Experimental Observation of Shear Thickening Oscillation in Dilatant Fluid

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What is Dilatant Fluid?

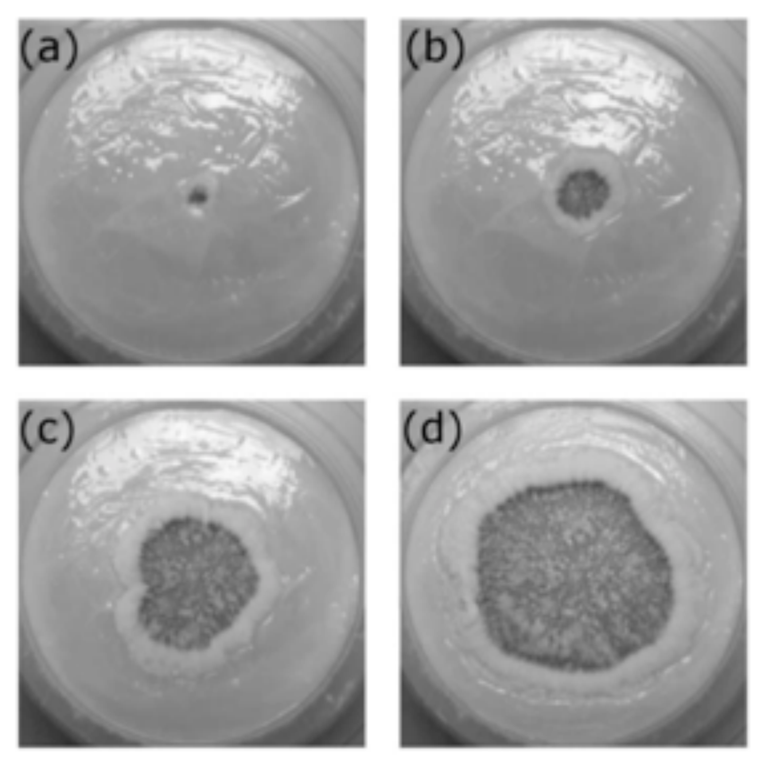
A typical example:

Dense mixture of starch and water.

(starch particles) ~ 10μm size

Peculiar features of Dilatant Fluid

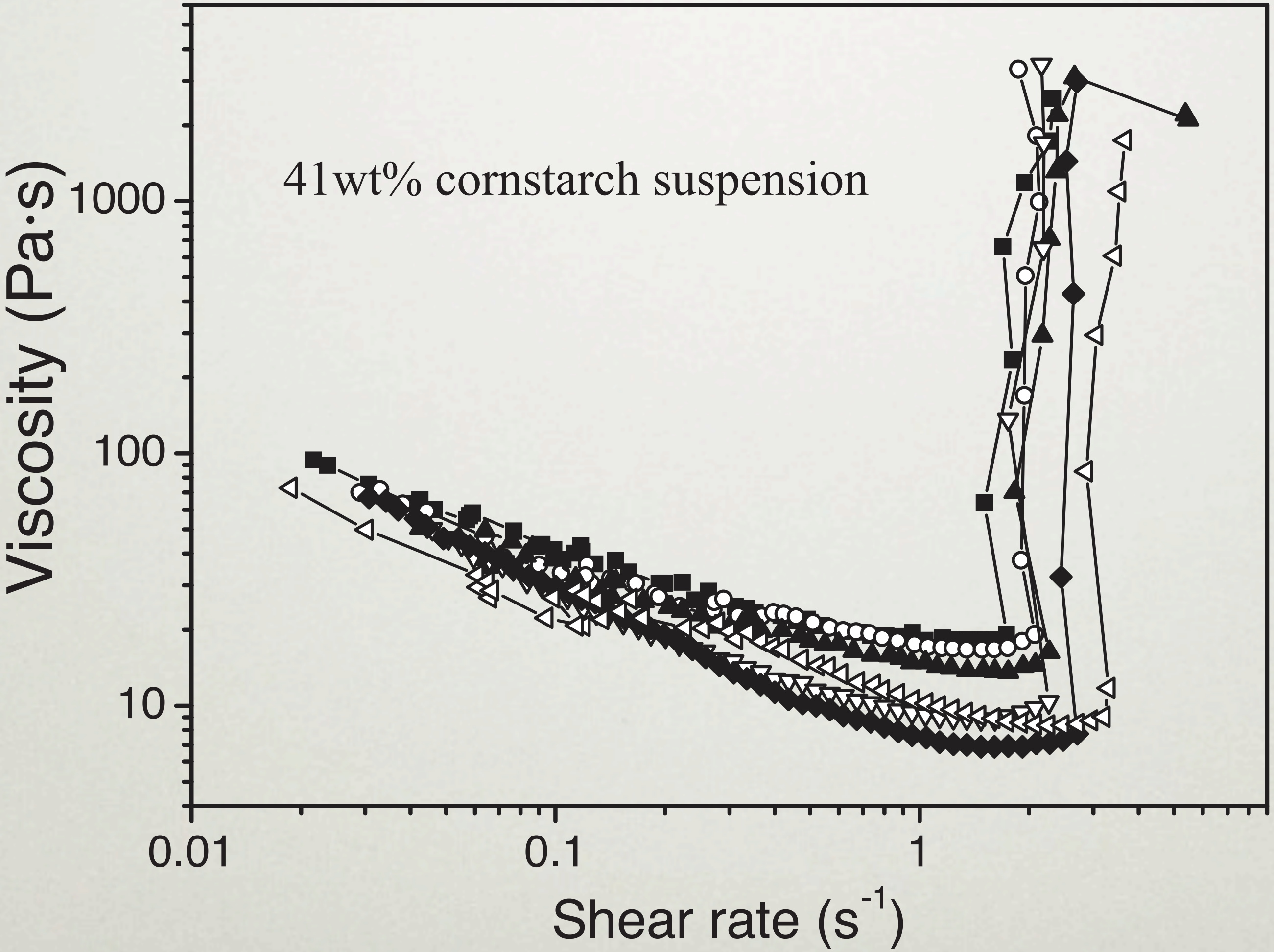
Persistent or expanding hall



Ebata, Tatsumi and Sano, PRE(2009)

Peculiar features of Dilatant Fluid

Jamming Transition



A. Fall, N. Huang, F. Bertrand, G. Ovarlez, D Bonn, PRL(2008)

Why it shear thickens?

A possible explanation

- Densely packed sand dilate upon deformation
- Coffee beans in vacuum bag is rigid because it cannot dilate due to the pressure.
- In the mixture, interstitial water surface could have particle size curvature.
  Pressure decreases due to the surface tension.

1. thickening is severe and instantaneous
2. relaxation after removal of the external stress is fast but not instantaneous.
3. thickened state is almost rigid and does not allow much elastic deformation
4. viscosity shows hysteresis
5. spontaneous oscillation due to shear thickening is observed.

Outline

- Fluid dynamics model of dilatant fluid

- Simulation of simple shear flows

  The present model reproduce basic nature of dilatant fluid and predicts shear thickening oscillation

- Experiment of Taylor-Couette flow

  We observed clear oscillations.

