Phase transition in peristaltic transport of granular particles

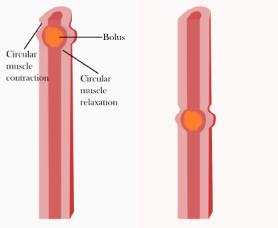
Naoki Yoshioka Hisao Hayakawa

Yukawa Institute for Theoretical Physics, Kyoto University

Physics of Granular Flows

Outline

Peristaltic transport



- Progressive wave of area contraction/expansion.
- Biological systems
  - esophagus
  - small intensine
  - ureters
- Peristaltic Pump
  - blood, corrosive fluids, foods, ...
  - preventing the transported fluid from their mechanical parts.

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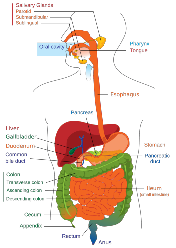

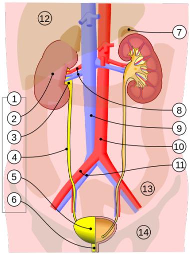

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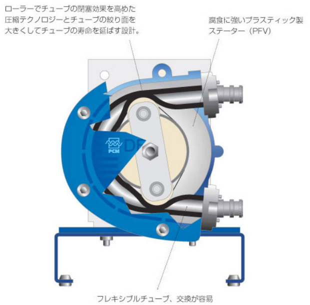

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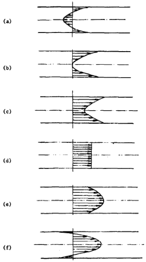
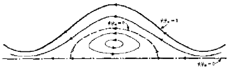


Zien and Ostrach, J. Biomech. **3**, 63 (1970)



Shapiro *et al.*, JFM **37**, 799 (1969)

- Newtonian fluids
  - Stokes approximation
    - assuming some of parameters are zero or small
  - reflux and trapping w/ pressure difference
  - width at bottlenecks v.s. flow rate
- Non-Newtonian fluids
  - many studies,
    e.g., Maxwell fluids,
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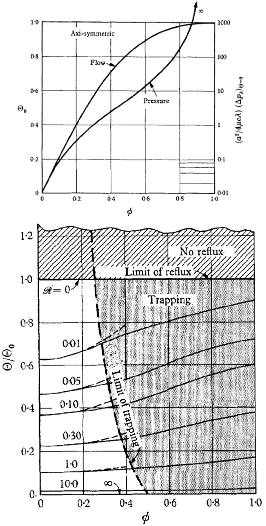
- Particles
  - one particle in fluids
  - dilute particles in fluids

Previous studies



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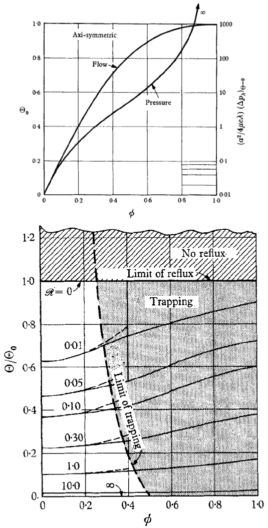

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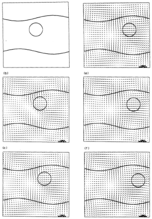
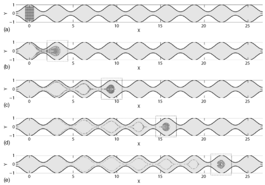


Fauci, Computers Fluids **21**, 583 (1992)



Jiménez-Lozano *et al.*, PRE **79**, 041901

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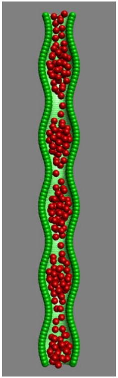

Hou *et al.*, PRL **91**, 204301 (2003).

Peristaltic transport of many particles.

- For example,
  - boluses/chymes in esophagus/intensine
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  - pumping corrosive sands, foods
- Efficiency of pumping?
- Particles might jam at bottleneck
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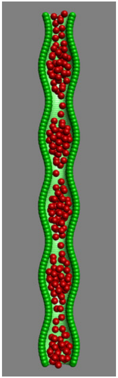

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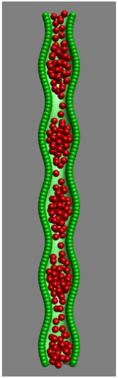

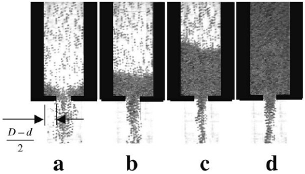

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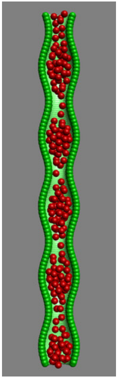

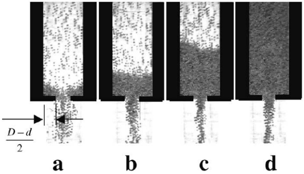

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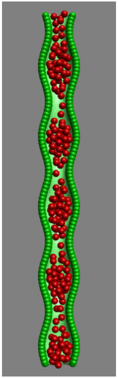

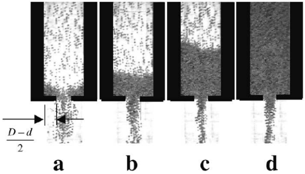

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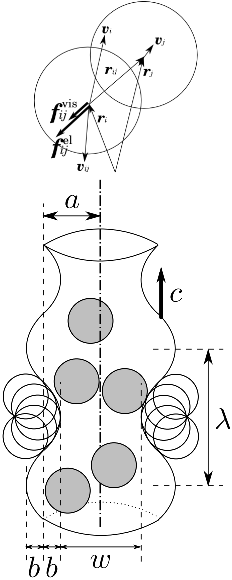
Peristaltic flow of frictionless granular particles



- Monodisperse dissipative particles $\Pi = \Pi_{\mathrm{p}} \cup \Pi_{\mathrm{w}}$, w/o gravity & fluid.
- Spring and viscous force at contact;
  $$\boldsymbol{f}_{ij}^{\mathrm{el}} = k\xi_{ij}\Theta(\xi_{ij})\boldsymbol{n}_{ij},$$
  $$\boldsymbol{f}_{ij}^{\mathrm{vis}} = -\eta(\boldsymbol{v}_{ij}\cdot\boldsymbol{n}_{ij})\Theta(\xi_{ij})\boldsymbol{n}_{ij},$$
- Particles in a tube, Π_{p};
  $$m\frac{\mathrm{d}^2\boldsymbol{r}_i}{\mathrm{d}t^2} = \sum_{j\in\Pi\setminus\{i\}} (\boldsymbol{f}_{ij}^{\mathrm{el}} + \boldsymbol{f}_{ij}^{\mathrm{vis}}).$$
- Particles embedded on a tube, Π_{w};
  $$\boldsymbol{r}_i = \left(r_i(t)\cos\phi_i, r_i(t)\sin\phi_i, \zeta_i\right),$$
  $$r_i(t) = a + b\sin\left(\frac{2\pi}{\lambda}(ct+\zeta_i)\right).$$

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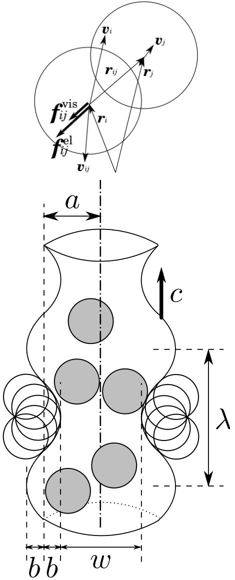

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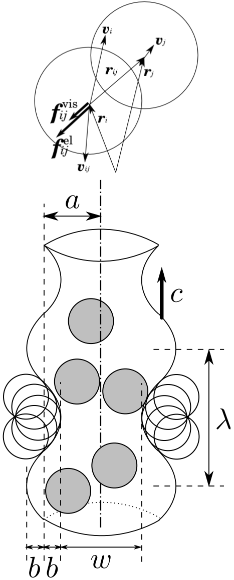

