Granular impact drag force and its material-dependent scaling

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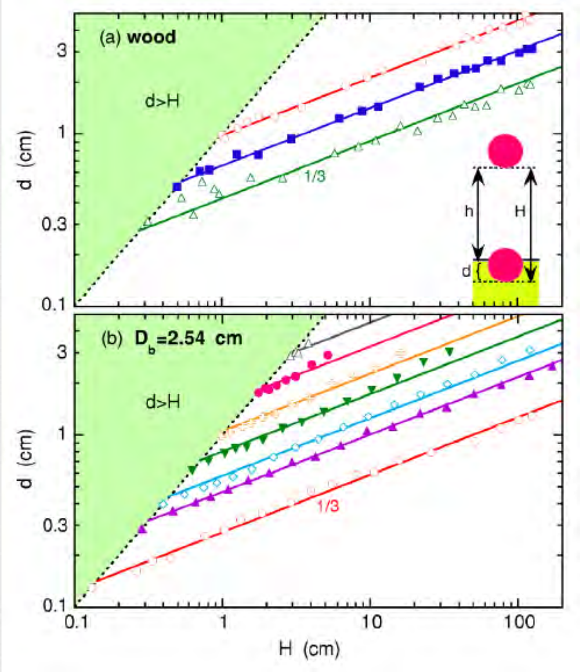
Impact!

- Fundamental process from planetary scale to our everyday
- Fundamental physics of granular matter
  - Solid - fluid - solid transition
  - Response to disturbance

Contradictory previous works?

