Collective dynamics and patterns of rapid granular fluid and amplitude equation

Physics of Granular Flows
YITP, Kyoto University, Japan
June 24 - 6 July 2013

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2 July 2013

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Outline of Talk

Part 1

- Introduction: collective dynamics & patterns
- Amplitude equation in granular fluid

Part 2

- Problem description: 3D Couette flow
- Weakly nonlinear analysis: Amplitude Eqn.
- Numerical methods
- Results
- Conclusions

Introduction

When a system is driven by external forcing, it becomes unstable beyond some critical value of the control parameter

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Spatio-temporal instabilities arises

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Collective dynamics of unstable system, far from equilibrium, yields patterned states

(Cross & Hohenberg, Rev Mod Phys 1993)

<u>Classification depending on external forcing</u>

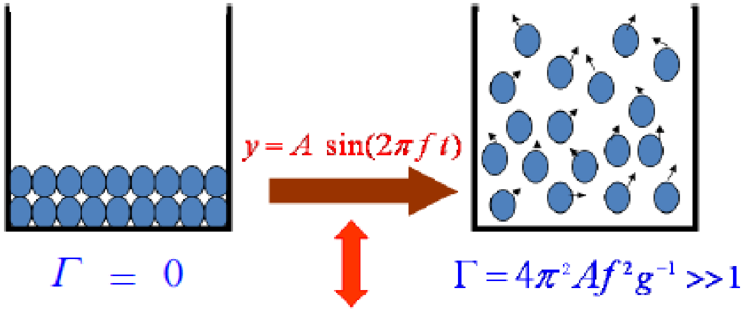
Aranson & Tsimring, Rev Mod Phys. 2006

Vibration

Gravity

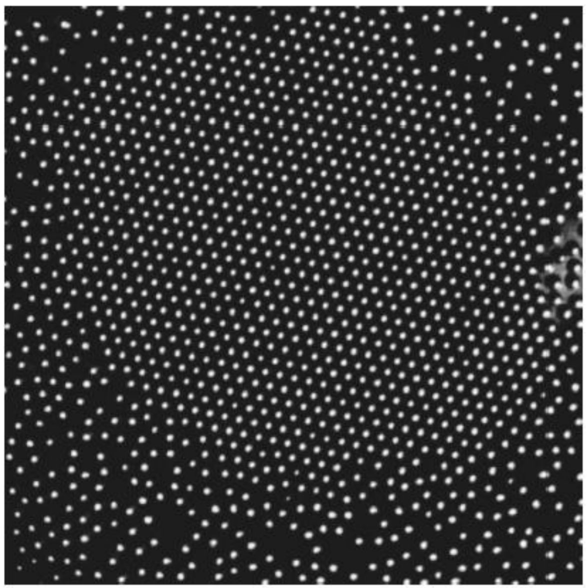
Shear

Patterns in vibration driven granular matter



Phenomena

*Clustering
*Surface wave
*Localized structure
*Convection

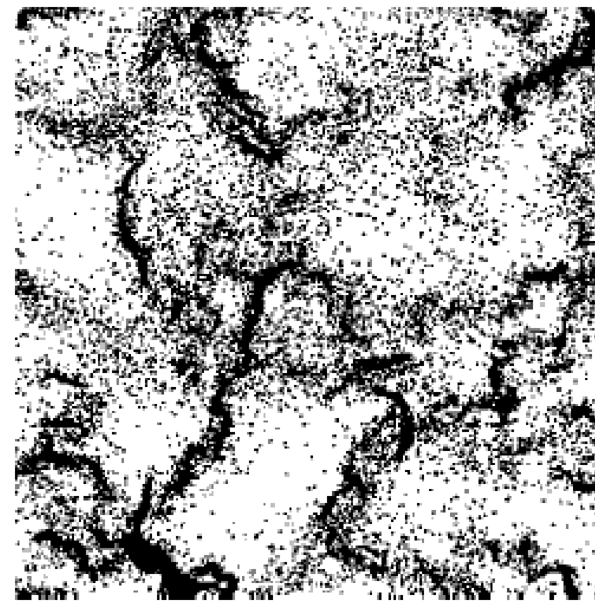


Uniform configuration to

↓

Clustering

Coexistence of
dilute and dense region

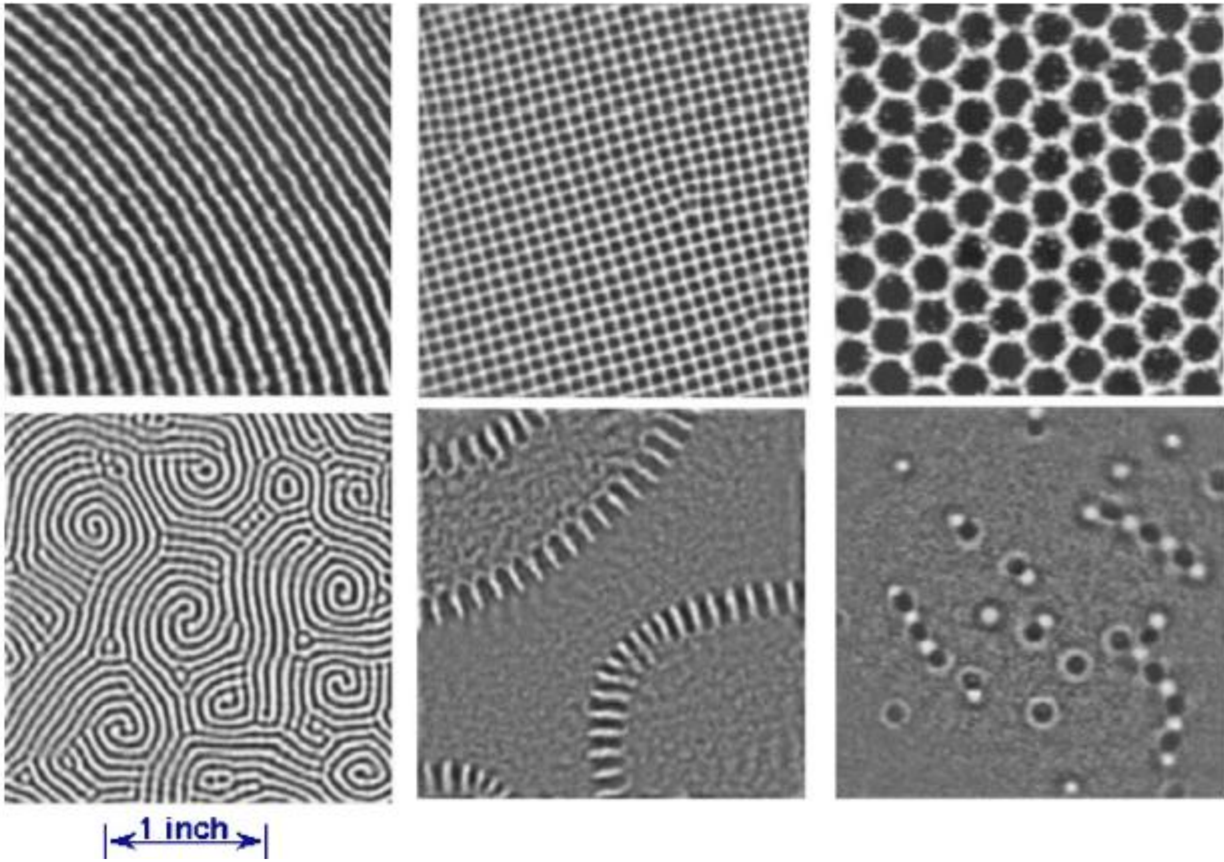


Top view of a submonolayer of grains on a vibrated plate
(Olafsen and Urbach, 1998)

Freely cooling granular gas
(Goldhirsch and Zanetti, 1993)

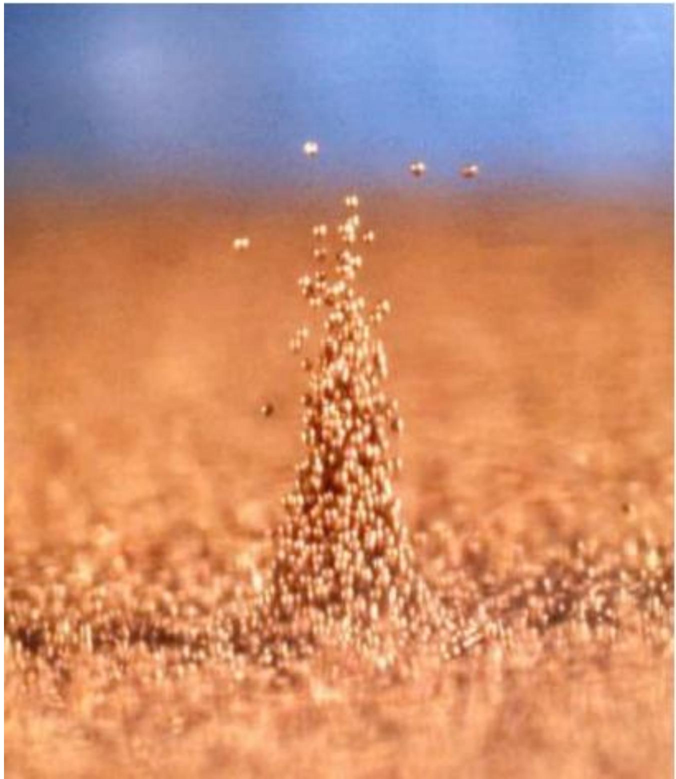
Uniform granular layer to → surface wave and localized structure

Surface wave



Localized structure

oscillon



Pattern for various values of frequency and acceleration:

stripes, squares, hexagons,

spiral, interfaces, and oscillons

(Umbanhower et al. 1996)

Non-uniform Configuration → Convection

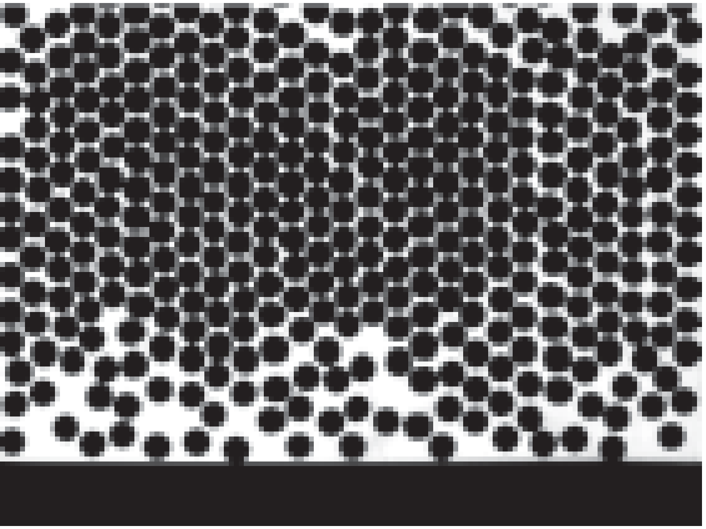
Bouncing state *(Hayakawa, Yue, & Hong, 1995)*

Density inversion *(Khain & Meerson, 2003)*

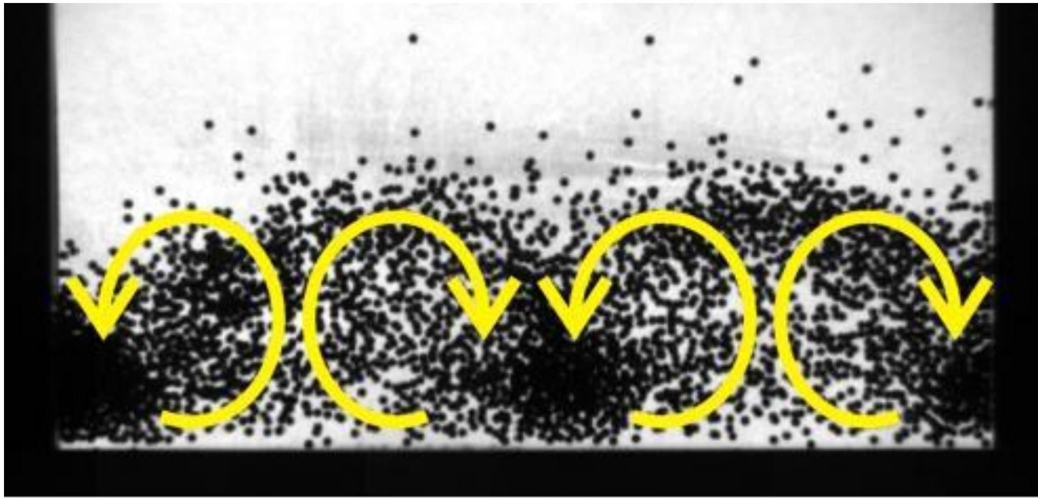
Leidenfrost state *(Eshuis et al 2010)*

→ **convection**

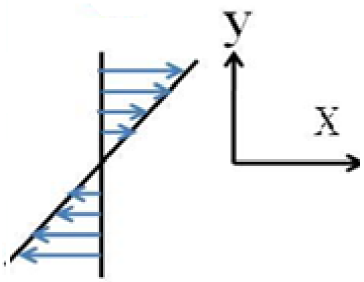
Leidenfrost ***(Eshuis et al 2005)***



→ **Convection** ***(Eshuis et al 2010)***



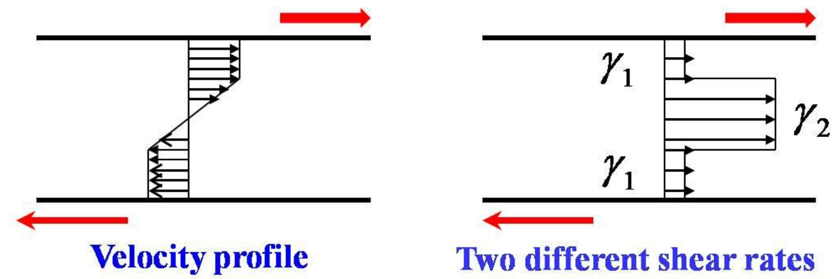
Patterns in shear driven granular matter