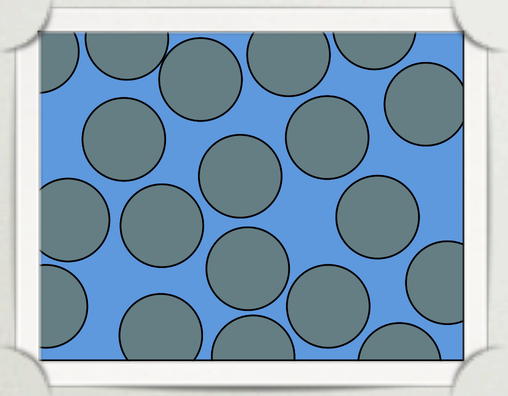
Fluid dynamics model for dilatant fluid

Modeling the dynamics of dilatant Fluid

1) Phenomenological description for shear thickening

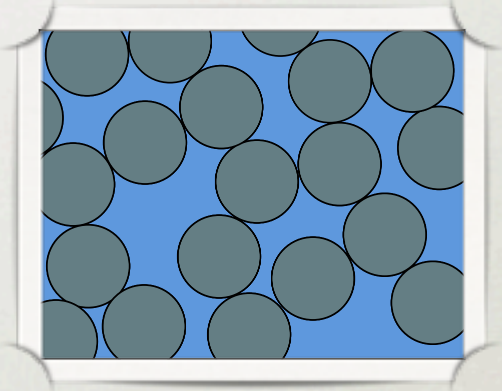
Introduce a state variable: $\phi(r,t)$

$\phi = 0$



under low stress

$\phi = 1$



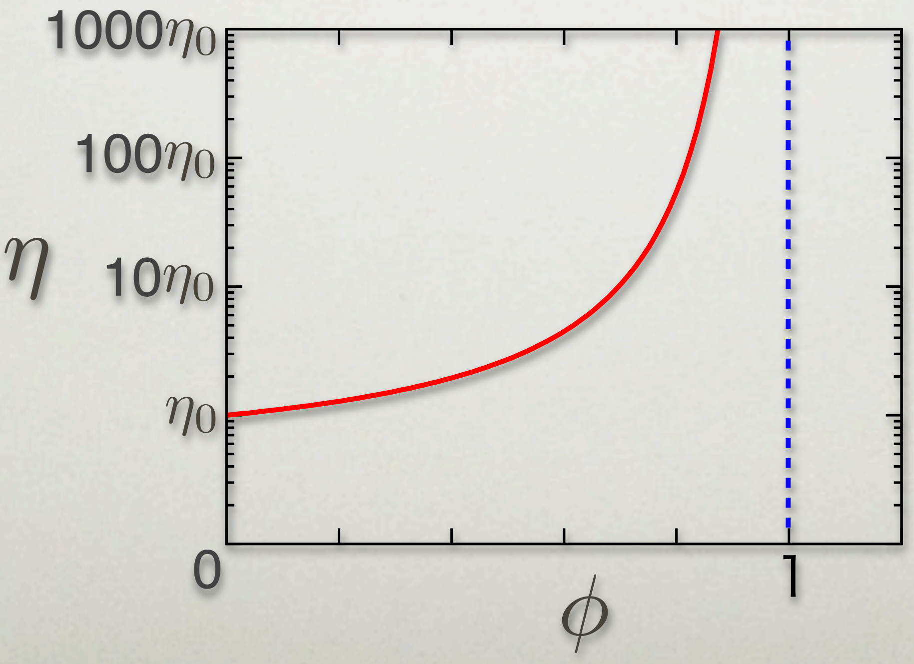
under high stress

Modeling the dynamics of dilatant Fluid

2) Viscosity is strongly increase func. of $\phi(r, t)$

We assume Vogel-Fulcher type divergence:

$$\eta(\phi) = \eta_0 \exp\left(\frac{\phi}{1-\phi}\right)$$



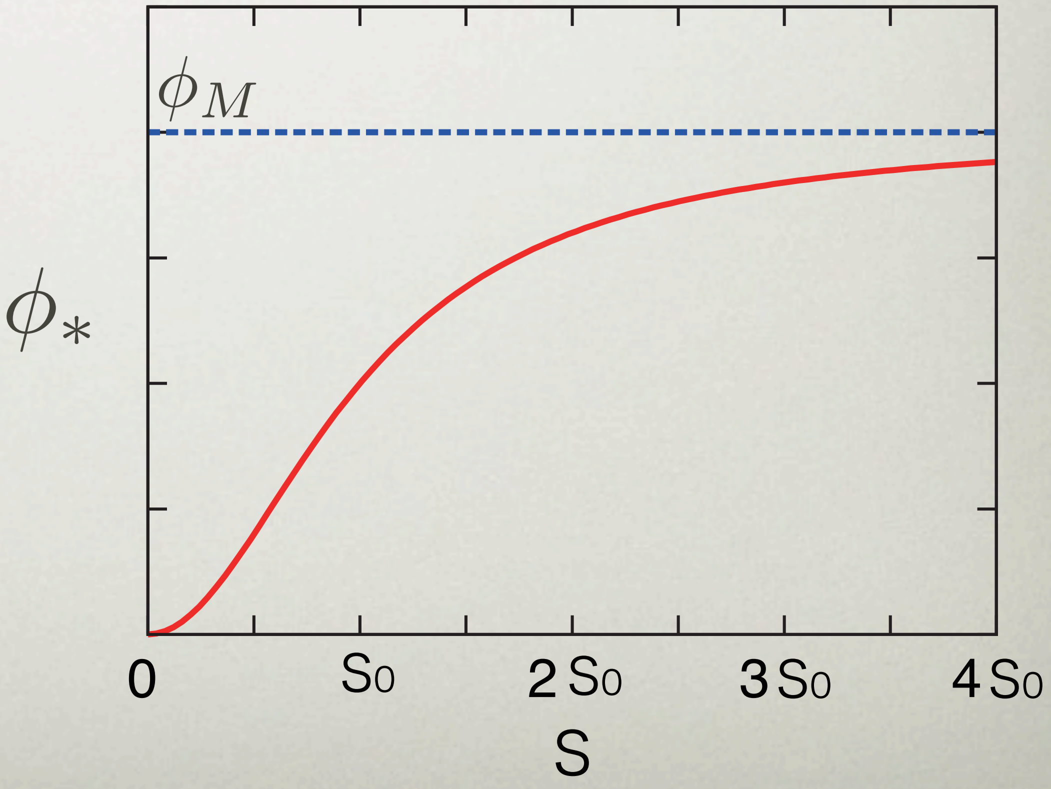
Modeling the dynamics of dilatant Fluid

3) State variable $\phi(r,t)$, in turn, depends on stress

Steady value $\phi_*(S) = \phi_M \dfrac{(S/S_0)^2}{1+(S/S_0)^2}$

where,

$$S = \sqrt{\frac{1}{2}\mathrm{Tr}(\hat{\dot{\sigma}}\hat{\dot{\sigma}})}$$



Model Equations

$$\frac{D\phi(\boldsymbol{r},t)}{Dt} = -\frac{1}{\tau}\Big\{\phi(\boldsymbol{r},t) - \phi_*(S(\boldsymbol{r}))\Big\},$$

Relaxation is driven by deformation (athermal)

$$\frac{1}{\tau} = \frac{1}{r}|\dot\gamma|$$

$|\dot\gamma|$: local shear rate $\quad |\hat{\dot\gamma}| = \sqrt{\frac{1}{2}\mathrm{Tr}(\hat{\dot\gamma}\hat{\dot\gamma})}$

r : dimansionless parameter

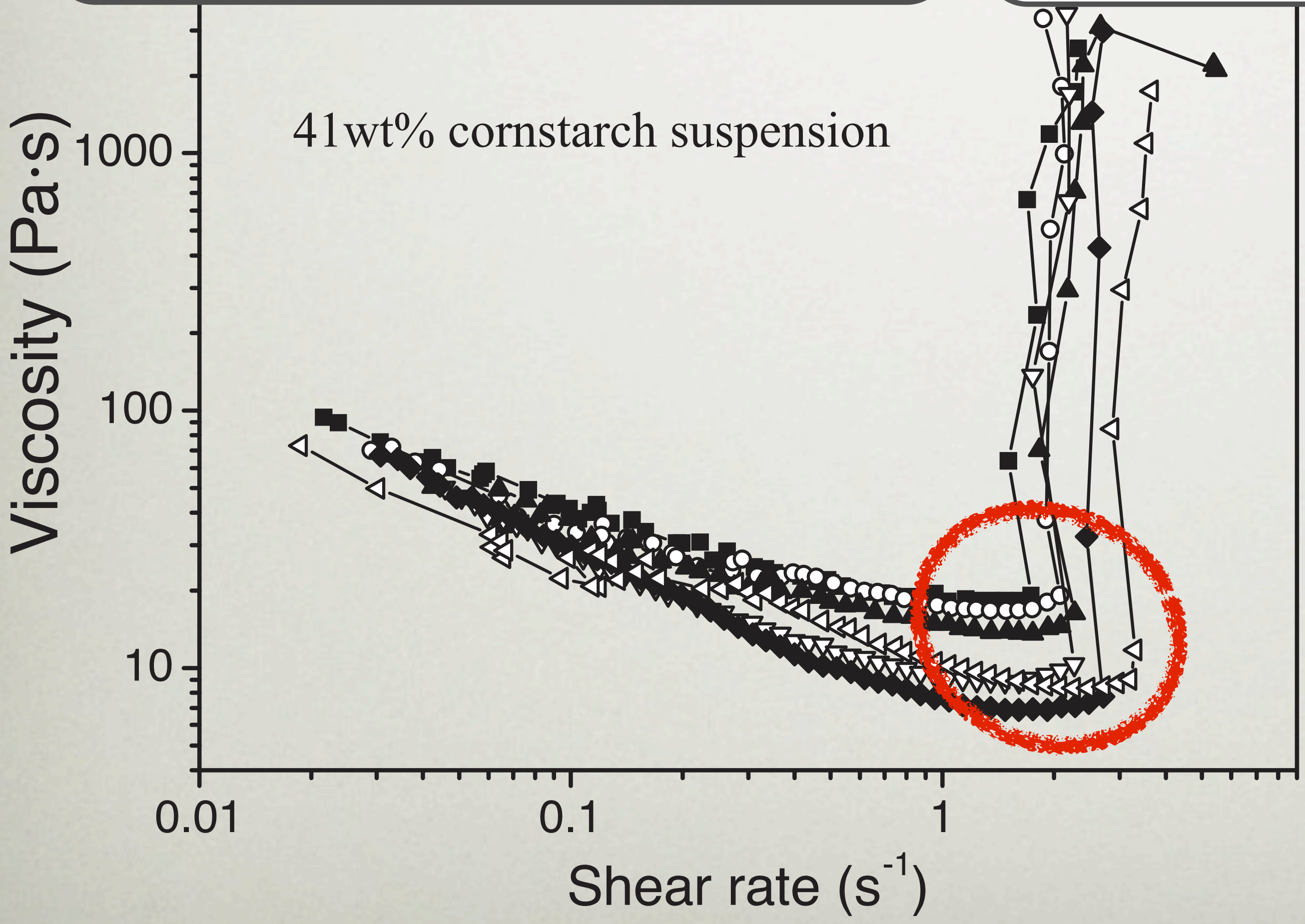
Incompressible Navier-Stokes eq.

$$\rho\frac{Dv_i}{Dt} = \frac{\partial}{\partial x_j}\left(-P\delta_{ij} + \sigma_{ij}\right) \qquad \sigma_{ij} = \eta(\phi)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

Length and time scale

$$\eta(\phi) = \eta_0 \exp\left(\frac{\phi}{1-\phi}\right)$$

$$\phi_*(S) = \phi_M \frac{(S/S_0)^2}{1+(S/S_0)^2}$$



$$\eta_0 \simeq 10\text{Pa}\cdot\text{s}$$

$$S_0 \simeq 50\text{Pa}$$

Fall, Huang, Bertrand, Ovarlez, Bonn, PRL (2008).