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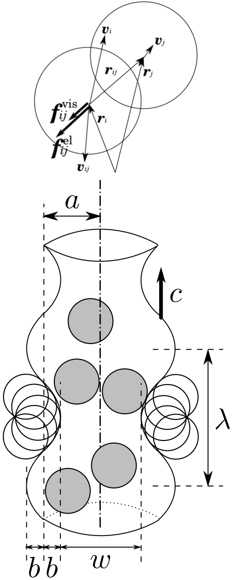

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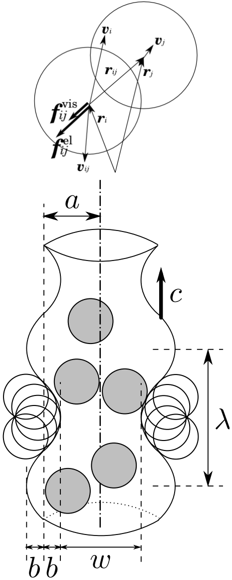

- Scaled by
  - mass m,
  - diameter d,
  - $\sqrt{k/m}$
- $a = 1.5,\ \lambda = 10,\ \eta = 5.48 \times 10^{-3}$
- restitution coefficient $e = \exp\left(-\pi\eta/\sqrt{2-\eta^2}\right) \simeq 9.88 \times 10^{-1}$
  - particles are almost elastic
- Control parameters
  - width at a bottleneck $w \equiv 2(a-b)$
  - strain rate $\dot{\epsilon} \equiv c/\lambda$
  - volume fraction at $b = 0$, $\bar{\rho} \equiv N/6a^2L$

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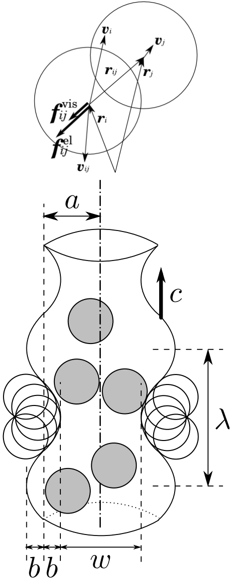

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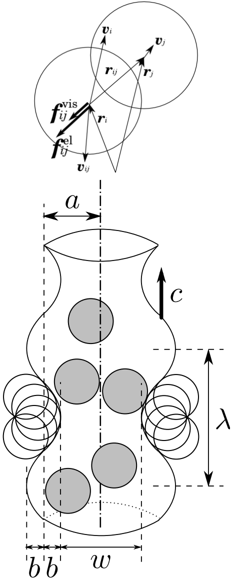

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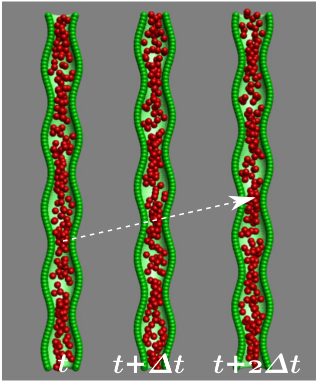
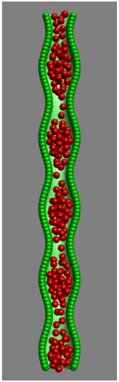
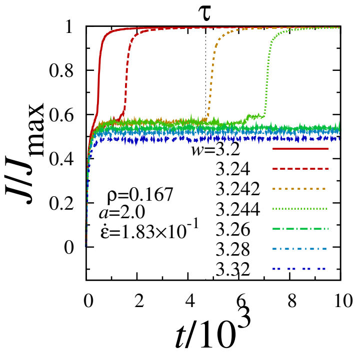




- unjammed flow $\rightarrow$ jammed flow

Typical time evolution of mass flux



- Initial condition: $J = 0$.
- $J_{\max} \equiv Nc/L$.
- Large w
  - steady slow unjammed flow
- Small w
  - Transition
  - from unsteady unjammed flow
  - to steady fast jammed flow
- Transition at $w = w_{\mathrm{c}}$.

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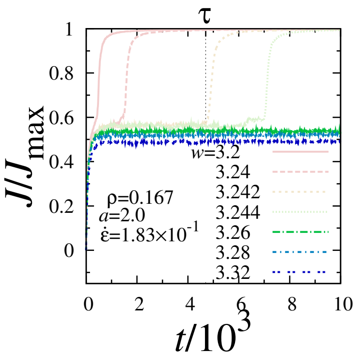

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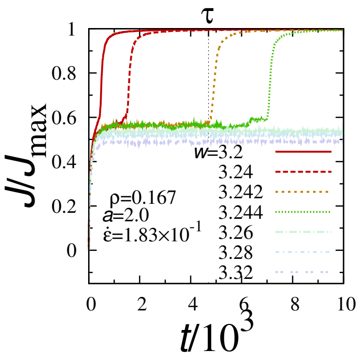

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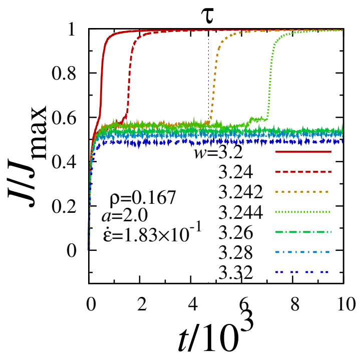

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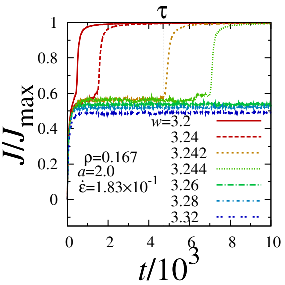
Transition time and its fluctuation



- Time $t = \tau$ at which the transition occurs.
- τ depends on w.
- Diverges at $w = w_c(\dot{\epsilon})$;
  - $\tau \sim (w_c - w)^{-1}$
- Transition time τ
  - $\tau \sim \dot{\epsilon}^{-7/2} f\left((w_c - w)/\dot{\epsilon}^{3/2}\right)$, $f(x) \sim x^{-1}$ for $x \sim 1$.
- $\chi_\tau \equiv \langle \tau^2 \rangle - \langle \tau \rangle^2$
  - $\chi_\tau \sim (w_c - w)^{-3}$
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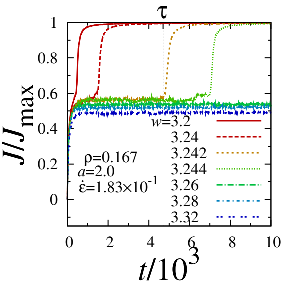

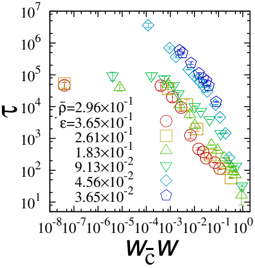

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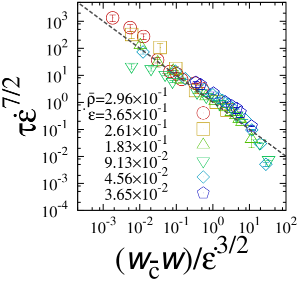

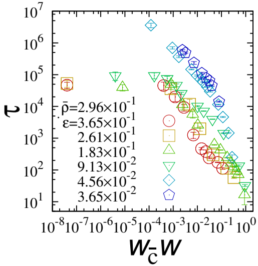

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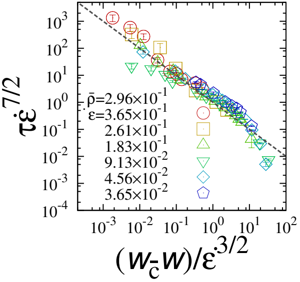
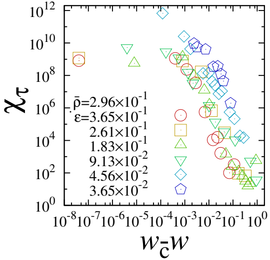
  - $\tau \sim \dot{\epsilon}^{-7/2} f\left((w_c - w)/\dot{\epsilon}^{3/2}\right)$, $f(x) \sim x^{-1}$ for $x \sim 1$.
- $\chi_\tau \equiv \langle \tau^2 \rangle - \langle \tau \rangle^2$
  - $\chi_\tau \sim (w_c - w)^{-3}$
  - $\chi_\tau \sim \dot{\epsilon}^{-7} g\left((w_c - w)/\dot{\epsilon}^{3/2}\right)$, $g(x) \sim x^{-3}$ for $x \sim 1$.