Shearbanding

Phenomena

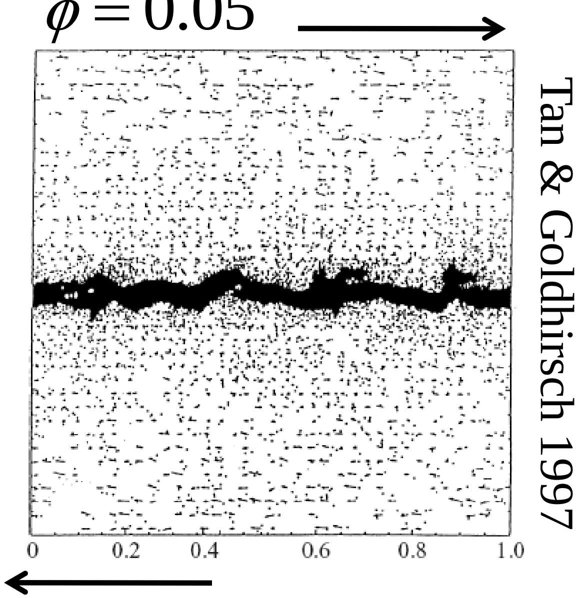
*Shearbanding
*Segregation
*Density wave
*Clustering



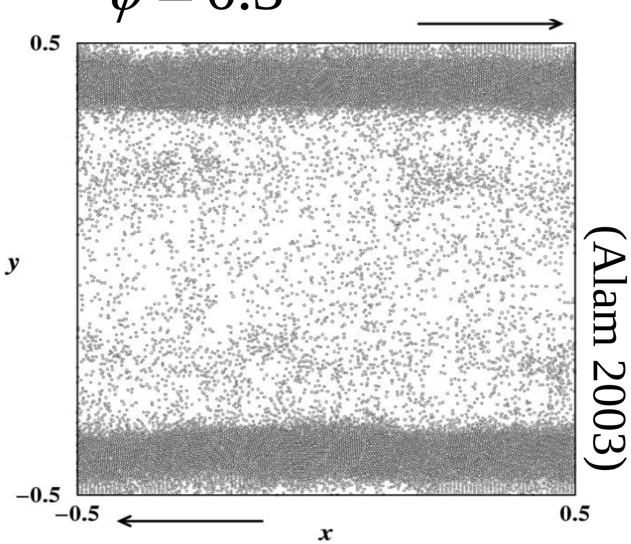
Gradient Banding:

Bands of different shear rates, along the gradient direction, coexist

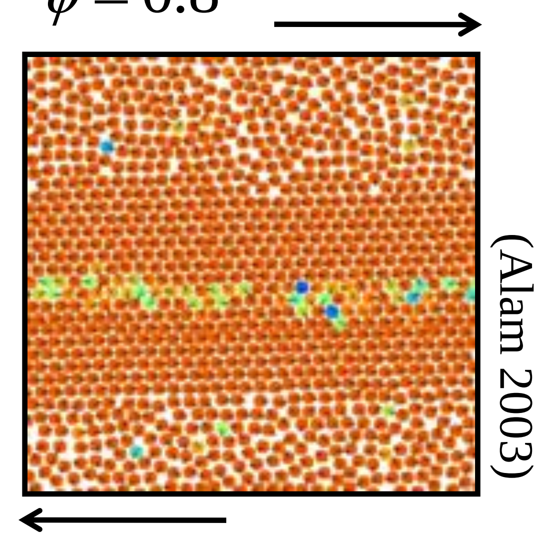
$\phi = 0.05$



$\phi = 0.3$



$\phi = 0.8$



Shear-Banding in ‘Dense’ Granular Flow

(Savage & Sayed 1984; Mueth et.al. 2000)

Shear-bands are narrow and localized near moving boundary.

Circular Couette geometry

Mueth et al. 2000

Fast particles (yellow) near the inner wall appear to move smoothly while the orange and red particles display more irregular and intermittent motion

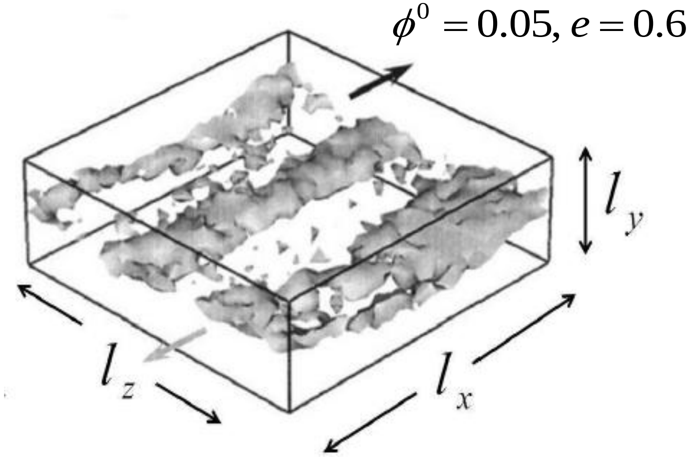
Vorticity Banding:

Bands of different shear stress, along the vorticity direction, coexist

Three bands of particles along the vorticity direction

Conway & Glasser, (2004)

$\phi^0 = 0.05, e = 0.6$



Clustering & Density wave

Conway & Glasser, (2004)

Reason

Inelastic collisions

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Fluctuations

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Generate regions of high density

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Instability

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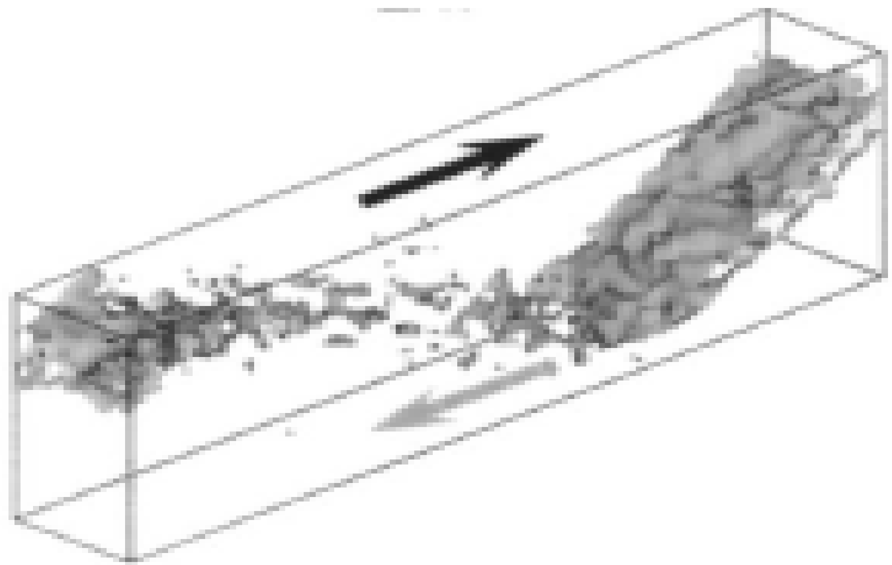
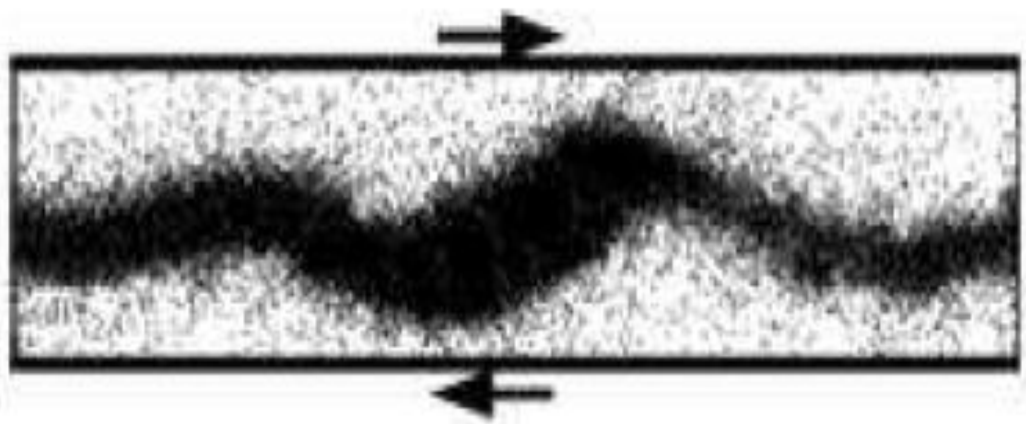
Dense cluster, plug..





Amplitude (order parameter) equation

Describe the slow modulation in space and time near the onset of instability

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Gives a qualitative insight of pattern formation

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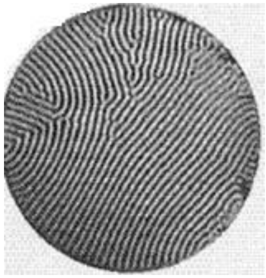
- Growth of the perturbation about the spatially uniform state
- Saturation of the growth by nonlinearity
- Dispersion and effect of spatial distortions

$$\frac{\partial A}{\partial t} + a_1 \frac{\partial A}{\partial x} + a_2 \nabla^2 A = g(A, t, x, \cdots)$$

Order Parameter model for Granular patterns

Vibration



Complex Ginzburg-Landau Equation

Patterns in vibrated bed can be predicted by the complex **Ginzburg-Landau Eqn** (**'çubic'**)

(Tsimring and Aranson 1997, PRL)

Phenomenological model

$$\frac{\partial \psi}{\partial t} = \gamma\psi^* - (1 - i\omega)\psi + (1 + ib)\nabla^2\psi - \psi|\psi|^2 - \rho\psi$$

$$\frac{\partial \rho}{\partial t} = \alpha\nabla.(\rho\nabla|\psi|^2) + \beta\nabla^2\rho$$

ψ : complex amplitude of subharmonic pattern (order parameter)

ρ: thickness of the granular layer

$\gamma\psi^*$: parametric driving

γ : normalized amplitude

ω:frequency of driving

b:ratio of dispersion to diffusion

Phenomena

*Turbulance

*Nonlinear Waves,

*Phase transitions,

*Superconductivity,

*Superfluidity, etc.

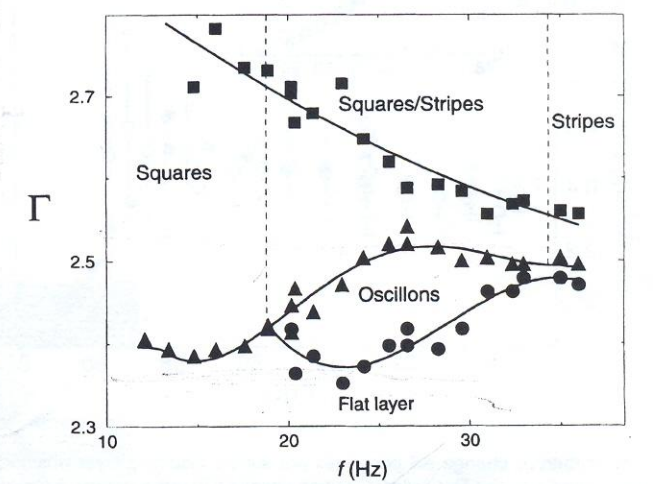


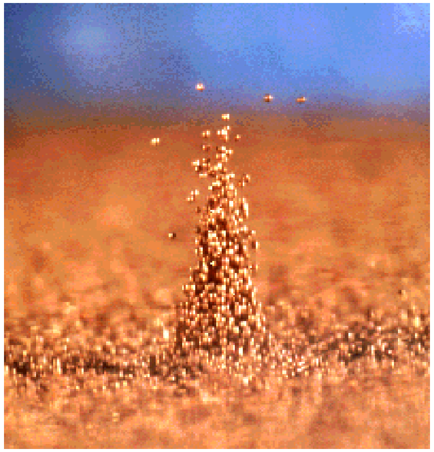
FIG. 2 Diagram showing the stability regions for different states, as a function of f and Γ, for increasing Γ (squares) and decreasing Γ (triangles and circles). The transitions from the flat layer to squares and stripes are hysteretic, but the hysteresis is much smaller for stripes. Oscillons are observed for layers greater than 13 particles deep in a range of f which increases with increasing depth. For thinner layers, the phase diagram is similar but without the oscillon region.

