Driven Granular Fluids: Collective Effects

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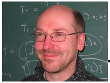
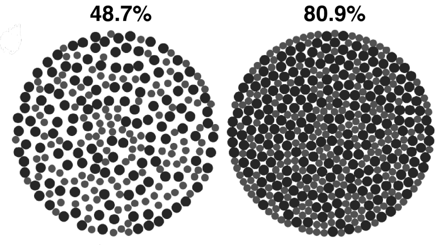
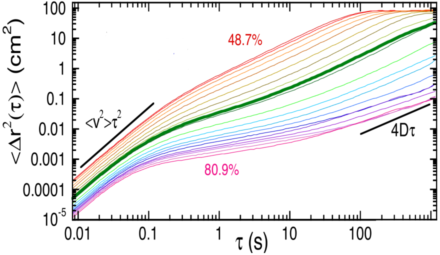
Acknowledgment

-  Annette Zippelius
-  Andrea Fiege
-  Timo Aspelmeier
-  Matthias Sperl
-  Iraj Gholami

A Sandstorm for Experimental Physicists





Abate & Durian, PRE **74** 2006

- Steel balls ($\sim 7\,\mathrm{mm}\,\varnothing$) on a sieve
- Driven by air flow
- Measurement of mean square displacement
  $\delta r^2(t) = \langle [\boldsymbol{r}(t) - \boldsymbol{r}(0)]^2 \rangle$

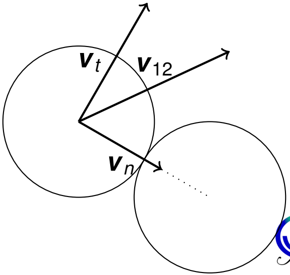
Inelastic, Smooth, Hard Spheres: A Model for Granular Particles

Hard Spheres completely characterized by

- Mass m
- Radius a
- Coefficient of restitution $\epsilon \in [0, 1]$

Collision law

$$\boldsymbol{v}'_n = -\epsilon\, \boldsymbol{v}_n,$$
$$\boldsymbol{v}'_t = \boldsymbol{v}_t$$



Energy Loss on average per collision

$\Delta E \propto 1 - \epsilon^2$

A Sandstorm for Theoretical Physicists

Random Force $\xi_i(t)$, gaussian distributed

- Average $\langle \xi_i \rangle = 0$
- Driving power $P_D = \langle \xi_i^2 \rangle$

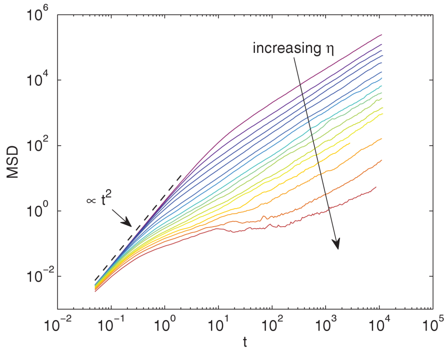
Stationary State as a balance between driving & dissipation

Event Driven Simulations 10 000 particles

Bidisperse to avoid crystallization

Coefficient of Restitution $\epsilon = 0.9$

Area Fraction $\eta = 0.1$–0.81



I. Gholami *et al.* PRE **84** 2011

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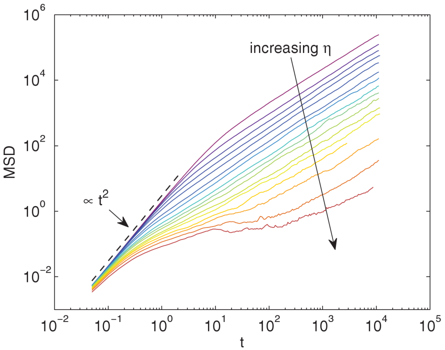