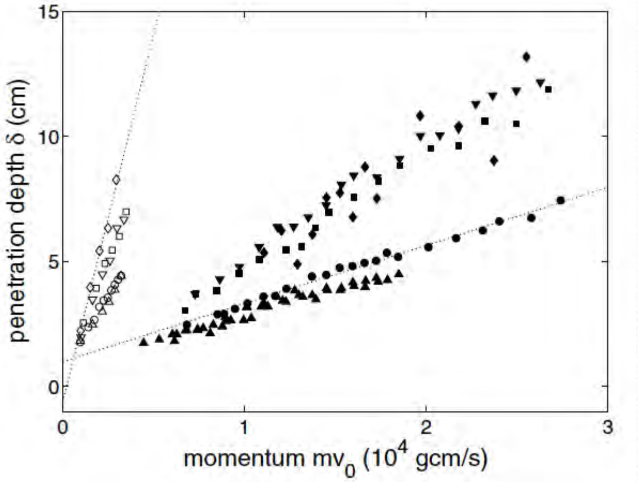


Uehara et al, PRL 2003,
Ambroso et al, PRE 2004

$$d \sim H^{1/3}$$

$$\Downarrow$$

$$F \sim |z|^{\alpha}|v|^{\frac{4-2\alpha}{3}}$$

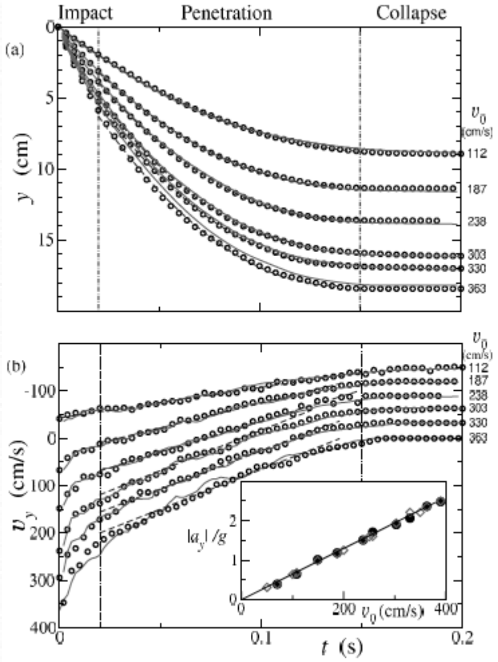


de Bruyn et al, CJP 2004

$$d = \alpha + c v_0$$

$$\Downarrow$$

$$F \sim F_0 + C|v|$$

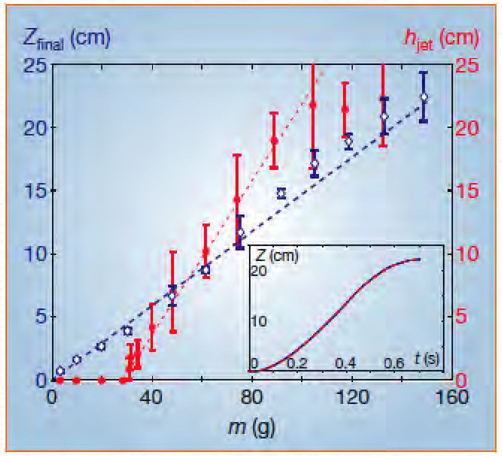


Ciamarra et al, PRL 2004

$$t_{stop} = \text{const.}$$

$$\Downarrow$$

$$F \sim |v_0|$$



Lohse et al, Nature 2004

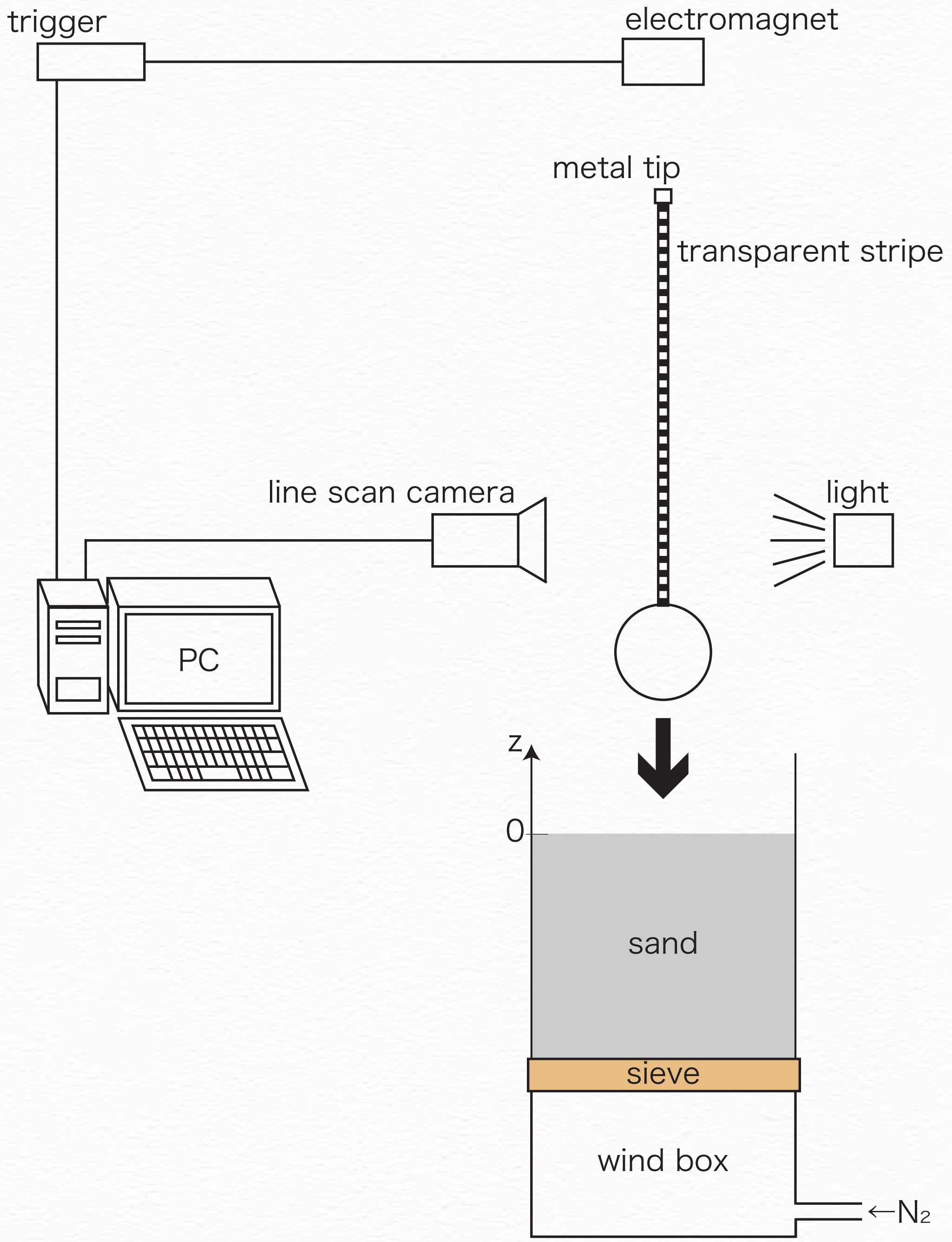
$$d \sim m$$
$$(v_0 = 0)$$

$$\Downarrow$$

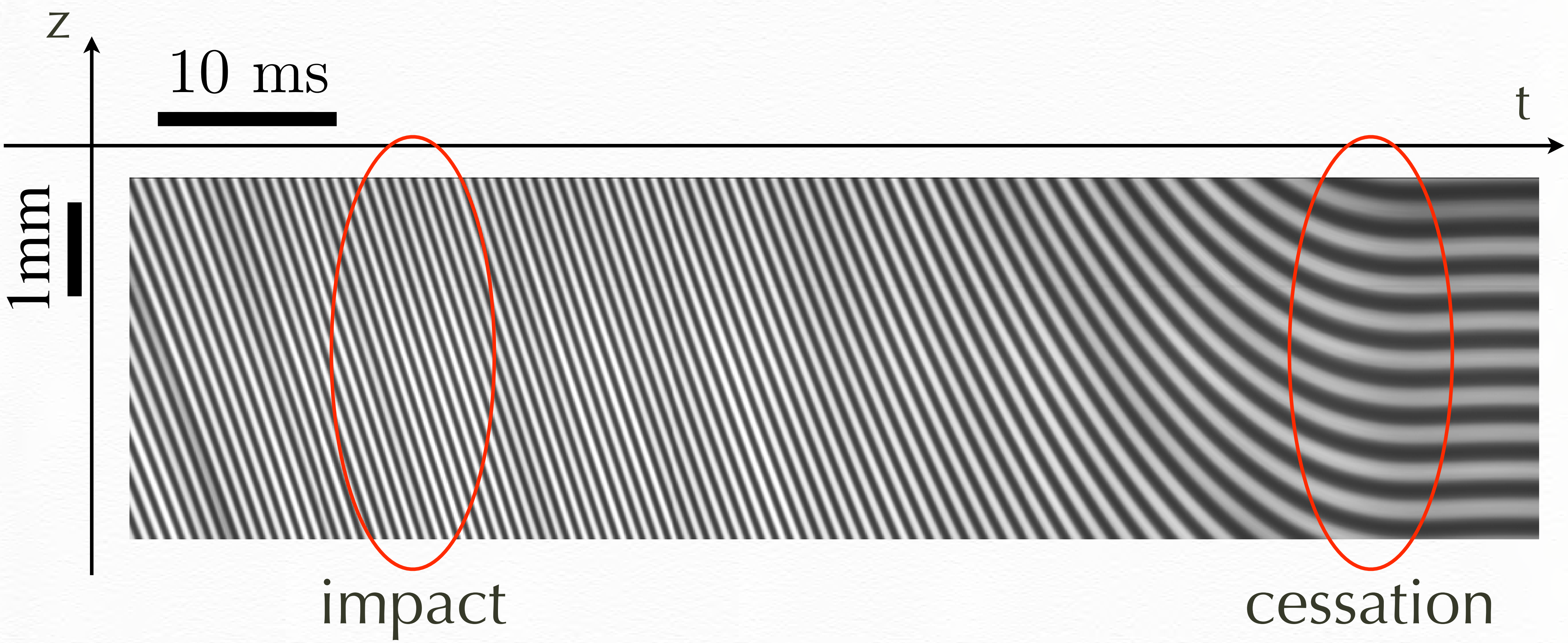
$$F \sim |z|$$

Experimental Apparatus

- Sand is fluidized before each impact.
- Free fall is triggered by an electromagnet holder.
- Dropping transparent stripe is captured by a line-scan camera.