Transition time and its fluctuation





- Time $t = \tau$ at which the transition occurs.
- τ depends on w.
- Diverges at $w = w_\mathrm{c}(\dot{\epsilon})$;
  - $\tau \sim (w_\mathrm{c} - w)^{-1}$
- Transition time τ
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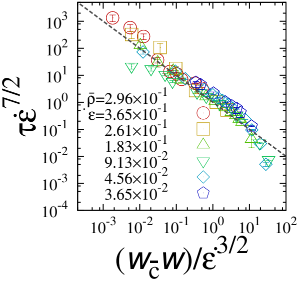

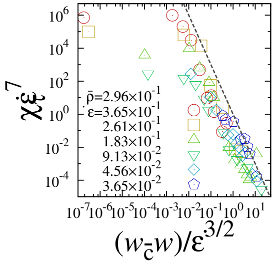

- Time $t = \tau$ at which the transition occurs.
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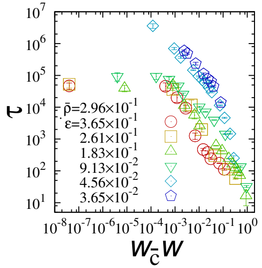
w-dependence of flux



- Estimating w_c, using the relation $\tau \sim (w_c - w)^{-\alpha}$.
- Mass flux $J/J_{\max}$, where $J_{\max} \equiv Nc/L$.
  - fast jammed flow for $w < w_c(\dot{\epsilon})$.
  - slow unjammed flow for $w > w_c(\dot{\epsilon})$.
  - jumps at $w = w_c$.
  - No such discontinuity has observed in previous studies ($\phi = 1 - w/2a$)
- w_c linearly decreases as $\dot{\epsilon}$, $w_c \simeq -3.75\dot{\epsilon} + w_{\max}$.
  - but behavior for $\dot{\epsilon} \lesssim 1.0 \times 10^{-2}$ is not well understood

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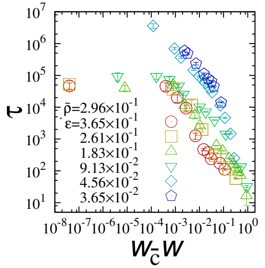

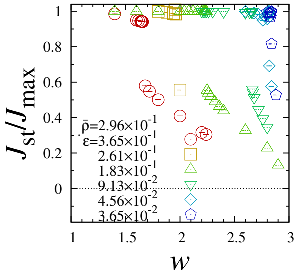

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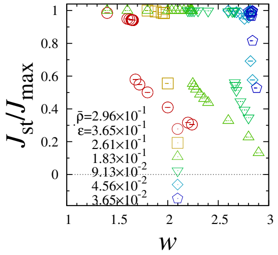

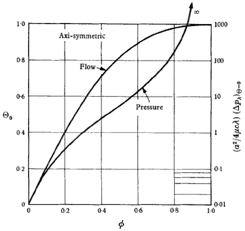

Shapiro *et al.*, JFM **37**, 799 (1969)

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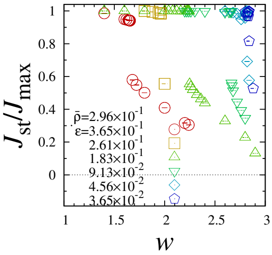

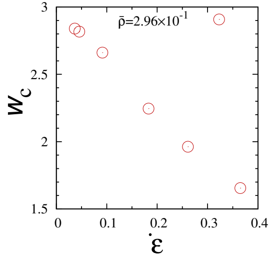

- Estimating $w_{\rm c}$, using the relation $\tau \sim (w_{\rm c} - w)^{-\alpha}$.
- Mass flux $J/J_{\rm max}$, where $J_{\rm max} \equiv Nc/L$.
  - fast jammed flow for $w < w_{\rm c}(\dot{\epsilon})$.
  - slow unjammed flow for $w > w_{\rm c}(\dot{\epsilon})$.
  - jumps at $w = w_{\rm c}$.
  - No such discontinuity has observed in previous studies ($\phi = 1 - w/2a$)
- $w_{\rm c}$ linearly decreases as $\dot{\epsilon}$, $w_{\rm c} \simeq -3.75\dot{\epsilon} + w_{\rm max}$.
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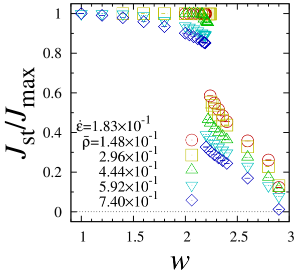
Density dependence



- Fixing $\dot{\epsilon}$ and changing $\bar{\rho}$
- Normalised flux $J/J_{\max}$ decreases as ρ.
- $w_c(\dot{\epsilon})$ is almost constant for ρ.
- $\alpha \simeq 1$ $[\tau \sim (w_c - w)^{-\alpha}]$ for $0.15 \lesssim \rho \lesssim 0.60$.

- Changing density ρ affects only $J/J_{\max}$.

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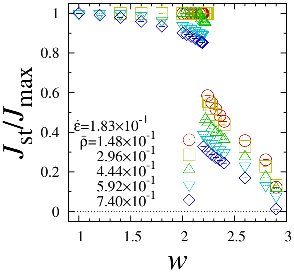

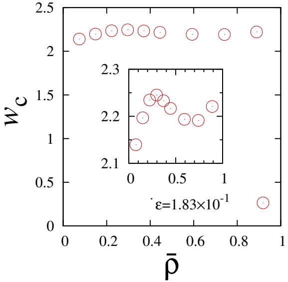

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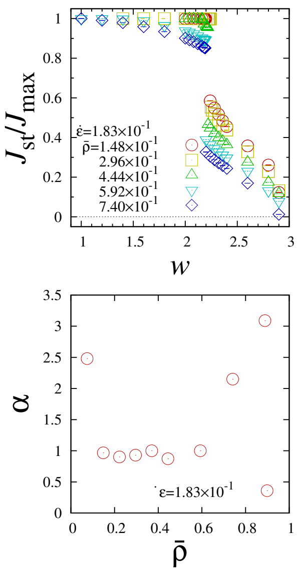

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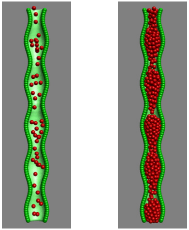

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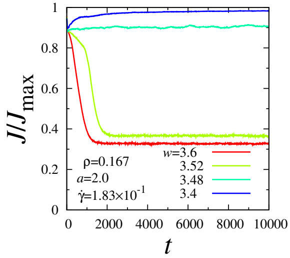
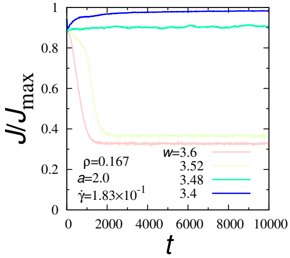
Hysteresis



- Initial condition: $J = J_{\max}$.
- Small w
  - steady jammed flow
- Large w
  - Transition
  - from unsteady jammed flow
  - to steady unjammed flow
- Transition at $w = w_c{}' \neq w_c$.
  - First-order transition

Hysteresis



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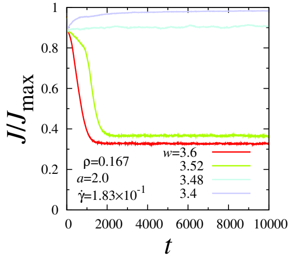

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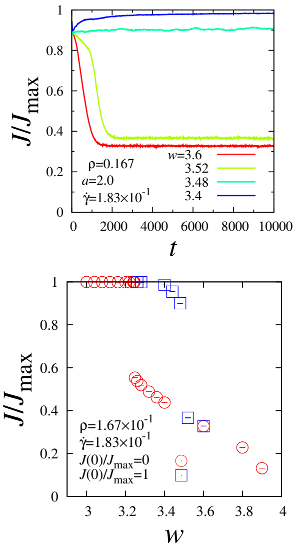