I. Gholami *et al.* PRE **84** 2011

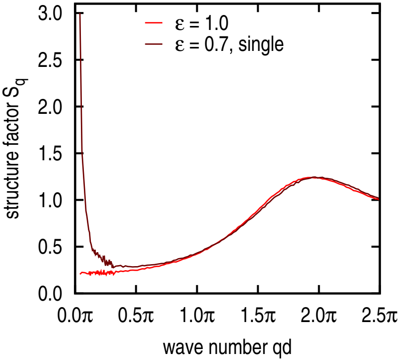
Outline

1. Static Structure & Momentum Conservation
2. Long-Time Tails
3. The Granular Glass Transition[1]

[1] See also the Lecture on Friday

Static Structure: A Surprise[2]

- Strong increase for $k \to 0$
- Highly Correlated on large Length Scales
- Implies so called *Giant Number Fluctuations*



- Volume Fraction $\varphi = 0.2$
- $N = 50 \times 400\,000$

[2] Kranz, Fiege, Zippelius, in preparation

A Toy Model

$$\partial_t h(\boldsymbol{r}, t) = \eta \nabla^2 h(\boldsymbol{r}, t) + \xi(\boldsymbol{r}, t)$$

Correlation Function $C(k) = \left\langle \hat{h}(-k)\hat{h}(k) \right\rangle \simeq \frac{\langle \hat{\xi}(-k)\hat{\xi}(k) \rangle}{\eta k^2}$ Grinstein *et al.*, PRL **64** 1990

Random Force $\langle \xi(r)\xi(r') \rangle \propto \delta(r - r') \Rightarrow \left\langle \hat{\xi}(-k)\hat{\xi}(k) \right\rangle = 1.$

Equilibrium $\left\langle \hat{\xi}(-k)\hat{\xi}(k) \right\rangle \propto \eta k^2$ due to FDT

Local Pairs $\langle \xi(r)\xi(r') \rangle \propto \Theta(r - r' - \ell) \Rightarrow \left\langle \hat{\xi}(-k)\hat{\xi}(k) \right\rangle \propto \ell^2 k^2.$

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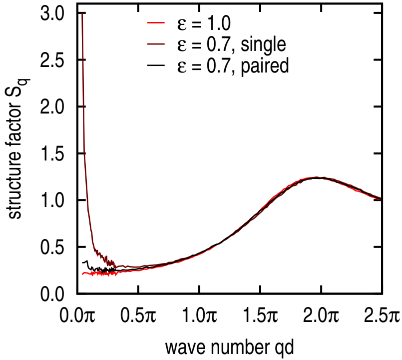
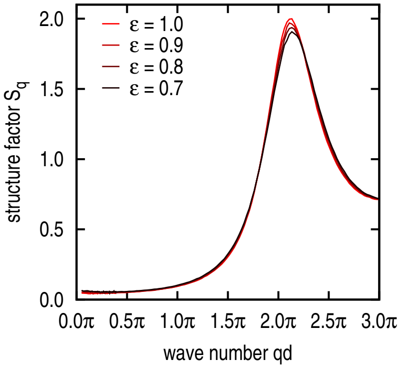
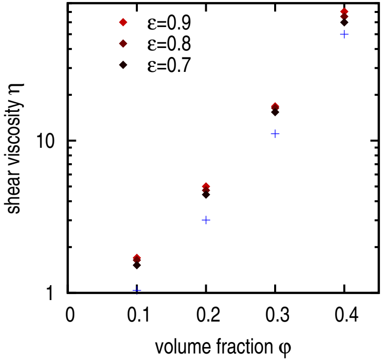
Divergence under Control