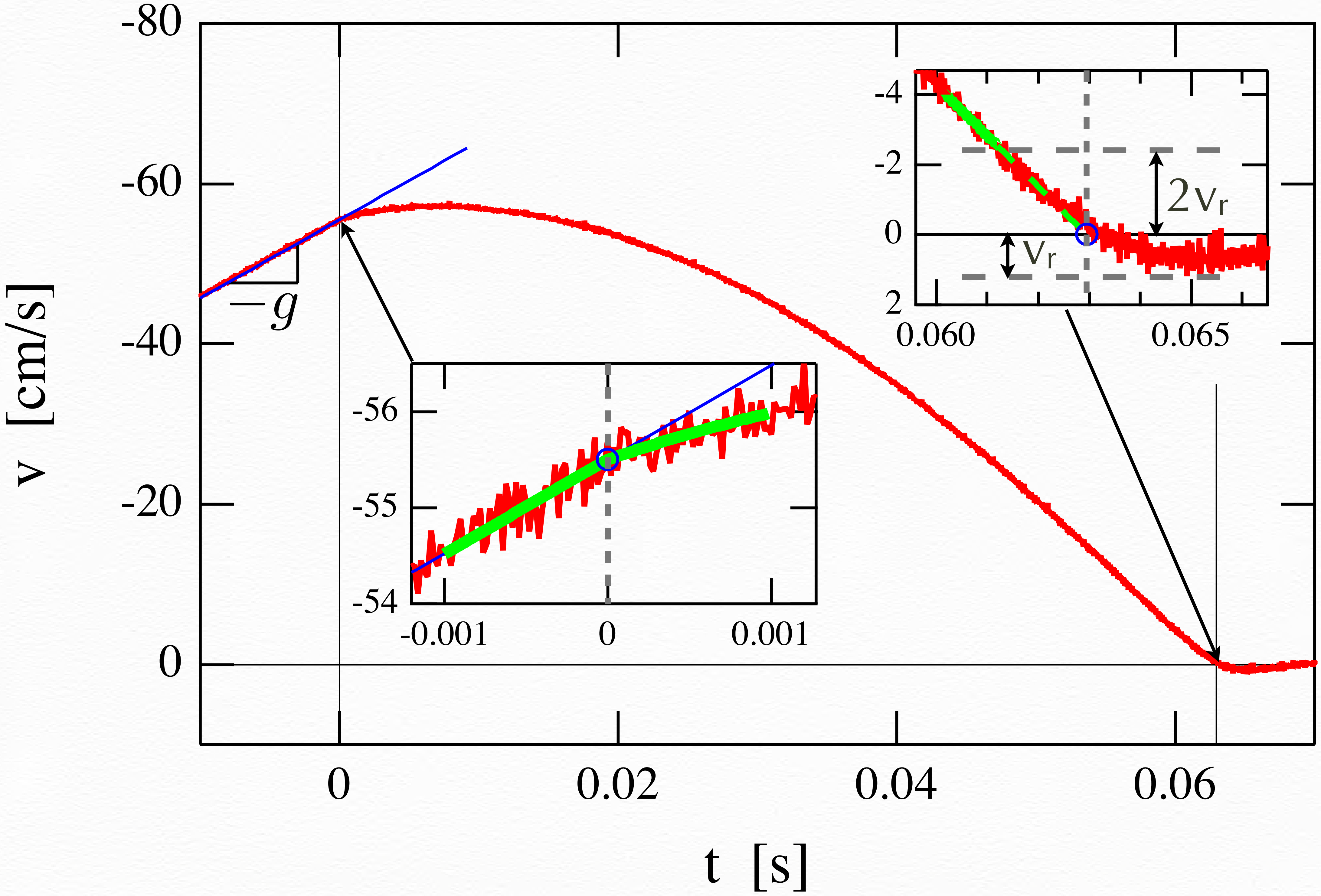


Raw data



$$z(t),\ v(t),\ a(t)$$

impact & stop time

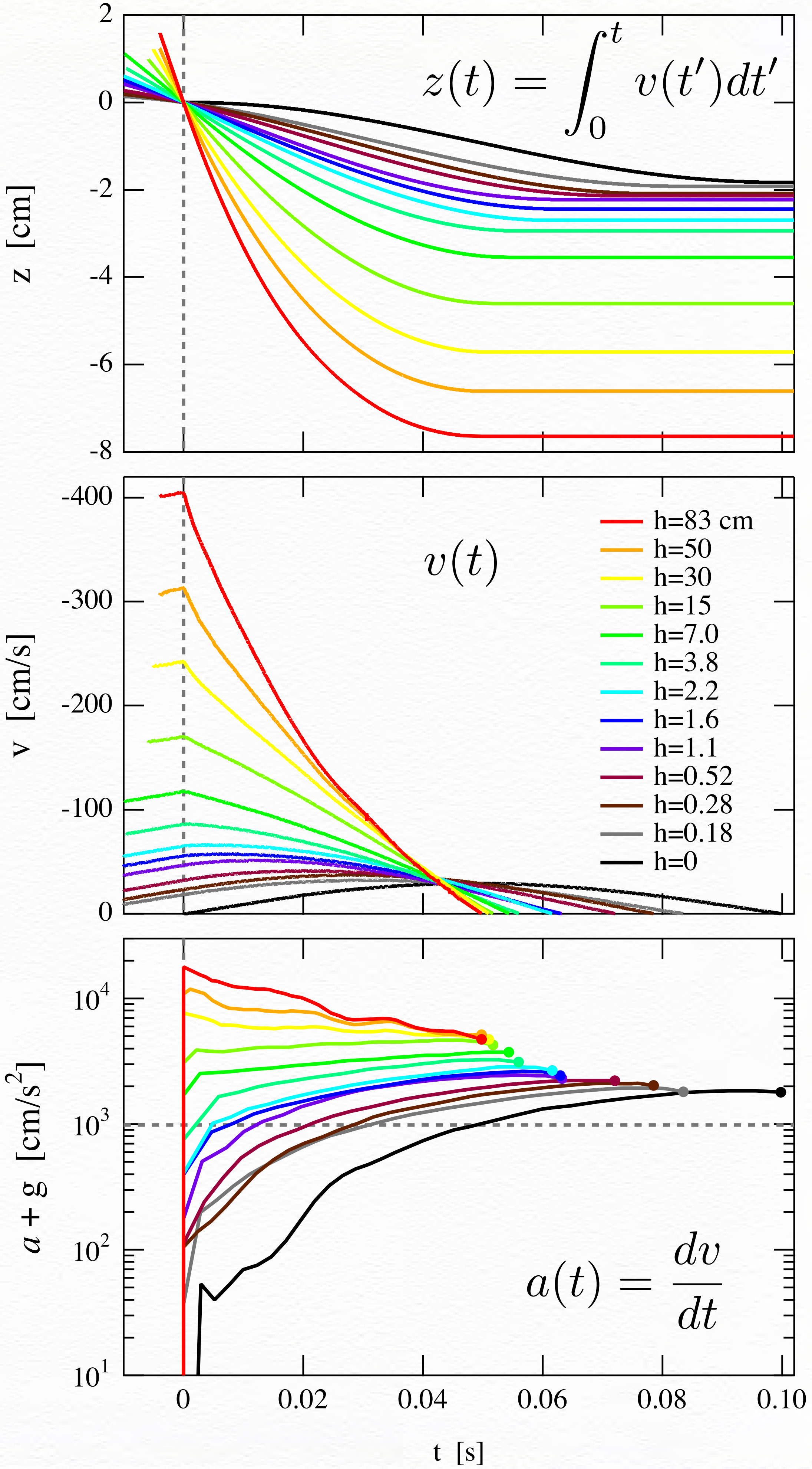


$$v = H(t-t_0)\{v_0+g(t-t_0)\}+H(t_0-t)\{v_0+(g-v_0^2/d1)(t-t_0)+([v_0^3-2gd_1v_0^2]/d_1^2)(t-t_0)^2\}$$

Various drop heights :

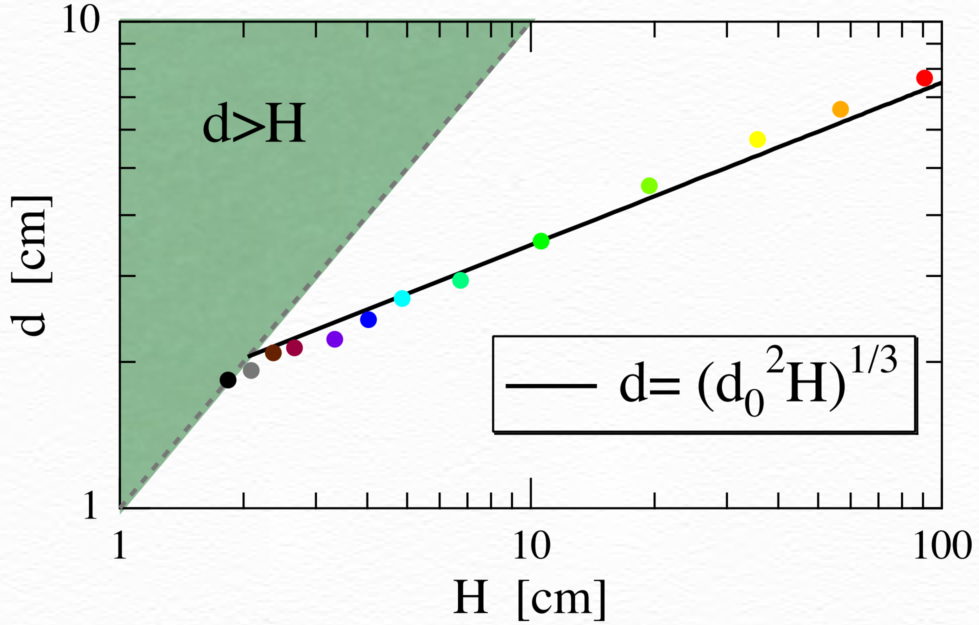
- Low speed impact takes longer time.
- Velocity is NOT linear function of time.
- Acceleration shows discontinuity at the stopping point.