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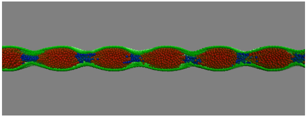
Peristaltic flow of frictional granular particles



- Polydisperse granular particles
  - diameter d_i, $0.8 \leq d_i/d^* \leq 1.0$
  - mass $m_i = m^*(d_i/d^*)^3$
  - no gravity, no ambient fluid
- $\boldsymbol{f}_{ij} = (f^{\rm n}_{ij}\boldsymbol{n}_{ij} + \boldsymbol{f}^{\rm t}_{ij})\Theta(\xi_{ij})\Theta(f^{\rm n}_{ij})$
  - $f^{\rm n}_{ij}$: Hertz force w/ damping term

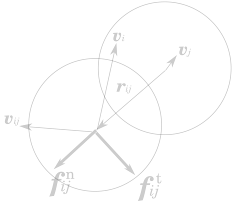
$$f^{\rm n}_{ij} = \frac{2Y\sqrt{R_{ij}}}{3(1-\nu^2)}\left(\xi_{ij}^{3/2} - A\sqrt{\xi_{ij}}v^{\rm n}_{ij}\right)$$

  - $\boldsymbol{f}^{\rm t}_{ij}$: tangential force

$$\boldsymbol{f}^{\rm t}_{ij} = \begin{cases} \tilde{\boldsymbol{f}}^{\rm t}_{ij} & \text{if } \left|\tilde{\boldsymbol{f}}^{\rm t}_{ij}\right| < \mu_{\rm s} f^{\rm n}_{ij} \\ \mu_{\rm k} f^{\rm n}_{ij}\boldsymbol{t}_{ij} & \text{otherwise} \end{cases}$$

$$\tilde{\boldsymbol{f}}^{\rm t}_{ij} = -k^{\rm t}\boldsymbol{u}^{\rm t}_{ij} - \eta^{\rm t}\boldsymbol{v}^{\rm t}_{ij}$$

  - Linear spring and no tangential force in our previous model



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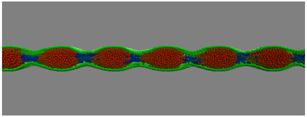

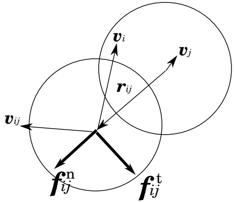

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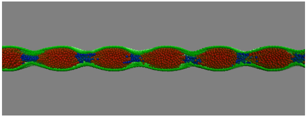
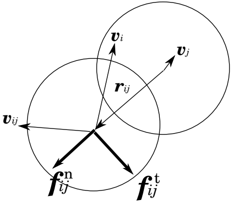
Peristaltic flow of frictional granular particles





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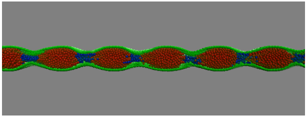

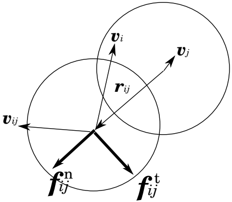

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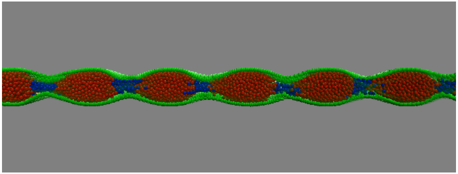
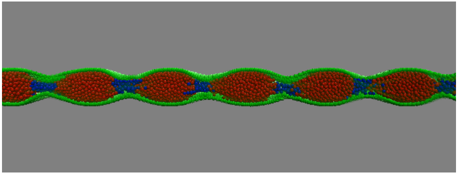
Peristaltic tube



- Monodisperse particles embedded in a tube's wall
- "Particle-Wall" interactions
  - $\boldsymbol{f}_{ij} = (f_{ij}^{\mathrm{n}} \boldsymbol{n}_{ij} + \boldsymbol{f}_{ij}^{\mathrm{t}})\Theta(\xi_{ij})\Theta(f_{ij}^{\mathrm{n}})$
    - neglecting rotation
  - $d_{\mathrm{w}} = d^*$, $m_{\mathrm{w}} = 0.1m^*$
- "Wall-Wall" interactions
  - Linear spring force w/ natural length l
- Peristaltic external force $\boldsymbol{f}_i = (f_i^{\mathrm{p}} \cos\phi_i, f_i^{\mathrm{p}} \sin\phi_i, 0) + \boldsymbol{f}_i^{\mathrm{keep}}$
  - $f_i^{\mathrm{p}} = f^{\mathrm{p}} \sin\big(2\pi(z_i - ct)/\lambda\big)$
  - $\boldsymbol{r}_i = (r_i \cos\phi_i, r_i \sin\phi_i, 0)$, $r_i = a + b\sin\big(2\pi(z_i - ct)/\lambda\big)$ in our previous "strain-controlled" model

Peristaltic tube



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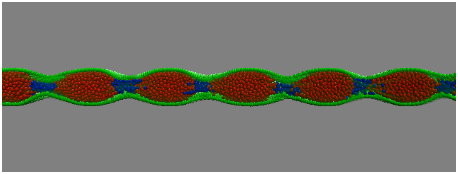

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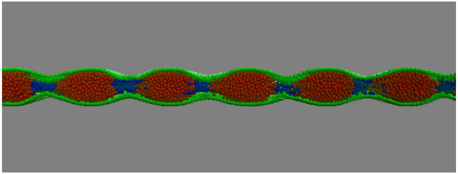

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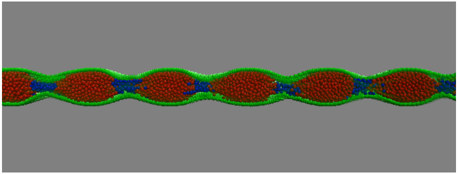
- "Wall-Wall" interactions
  - Linear spring force w/ natural length l
- Peristaltic external force $\boldsymbol{f}_i = (f^{\mathrm{p}}_i \cos\phi_i, f^{\mathrm{p}}_i \sin\phi_i, 0) + \boldsymbol{f}^{\mathrm{keep}}_i$
  - $f^{\mathrm{p}}_i = f^{\mathrm{p}} \sin\big(2\pi(z_i - ct)/\lambda\big)$
  - $\boldsymbol{r}_i = (r_i \cos\phi_i, r_i \sin\phi_i, 0)$, $r_i = a + b\sin\big(2\pi(z_i - ct)/\lambda\big)$ in our previous "strain-controlled" model

Peristaltic tube



- Monodisperse particles embedded in a tube's wall
- "Particle-Wall" interactions
  - $\boldsymbol{f}_{ij} = (f^{\mathrm{n}}_{ij}\boldsymbol{n}_{ij} + \boldsymbol{f}^{\mathrm{t}}_{ij})\Theta(\xi_{ij})\Theta(f^{\mathrm{n}}_{ij})$
    - neglecting rotation
  - $d_{\mathrm{w}} = d^*$, $m_{\mathrm{w}} = 0.1m^*$
- "Wall-Wall" interactions
  - Linear spring force w/ natural length l
- Peristaltic external force $\boldsymbol{f}_i = (f^{\mathrm{p}}_i \cos\phi_i, f^{\mathrm{p}}_i \sin\phi_i, 0) + \boldsymbol{f}^{\mathrm{keep}}_i$
  - $f^{\mathrm{p}}_i = f^{\mathrm{p}} \sin\big(2\pi(z_i - ct)/\lambda\big)$
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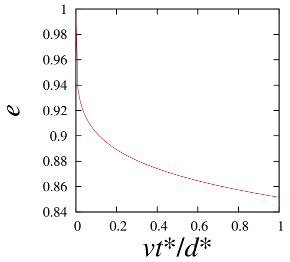
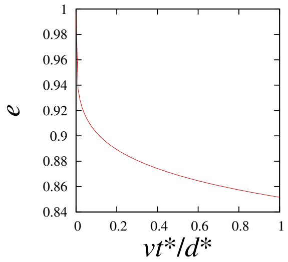
Parameters, etc.



