & Kremer 2011)

⇓

14-Moment Theory!

⇓

Transport coefficients from Series expansion?

Frictional Granular Matter

♣ **Particles are rough and frictional**

♣ **Rotary inertia is important**

(Dahler, Condiff 1960; Kanatani 1979)

@ Additional Hydrodynamic Fields

- $$\Omega(\mathbf{x},t) = <\boldsymbol{\omega}> = \frac{1}{n}\int \boldsymbol{\omega} f^{(1)}(\mathbf{z},\mathbf{c},\boldsymbol{\omega};t)\mathrm{d}\mathbf{c}\mathrm{d}\boldsymbol{\omega}$$

- $$\theta(\mathbf{x},t) = \frac{I}{md} < (\boldsymbol{\omega}-\boldsymbol{\Omega})\cdot(\boldsymbol{\omega}-\boldsymbol{\Omega}) >$$

@ Angular Momentum Equation

$$nI\frac{D\boldsymbol{\Omega}}{Dt} = -\boldsymbol{\nabla}\cdot\mathbf{L} + \boldsymbol{\Psi}$$

- $$\boldsymbol{\Psi} = -2\mu_r\left(2\boldsymbol{\Omega} - \boldsymbol{\nabla}\times\mathbf{u}\right) \neq 0\,!$$