(1″ steel ball & glass beads)

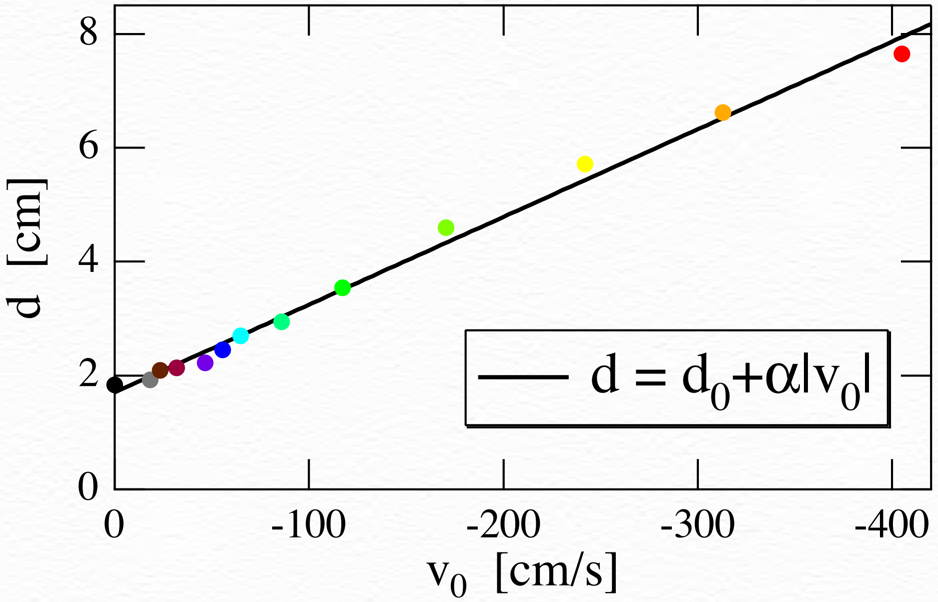


Empirical laws & our new data

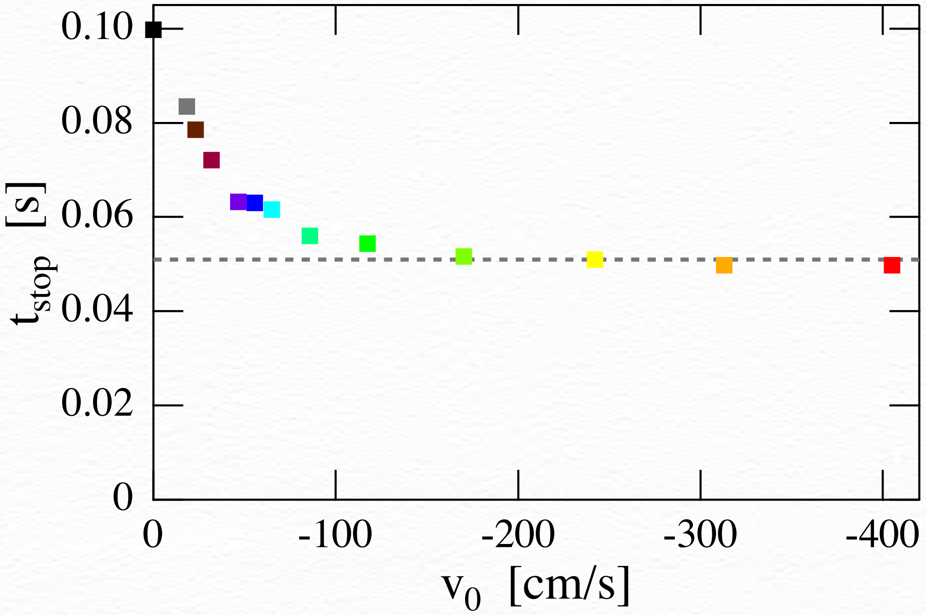
$$d/d_0 = (H/d_0)^{1/3}$$



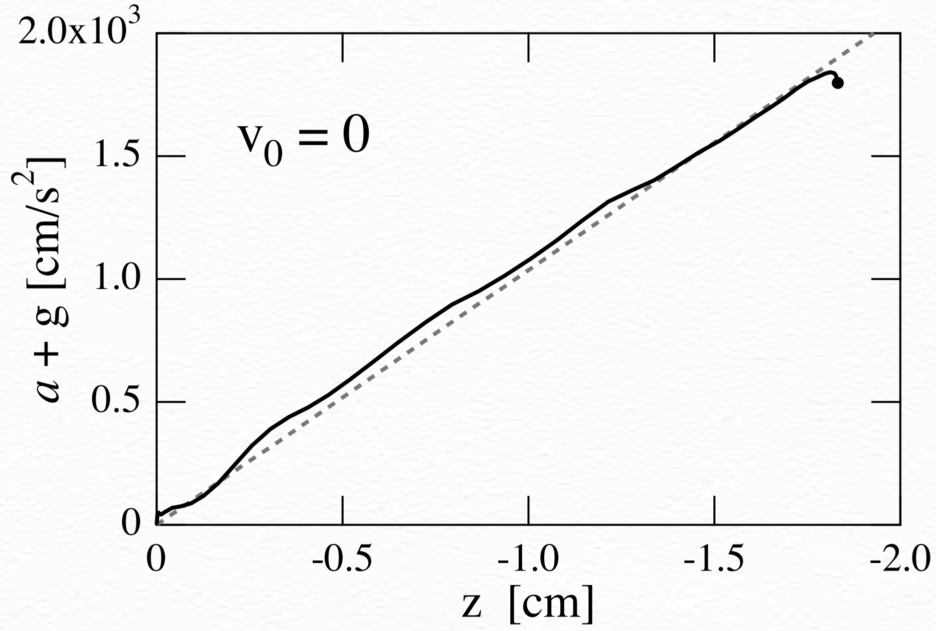
$$d = d_0 + \alpha|v_0|$$



Constant stop time



Coulomb friction



The data are completely consistent with empirical laws.

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What is the stopping force?

Stopping force model

$$\Sigma F (= ma) = -mg + f(z) + m\frac{v^2}{d_1}$$

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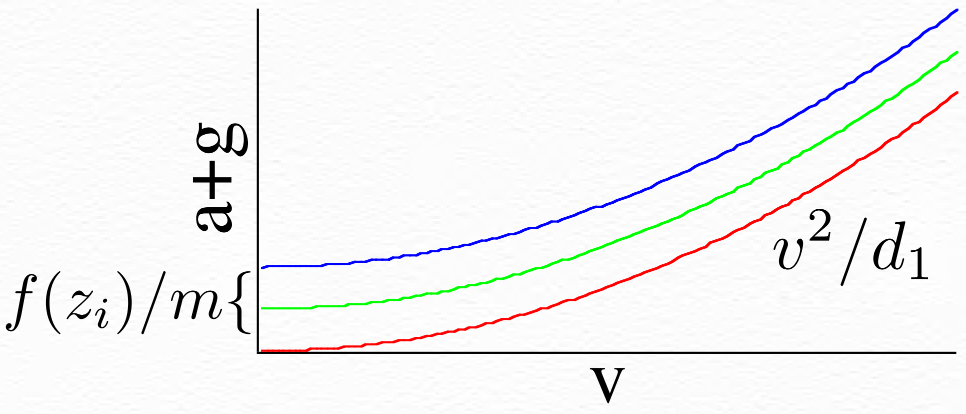
$$a + g = \frac{f(z)}{m} + \frac{v^2}{d_1}$$