- $t^* \equiv \sqrt{m^*/Yd^*}$
- Parameters
  - $a = 3.5d^*$, $\lambda \simeq 20.0d^*$
  - $A = 0.1t^*$, $\nu = 0.5$, $k^{\rm t} = 1.0Yd^*$, $\eta^{\rm t} = 0.1Yd^*t^*$, $\mu_{\rm s} = 0.5$, $\mu_{\rm k} = 0.4$
- Restitution coeff. $(d_i = d^*, m_i = m^*)$
  $e \simeq 0.85$ for $v \simeq d^*/t^*$
  - Müller and Pöschel, PRE (2011)
- Control parameters
  - amplitude of peristaltic force $f^{\rm p}$
  - peristaltic speed c
  - number of particles N

Parameters, etc.



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  - peristaltic speed c
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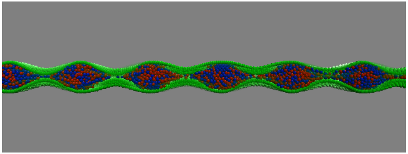
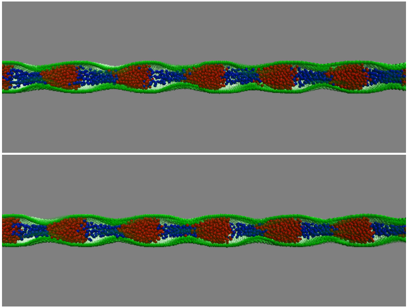
Snapshots

$N/V_0 = 7.10 \times 10^{-1}/d^{*3}, c/\lambda = 4.01 \times 10^{-3}/t^*$
$f^{\mathrm{p}} = 0.005 Y d^{*2}$

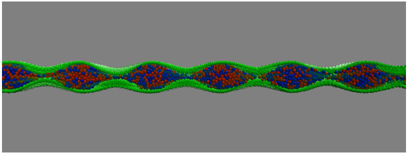


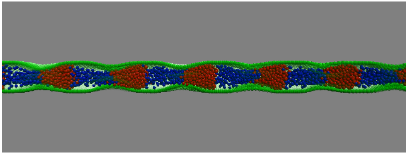


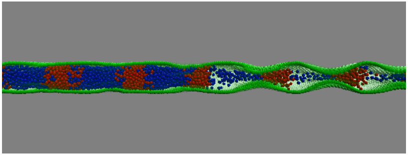
Blue: $\Leftarrow$, Red: $\Rightarrow$

Snapshots

$$N/V_0 = 7.10 \times 10^{-1}/d^{*3}, c/\lambda = 4.01 \times 10^{-3}/t^*$$
$$f^{\mathrm{p}} = 0.004 Y d^{*2}$$



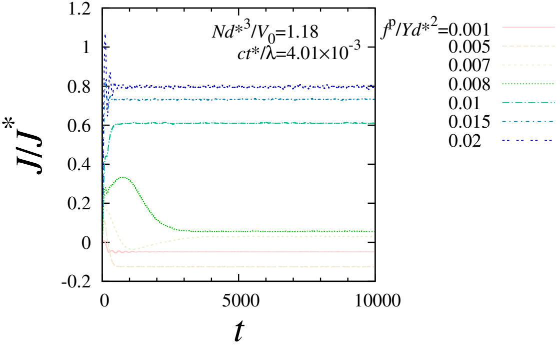




Blue: $\Leftarrow$, Red: $\Rightarrow$

Time evolution of averaged flow rate

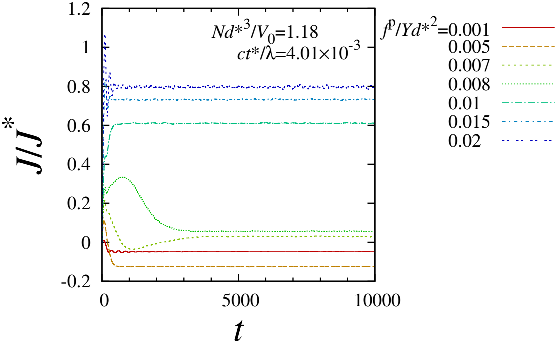
$$J/t^* = \sum_i v_{zi}/L, J^*/t^* = Nc/L$$



- Transitions exist for certain f^{p}'s
  - from a jammed flow to a unjammed flow
    - because of stress-contrlled walls
  - different transition which is found in the previous models
- J can be negative for small f^{p}'s

Time evolution of averaged flow rate

$$J/t^* = \sum_i v_{zi}/L, J^*/t^* = Nc/L$$



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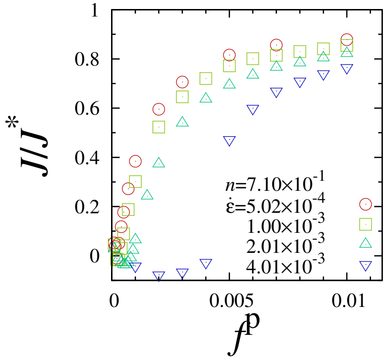
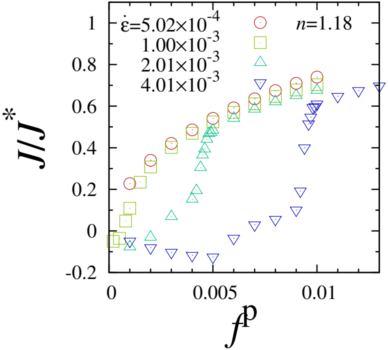
Stationary flow rate

$$n \equiv Nd^{*3}/V_0,\ \dot{\epsilon} \equiv ct^*/\lambda$$

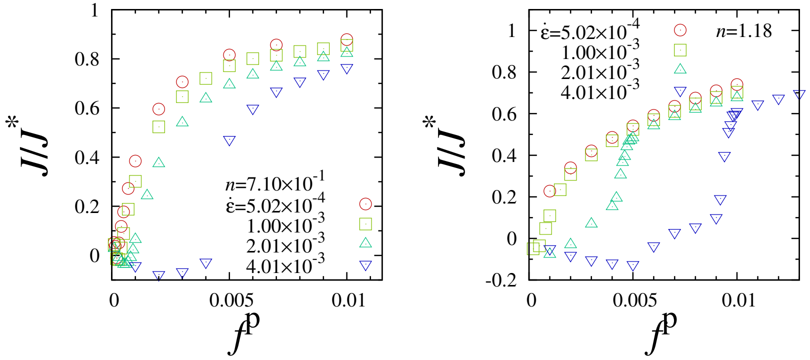




- Discontinuous transition for large c's
- No transition? or continuous transition? for small c's
- Negative J's for small f^{p}'s

Stationary flow rate

$$n \equiv N d^{*3}/V_0,\ \dot{\epsilon} \equiv c t^*/\lambda$$



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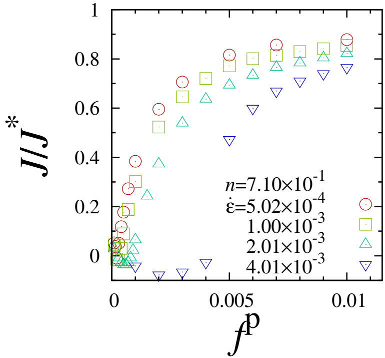