Parameters and scales

Thickening stress: $S_0 \approx 50\text{Pa}$

Relaxed state viscosity: $\eta_0 \approx 10\text{Pa} \cdot \text{sec.}$

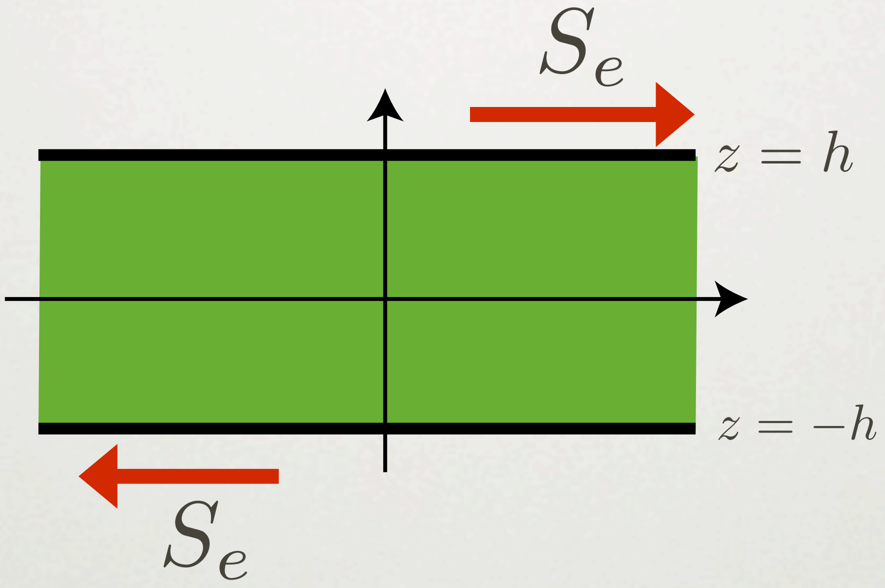
Density: $\rho \approx 10^3\text{kg}/\text{m}^3$

Time Scale $\tau_0 = \dfrac{\eta_0}{S_0}$ **Length Scale** $\ell_0 = \sqrt{\dfrac{\eta_0}{\rho}\tau_0}$

For 41wt% cornstarch suspension

$\tau_0 \approx 0.2\text{sec.}$ $\ell_0 \approx 5\text{cm}$

Simple Shear Flow of Dilatant Fluid

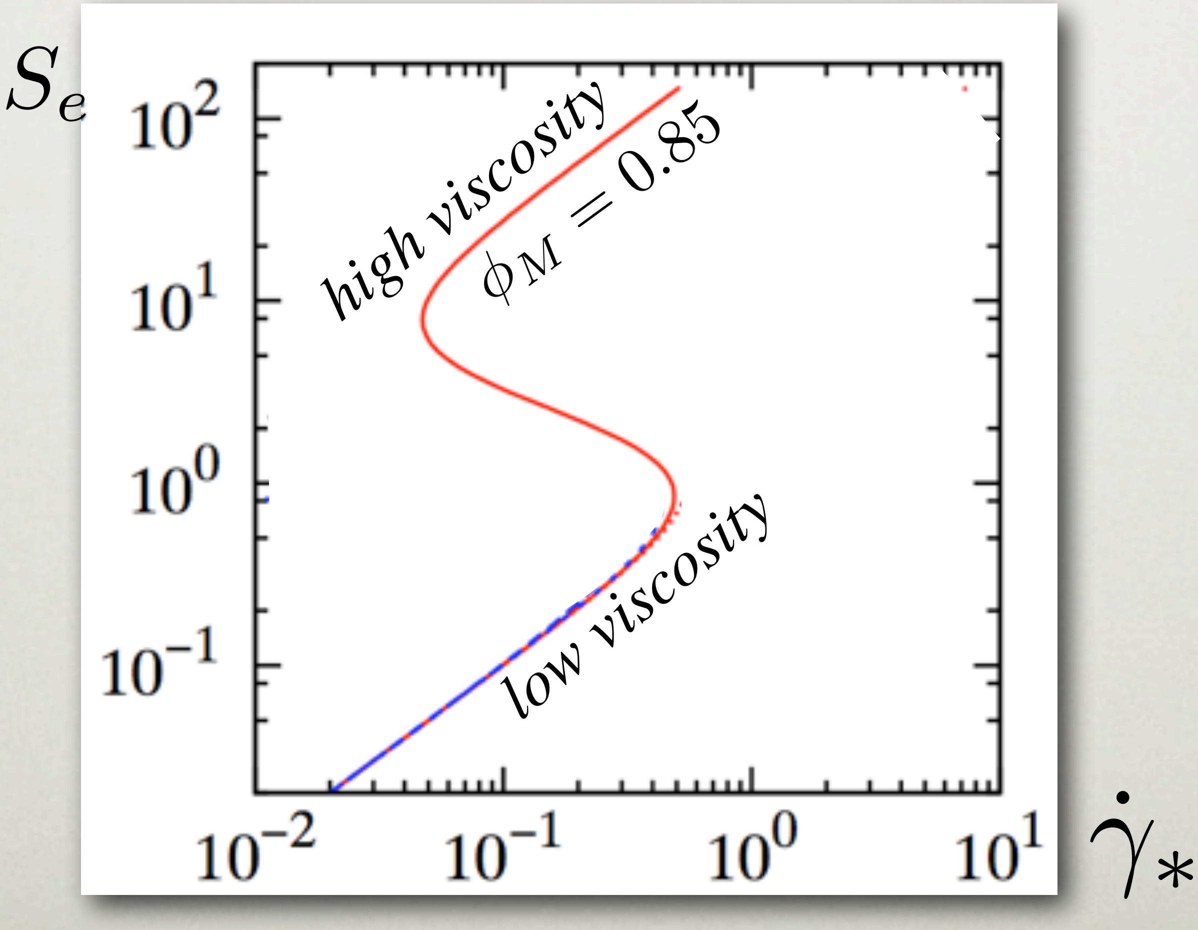


Boundary condition $\left. S(z,t) \right|_{z=\pm h} = S_e$

Simple Shear Flow of Dilatant Fluid

Steady State Solution of the Model Equation

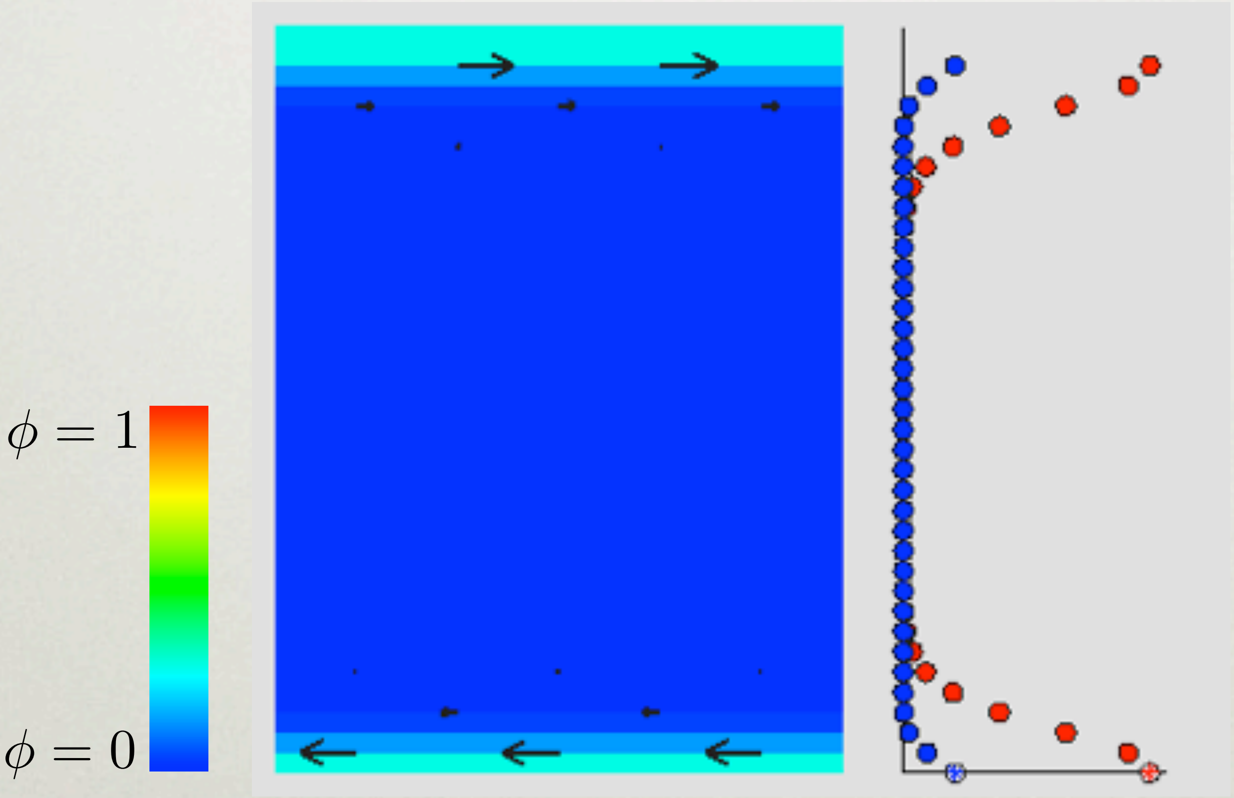
$$S_e = \eta\big(\phi_*(S_e)\big)\dot{\gamma}_*$$