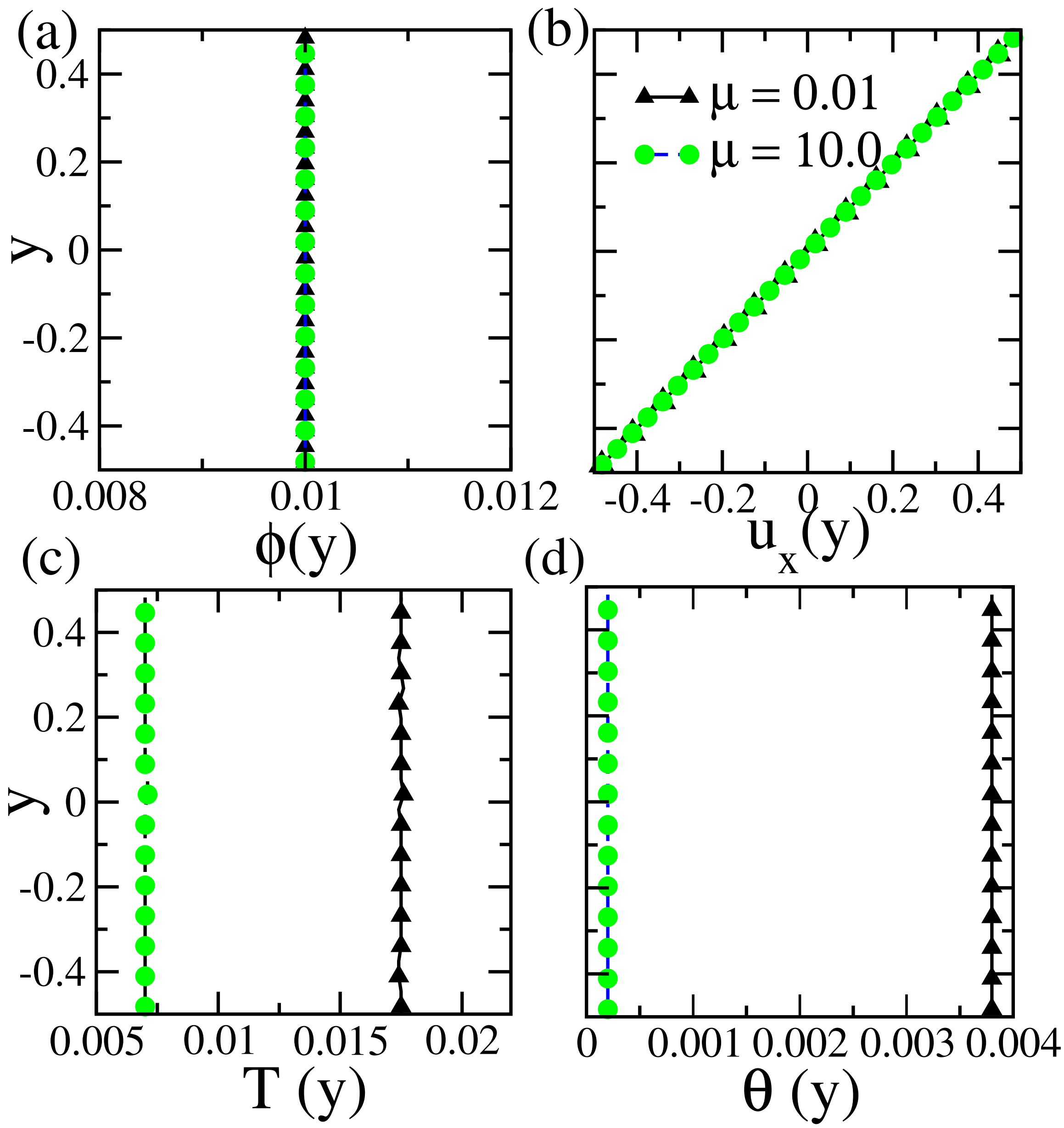
@ Rheological Model

- *Towards a Micropolar Continuum (Eringen 1964; Kanatani 1979; Mitarai, Hayakawa & Nakanishi 2002)*

Frictional Granular Matter

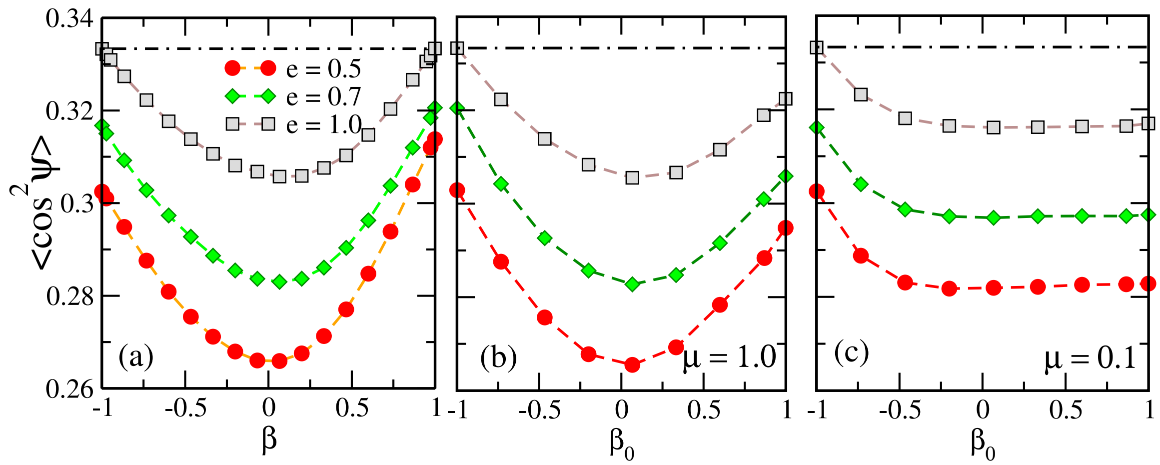
- *Instabilities in Shear Flow (Gayen & Alam, JFM, 2006)*
- *Exchange of energy between translational and rotational modes*

♣ Uniform Shear Flow (with Coulomb friction)