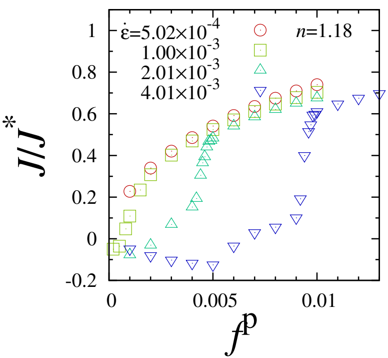

- Discontinuous transition for large c's
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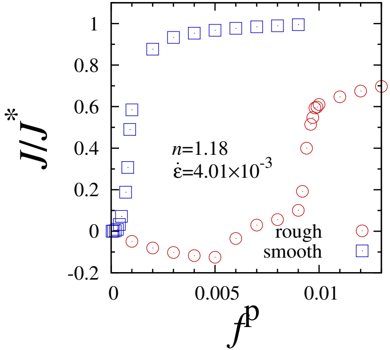
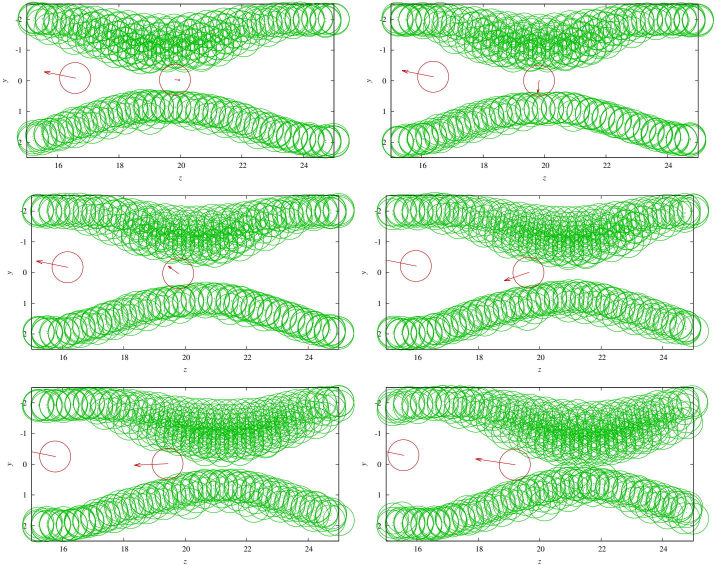
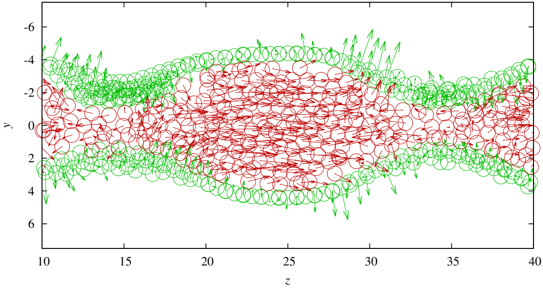
Negative J—rough v.s. smooth

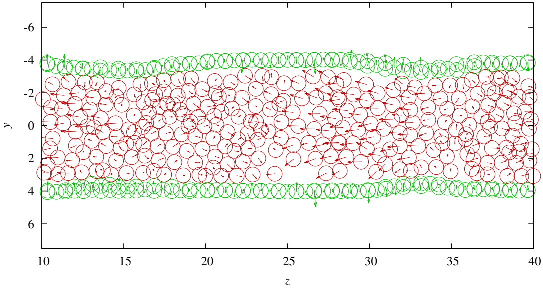
$n \equiv Nd^{*3}/V_0$, $\dot{\epsilon} \equiv ct^*/\lambda$



- No negative J's for smooth granular particles?
  - because of friction?


-1
0
1
y
16
18
20
22
24
z


-6
-4
-2
0
2
4
6
y
10
15
20
25
30
35
40
z


-6
-4
-2
0
2
4
6
y
10
15
20
25
30
35
40
z

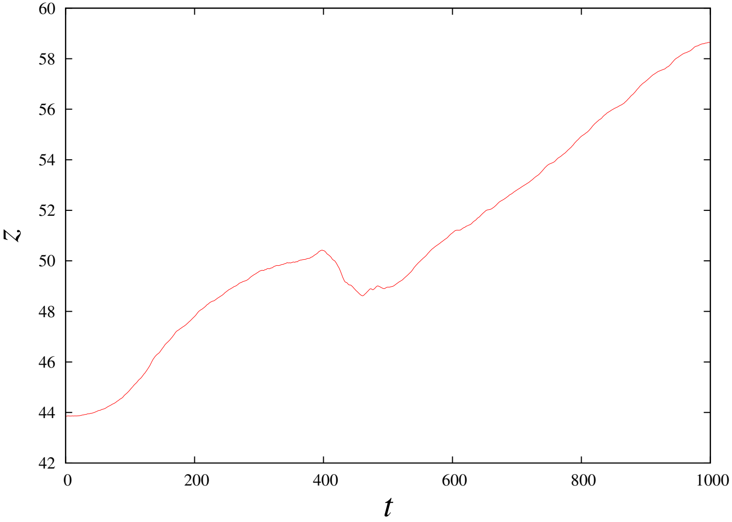
Summary

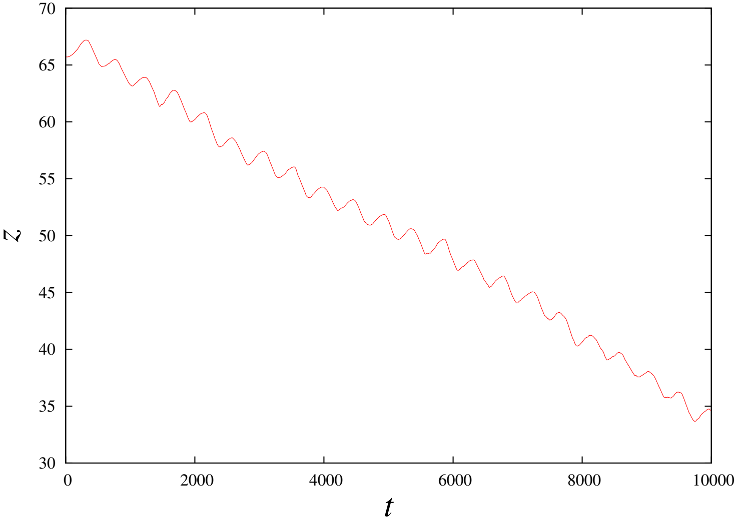
Peristalsis transport of granular particles

- Frictionless case
  - Discontinuous transition between jammed flow and unjammed flow
  - Scaling relationships

  N.Y. and H. Hayakawa, Phys. Rev. E **85**, 031302 (2012).

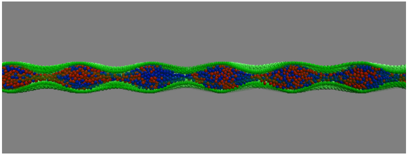
- Frictional case
  - Discontinuous transition between jammed flow and "unjammed flow"
    - this unjammed flow is different from that in frictionless case
  - Back flow


60
58
56
54
52
50
48
46
44
42
z
0
200
400
600
800
1000
t

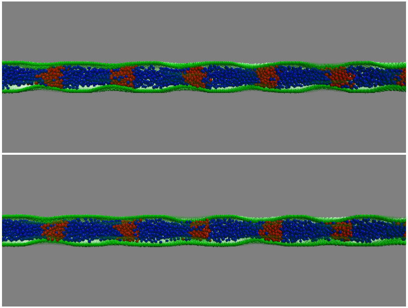

70
65
60
55
50
45
40
35
30
z
0
2000
4000
6000
8000
10000
t

Negative J

$N/V_0 = 7.10 \times 10^{-1}/d^{*3}, c/\lambda = 4.01 \times 10^{-3}/t^{*}$
$f^{\mathrm{p}} = 0.002 Y d^{*2}$

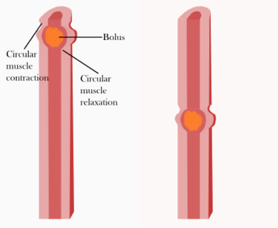


rotation
smooth



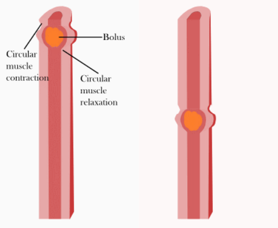
Blue: $\Leftarrow$, Red: $\Rightarrow$

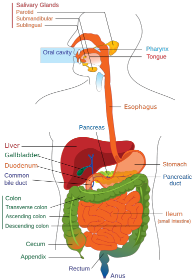
Peristaltic transport



- Progressive wave of area contraction/expansion.
- Biological systems
  - esophagus
  - small intensine
  - ureters
  - vasomotion (spontaneous oscillation) of small blood vessels
- Peristaltic Pump
  - blood, corrosive fluids, foods, ...
  - preventing the transported fluid from their mechanical parts.