Shear flow in the unstable branch

Flow oscillates spontaneously under constant stress

$$\phi_M = 1.0, h = 3.0, S_e = 1.0, r = 0.1$$



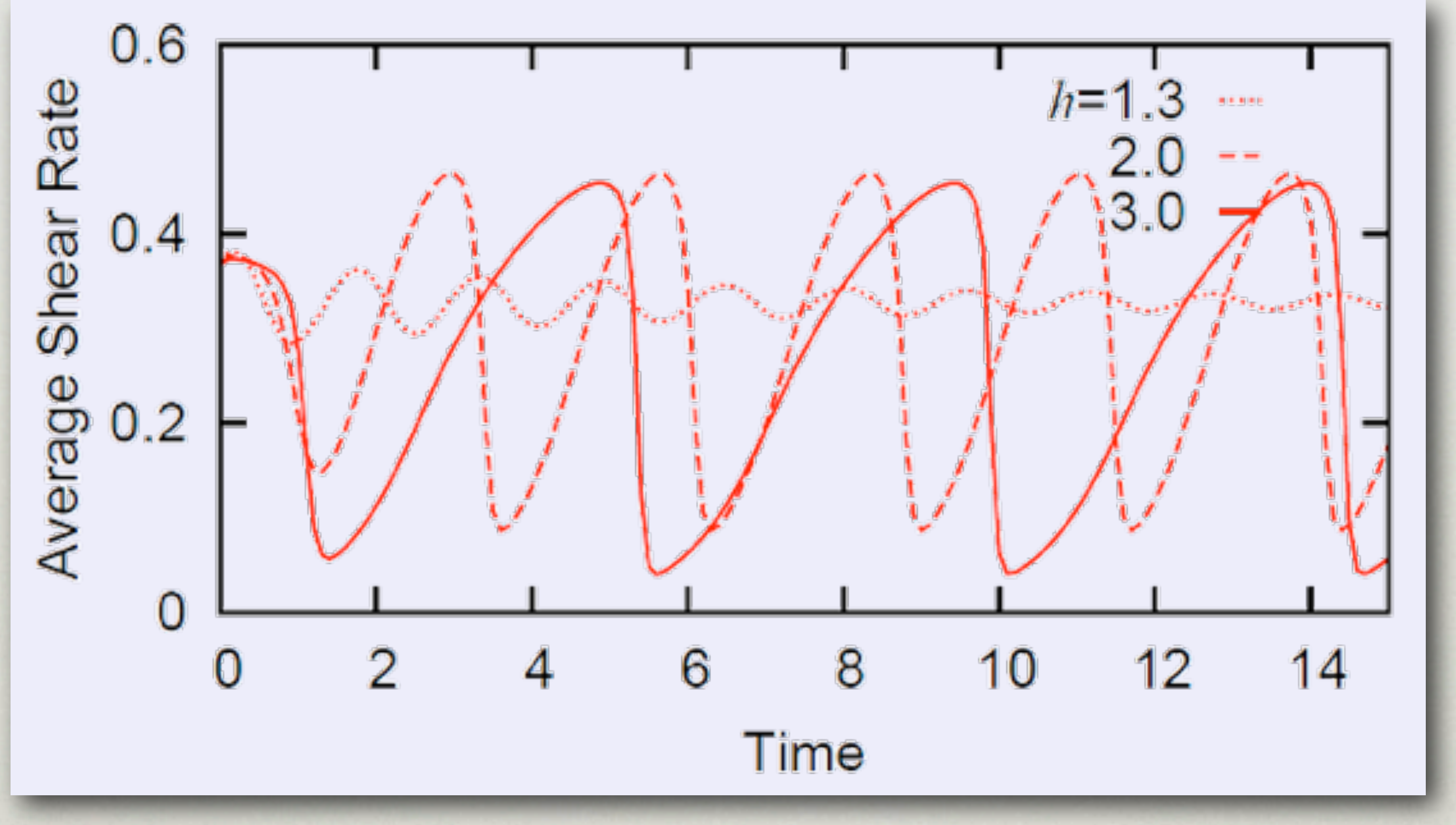
🔴 ϕ parameter

🔵 shear rate

Shear flow in the unstable branch

Saw-tooth like wave

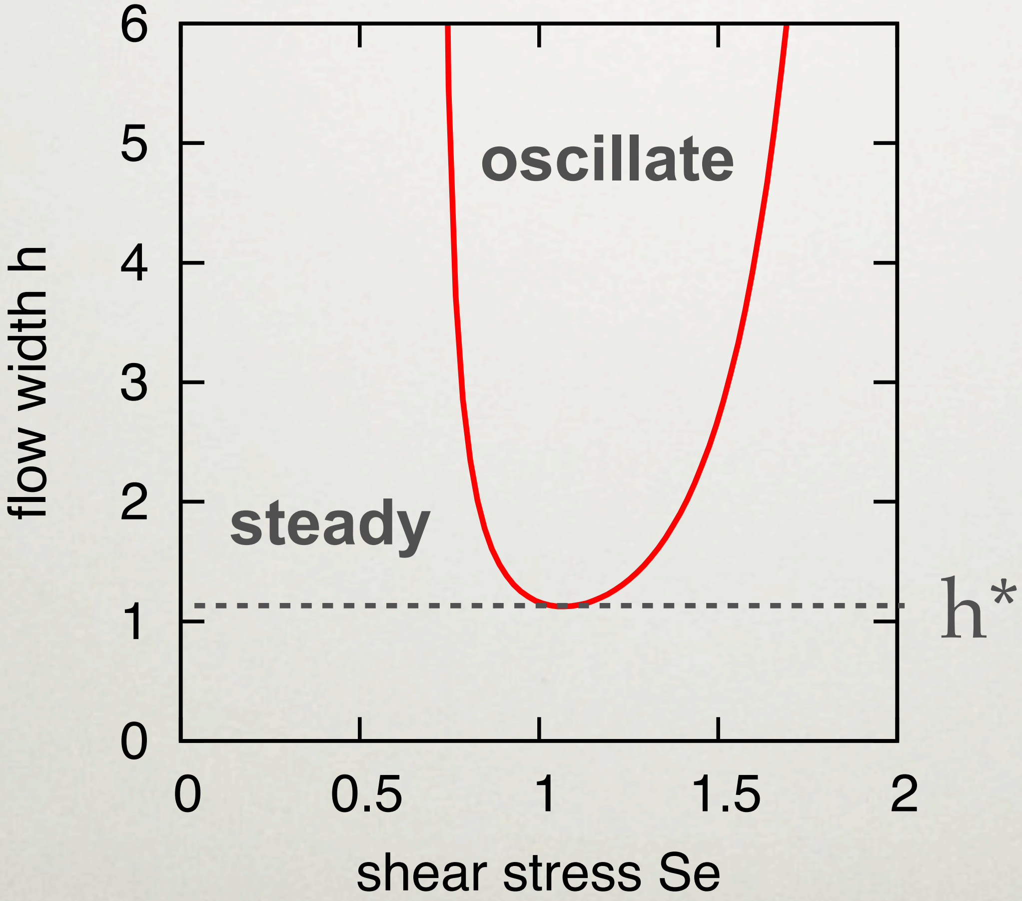
--- $\dot{\gamma}$ moderately increases and suddenly drops



- ✦moderate increase and sudden drop

Shear flow in the unstable branch

a State Diagram for steady and oscillatory region



~5cm for
41wt% suspension.