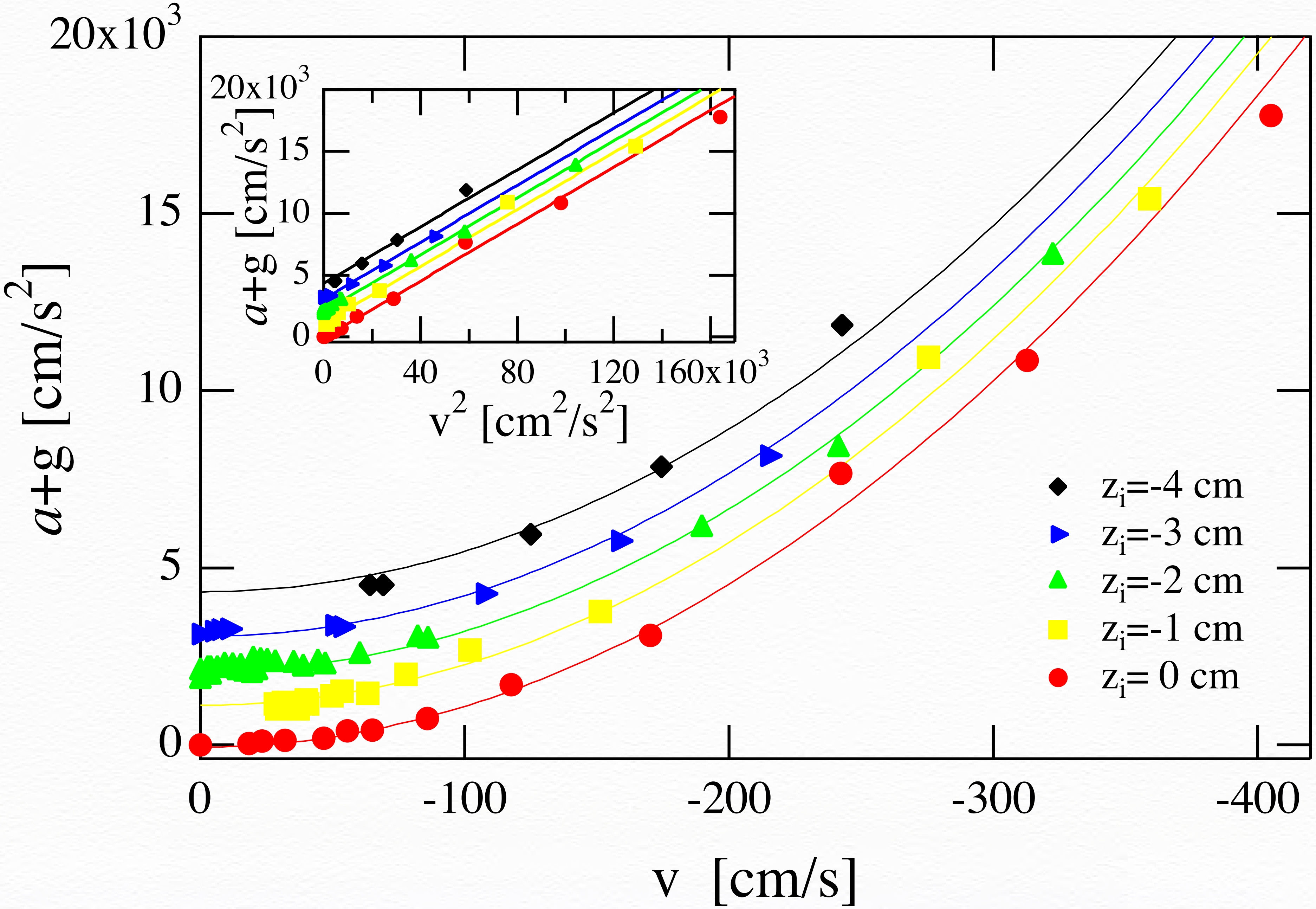
d_1 : independent of depth z

$f(z)$: independent of velocity v

Inertial drag

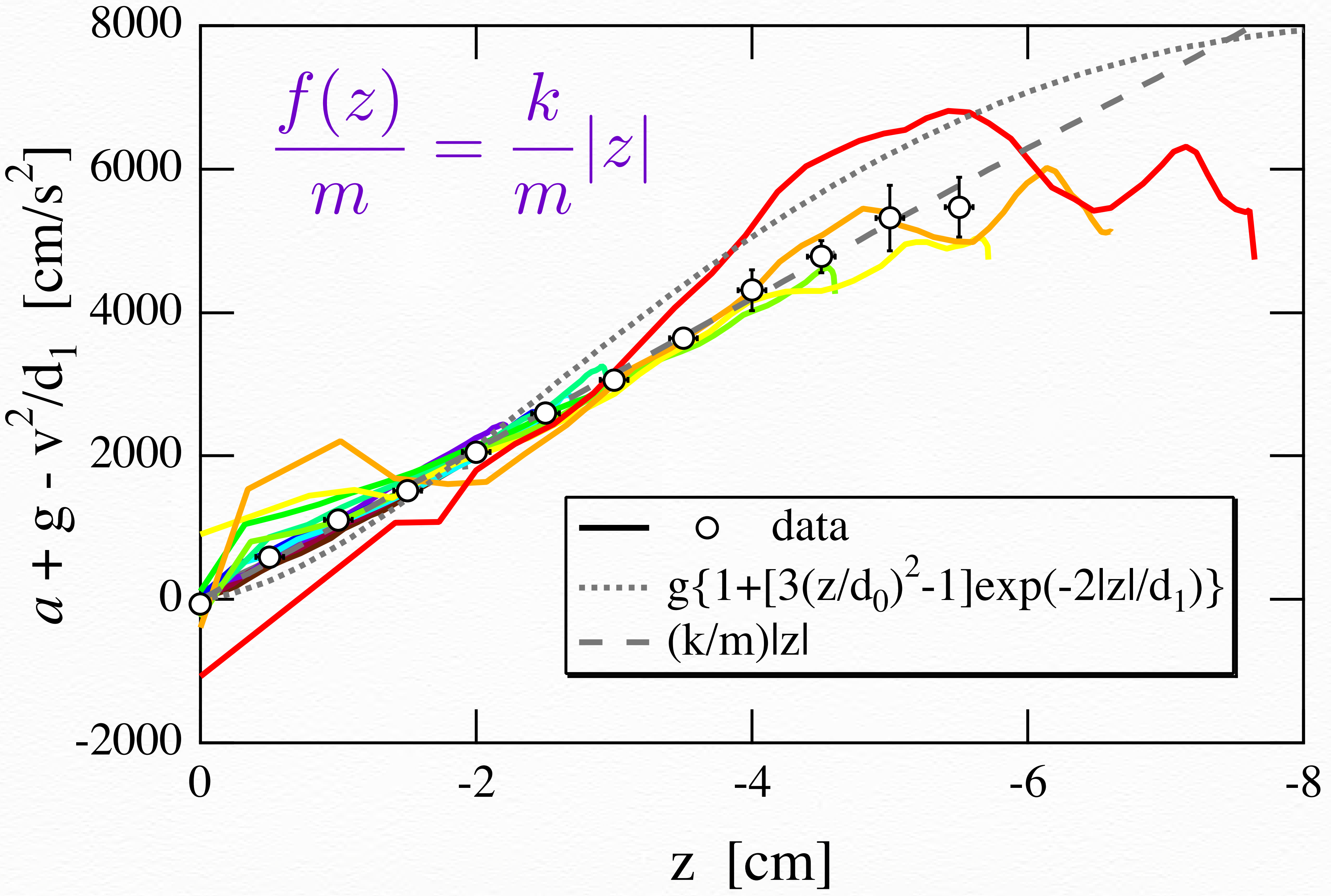
$$a + g = (1/d_1)v^2 + f(z_i)/m$$

$$z_i = \{0, -1, -2, -3, -4 \text{ [cm]}\}$$



Friction force

$$f(z)/m = a + g - (1/d_1)v^2$$



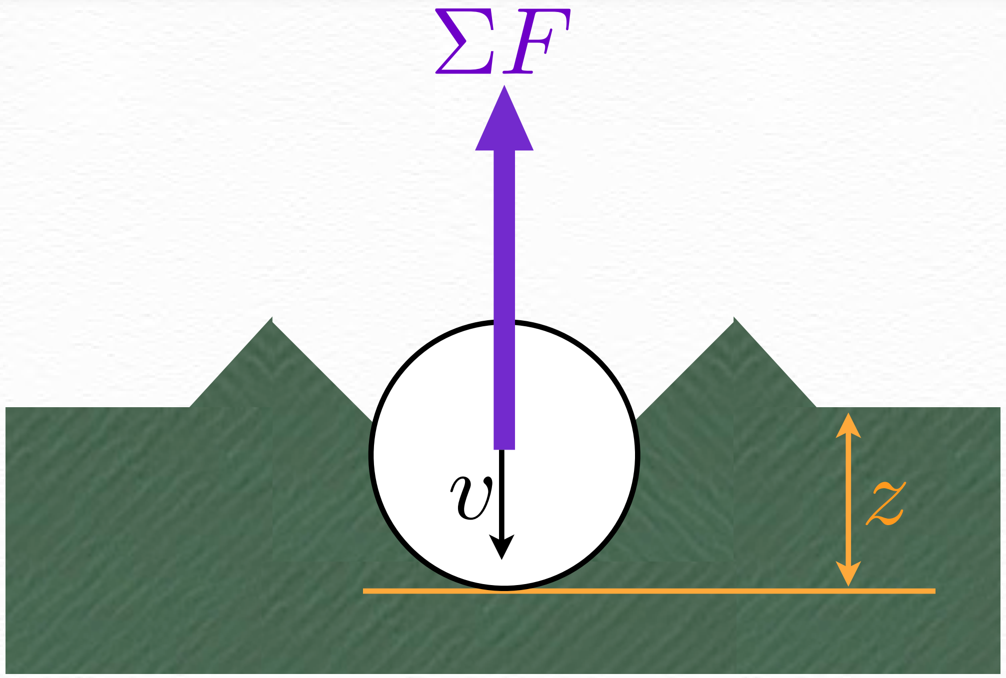
Unified Force law

$$\Sigma F = -mg + k|z| + m\frac{v^2}{d_1}$$

gravitational force

velocity dependent inertial drag
(depth independent)

depth proportional frictional drag
(velocity independent)



H. Katsuragi & D.J. Durian (2007)

Solving equation

$$K_e = \frac{1}{2}mv^2 \qquad \frac{dK_e}{dz} = mv\frac{dv}{dz} = m\frac{dv}{dt}$$

$$\frac{dK_e}{dz} = -mg + \frac{2}{d_1}K_e - k|z|$$