Generalized Swift Hohenberg Equation

Generalized Swift-Hohenberg equation describes primary pattern forming bifurcation: square, strips and oscillons ***(Crawford and Riecke 1999)***

$$\frac{\partial \psi}{\partial t} = R\psi - \left(\nabla^2 + 1\right)^2 \psi + N(\psi)$$

$$N(\psi) = b_1\psi^3 - b_2\psi^5 + \varepsilon\nabla.(\nabla\psi)^3 - \beta_1\psi(\nabla\psi)^2 - \beta_2\psi^2\nabla^2\psi$$



(Swinney 1996)

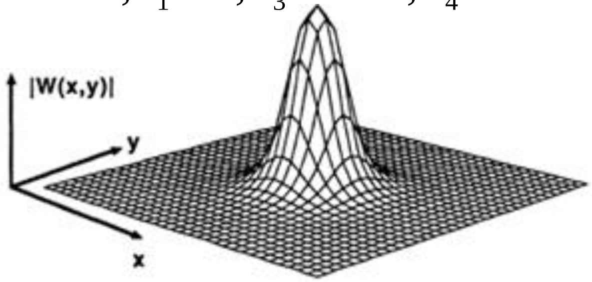
The magnitude of epsilon describes squares, strips, hexagons, oscillons, etc. patterns

Subcritical Complex Ginzburg-Landau Equation ('quintic')

$$\frac{\partial A}{\partial t} = \varepsilon A + (1 + ic_1)\nabla^2 A + (1 - ic_3)A|A|^2 - (1 - ic_4)A|A|^4$$

$\varepsilon = -0.192,\ c_1 = 4,\ c_3 = -0.3,\ c_4 = 0.4$

Localized pulse solution, amplitude surface



(Thual and Fauve, J. Phys. 1988)

Stuart-Landau Equation

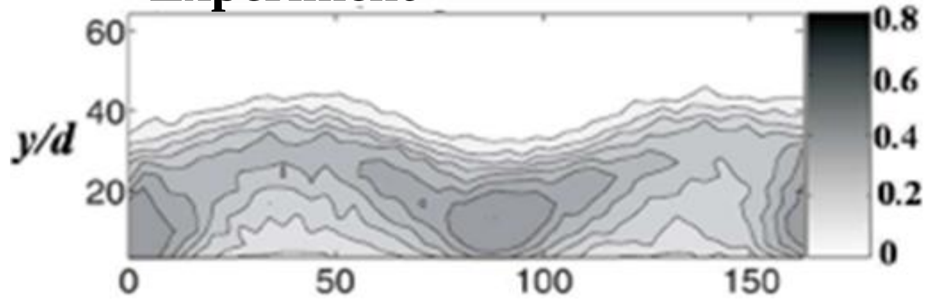
Shukla P., Meer D. V., Lohse D. & Alam M., 2013, Preprint

$$\frac{dA}{dt} = \sum_{j=0}^{\infty} c^{(2j)} A|A|^{2j}$$

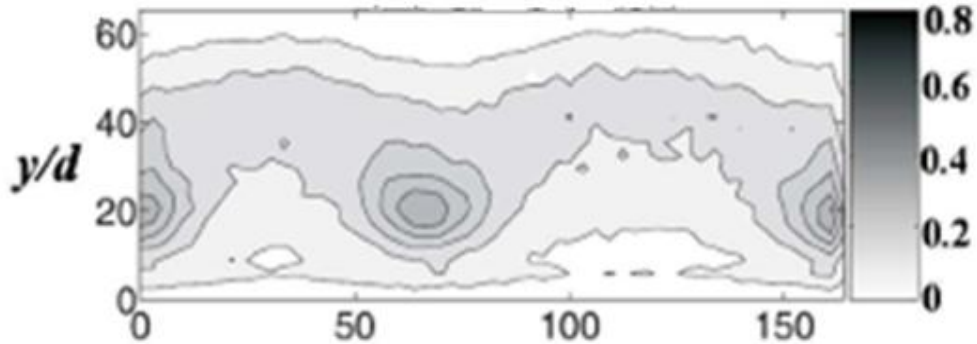
Leidenfrost State leads to convection via a supercritical bifurcation

$$F = 6.2, L = 164, a = 4.0mm,$$
$$f = 52Hz \text{ or } (S = 174)$$

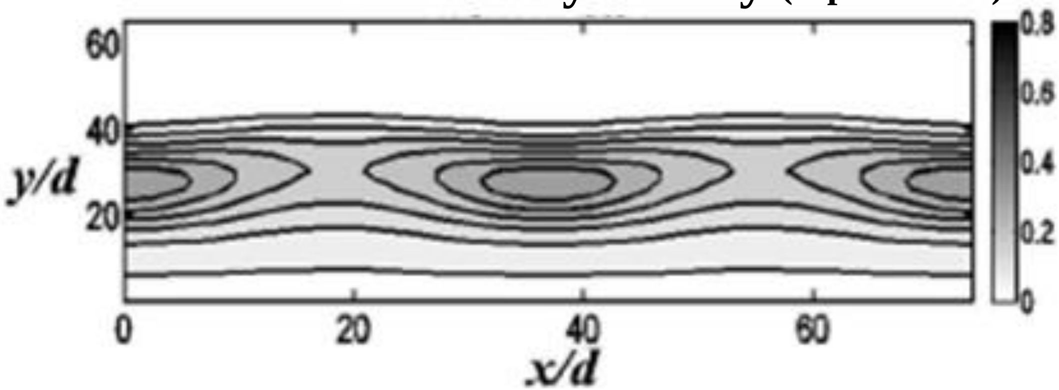
Experiment *(Eshuis et al 2010)*



Particle Simulation *(Eshuis et al 2010)*



Nonlinear Stability Theory (supercritical)



Shear

Complex Ginzburg-Landau (CGL) Equation

Slow evolution of the spatial structure of shearband using two dimensional CGL (*Saitoh K. & Hayakawa H., 2011)*

Unbounded domain

Stuart-Landau ('order parameter') Equation

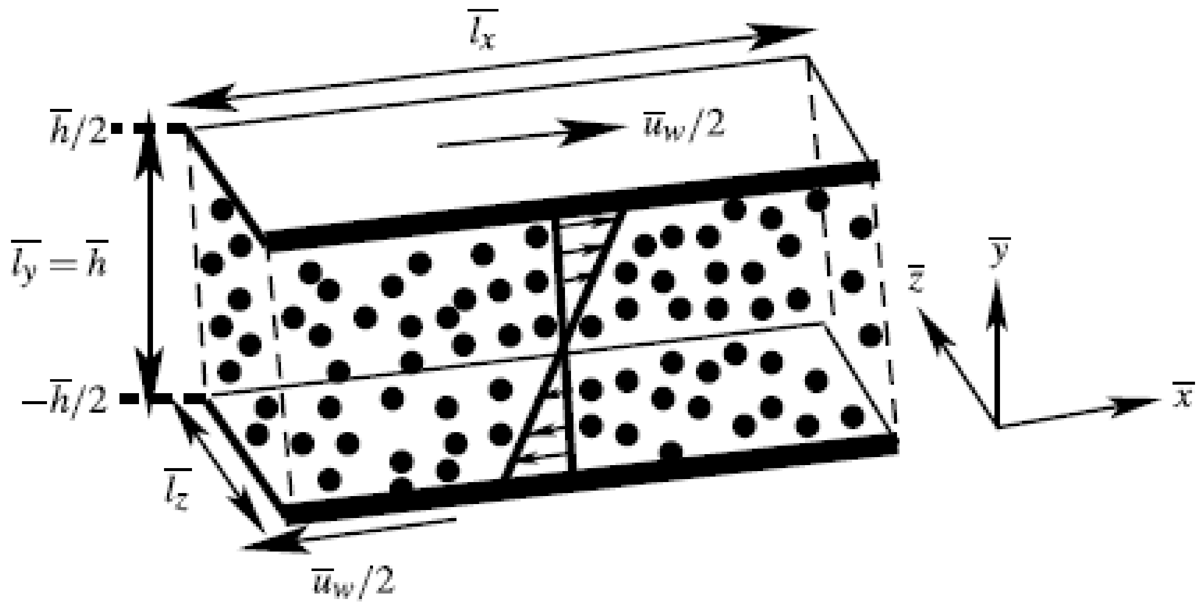
$$\frac{dA}{dt} = c^{(0)}A + c^{(2)}A|A|^2 + \cdots$$

Bounded domain

Instability	Form of perturbation	References
Pure transverse (gradient banding)	$\frac{\partial}{\partial y}(\cdot) \neq 0,\ k_x = 0$	*Shukla & Alam PRL 2009; Shukla & Alam, JFM, 2011a*
2D long-wave stationary and travelling	$\frac{\partial}{\partial y}(\cdot) \neq 0,\ k_x \sim 0$	*Shukla & Alam, JFM, 2011b*
2D dominant stationary and travelling	$\frac{\partial}{\partial y}(\cdot) \neq 0,\ k_x \sim O(1)$	*Shukla & Alam, JFM, 2011b*
Pure spanwise (vorticity banding)	$\frac{\partial}{\partial y}(\cdot) = 0,\ k_x = 0,\ k_z \neq 0$	*Shukla & Alam, JFM, 2013*
Three-dimensional (3D)	$\frac{\partial}{\partial y}(\cdot) \neq 0,\ k_x \neq 0,\ k_z \neq 0$	*Alam & Shukla, JFM, 2013*

Problem Description

Schematic diagram of 3D plane Couette flow



The plane Couette flow is unsable due to various stationary and travelling wave instabilities

Scaling: Gap between the walls, Average velcocity, and inverse of total shear rate

Granular Hydrodynamic Equations

(Savage, Jenkins, Goldhirsch, …)

Balance Equations

$$\frac{D\rho}{Dt} = -\rho\nabla \cdot \mathbf{u}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla \cdot \mathbf{\Sigma}$$

$$\frac{3}{2}\rho\frac{DT}{Dt} = -\nabla \cdot \mathbf{q} - \mathbf{\Sigma} : \nabla\mathbf{u} - \mathcal{D}$$

- $\rho = \rho_p\phi$: Bulk density
  ρ_p : Particle density
  ϕ : Volume fraction
- $\mathbf{u}$: Bulk velocity
- T : Granular temperature

Navier-Stokes Order Constitutive Model

Stress Tensor

$$\mathbf{\Sigma} = [p(\phi,T) - \zeta(\phi,T)\nabla \cdot \mathbf{u}]\,\mathbf{I} - 2\mu(\phi,T)\mathbf{S}$$

$$\mathbf{S} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T) - \frac{1}{3}(\nabla \cdot \mathbf{u})\,\mathbf{I}$$

Granular Heat Flux

$$\mathbf{q} = -\kappa(\phi,T)\nabla T$$

Dissipation term or sink of energy

$$\mathcal{D} = \frac{\rho_p}{d} f_5(\phi,e)T^{3/2} \sim (1-e^2)$$

Uniform shear flow

- Steady
- Fully developed
- Parallel
- Unidirectional

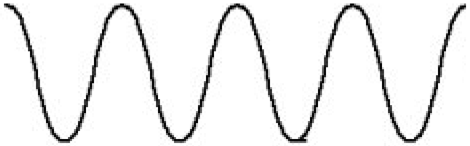
No Slip & Zero heat flux B.C.

Uniform Shear Solution

$$\phi^0(y) = const. \quad T^0(y) = const. = \frac{f_2(\phi^0)}{f_5(\phi^0)}$$

$$u^0(y) = y$$

Base Flow | Perturbation

Uniform shear flow (USF) +  $= X$

$$X^0 + X' = X$$

Disturbance Eqns

$$\frac{\partial X'}{\partial t} = LX' + N_2 + N_3 + \ldots$$

Linear problem

$$\frac{\partial X'}{\partial t} = LX'$$

➡

Normal mode sol

$$LX^{[1;1]} = c^{(0)} X^{[1;1]}$$

Weakly Nonlinear Analysis

Slow/ Active/ Unslaved	Fast/ Passive/ Slaved
Growing or neutrally stable	Decaying mode
Amplitudes are independently determined	…dependent

Bounded System

Discrete spectrum

Slow modes with positive, zero, or slightly negative growth rates

Amplitude Eq. (ODE)

Landau 1944, Stuart & Watson, 1960

Unbounded System

Continuous spectrum

Slow modes: slowly varying envelope of fast varying patterns.

Envelope Eq. (PDE)

Newell & Whitehead, 1969

Amplitude equation

Derivation of Amplitude Equation

Notation: Slow mode: ***S*** ; Fast Mode: ***F***

Coordinate of ‘***S***’: amplitude of discrete modes of ‘***S***’

⇓

Amplitude Order Parameter

(gives degree of order/disorder, and structure of the system)

Near the onset, the amplitudes of the passive modes in the set ‘***F***’ quickly relax to a manifold, called the center manifold ‘***F* = *F*(*S*)**’.

On center manifold, the amplitudes will evolve on a time scale proportional to inverse of linear growth rate.

Amplitude eqn.

$$\frac{dS}{dt} = G(S)$$

Easier to solve than original microscopic eqns.

Coefficient contains relevant information about the system

$G(S)$ → Product of active amplitude

Separation of mode $X' = S + F$

Center manifold $F = F(S)$

Amplitude equation $\frac{dS}{dt} = G(S)$

Newell, Passot & Lega, Annu. Rev. Fluid Mech. 1993

Center Manifold Reduction

(Carr 1981; Shukla & Alam, PRL 2009)

Separation of mode $X' = S + F$

Center manifold $F = F(S)$

Amplitude equation $\frac{dS}{dt} = G(S)$

Step 1

$$\left(\frac{\partial}{\partial t} - L\right) X' = \sum_{j\geq 2} N_j$$

Disturbance Eqn

$$S = A X^{[1;1]} E + \tilde{A} \tilde{X}^{[1;1]} \tilde{E}$$

$$E = e^{i\vec{k}.\vec{x} + c^{(0)}t}$$

- Inner product with adjoint eigenfunction of the linear problem

Step 2

$$\left(\frac{\partial}{\partial t} - L\right) S = \sum_{j\geq 2} N_j \;\Rightarrow\; \left(\frac{\partial}{\partial t} - c^{(0)}\right) S = \sum_{j\geq 2} N_j$$

$$\left(\frac{\partial}{\partial t} - L\right) F = \sum_{j\geq 2} N_j \;\Rightarrow\; \left(\alpha c^{(0)} - L\right) F = g(S)$$

- Separating the like-power terms in amplitude,
- Yields an amplitude eqn

Eigenvalue

$$c^{(0)} = a^{(0)} + i b^{(0)}$$

Step 3

$$\frac{dA}{dt} = c^{(0)} A + c^{(2)} A|A|^2 + \cdots$$

First Landau Coeff

$$c^{(2)} = a^{(2)} + i b^{(2)}$$

→ **Stuart-Landau Equation**

Determination of Landau Coefficients

All fast modes are determined algebraically as a balance between each linearly decaying fast mode and its regeneration by nonlinear interactions involving members of S.