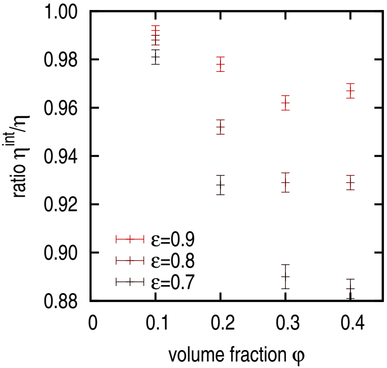




- Volume Fraction $\varphi = 0.2$
- Volume Fraction $\varphi = 0.4$

Insight can be used for Measurements





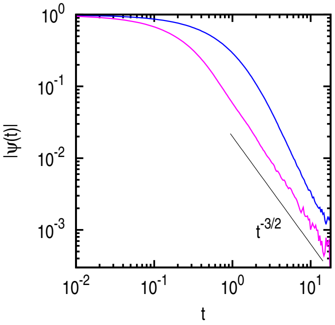
Long-Time Tails

The Velocity Autocorrelation Function

- $\psi(t) = \langle \boldsymbol{v}_s(0) | \boldsymbol{v}_s(t) \rangle$

Long-Time Tails $\psi(t) \propto t^{-\alpha}$
(instead of exponential decay)

- In 3D elastic & inelastic hard spheres have $\alpha = 3/2$



Equation of Motion

$$\partial_t \psi(t) + \omega_E \psi(t) + \int_0^t d\tau M(t-\tau)\psi(\tau) = 0$$

Collision Frequency ω_E

Memory Kernel $M(t)$

Incoherent Scattering Function $\phi_s(k, t)$ contains more information about the tagged particle

Mode-Coupling Approximation[3]

- Consider coupling of tagged particle to
  - Collective Density Modes $\phi(k,t)$
  - Longitudinal Current Modes $\phi_L(k,t)$
  - Transverse Current Modes $\phi_T(k,t)$
- Transverse Mode yields slowest decay (in 3D)

$$M(t \to \infty) = M_T(t \to \infty) \propto (1+\varepsilon)^2 \int_0^\infty dk j_0''^2(k)\phi_T(k,t)\phi_s(k,t)$$

and indeed $\psi(t \to \infty) \propto t^{-3/2}$

- Situation in 2D is very subtle

[3] Kranz, Zippelius, in preparation

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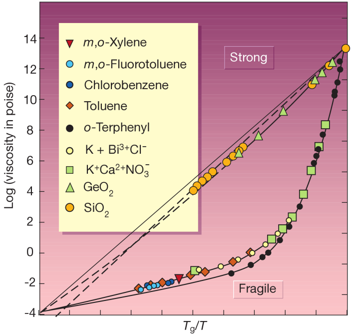
The Glass Transition

The Glass Transition

Amorphous Solid from either

1. Supercooled Melt
2. Supersaturated Suspension
3. Dense Granular Fluid?

- No Static Order Parameter



Debenedetti & Stillinger, Nature 2001

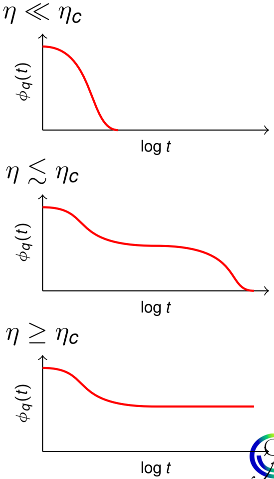
Order Parameter: Plateau of the Scattering Function

Scattering Function $\phi(q,t) = \langle \rho_q^*(\tau)\rho_q(\tau+t)\rangle$
independent of τ

Density $\rho(\boldsymbol{r},t) = \sum_i \delta(\boldsymbol{r}_i(t) - \boldsymbol{r})$

Order Parameter $f_q = \phi(q, t \to \infty)$

- Fluid: $f_q = 0$
- Glass: $f_q > 0$



Equation of Motion[4]

$$(\partial_t^2 + q^2 v_q^2)\phi(q,t) + \int_0^t d\tau M(q,t-\tau)\partial_\tau \phi(q,\tau) = 0$$

Speed of Sound v_q

Memory Kernel $M(q,t)$

[4] Kranz,Sperl,Zippelius, PRL **104**, 225701 (2010); PRE **87**, 022207 (2013)

Mode-Coupling Approximation

$$(\partial_t^2 + q^2 v_q^2)\phi(q,t) + \int_0^t d\tau M(q, t-\tau)\partial_\tau \phi(q,\tau) = 0$$