linearized!

Clark & Behringer, EPL (2013)

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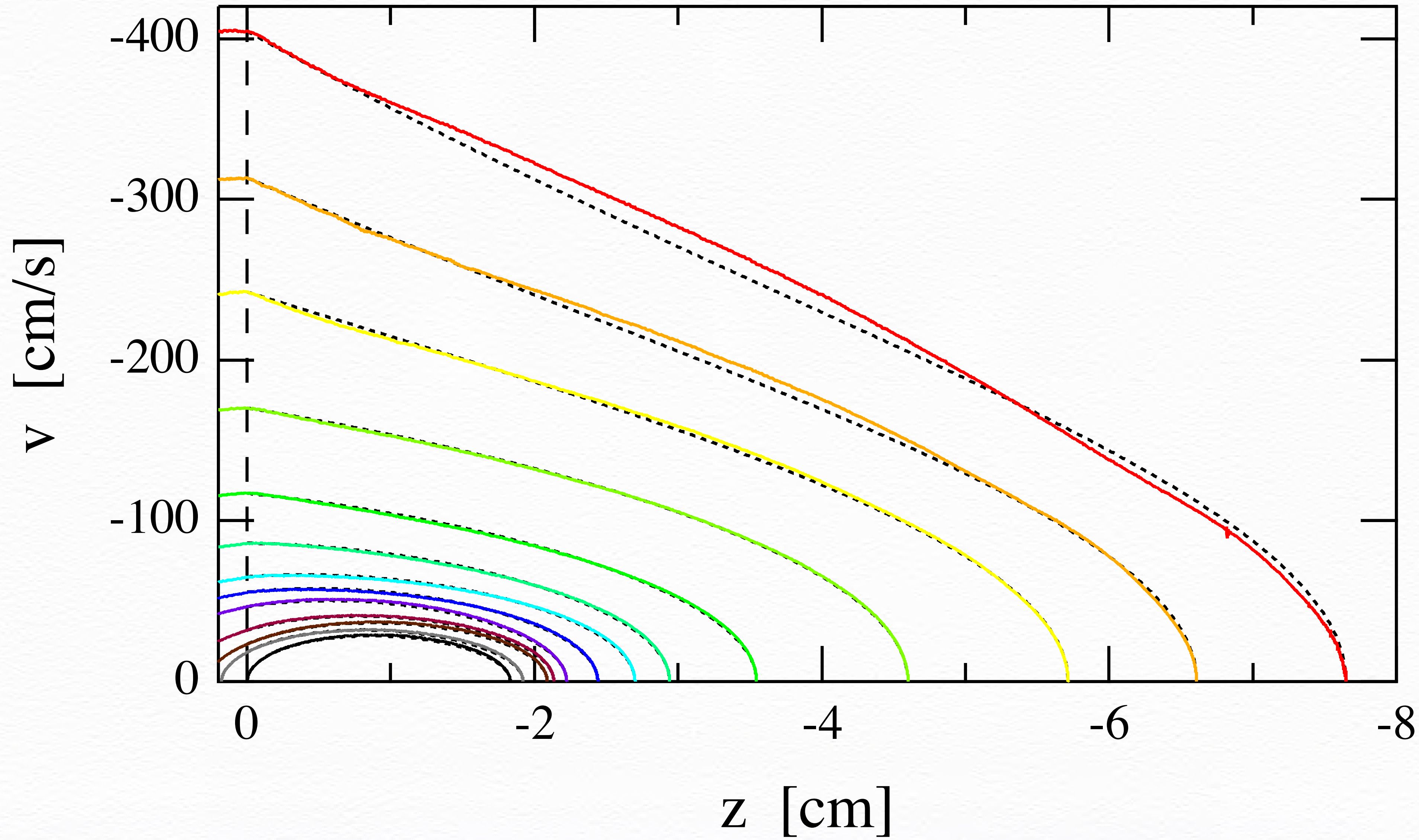
$$v = -\left[v_0^2 e^{-\frac{2|z|}{d_1}} - \frac{kd_1|z|}{m} + (1 - e^{-\frac{2|z|}{d_1}})\left(gd_1 + \frac{kd_1^2}{2m}\right)\right]^{1/2}$$

Katsuragi & Durian, PRE (2013)

v(z) & force model

$$v = -\left[v_0^2 e^{-\frac{2|z|}{d_1}} - \frac{kd_1|z|}{m} + (1 - e^{-\frac{2|z|}{d_1}})\left(gd_1 + \frac{kd_1^2}{2m}\right)\right]^{1/2}$$

$$d_1 = 8.7 \text{ cm} \quad k/m = 1040\ s^{-2}$$



Universality

Two parameters d_1 and k/m determine the dynamics.

For steel ball vs glass beads:

$$d_1 = 8.7 \text{ cm}$$

$$k/m = 1040 \ s^{-2}$$

How can we predict these values for other material impacts?