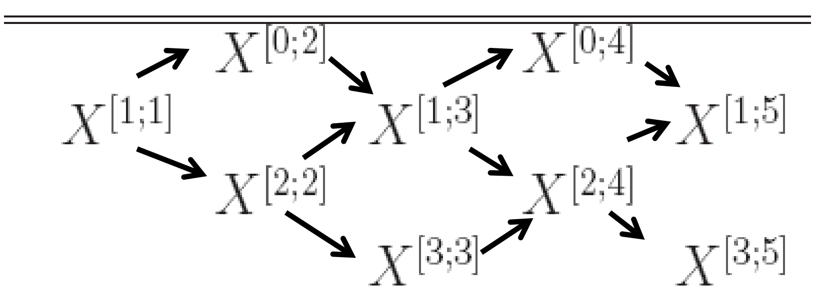
$$c^{(2)} = \frac{\langle Y, N_2(X^{[0;2]}, \boldsymbol{X}^{[1;1]}) + N_2(X^{[2;2]}, \tilde{\boldsymbol{X}}^{[1;1]}) + N_3(\boldsymbol{X}^{[1;1]}, \tilde{\boldsymbol{X}}^{[1;1]}, \boldsymbol{X}^{[1;1]})\rangle}{\langle Y, \boldsymbol{X}^{[1;1]}\rangle}$$

$$\left(\frac{\partial}{\partial t} - L\right) F = \sum_{j \geq 2} N_j$$

Enslaved Equation

represents all non-critical modes

Regeneration of F mode by nonlinear interactions of members of S



Amplitude Expansion Method

(Stuart & Watson 1960; Shukla & Alam, JFM 2010a)

$$X' = \sum X^{(k)}(y, A)\, e^{ik\theta} + c.c.$$

$$X^{(k)}(y) = A^{(k)} X^{[k;n]}(y)$$

$$A^{-1}\frac{dA}{dt} = a^{(0)} + A a^{(1)} + A^2 a^{(2)} + \cdots$$

$$\frac{\partial\theta}{\partial t} = \frac{\partial\Theta}{\partial t} = \omega + \frac{d\omega}{dA}\left(t\frac{dA}{dt}\right) = b^{(0)} + A b^{(1)} + A^2 b^{(2)} + \cdots$$

Co-ordinate Transformation

$$\theta(x,z,\omega,t) = k_x x + k_z z + \Theta(\omega,t)$$

$$\Theta(\omega,t) = \omega(A)t$$

$$A = A(t)$$

$$y = y$$

A : Real amplitude

$c^{[n-1]}$ Landau coefficient

$$L_{kn} X^{[k;n]} = -c^{[n-1]} X^{[1;1]} \delta_{k1} + G_{kn}$$

$$c^{[n-1]} = a^{[n-1]} + i b^{[n-1]}$$

$$G_{kn} = -\left(m a^{[n-m]} + ikb^{[n-m]}\right) X^{\{k;n\}} + E_{kn}/(1+\delta_{k0}) + F_{kn}$$

$$L_{kn} = (na^{(0)} + ikb^{(0)})I - L_k$$

Solution Procedure:

$$L_{11} X^{[1;1]} = c^{(0)} X^{[1;1]}$$

$$L_{02} X^{[0;2]} = G_{02},$$

$$L_{22} X^{[2;2]} = G_{22}, \quad L_{13} X^{[1;1]} = -c^{(2)} X^{[1;1]} + G_{13}$$

Solvability Condition

$$c^{[n-1]} = \frac{\int_{-1/2}^{1/2} G_{1n} Y dy}{\int_{-1/2}^{1/2} X^{[1;1]} Y dy}$$

Equilibrium Amplitude and Bifurcation

$$\frac{dZ}{dt} = c^{(0)}Z + c^{(2)}Z|Z|^2 + c^{(4)}Z|Z|^4 + \dots$$

Cubic Landau Eqn $Z = Ae^{i\theta}$

$$\frac{dA}{dt} = a^{(0)}A + a^{(2)}A^3,$$

Real amplitude eqn.

$$\frac{d\theta}{dt} = b^{(0)} + b^{(2)}A^2$$

Phase eqn.

Cubic Solution

$$\frac{dA}{dt} = 0 \quad \Rightarrow \quad A = 0, \quad A = \pm\sqrt{-\frac{a^{(0)}}{a^{(2)}}}$$

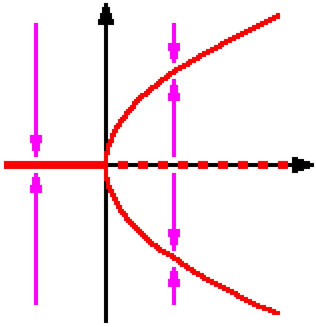
Supercritical Bifurcation $a^{(0)} > 0, a^{(2)} < 0$

Subcritical Bifurcation $a^{(0)} < 0, a^{(2)} > 0$

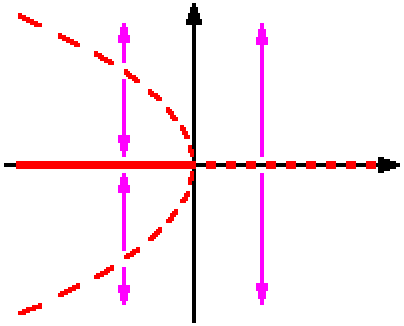
$b^{(0)} = 0$ Pitchfork

$b^{(0)} \neq 0$ Hopf

Supercritical



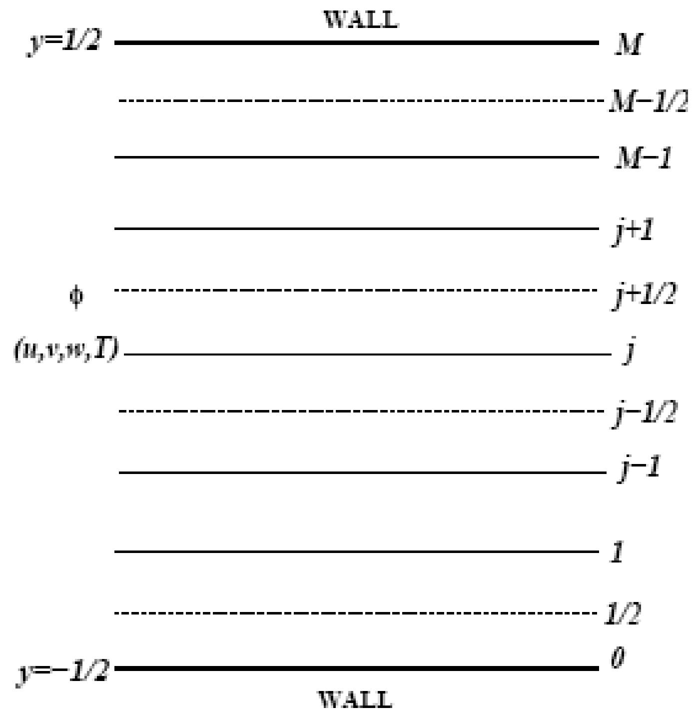
Subcritical



Numerical Methods

Discretization

Chebyshev spectral collocation method with staggered grid ***(Canuto et al, 1988; Alam & Nott 1998, Shukla & Alam 2011b)***



Gauss Lobatto point

$$\xi_i = \cos\left(\frac{i\pi}{M}\right)$$

Physical grid to spectral grid

$$\xi = 2y \in (-1, 1)$$

Gauss point

$$\xi_{i+1/2} = \cos\left(i + \frac{1}{2}\right)\frac{\pi}{M}, \; i = 0, \ldots M-1$$

Lagrange Interpolation Matrices

$$\mathcal{P}_j(\xi_k) = \frac{(-1)^{j+k}\sqrt{(1-\xi_j^2)}}{M(\xi_k - \xi_j)},$$

$$\mathcal{Q}_j(\xi_{k+1/2}) = \frac{(-1)^{k+j+1}\sqrt{1-\xi_{k+1/2}^2}}{c_j M(\xi_{k+1/2} - \xi_j)}$$

GL point to G point

$$(u, v, w, T)(\xi_{j+1/2}) \rightarrow (u, v, w, T)(\xi_j)$$

$$\phi(\xi_j) \rightarrow \phi(\xi_{j+1/2})$$

Types of Matrix Problem

Analytical solution exists!
for shearbanding instability
(Shukla & Alam 2011a, Shukla & Alam 2013)

Type 1

$O(A)$

$LX^{[1;1]} = c^{(0)} X^{[1;1]}$

Type 2

$L_{22} X^{[2;2]} = G_{22}$

$O(A^2)$

$L_{02} X^{[0;2]} = G_{02}$

Type 3

$O(A^3)$

$L_{13} X^{[1;3]} = c^{(2)} X^{[1;1]} + G_{13}$

Type 1

Generalized Eigenvalue problem
Order one in amplitude

$$AX = c^{(0)} BX$$

QZ Algorithm

Type 2

Inhomogeneous Equation
Even order in amplitude

$$AX = b$$

Singular Value Decomposition

Type 3

SL problem
(Inhomogeneous equation with solvability condition)
Odd order in amplitude

$$AX = cX + b$$

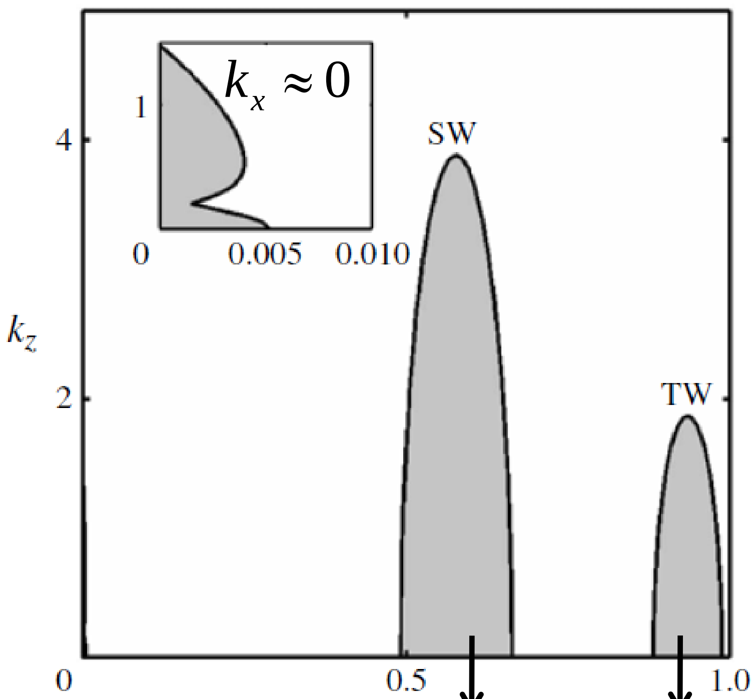
Gauss-Chebyshev Quadrature & Singular Value Decomposition

Results

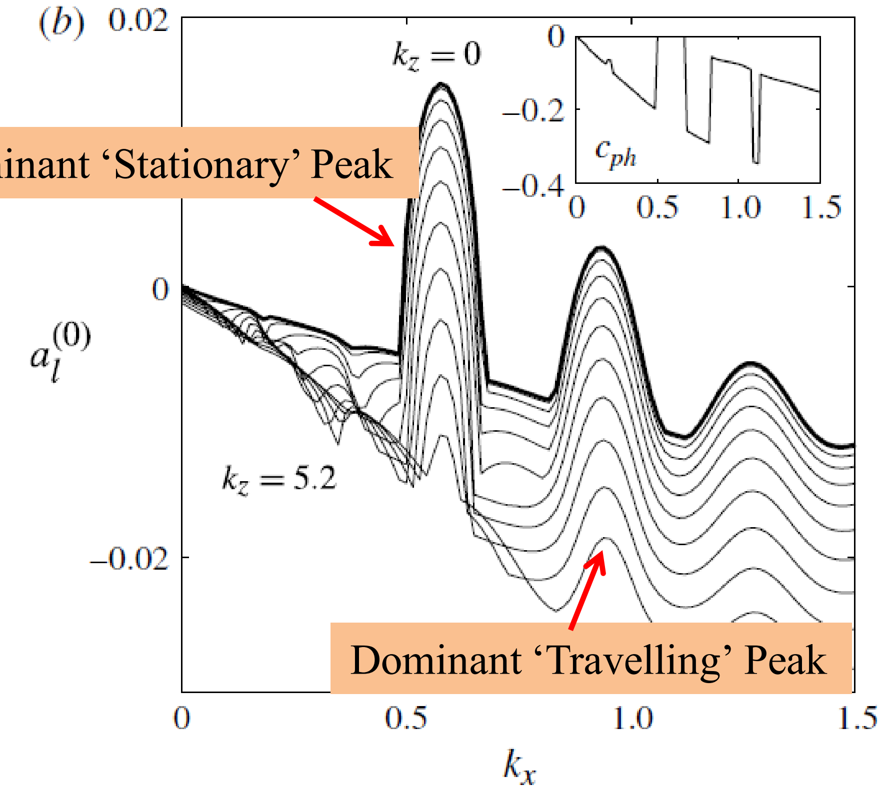
Moderate density

$\phi^0 = 0.2, H = 100, e = 0.8$

Instability Contours



$k_x = 0.55$ $k_x = 0.95$



The Growth rates of both dominant SW & TW Instabilities reach maximum for 2D & decreases with increasing span-wise wave number at any value of stream-wise wave number

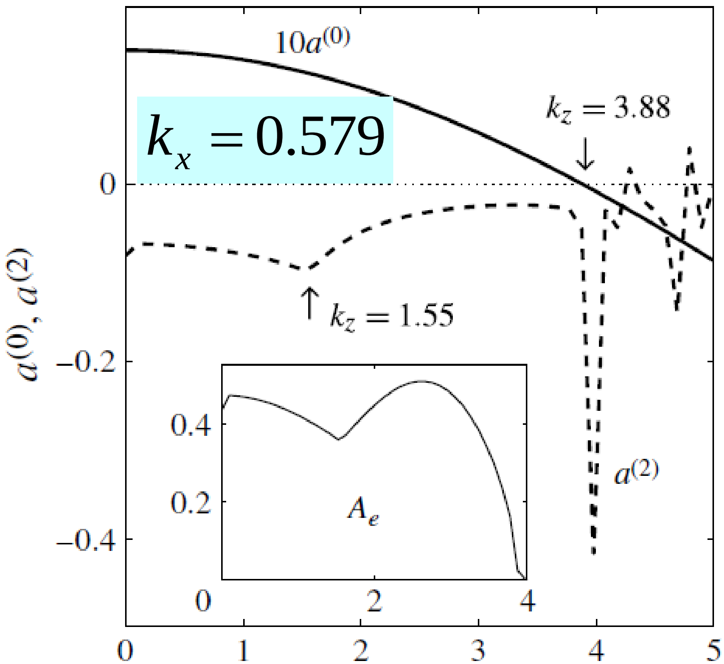
At moderate density, GPCF admits

- Dominant stationary instabilities
- Dominant travelling instabilities
- Long wave instabilities

Effect of 3D perturbations on nonlinear saturation of these modes?