- Interpretation as interacting undamped sound waves

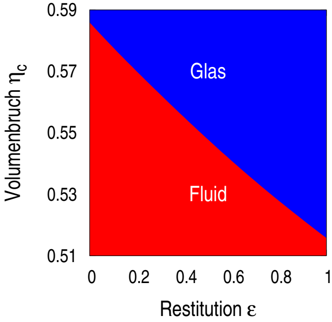
$$M(q,t) \approx \sum_{\boldsymbol{q}=\boldsymbol{k}+\boldsymbol{p}} \mathcal{V}_{\boldsymbol{qkp}} \mathcal{W}_{\boldsymbol{qkp}} \phi(k,t)\phi(p,t)$$

Loss of Detailed Balance implies

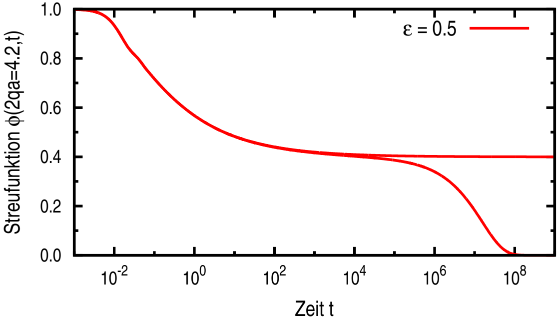
- Rate of creation $\mathcal{V}_{\boldsymbol{qkp}} \neq$ rate of annihilation $\mathcal{W}_{\boldsymbol{qkp}}$
- Can still be calculated explicitly

The Granular MCT Glass Transition

- Percus-Yevick static structure factor
- Iterative Numerical Solution
- Standard Discretization Parameters



Nonuniversal Dynamics



- Critcial exponents a, b depend on ε.

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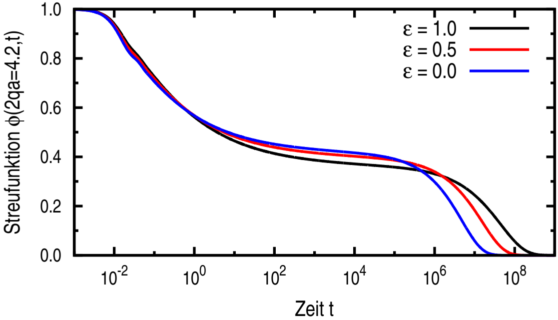

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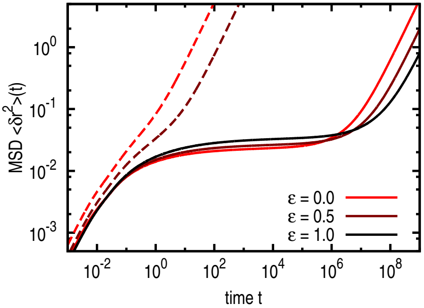
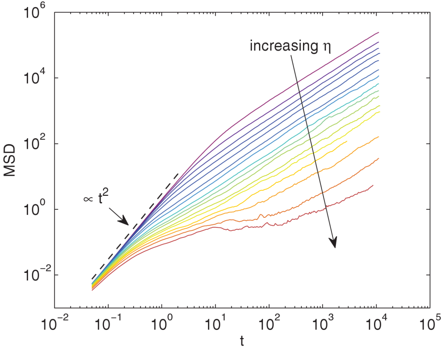
The Mean Square Displacement[5]





[5] Sperl,Kranz,Zippelius, EPL **98**, 28001 (2012)

Work in Progress

- Understand & Use Integration Through Transients (with Matthias Fuchs/Sperl)
- Simulation Results for the Granular Glass Transition (with Stefan Luding, Vitaliy Ogarko)
- Active Particles/Mobile Cells

Summary

- Momentum Conservation matters
- Violations of FDT may be useful
- Long-Time Tails are the same as in Equilibrium
- There is a Granular Glass Transition
- It has nontrivial Properties

Thank you for your attention

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