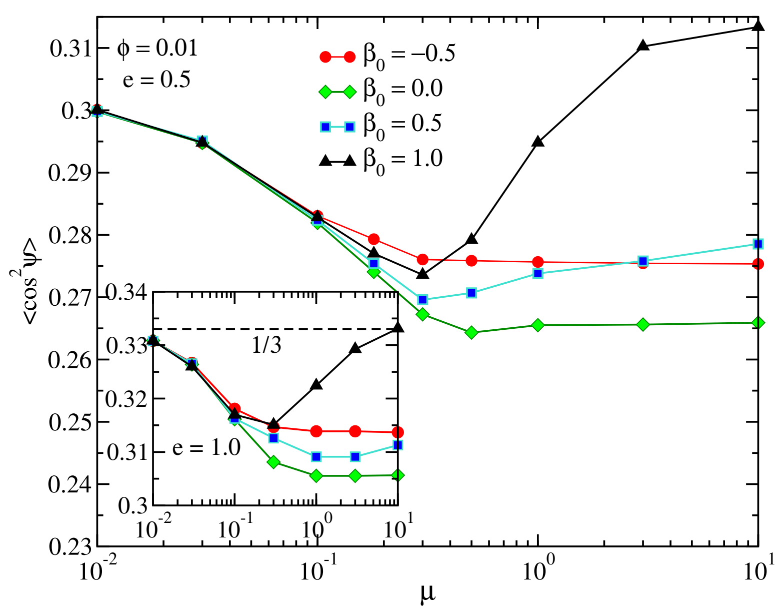
Frictional Granular Matter

♣ Orientational Correlation: Effect of friction



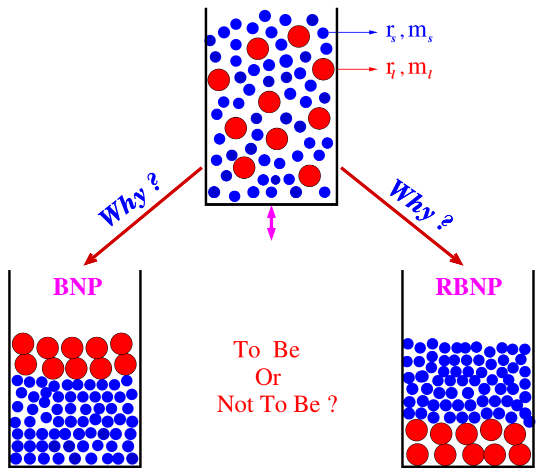
$$\Lambda(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{(C_i\cdot\Omega_i)^2}{(C_i^2\Omega_i^2)} = \frac{1}{N}\sum_{i=1}^{N}\cos^2\Psi_i \equiv \cos^2\Psi.$$

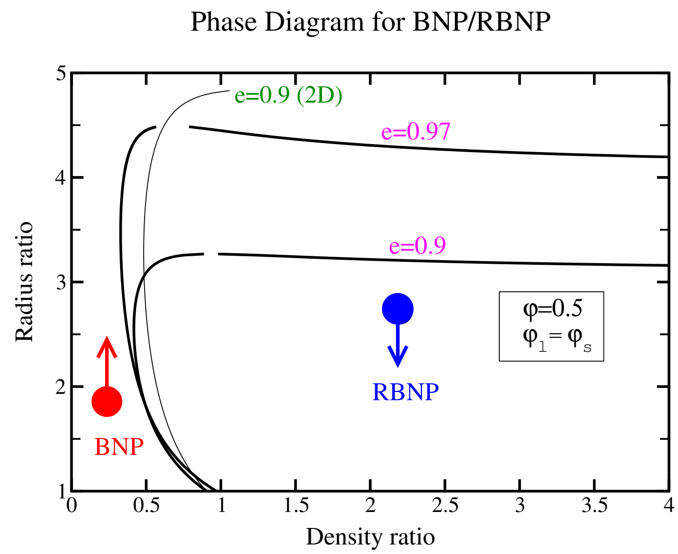
- *Gayen & Alam (PRL 2008, PRE 2011)*



Segregation in Granular Mixtures

♣ Hydrodynamic Theory for Brazil Nut Segregation





$$m_l \frac{\mathrm{d}u_l^r}{\mathrm{d}t} = \left[m_s \left(\frac{Z_l}{Z_s} \frac{T_l}{T_s} \right) - m_l \right] \mathrm{g} - \frac{4K_{ls}T}{r_{ls}} \left(\frac{2m_l m_s}{\pi m_{ls} T} \right)^{1/2} u_l^r + \left[m_s \left(\frac{Z_l}{Z_s} \frac{T_l}{T_s} \right) - m_l \right] \frac{\mathrm{d}u_s}{\mathrm{d}t}$$

@ Competition between buoyancy and geometric forces

- *Alam, Trujillo & Herrmann (Jl. of Statistical Physics, 2006)*
- *Trujillo, Alam & Herrmann (Europhysics Letters, 2003)*

Acknowledgement

♣ **JNCASR & Students@Alam-Lab, Bangalore**

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♣ **YITP & Prof. Hisao Hayakawa**