Originate mainly from 2D instabilities

Dominant Stationary Wave (SW) Instabilities



$k_x = 0.579$

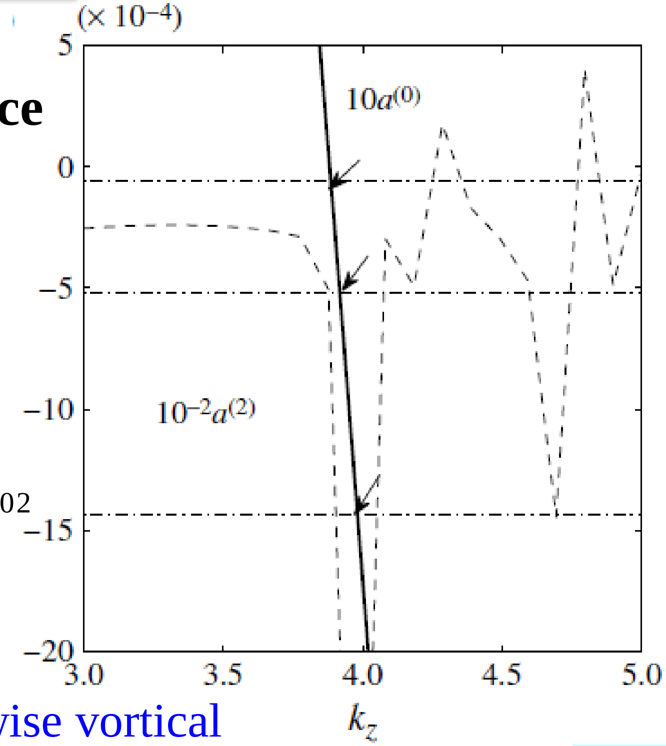
Mean-Flow Resonance

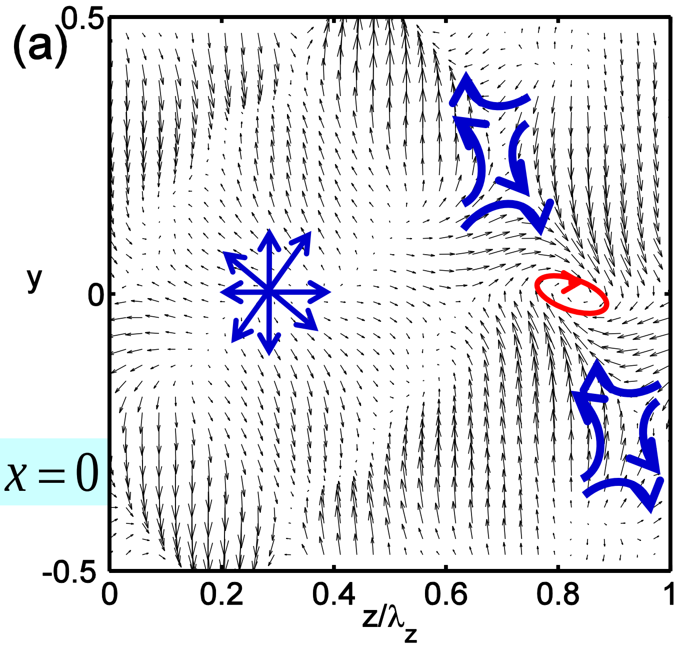
$$L_{02} X^{[0;2]} = G_{02}$$

$$X^{[0;2]} =$$

$$L_{02}^{-1} G_{02} = (2a^{(0)} - L_0)^{-1} G_{02}$$

$$L_0 = L(k_x = 0, k_z = 0)$$





$x = 0$

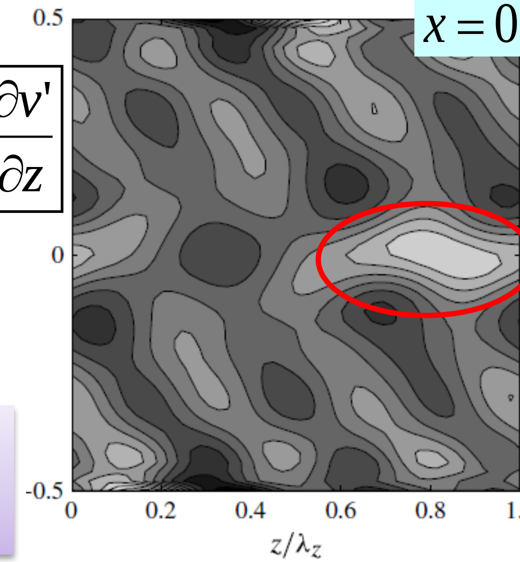
$k_z = 1$

Observation

- Saddle Nodes
- Closed Orbit
- Star Nodes (Sources & Sinks)
- Imperfect Saddles (near the wall)

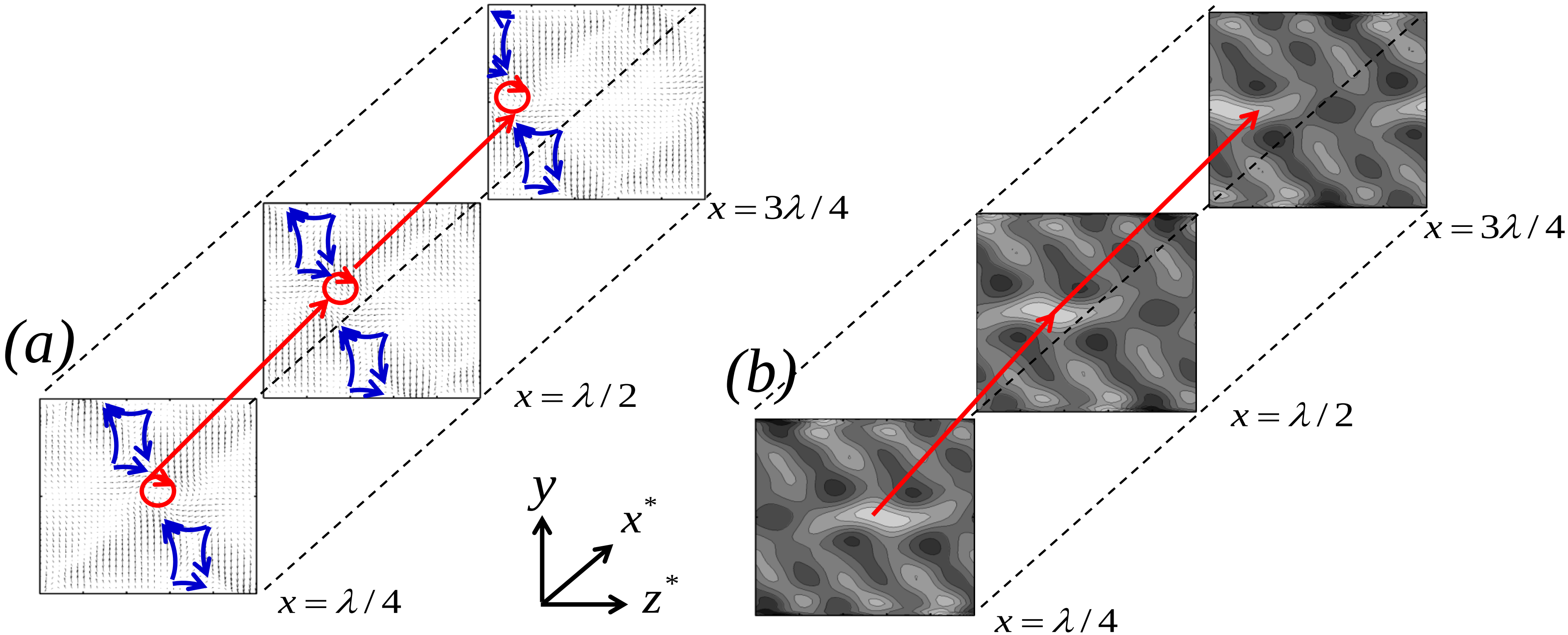
Streamwise vortical structures

$$\Omega_x = \frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z}$$

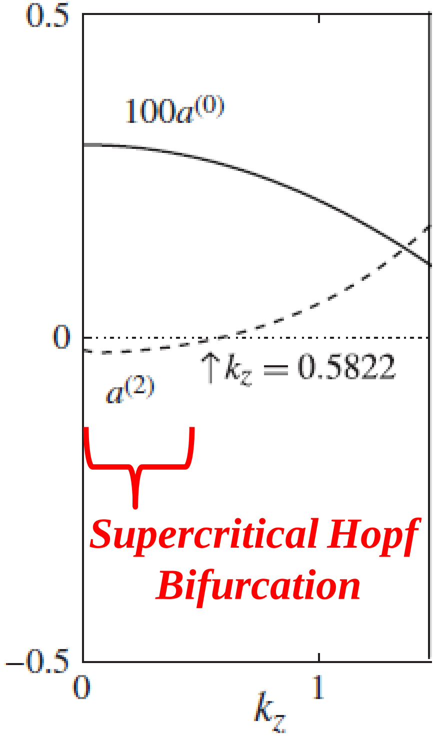


$x = 0$

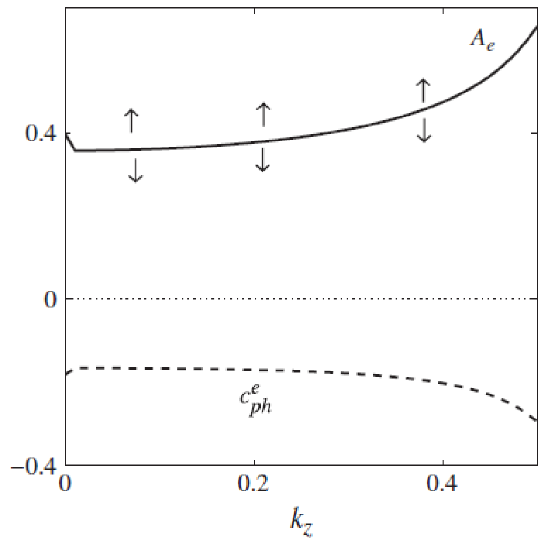
All fixed points are dynamically attached


(a)
x = λ / 4
x = λ / 2
x = 3λ / 4
y
x*
z*
(b)
x = λ / 4
x = λ / 2
x = 3λ / 4

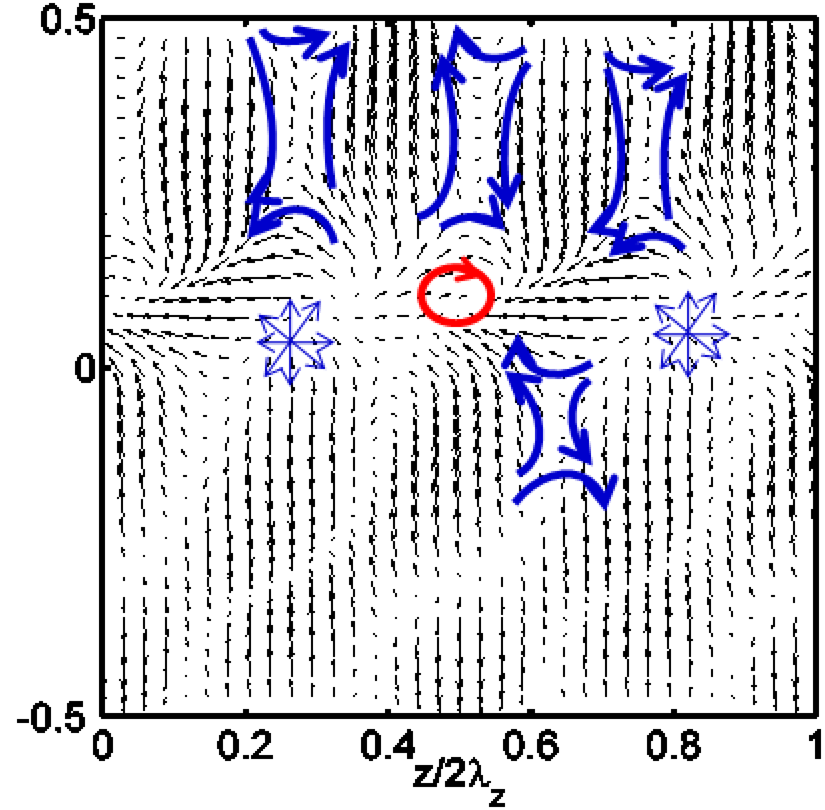
Dominant Travelling Wave (TW) Instabilities