Experiment with starch-water mixture

Parameters and scales

Thickening stress: $S_0 \approx 50\mathrm{Pa}$

Relaxed state viscosity: $\eta_0 \approx 10\mathrm{Pa} \cdot \mathrm{sec.}$

Density: $\rho \approx 10^3\mathrm{kg}/\mathrm{m}^3$

Time Scale $\tau_0 = \dfrac{\eta_0}{S_0}$ **Length Scale** $\ell_0 = \sqrt{\dfrac{\eta_0}{\rho}\tau_0}$

For 41wt% cornstarch suspension

$\tau_0 \approx 0.2\mathrm{sec.}$ $\ell_0 \approx 5\mathrm{cm}$

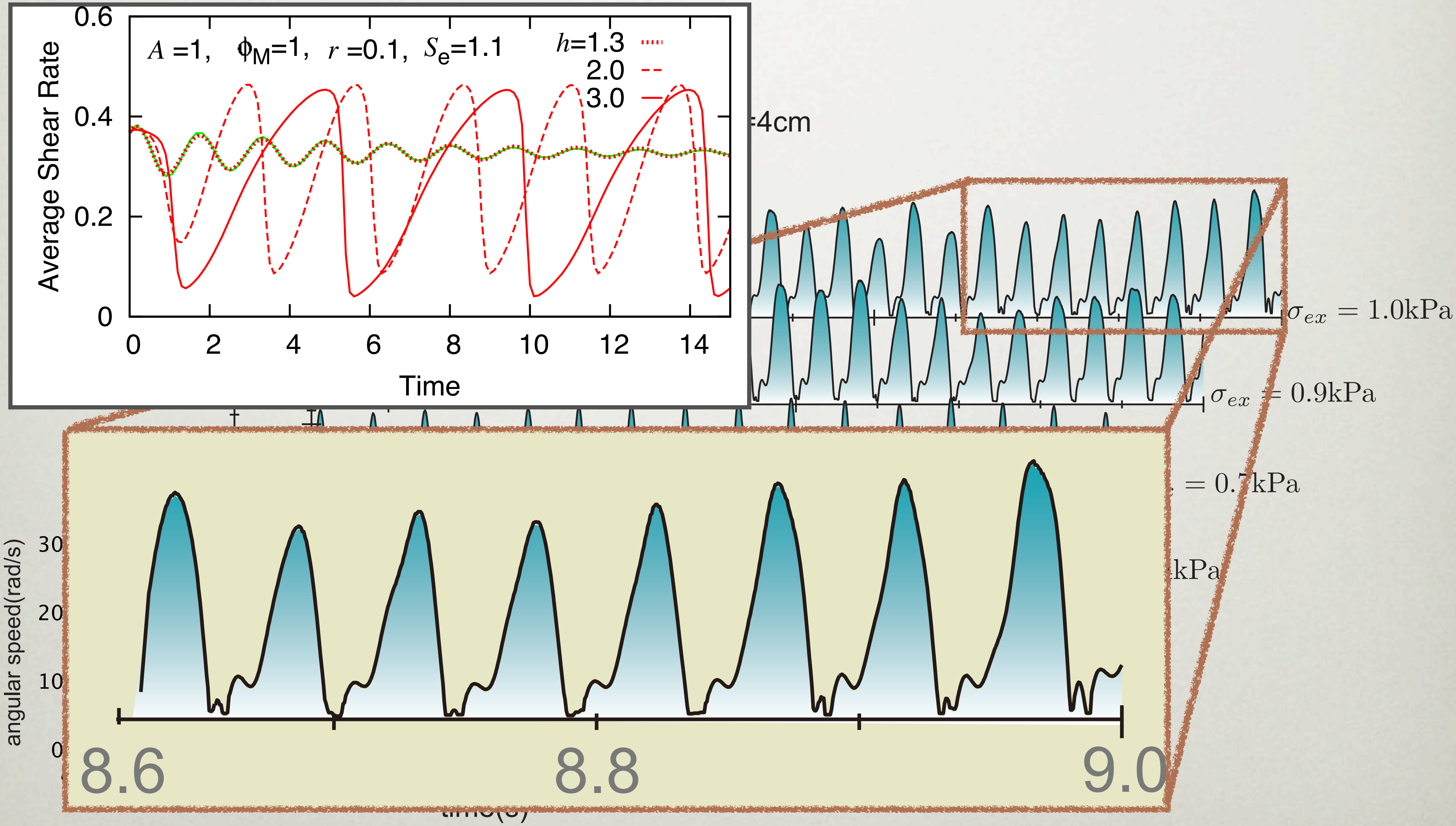
Experimental Setup

tarch

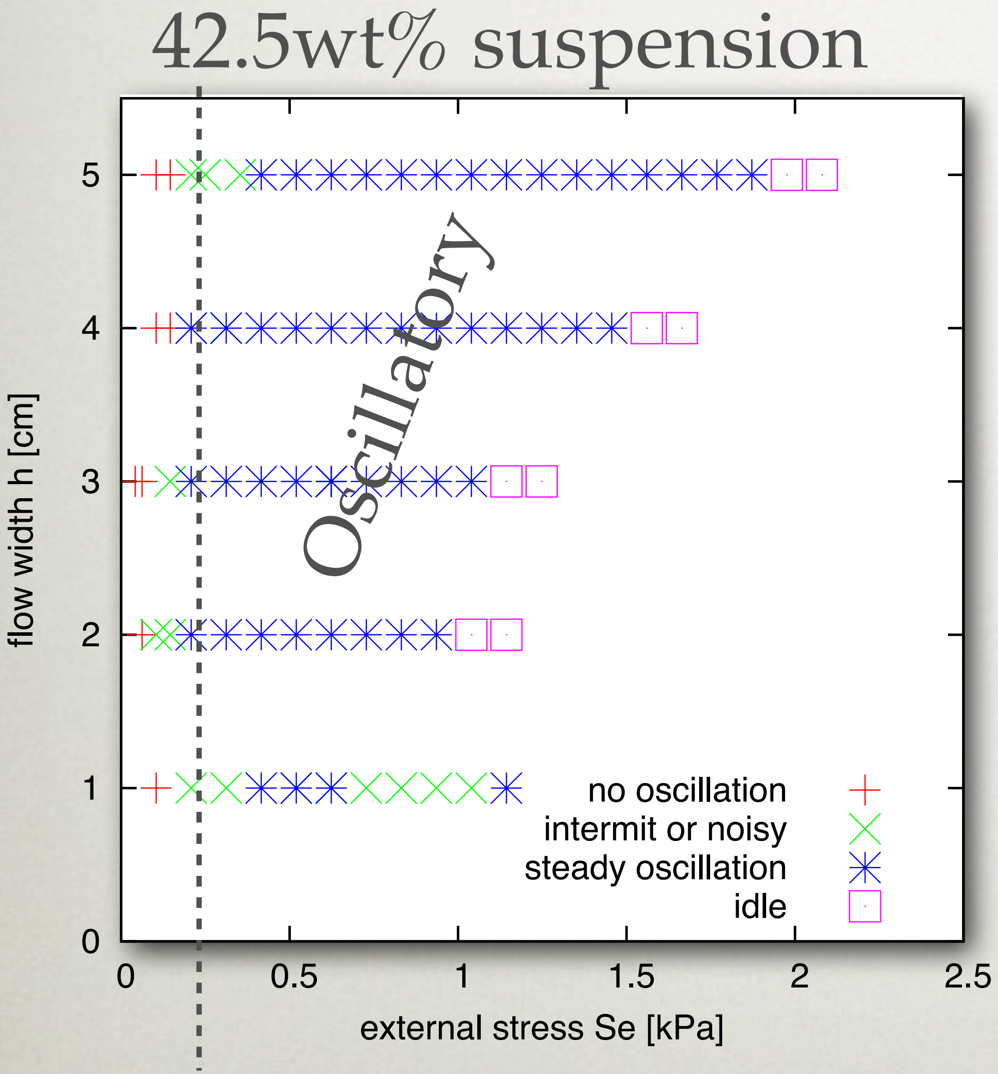
starch-