Peristaltic transport





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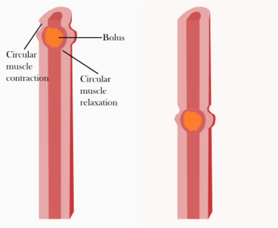

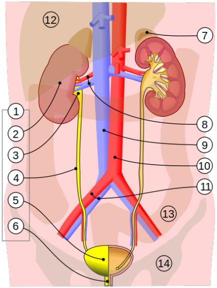

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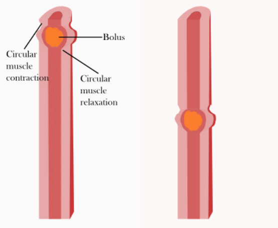

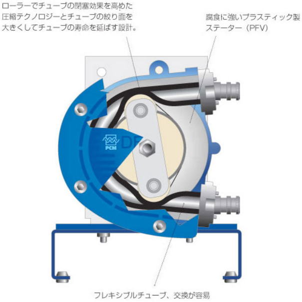

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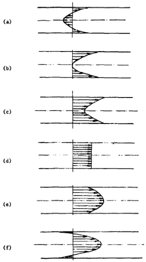
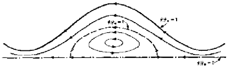
Previous studies



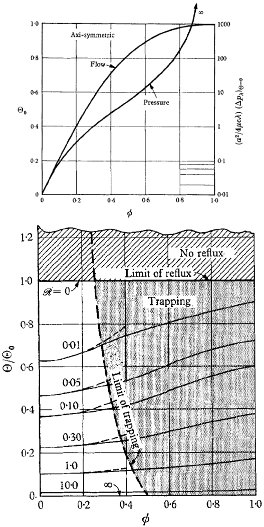
Zien and Ostrach, J. Biomech. **3**, 63 (1970)



Shapiro *et al.*, JFM **37**, 799 (1969)

- Newtonian fluids
  - Stokes approximation
    - assuming some of parameters are zero or small.
  - reflux, trapping.
- Non-Newtonian fluids
  - many studies,
    e.g., Maxwell fluids,
    third-order fluids,
    power-law fluids, ...
- Particles
  - one particle in fluids
  - dilute particles in fluids

Previous studies



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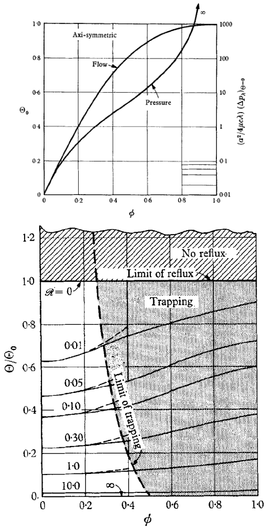

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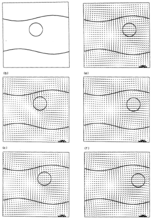
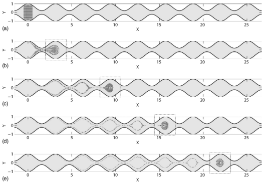


Fauci, Computers Fluids **21**, 583 (1992)



Jiménez-Lozano *et al.*, PRE **79**, 041901

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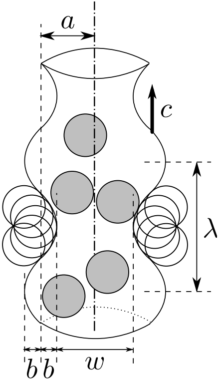
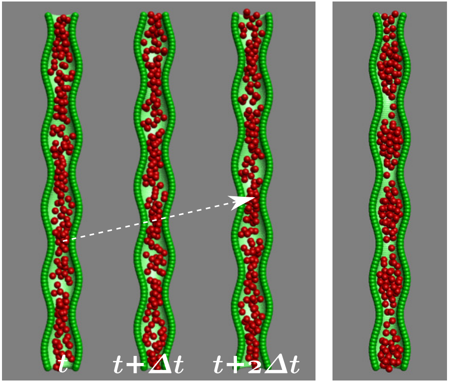
Previous results—snapshots

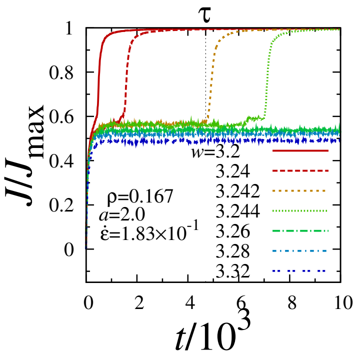
N. Y. and H. H., Phys. Rev. E **85**, 031302 (2012).

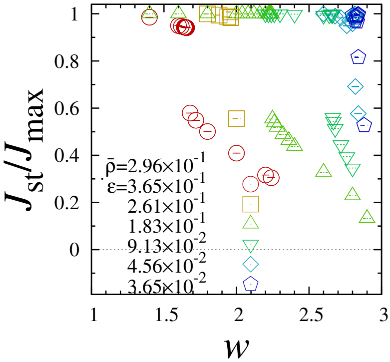




- Peristaltic transport of smooth dissipative particles
- Strain-controlled peristaltic motion
- Unjammed flow $\rightarrow$ Jammed flow

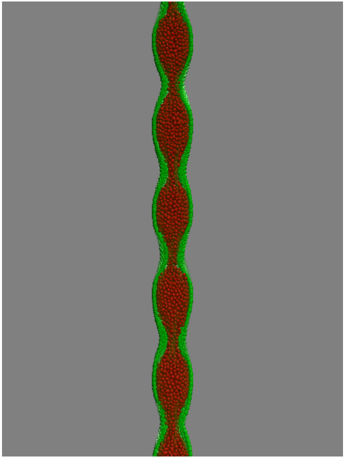
Previous results—flow rate





- Large w ⇒ steady slow unjammed flow
- Small w ⇒ steady fast jammed flow
- Discontinuous transition at $w = w_{\mathrm{c}}$.

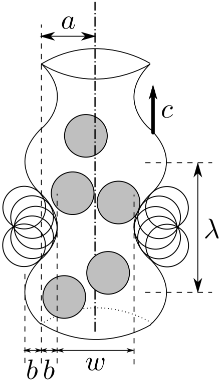
Objectives



Peristaltic transport of frictional granular particles

- More realistic systems
  - rough v.s. smooth
  - stress- v.s. strain-controlled
- slow peristaltic speed

Model—granular particles

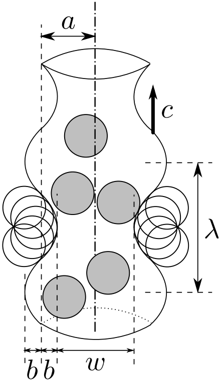


- Polydisperse granular particles w/o gravity & fluid
  - diameter d_i, $0.8 \leq d_i/d^* \leq 1.0$
  - mass $m_i = m^*(d_i/d^*)^3$
- $\boldsymbol{f}_{ij} = (f_{ij}^{\mathrm{n}}\boldsymbol{n}_{ij} + \boldsymbol{f}_{ij}^{\mathrm{t}})\Theta(\xi_{ij})\Theta(f_{ij}^{\mathrm{n}})$
  - $\boldsymbol{n}_{ij} = \boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|, \boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j,$
  - $\xi_{ij} = (d_i + d_j)/2 - |\boldsymbol{r}_{ij}|,$
- Hertzian contact force w/ damping term

  $$f_{ij}^{\mathrm{n}} = \frac{2Y\sqrt{R_{ij}}}{3(1-\nu^2)}\left(\xi_{ij}^{3/2} - A\sqrt{\xi_{ij}}v_{ij}^{\mathrm{n}}\right)$$

  - $v_{ij}^{\mathrm{n}} = \boldsymbol{v}_{ij} \cdot \boldsymbol{n}_{ij},\ \boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j,$
  - $R_{ij} = d_i d_j / 2(d_i + d_j)$