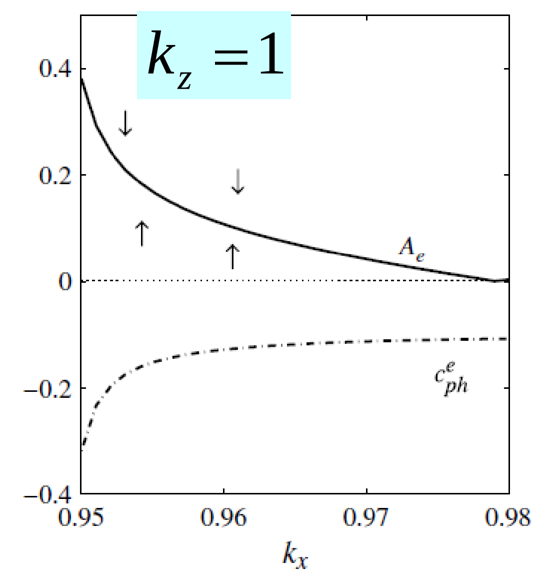


$$c_{ph}^{e} = c_{ph} - \frac{b^{(0)} A_e^2}{\sqrt{k_x^2 + k_z^2}}$$

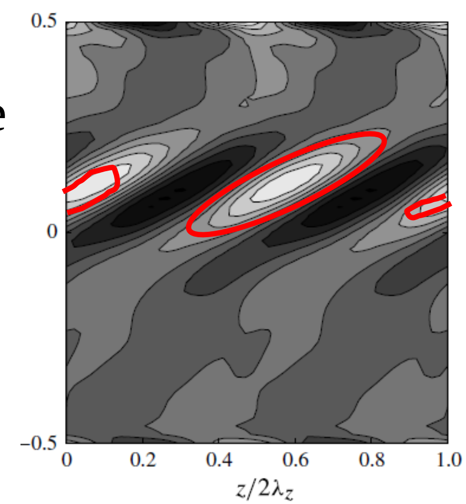


$k_x = 0.97, k_z = 1$





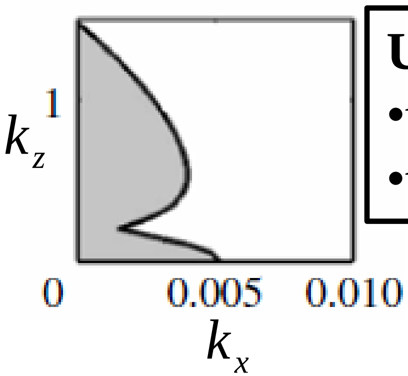
Nonlinear TW solutions at larger span-wise wavenumber



Observation

For dominant SW & TW instabilities , the cross stream motion is dominated by saddle type motions with a streamwise vortex

Long Wave (LW) Instabilities

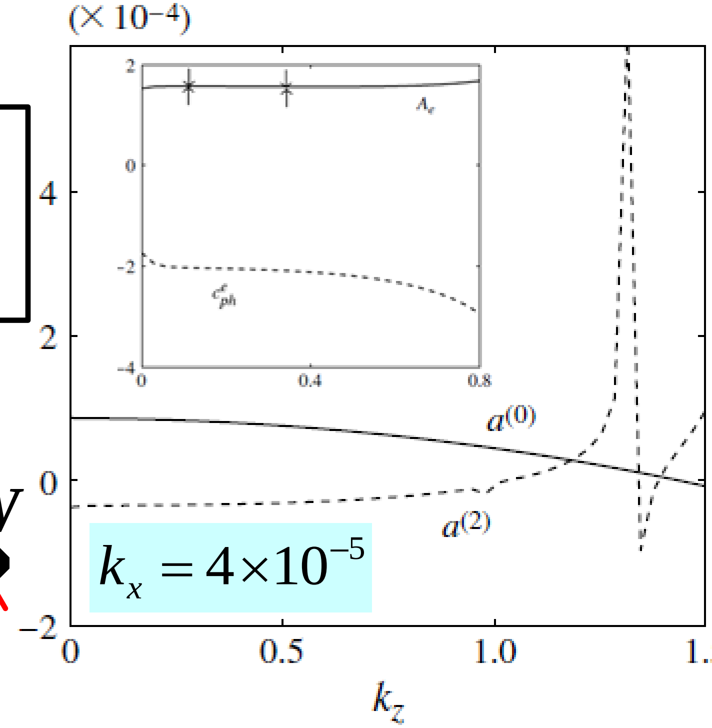


Unstable

- to *Gradient Banding* mode
- to long-wave length *2D SW* & *TW*

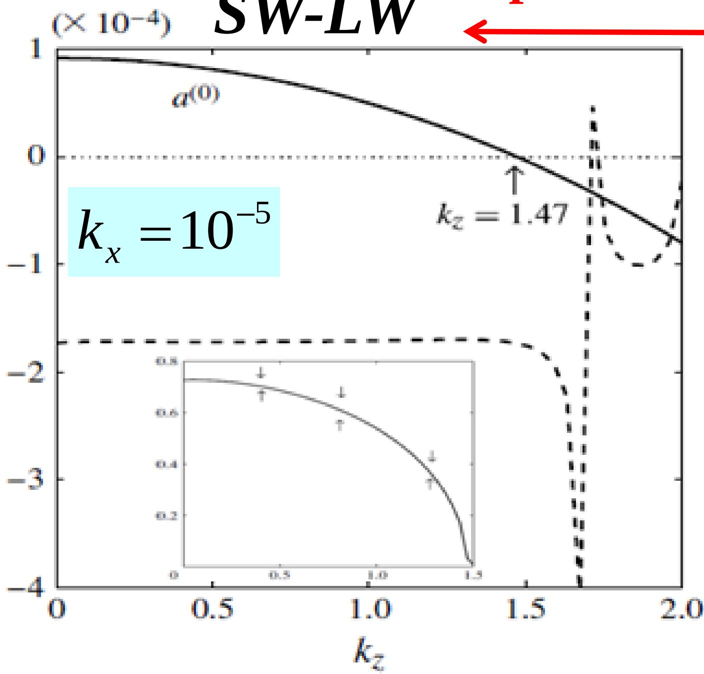
Effect of spanwise wavenumber on LW

TW-LW

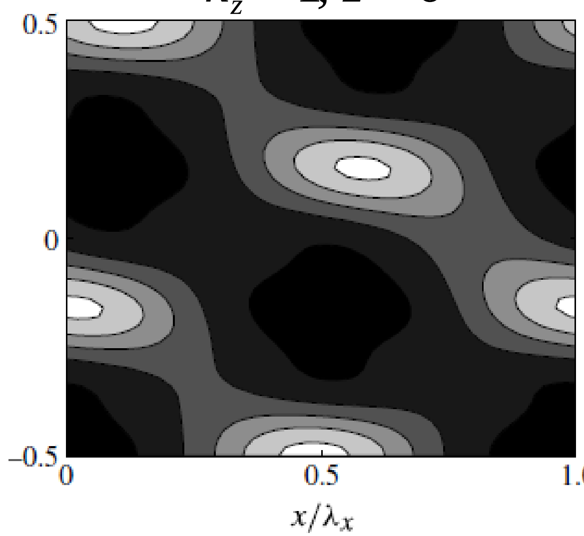


Supercritical Hopf Bifurcation

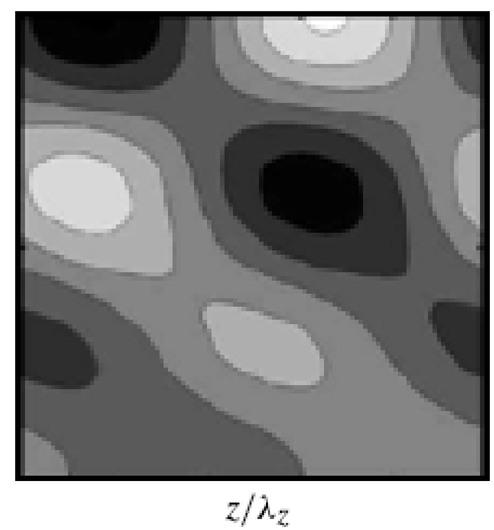
SW-LW



$k_z = 1, z = 0$

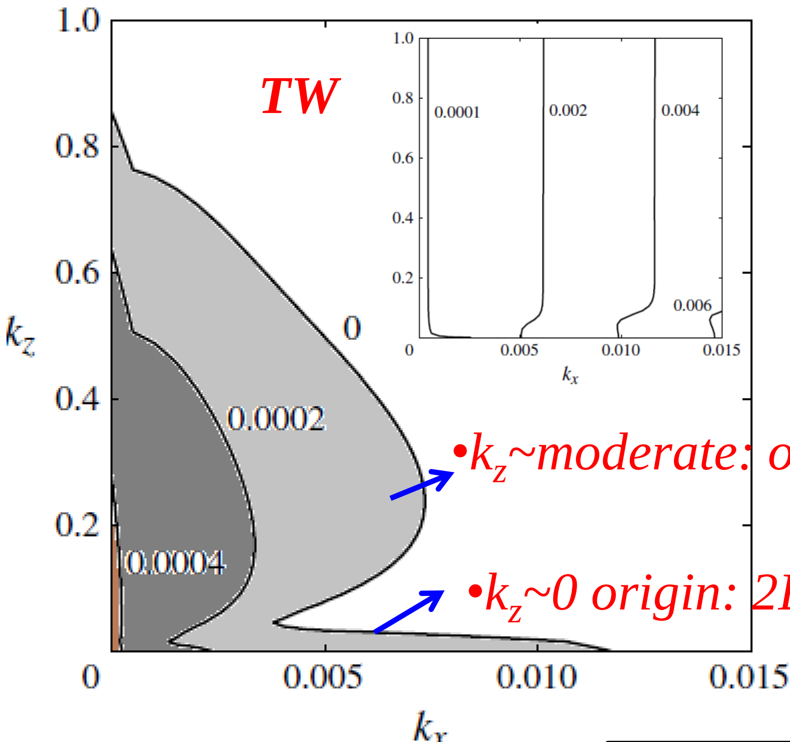


$k_z = 0.3, x = 0$



Instabilities & Patterns in dense flows

$\phi^0 = 0.4, H = 50, e = 0.8$



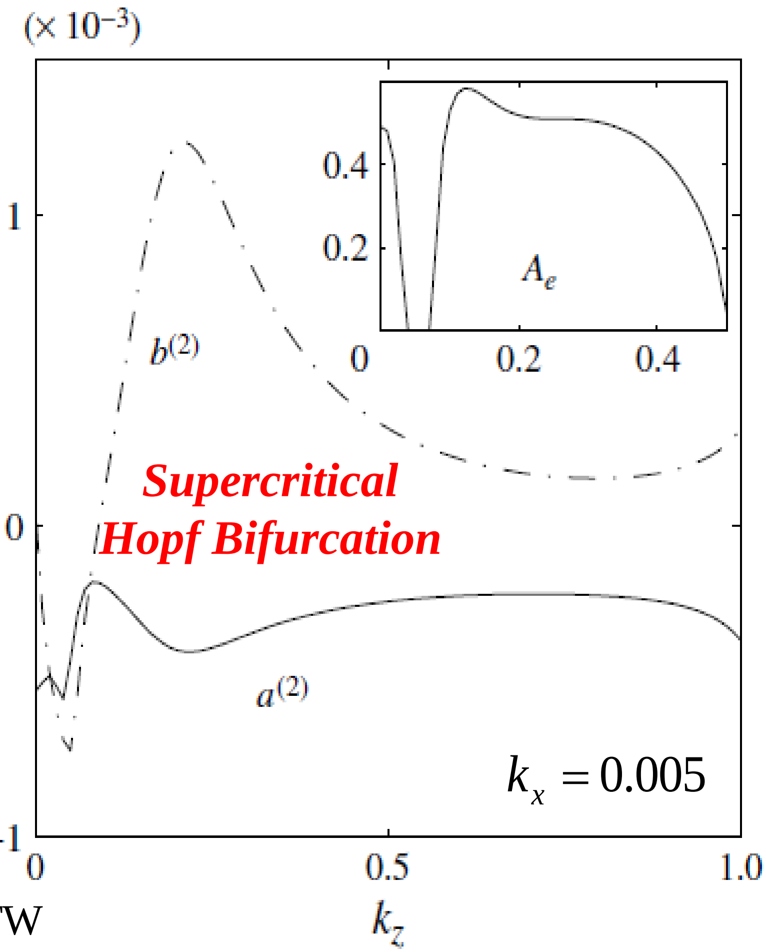
•k_z~moderate: origin 3D

•k_z~0 origin: 2D

Known to be unstable

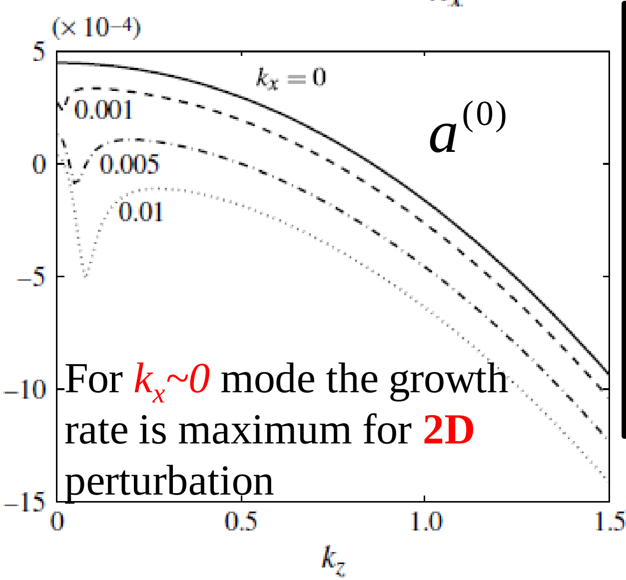
•to 2D perturbation for small range of stream-wise wavenumber

•Originated from *Gradient Banding* mode



Supercritical Hopf Bifurcation

$k_x = 0.005$



For k_x~0 mode the growth rate is maximum for **2D** perturbation

Observation

•**Orientation** & **structure** of particle cluster originating from **2D** is differ from **3D**