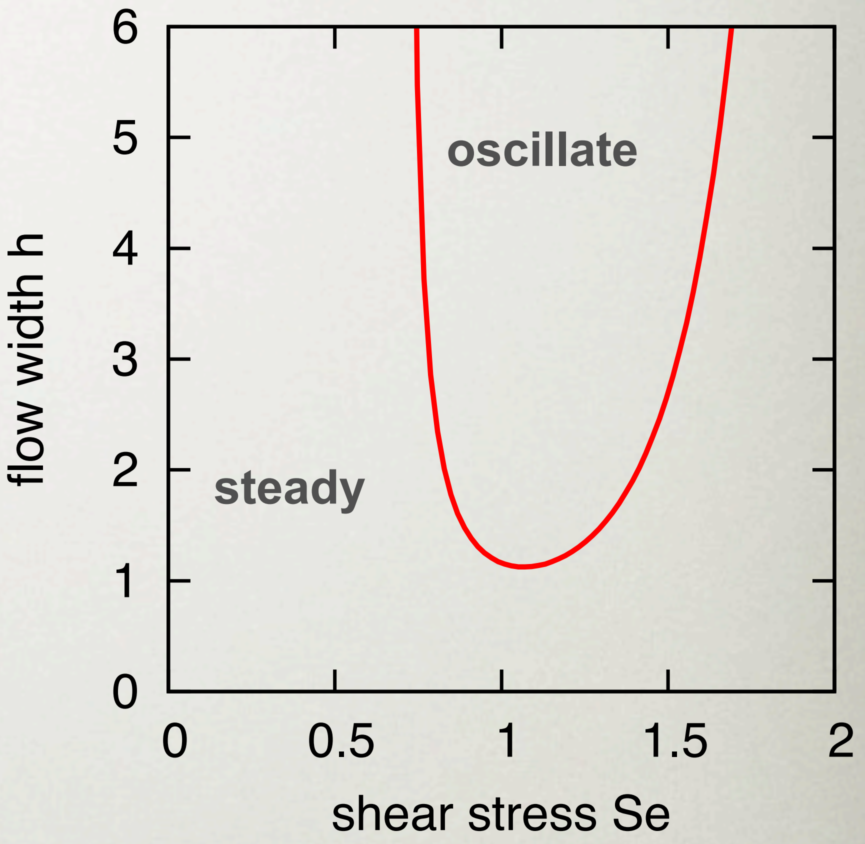
Oscillation: 1000fps movie

Angular speed of the center rod



A State diagram of the flow

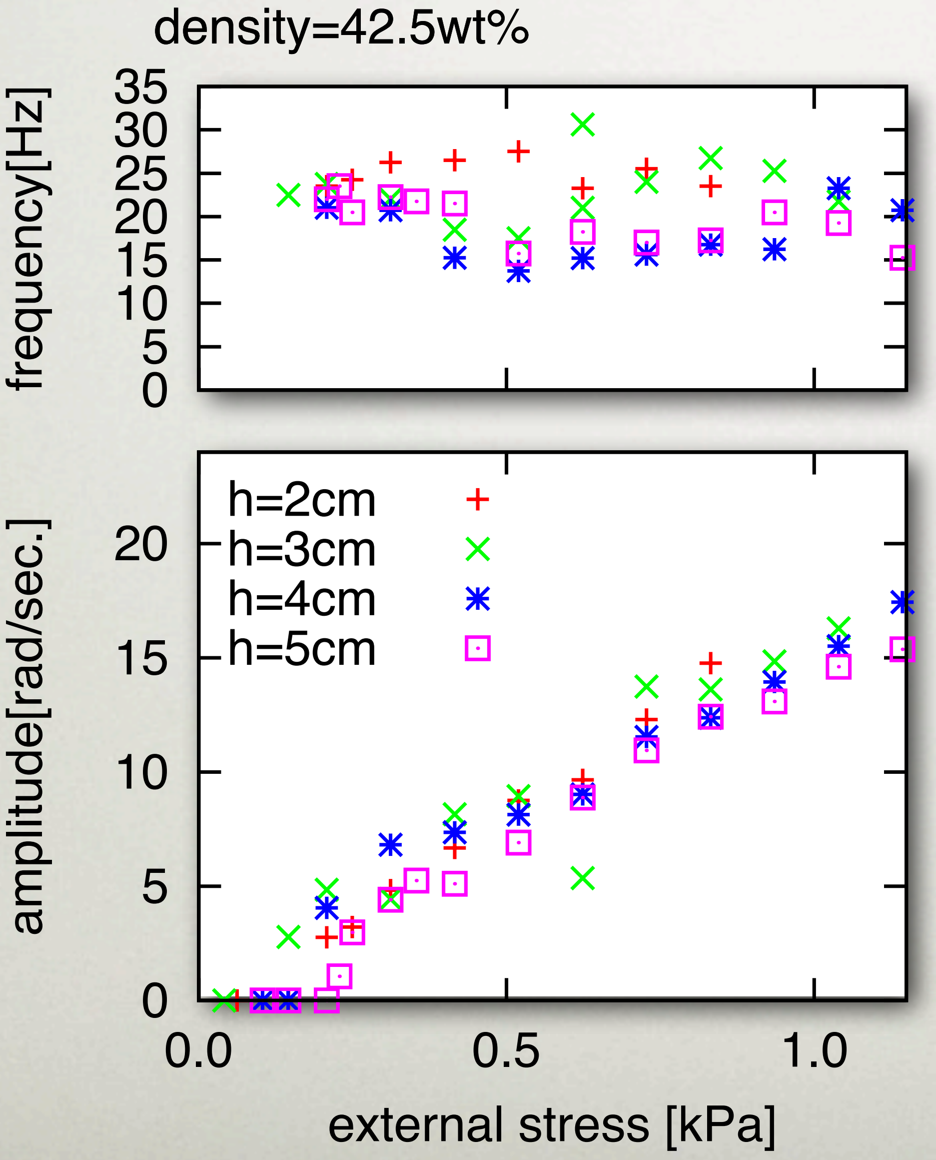




$$S_0 = 50Pa$$
$$\ell_0 = 5cm$$

Threshold stress. $S_e{}^* \simeq 0.1\mathrm{kPa}$

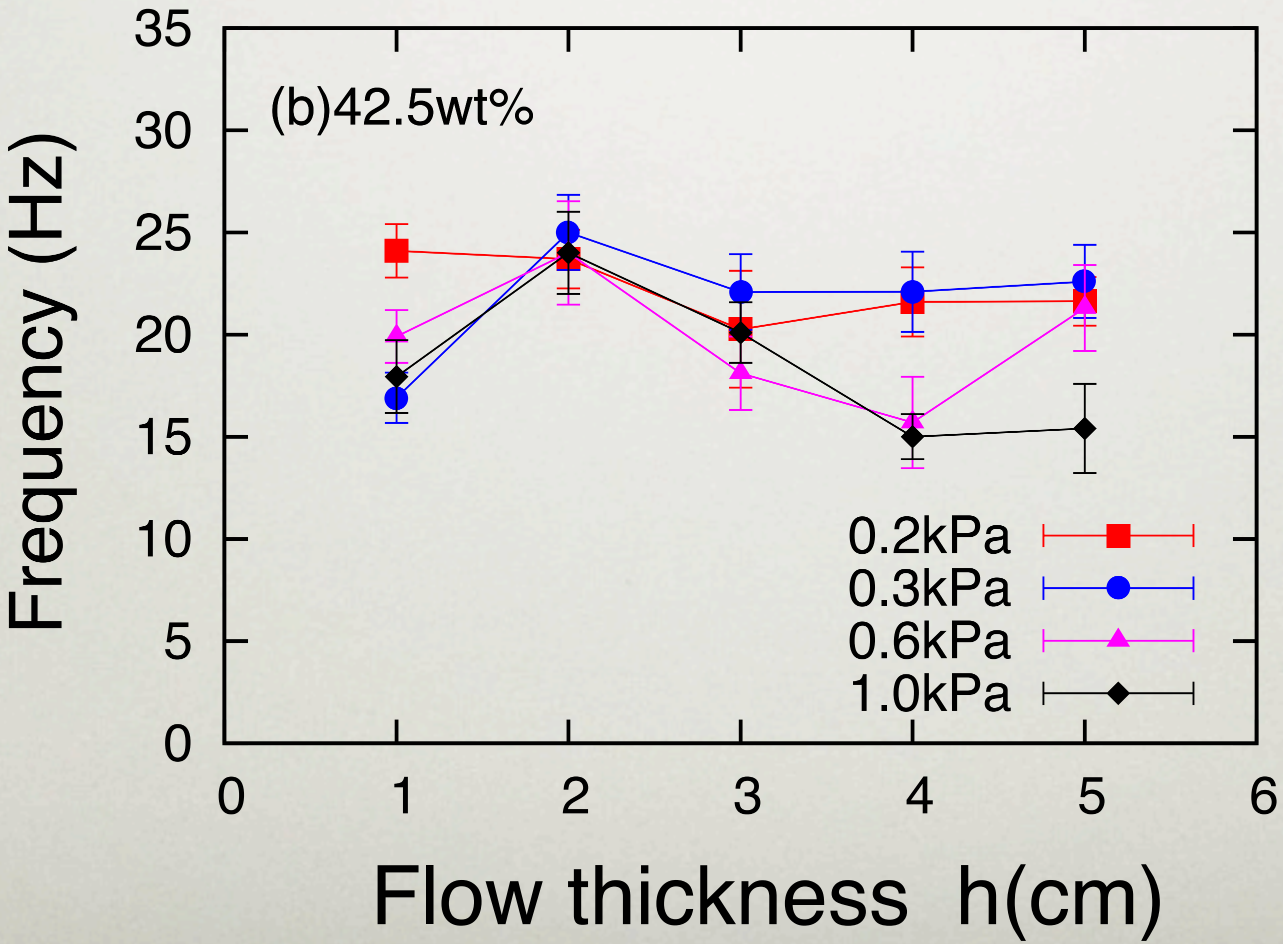
Stress dependence of freq. and amplitude



*Frequency stays almost constant near threshold

Frequency vs flow thickness

- No systematic dependence either on the thickness and shear stress
- Frequencies are **always around 20Hz** (twice the predicted value)