Expectation for inertial drag

$$\frac{m}{d_1}v^2 \sim \rho_g A v^2$$

(momentum transfer)

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$$d_1 \sim \alpha^{-1}\frac{\rho_p}{\rho_g}D_p$$

ρ_p : density of projectile
ρ_g : density of granular media
D_p : diameter of projectile

A : impact area
$\alpha = 3/2$ (ball), $4/\pi$ (cylinder)
(ratio of area and volme factor)

Expectation for friction force

$$k|z| \sim \mu g \rho_g |z| A$$

(hydrostatic pressure + Coulomb friction)

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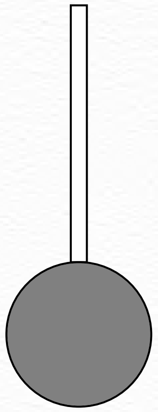
$$\frac{k}{m} \sim \alpha \mu g \frac{\rho_g}{\rho_p} \frac{1}{D_p}$$

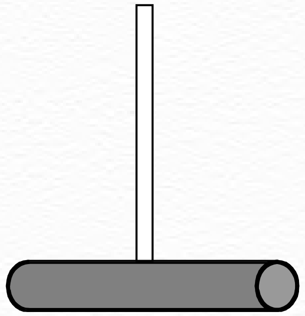
ρ_p : density of projectile
ρ_g : density of granular media
D_p : diameter of projectile

A : impact area
$\mu = \tan \theta_r$: friction coefficient
$\alpha = 3/2$ (ball), $4/\pi$ (cylinder)
(ratio of area and volme factor)

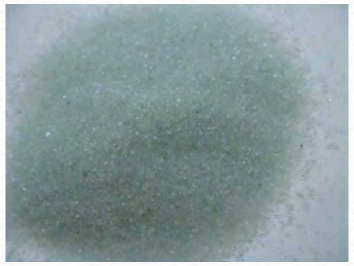
Various projectiles and sand



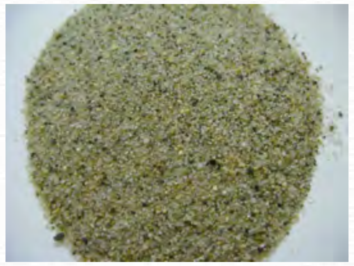
1″ Tungsten, steel,polymer, wood
2″ - 1/8″ steel



1/2″ - 1/4″ diameter
2″ - 6″ length aluminum



glass beads



beach sand

rice

sugar

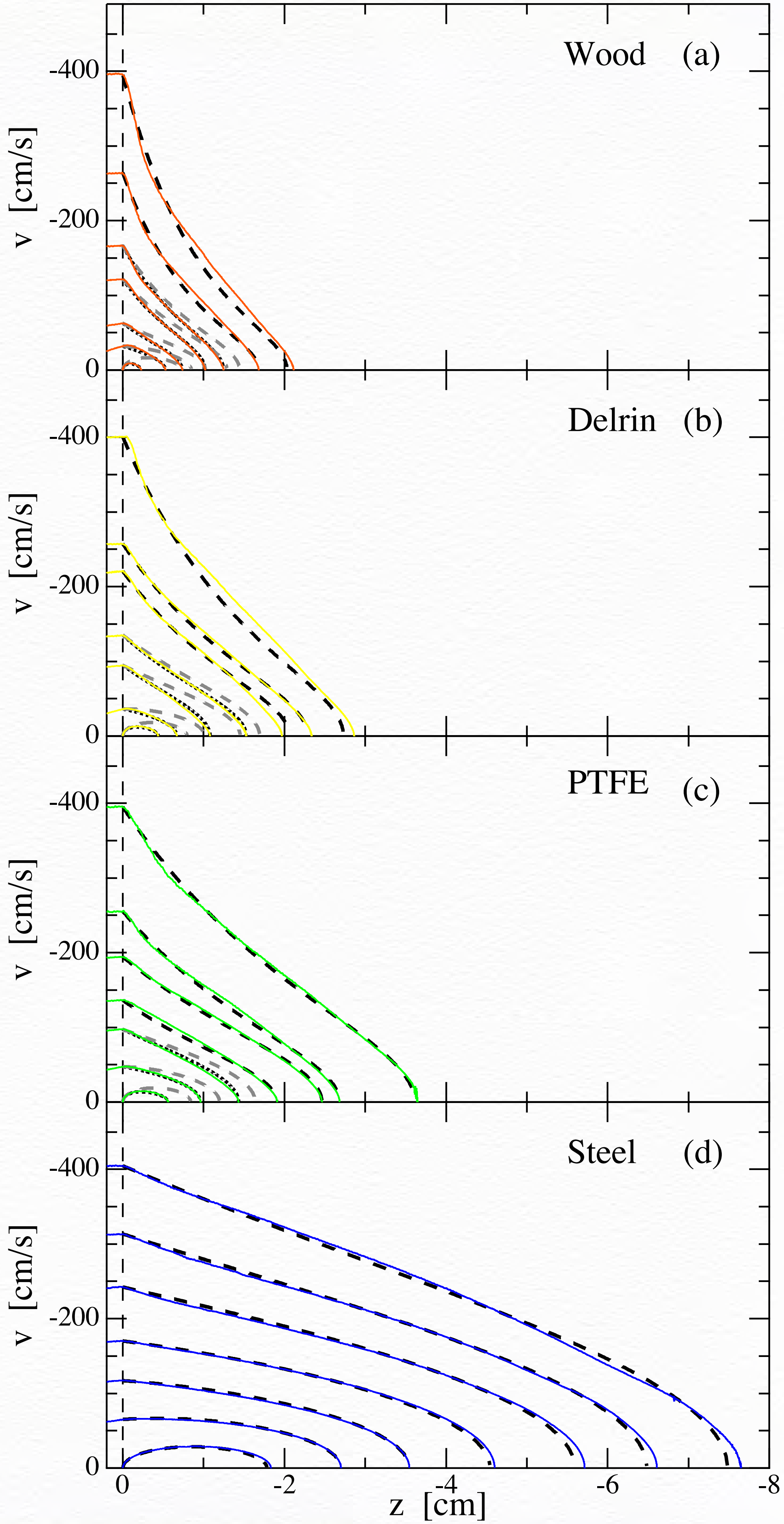
Limited fitting of v(z)

by fixed d_1 and k/m

$$v = -\left[v_0^2 e^{-\frac{2|z|}{d_1}} - \frac{k d_1 |z|}{m} + (1 - e^{-\frac{2|z|}{d_1}}) \left(g d_1 + \frac{k d_1^2}{2m} \right) \right]^{1/2}$$