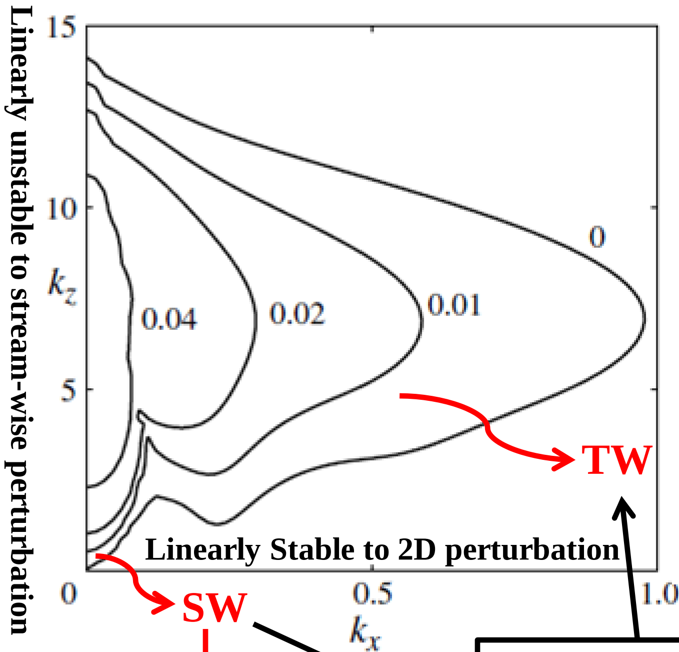
•**Saddle** node type motion

•patterns are differ from previous **LW**

Small k_x ~0.001 also gives Supercritical & subcritical TW

Purely 3D instability in dilute flows

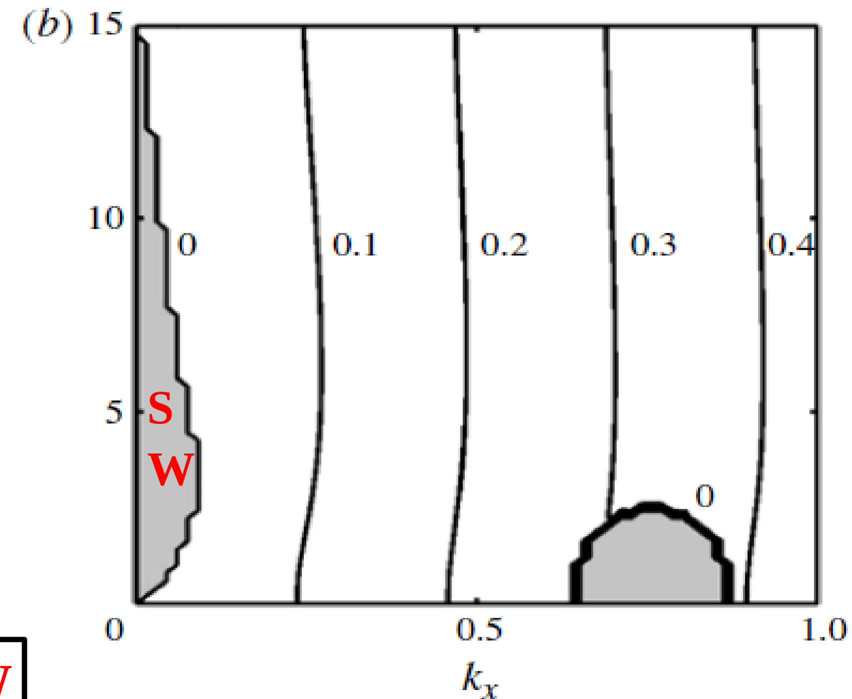
$\phi^0 = 0.05, H = 100, e = 0.8$

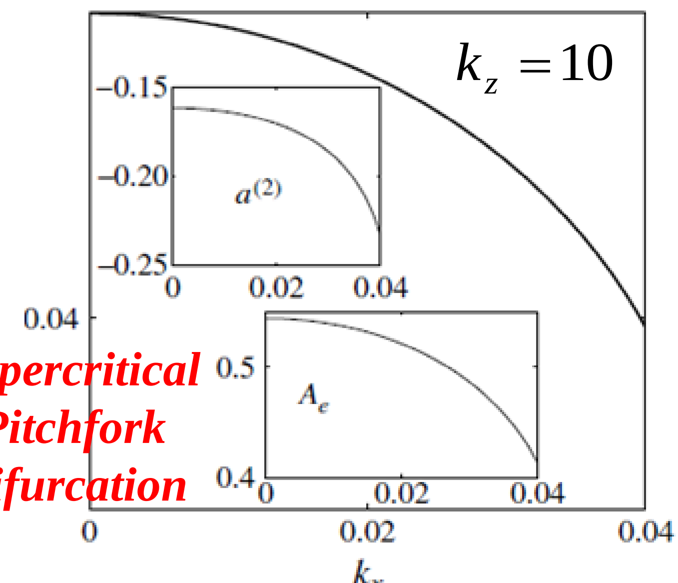


Originated from ***'pure span-wise'*** **perturbation**

Resposible for ***vorticity*** *banding*

Modal evolution of **SW** instability with k_x gives birth to new **3D TW** instability





Stationary Wave Instabilities (SW)

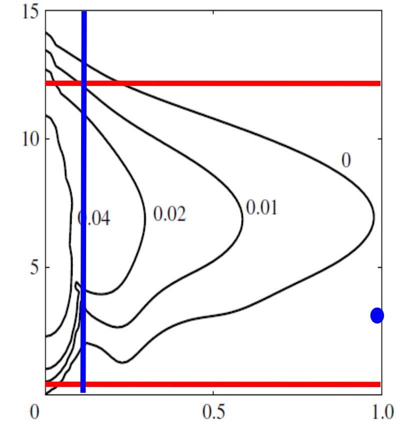
Equilibrium solution does not exist for small values of $\boldsymbol{k_z}$ *(=< 0.5)*

Travelling Wave (TW) Instabilities

$k_x = 0.1$

$k_z = 1.273$

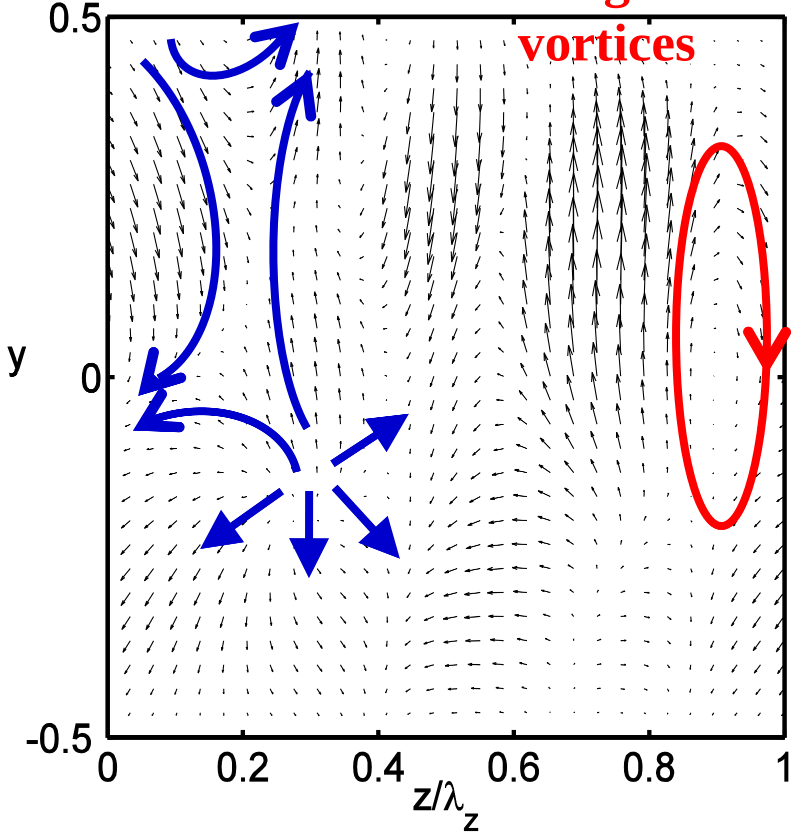
Subcritical Hopf → ***Supercritical Hopf***



Particle clusters are oriented horizontally along *z*-direction

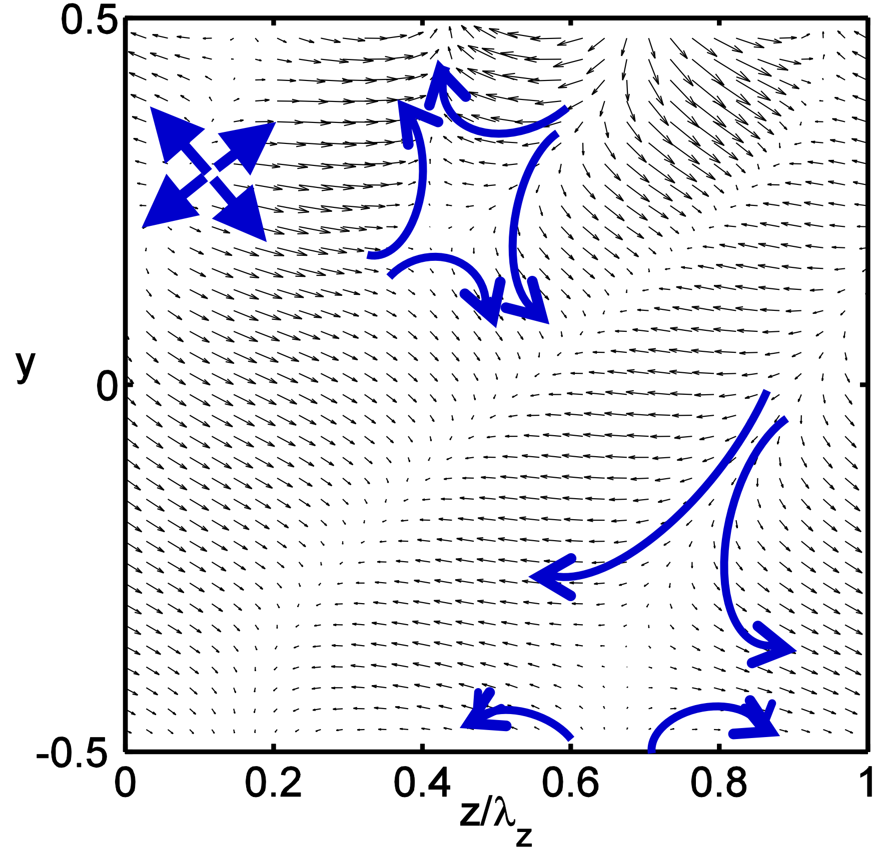
$k_z = 0.2$

Large scale vortices



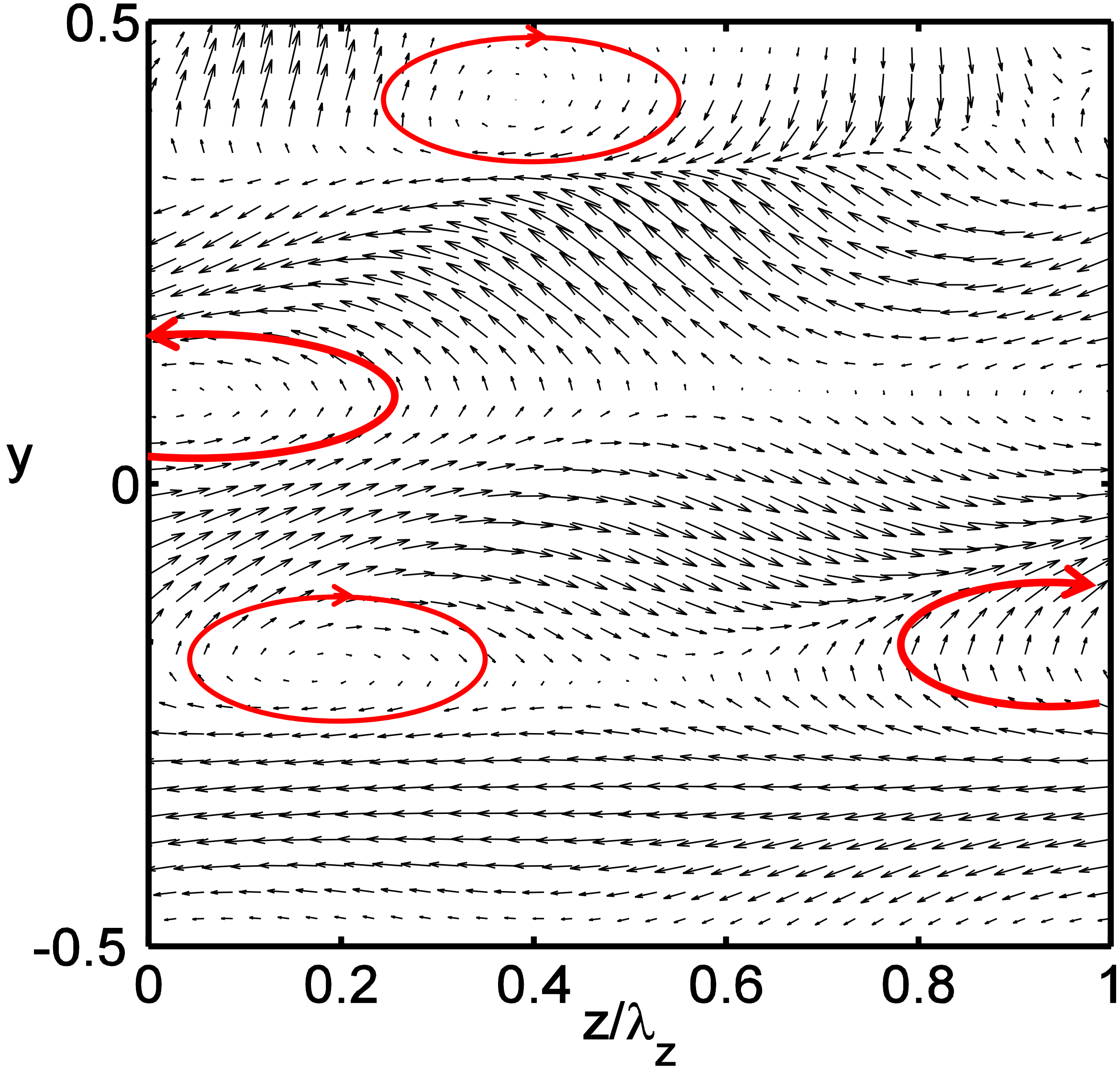
$k_z = 12$

Particle clusters are oriented at some oblique angle

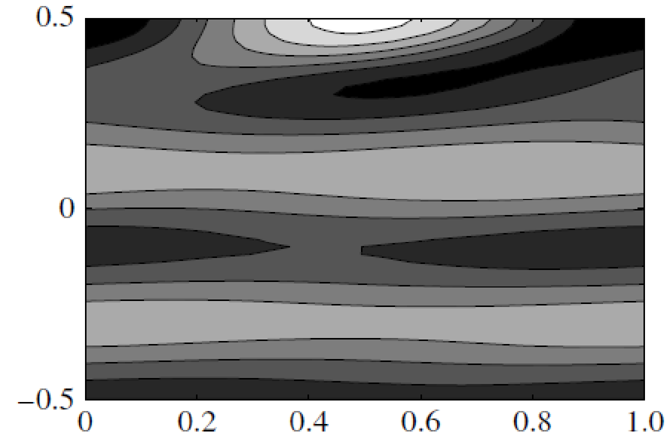


Subcritical TW patterns

$k_x = 1.0, k_z = 1.3$



Vortices repeat along the periodic *z*-direction; forming an array of vortices with saddle between them