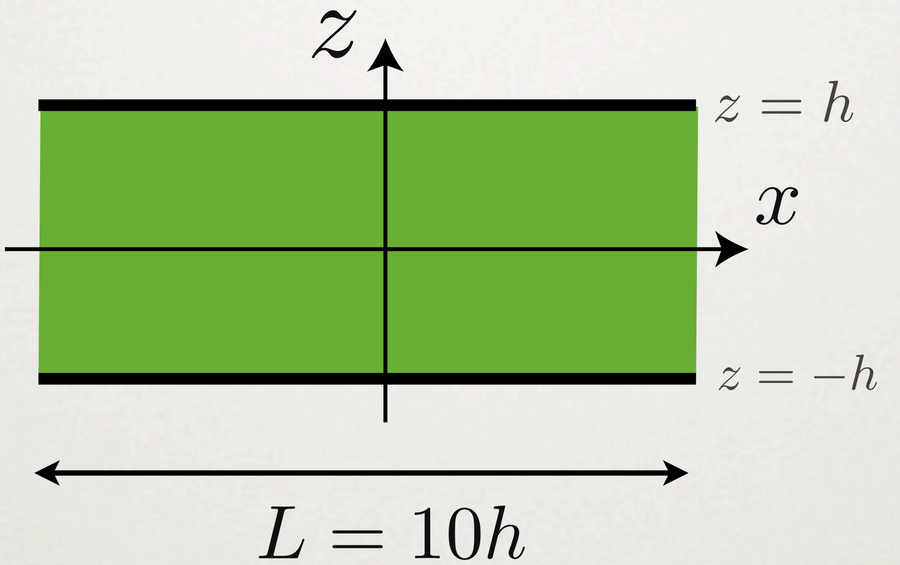


Experimental observation

- About **20Hz** frequency.
- Oscillation starts with **Hopf bifurcation**.
- Frequency **does not depend on both *Se* and *h***

2D Simulations

— Maker and Cell (MAC) method

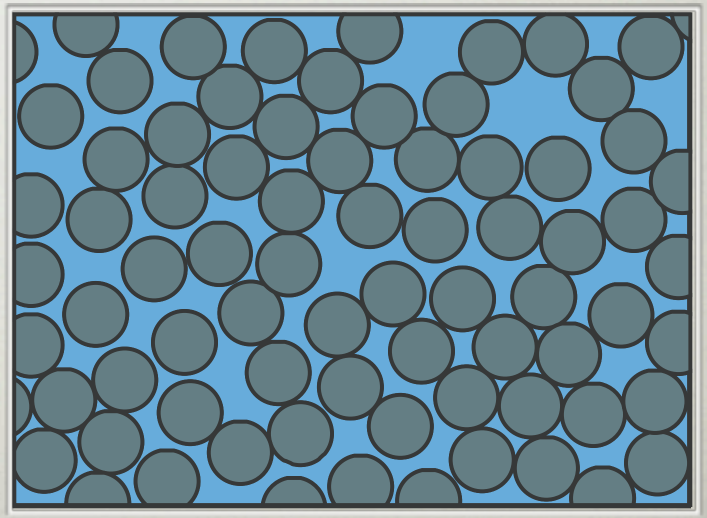


Initial condition:

$$v_i(\boldsymbol{r}, t = 0) = 0$$

$$\phi(\boldsymbol{r}, t = 0) = \xi_i$$

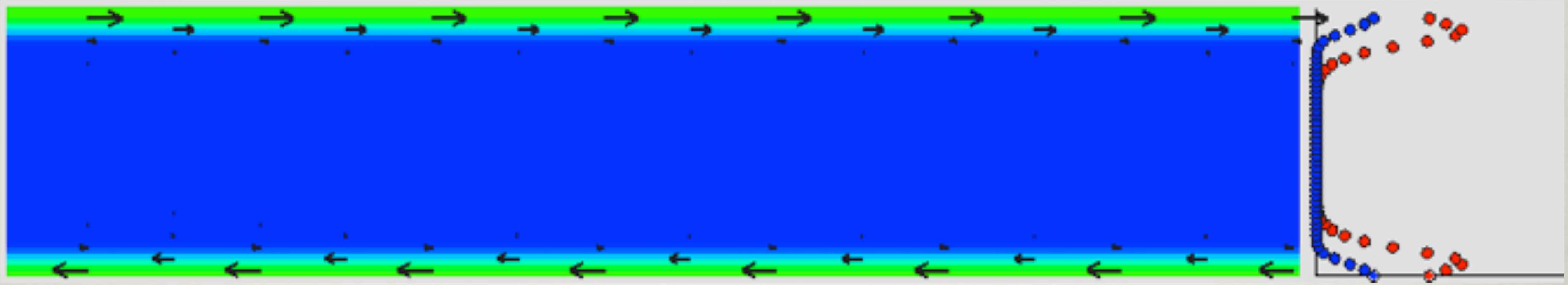
Initial noise: $|\xi_i| = 10^{-4}$

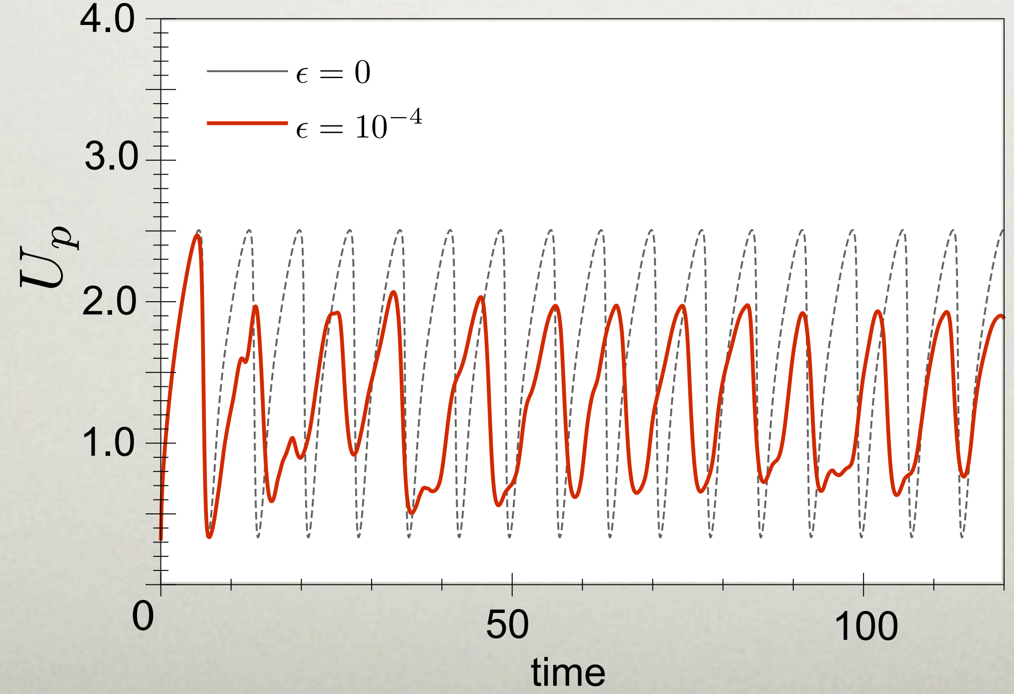


Inhomogeneous Oscillation

Small noise is given to initial ϕ

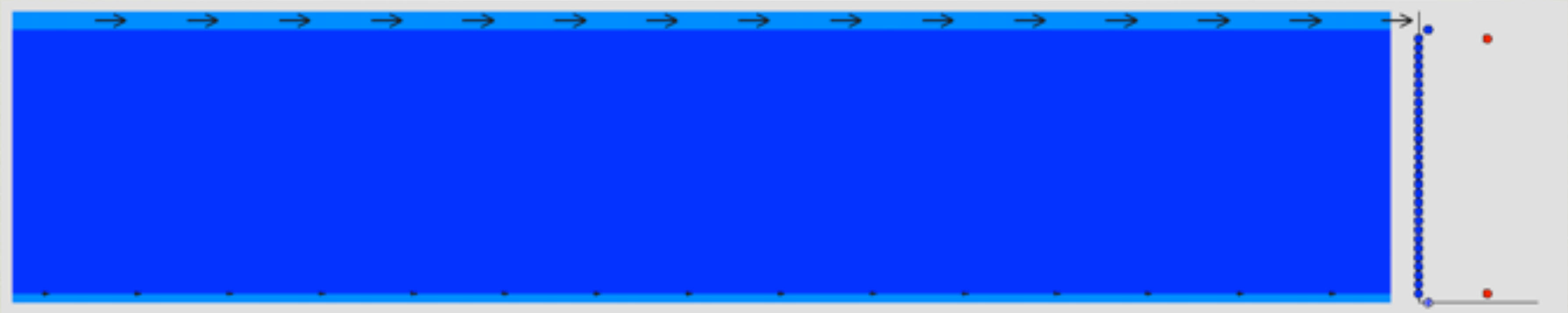
ϕ_M=0.85, Se=1.0

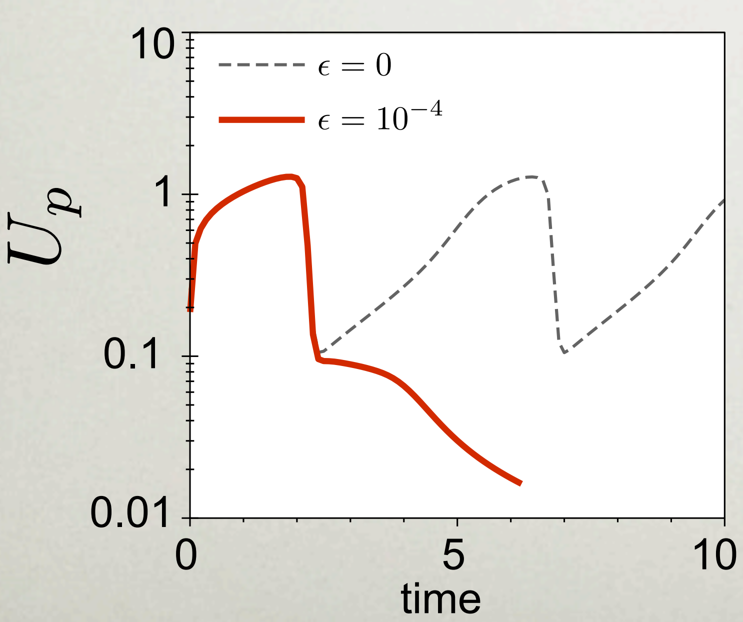


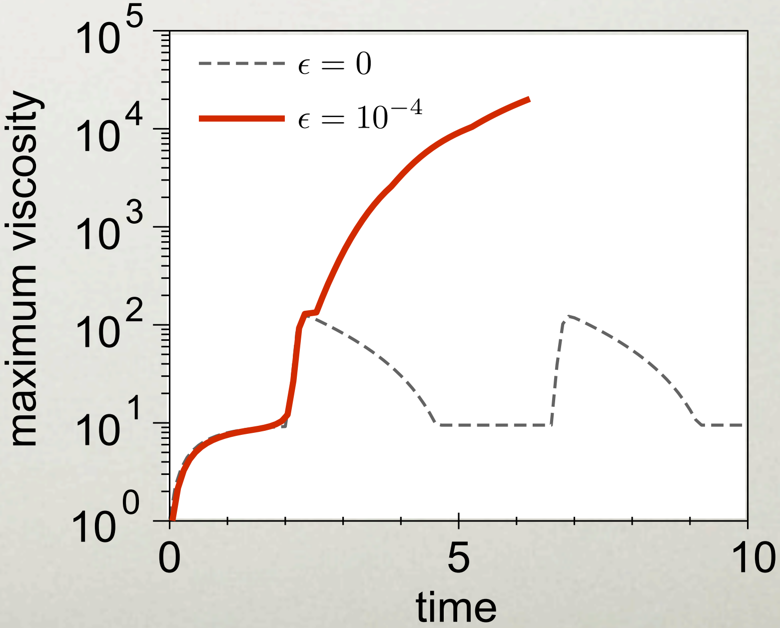


Jamming caused by instability

$$\phi_M = 1.0$$

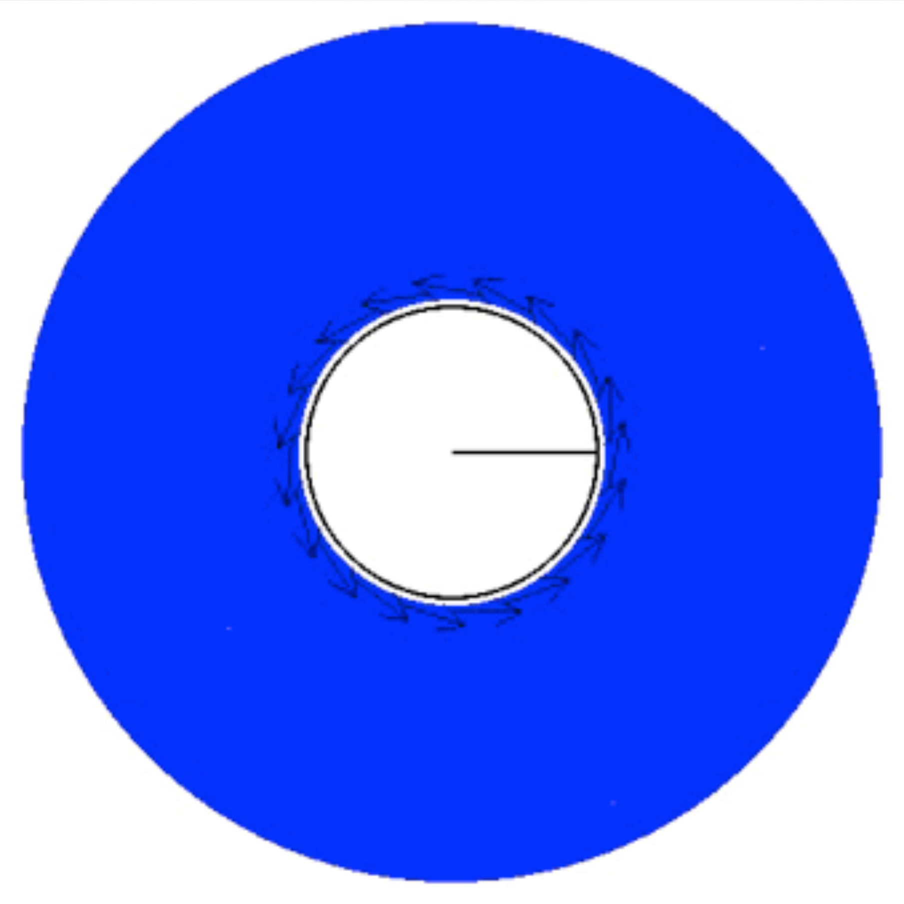






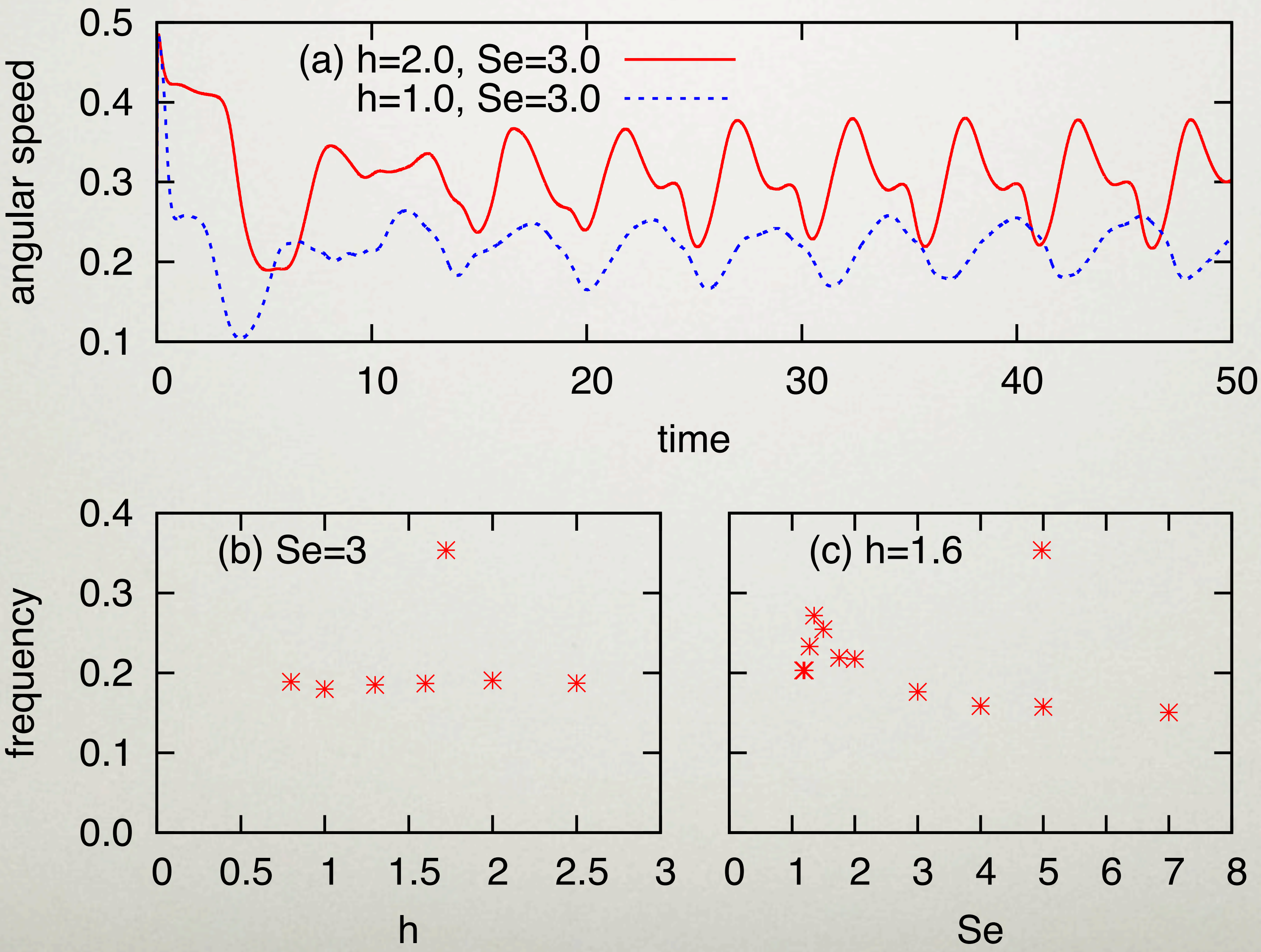
Inhomogeneous Oscillation

ϕ_M=0.85, r_{in}=1.0, r_{out}=3.0, S_e=2.0



Inhomogeneous Oscillation

h and S_e independent Frequency



summary and remarks

- We proposed phenomenological model
- the model predicts spontaneous oscillation
- the oscillation is also observed experimentally

and next...

- We’d like to confirm if the thickening band governs the oscillation.
  - measure pressure of the fluid (?)
  - measure off-center force acts on the rod (?)