Critical behavior of a domain wall collapse and Oscillon with gravity

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What is Oscillon?

- Let’s consider time evolution of scalar field.
  - scalar field + double well potential
    - EOM
      \[
      \frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = -V'(\Phi) \quad V(\Phi) = \frac{1}{4}(\Phi^2 - \sigma^2)^2
      \]
    - initial data : Gaussian bubble
      \[
      \Phi(t = 0, r) = 2\sigma e^{-r^2/R_0^2} - \sigma
      \]
      \[
      \Pi(t = 0, r) = 0
      \]
What is Oscillon?

- Oscillon = extremely long-lived time dependent solution of scalar field
  - typical initial data
    - Gaussian bubble
      \[ \Phi(t = 0, r) = (\Phi_c + \sigma)e^{-r^2/R_0^2} - \sigma \]
    - tanh bubble
      \[ \Phi(t = 0, r) = \frac{1}{2}[(-\sigma - \Phi_c) \tanh(r - R_0) - \sigma + \Phi_c] \]

\[ \Phi_c : \text{value of the scalar field at the bubble's core} \]
Life time vs bubble radius

- Oscillon = extremely long-lived time dependent solution of scalar field
- Oscillon naturally appears during the collapse of bubble.
- Definition of life time: $t_f$

$$E(t; r_0) = \int_0^{r_0} \left\{ \Phi^2 + (\nabla \Phi)^2 + V(\Phi) \right\} 4\pi r^2 dr$$

$$\frac{E(t_f; r_0)}{E(t = 0; r_0)} < \text{(some small value)}$$

$t = 0$ \hspace{1cm} $t = t_f$

![Diagram showing life time vs bubble radius](image)

1. About Oscillon

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Fine structure of Oscillon

- Fine structure of lifetime (Honda et al. 2002)
  - When we tune the initial parameter, resonance structure of lifetime appears.

- Critical behavior appears around the fine structure. (critical solution exist)

\[ t_f \sim \gamma \ln |R_0 - R_*| \]
What we want to do?

- We want to construct Oscillon with gravity.
  - Does it exist (not only in weak gravity case but also strong gravity case) ?
  - Its property ?
  - It may be one of the final state or intermediate state of gravitational collapse.

- What is interesting ?
  - Can Oscillon collapse to black hole ?
  - If critical behavior appears, there are special solutions associated with critical behavior.

➡️ We need to develop the numerical code for spherically symmetric spacetime.
System and initial data

- EOM
  \[ G_{\mu\nu} = 8\pi GT_{\mu\nu} \]
  \[ \nabla^2 \Phi = V'(\Phi) \quad V(\Phi) = \frac{1}{4}(\Phi^2 - 1)^2 \]
  \( G \): Newton constant

- initial data
  - momentary static bubble

\[
\begin{align*}
  \Phi(t = 0, r) &= -1 + 2e^{-r^2/R_0^2} \\
  \Pi(t = 0, r) &= 0 \\
  \tilde{\gamma}_{ij} dx^i dx^j &= dr^2 + r^2 d^2\Omega \\
  K_{ij} &= 0
\end{align*}
\]

- \( \phi \) is given from the solution of Hamiltonian constraint.

2.GBSSN formulation

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Our numerical code

- Our numerical code
  - It is written in C++.
  - GBSSN formulation - spherically symmetric case
  - free evolution
  - time integration : iterative Crank Nicolson scheme
  - spatial derivative : central difference
  - totally second order accuracy
  - We add 2nd order numerical dissipation term in each time evolution equation.
  - We use inhomogeneous grid
  - parallel computation by using Open MP
Kodama mass

- Kodama mass = locally conserved energy in the spherically symmetric system
  - definition
    - Kodama vector
      \[ K^A = \epsilon^{AB} \partial_B R \]
    (extend) (vector on 2-dim manifold charted by t and r)
      \[ K^\mu \] (vector on 4-dim manifold)
    - Kodama mass
      \[ S^{\mu} = T^{\mu}_{\nu} K^{\nu} \]
      \[ M(t; r_0) = \int_{\Sigma} S^t \alpha \sqrt{\gamma} dx^3 \]

\[ ds^2 = G_{AB} dx^A dx^B + R^2 d^2 \Omega \]

\[ \epsilon_{AB} = \sqrt{-G} \epsilon_{AB} \]

\[ \epsilon_{tr} = 1 \]
Kodama mass

- proof

\[ ds^2 = G_{AB}dx^A dx^B + r^2 d\Omega^2 = -\alpha^2(t, r)dt^2 + a^2(t, r)dr^2 + r^2 d\Omega^2 \]

\[ K^A = \epsilon^{AB} \partial_B r \]

\[ K^t = \epsilon^{tr} \partial_r r = -\frac{1}{\alpha a}, \quad K^r = 0 \]

\[ \epsilon_{AB} = \sqrt{-g} \epsilon_{AB} \]

\[ \epsilon_{tr} = 1 \]

\[ \epsilon_{tr} = \alpha a \]

\[ S^\mu = T^\mu_\nu K^\nu \]

\[ S^t = \frac{1}{\alpha^3 a} T_{tt} = \frac{1}{\alpha^3 a} G_{tt} = \frac{1}{\alpha^3 a} \left( \frac{2\alpha^2 \partial_r a}{ra^3} + \frac{\alpha^2}{r^2} - \frac{\alpha^2}{r^2 a^2} \right) \]

\[ S^r = \frac{1}{\alpha a^3} T_{rt} = \frac{1}{\alpha a^3} G_{rt} = -\frac{2\partial_t a}{r a a^4} \]

\[ \partial_\mu (\sqrt{-g} S^\mu) = 0 \]
Kodama mass

- conservation law of Kodama mass

\[ \nabla_\mu S^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} S^\mu) = 0 \]

\[ M(t = 0; r_0) + P(t = 0, t; r_0) = M(t; r_0) \]

\[
\begin{cases}
M(t; r_0) = \int_\Sigma S^t \alpha \sqrt{\gamma} dx^3 \\
P(t = 0, t; r_0) = \int_0^t dt \int_{\partial \Sigma} d^2 x \ n_\mu S^\mu
\end{cases}
\]

- We define the lifetime of Oscillon from Kodama mass in the case of Oscillon with gravity.
Oscillon with gravity (preliminary result)

- Some examples of numerical simulation

$$\Phi(t, r = 0)$$

Kodama mass

$G = 0$

$R_0 = 2.2750$

no Oscillon

$G = 10^{-5}$

$R_0 = 2.2750$

no Oscillon

3.Oscillon with gravity

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Oscillon with gravity (preliminary result)

\[ G = 10^{-4} \]
\[ R_0 = 2.2750 \]

\[ G = 10^{-3} \]
\[ R_0 = 2.2750 \]
Life time of Oscillon with gravity (preliminary result)

- initial bubble radius vs life time of Oscillon with gravity

\[ G = 10^{-4} \]
Life time of Oscillon with gravity (preliminary result)

\[ R_0 < R_\ast \]

\[ R_0 > R_\ast \]

3. Oscillon with gravity
Summary and future work

• Summary
  - Oscillon = long lifetime localized solution of scalar field on Minkowski background
  - It is the product of nonlinear effect of scalar field.
  - We construct the Oscillon with gravity.
  - We get sign of its fine structure in the life time.

• Future work
  - Can Oscillon collapse to black hole?
  - Can we check the critical behavior?
  - What is the properties of the critical solution?