Observation
Vortices are located around the local density maxima

Temperature pattern shows that vortices are born near the local minima of kinetic pressure

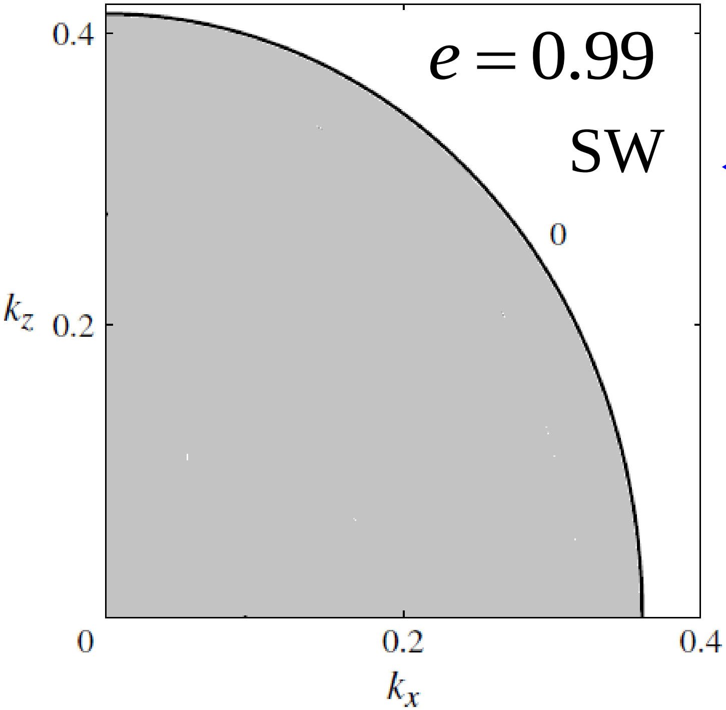
Connection !

Correlation of a vortex core with a low-pressure region in classical fluid

Anomalous 2D instability

$\phi^0 < 0.05$

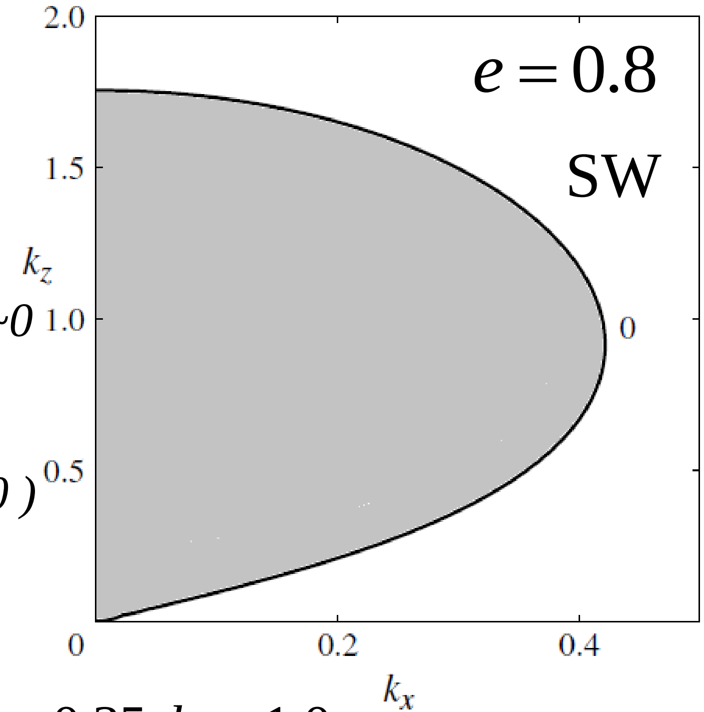
$\phi^0 = 0.01, H = 50$



Maximum growth rate occurs for $k_x=0$

Instability survives at k_z~0 in the quasi-elastic limit

Origin must be 2D ($k_z = 0$) perturbations



Observation

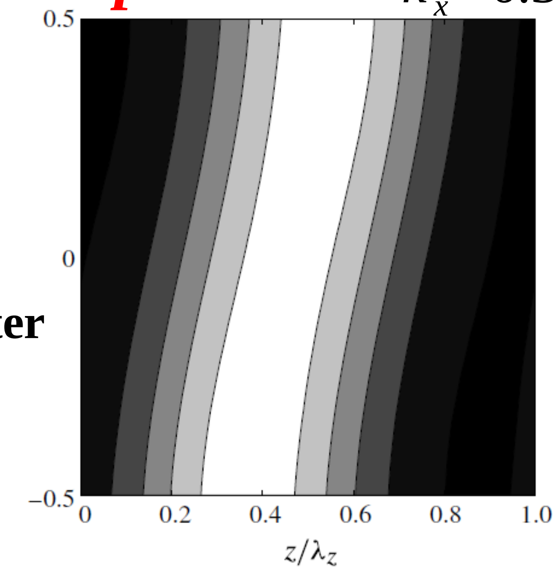
2D instability vanishes for large dissipation implies anomalous dependence on inelasticity

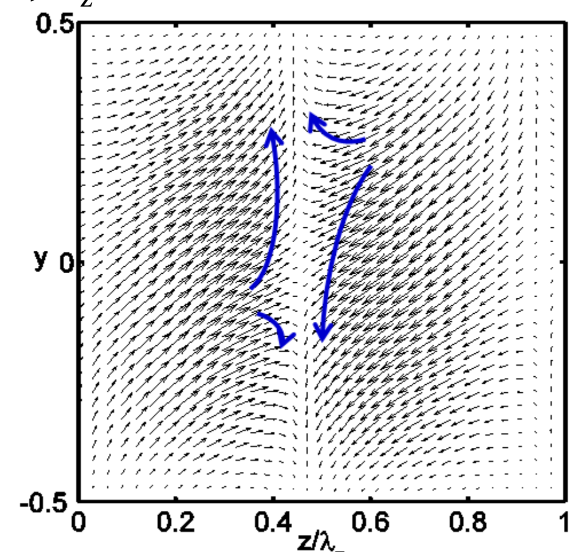
Columnar Structures of density cluster

Different from type of patterns

Origin ????

Supercritical $k_x = 0.35, k_z = 1.0$





Conclusions

Using NS level hydrodynamics of rapid granular fluid, *weakly nonlinear stability* of GPCF has been analyzed.

Dominant SW & TW, and LW instabilities are originated from 2D perturbations for 'moderately dense' to 'dense' flows.

Purely 3D SW & TW has it origin in 3D perturbation in 'dilute' flows

Nonlinear flow patterns of cross stream velocity have been analyzed in terms of the fixed point (saddle, source, sink, vortex……)

Local maximum of stream-wise vorticity gives the location of vortex

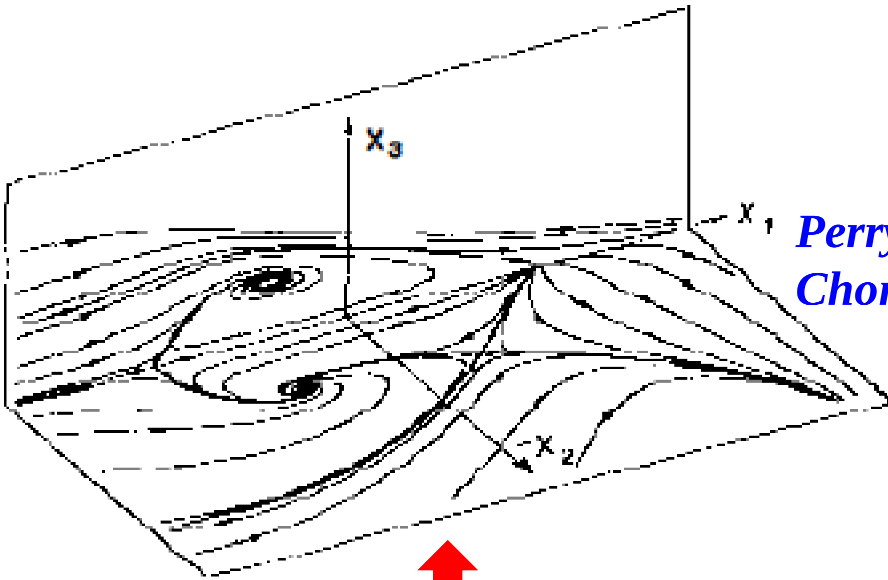
↓

Responsible for more inhomogeneous particle clustering in 3D flow

Outlook & future work

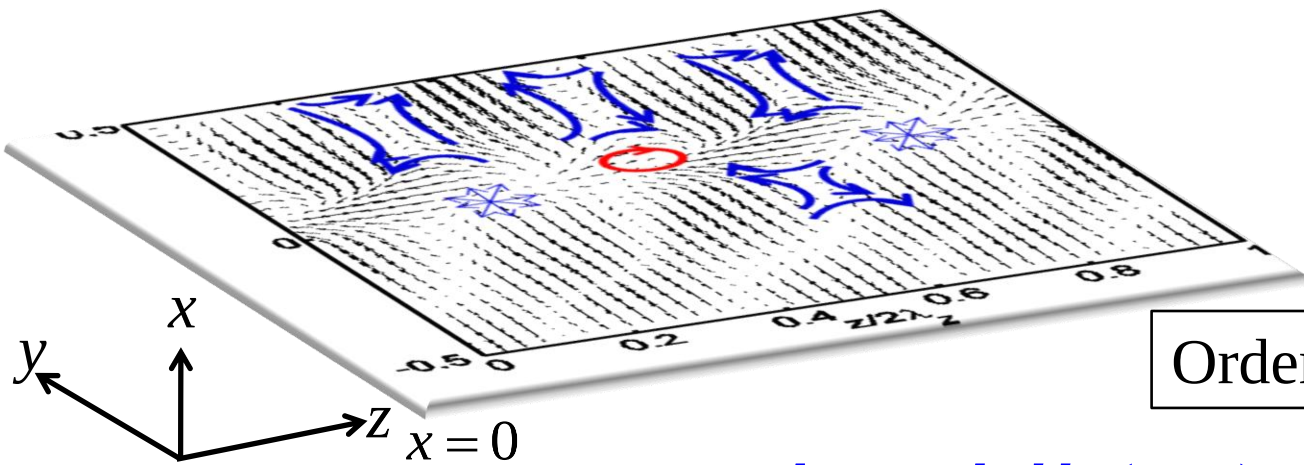
Three Dimensional Flow

Fixed point approach
using velocity gradient tensor



Perry & Chong (1987), Annu. Rev Fluid Mech.
Chong, et. al. (1990), Phys. Fluids

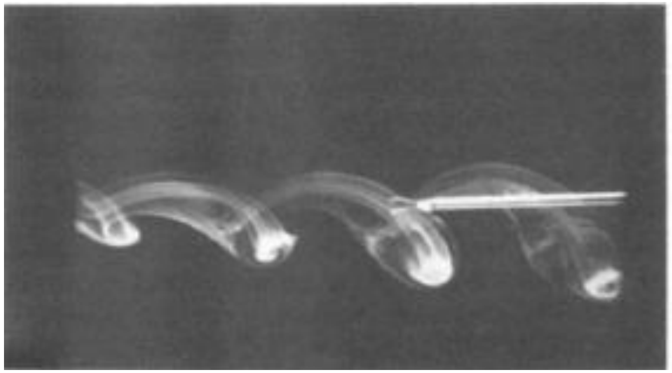
Connection?



Order parameter approach

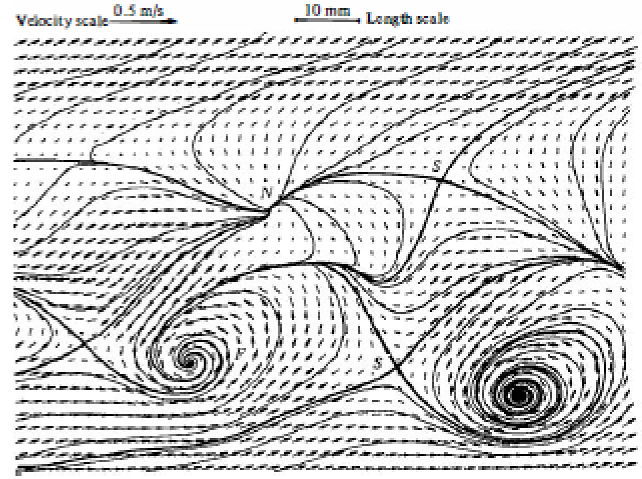
Alam & Shukla (2013) J. Fluid Mech., 716, 349-413

(Perry & Chong 1987)



Wake

up



Acknowledgments

Prof. Hisao Hayakawa

Prof. Meheboob Alam

Thank you