Bad fitting for all shallow impacts

Good fitting for all impacts



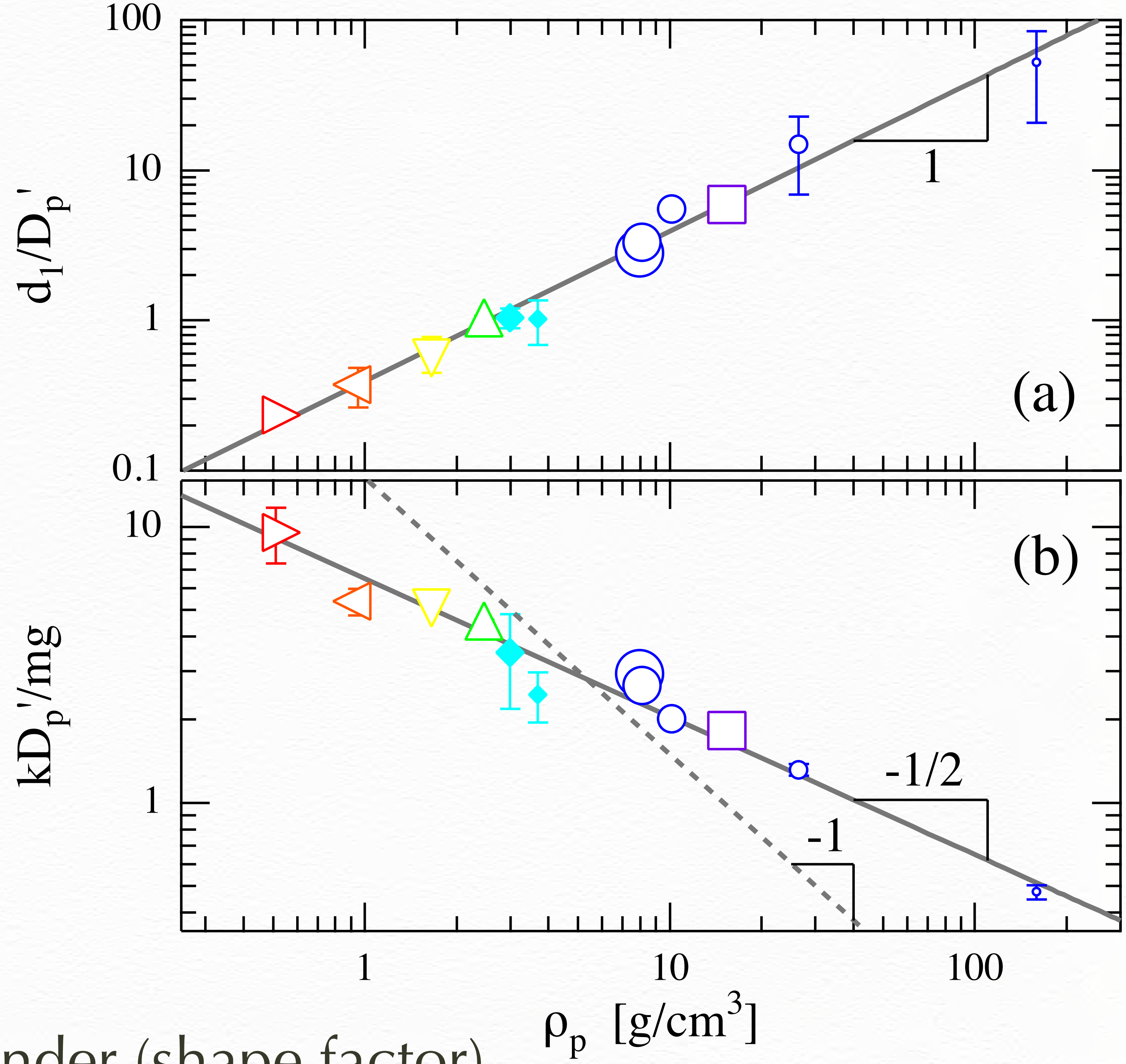
Scaling of parameters

Inertial parameter:

$$\frac{d_1}{D_p{}'} \sim \rho_p$$

Friction parameter:

$$\frac{kD_p{}'}{mg} \sim \rho_p{}^{-1/2}$$



$D_p{}' = 3\pi/8D_p$ for cylinder (shape factor)

Internal friction dependence

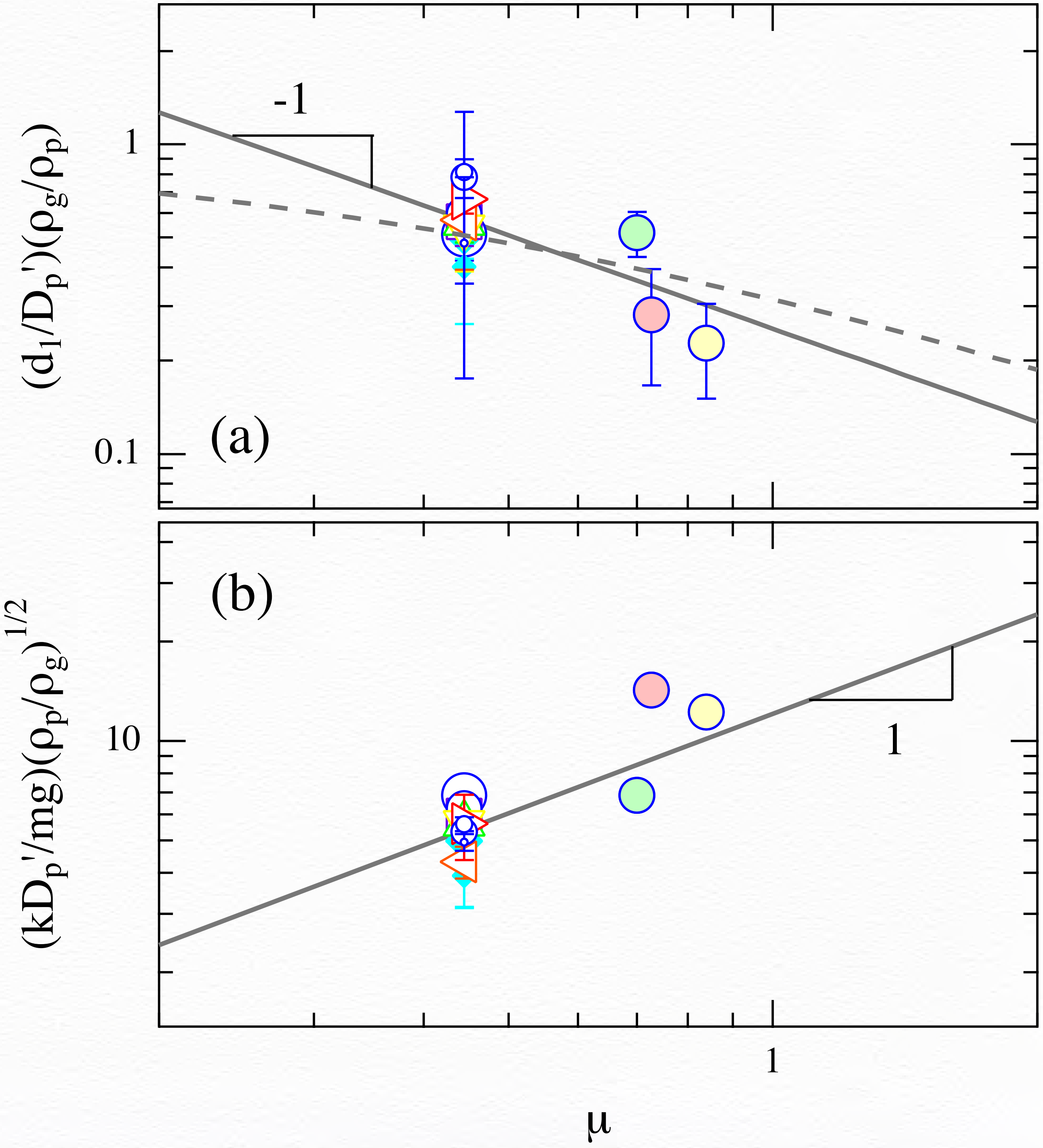
Inertial parameter

$$\frac{d_1}{D_p{}'} = \frac{0.25}{\mu}\frac{\rho_g}{\rho_p}$$

$$d_1 = 1/(1 + 2.2\mu)$$

Friction parameter

$$\frac{kD_p{}'}{mg} = 12\mu\left(\frac{\rho_g}{\rho_p}\right)^{1/2}$$



UNIFIED FORCE LAW

final form of the drag force:

$$\Sigma F = -mg + k|z| + m\frac{v^2}{d_1}$$

Scaling by material properties:

$$\frac{d_1}{D_p} = \frac{0.25}{\mu}\frac{\rho_p}{\rho_g}$$

$$\frac{k}{m}\frac{D_p}{g} = 12\mu\left(\frac{\rho_p}{\rho_g}\right)^{1/2}$$

H. Katsuragi & D.J. Durian, Phys. Rev. E 87, 052208 (2013)