Model—granular particles



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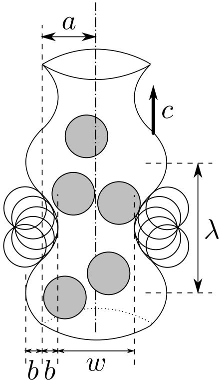
$$f_{ij}^{\mathrm{n}} = \frac{2Y\sqrt{R_{ij}}}{3(1-\nu^2)}\left(\xi_{ij}^{3/2} - A\sqrt{\xi_{ij}}v_{ij}^{\mathrm{n}}\right)$$

  - $v_{ij}^{\mathrm{n}} = \boldsymbol{v}_{ij}\cdot\boldsymbol{n}_{ij}, \boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j,$
  - $R_{ij} = d_i d_j/2(d_i + d_j)$

Model—granular particles



- Polydisperse granular particles w/o gravity & fluid
  - diameter d_i, $0.8 \le d_i/d^* \le 1.0$
  - mass $m_i = m^*(d_i/d^*)^3$
- $\boldsymbol{f}_{ij} = (f^{\mathrm{n}}_{ij}\boldsymbol{n}_{ij} + \boldsymbol{f}^{\mathrm{t}}_{ij})\Theta(\xi_{ij})\Theta(f^{\mathrm{n}}_{ij})$
  - $\boldsymbol{n}_{ij} = \boldsymbol{r}_{ij}/|\boldsymbol{r}_{ij}|, \boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j,$
  - $\xi_{ij} = (d_i + d_j)/2 - |\boldsymbol{r}_{ij}|,$
- Hertzian contact force w/ damping term

$$f^{\mathrm{n}}_{ij} = \frac{2Y\sqrt{R_{ij}}}{3(1-\nu^2)}(\xi_{ij}^{3/2} - A\sqrt{\xi_{ij}}v^{\mathrm{n}}_{ij})$$

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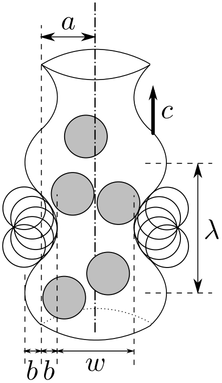
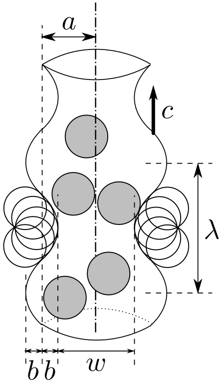


- $\boldsymbol{f}_{ij} = (f^{\mathrm{n}}_{ij}\boldsymbol{n}_{ij} + \boldsymbol{f}^{\mathrm{t}}_{ij})\Theta(\xi_{ij})\Theta(f^{\mathrm{n}}_{ij})$
- Cundall-Strack

$$\boldsymbol{f}^{\mathrm{t}}_{ij} = \begin{cases} \tilde{\boldsymbol{f}}^{\mathrm{t}}_{ij} & \text{if } \left|\tilde{\boldsymbol{f}}^{\mathrm{t}}_{ij}\right| < \mu_{\mathrm{s}} f^{\mathrm{n}}_{ij} \\ \mu_{\mathrm{k}} f^{\mathrm{n}}_{ij}\boldsymbol{t}_{ij} & \text{otherwise} \end{cases}$$

  - $\tilde{\boldsymbol{f}}^{\mathrm{t}}_{ij} = -k^{\mathrm{t}}\boldsymbol{u}^{\mathrm{t}}_{ij} - \eta^{\mathrm{t}}\boldsymbol{v}^{\mathrm{t}}_{ij}$
  - $\dot{\boldsymbol{u}}^{\mathrm{t}}_{ij} = \boldsymbol{v}^{\mathrm{t}}_{ij} - [(\boldsymbol{u}^{\mathrm{t}}_{ij}\cdot\boldsymbol{v}_{ij})/|\boldsymbol{r}_{ij}|]\boldsymbol{n}_{ij}$
  - $\boldsymbol{v}^{\mathrm{t}}_{ij} = (\boldsymbol{v}_{ij} - v^{\mathrm{n}}_{ij}\boldsymbol{n}_{ij}) + \dfrac{d_i - \xi_{ij}}{2}\boldsymbol{n}_{ij}\times\boldsymbol{\omega}_i - \dfrac{d_j - \xi_{ij}}{2}\boldsymbol{n}_{ji}\times\boldsymbol{\omega}_j$
  - $\boldsymbol{t}_{ij} = \tilde{\boldsymbol{f}}^{\mathrm{t}}_{ij}/\left|\tilde{\boldsymbol{f}}^{\mathrm{t}}_{ij}\right|$
- Solving eqs. of motion by Two-step Adams–Bashforth method

Model—granular particles



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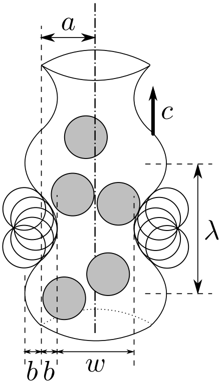
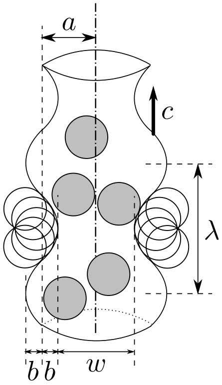
Peristaltic tube



- Monodisperse particles embedded in a tube's wall
- "Particle-Wall"
  - Hertzian force w/ damping term
    $\boldsymbol{f}_{ij} = (f^{\mathrm{n}}_{ij}\boldsymbol{n}_{ij} + \boldsymbol{f}^{\mathrm{t}}_{ij})\Theta(\xi_{ij})\Theta(f^{\mathrm{n}}_{ij})$
    - no rotation
  - diameter of "wall" particle $d_{\mathrm{w}}/d^* = 1.0$
  - mass of "wall" particle $m_{\mathrm{w}}/m^* = 0.1$
- "Wall-Wall"
  - Linear spring force w/ natural length l
    $\boldsymbol{f}_{ij} = -k(|\boldsymbol{r}_{ij}| - l)\boldsymbol{n}_{ij}$
- Peristaltic external force
  $\boldsymbol{f}_i = (f^{\mathrm{p}}_i \cos\phi_i, f^{\mathrm{p}}_i \sin\phi_i, 0) + \boldsymbol{f}^{\mathrm{keep}}_i$
  - $f^{\mathrm{p}}_i = f^{\mathrm{p}} \sin\left(\frac{2\pi}{\lambda}(z_i - ct)\right)$

Peristaltic tube



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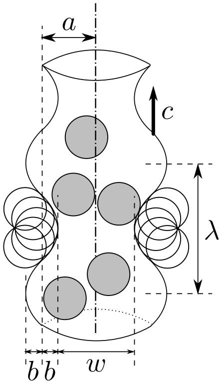

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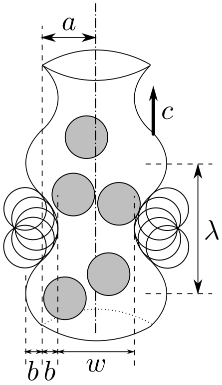

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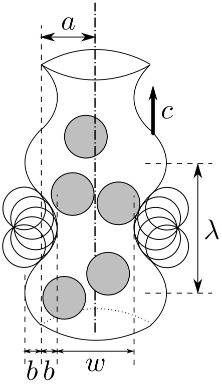
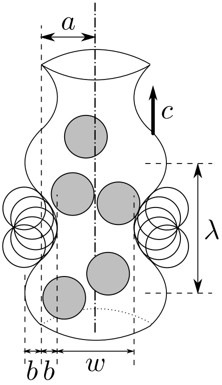
Parameters, etc.



- Scaled by
  - largest mass m^*,
  - largest diameter d^*,
  - $\sqrt{m^*/Yd^*}$
- Parameters
  - $a = 3.5,\ \lambda \simeq 20.0$
  - $A = 0.1,\ \nu = 0.5,\ k^{\mathrm{t}} = 1.0,\ \eta^{\mathrm{t}} = .1,$ $\mu_{\mathrm{s}} = 0.5,\ \mu_{\mathrm{k}} = 0.4$
- Control parameters
  - amplitude of peristaltic force f^{p}
  - strain rate $\dot{\epsilon} \equiv c/\lambda$
  - initial number density $n \equiv N/\pi a^2 L$

Parameters, etc.



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