~~Critical behavior of a domain wall collapse and~~ Oscillon with gravity

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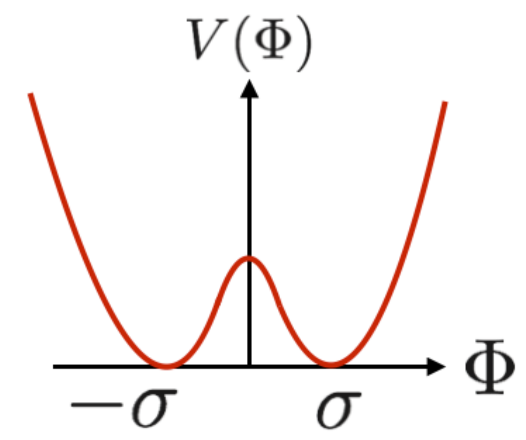
What is Oscillon?

- Let's consider time evolution of scalar field.
  - scalar field + double well potential
    - EOM

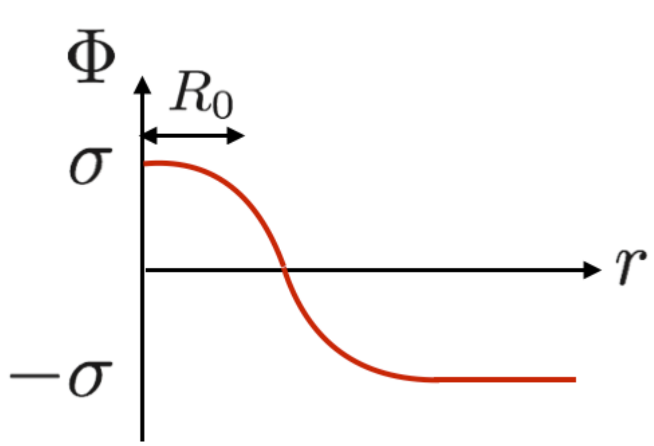


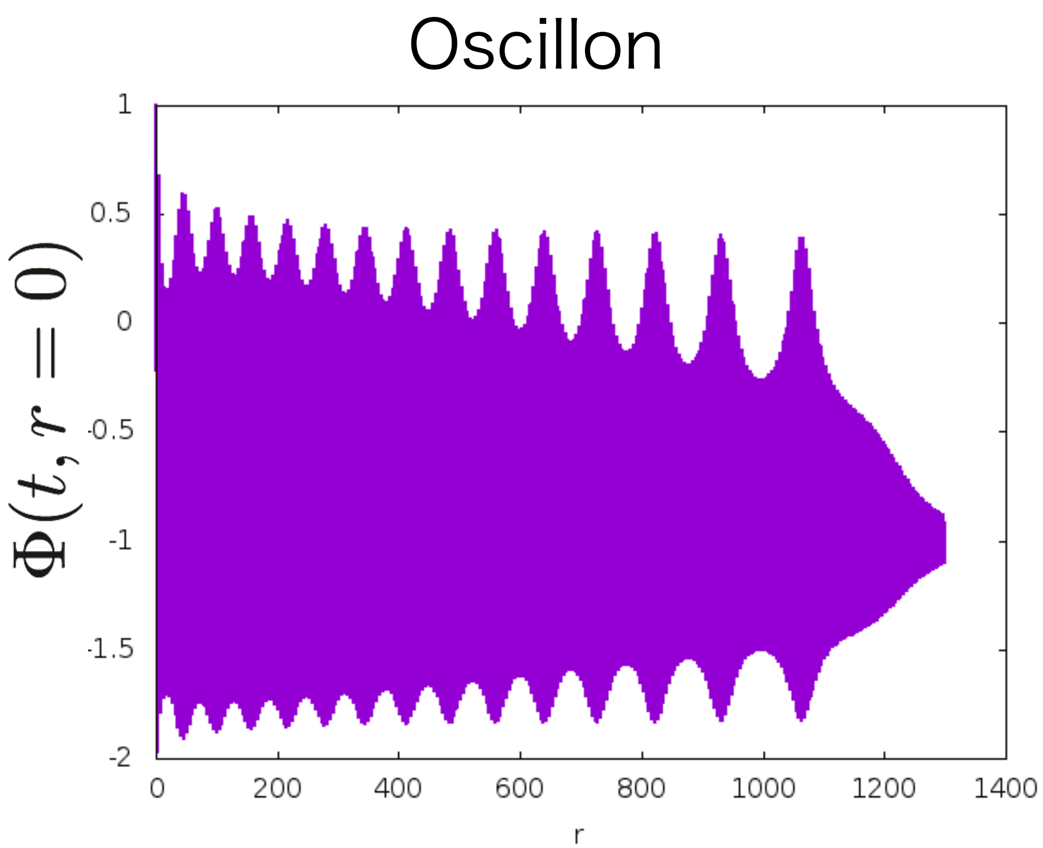
$$\frac{\partial^2}{\partial t^2}\Phi - \nabla^2\Phi = -V'(\Phi) \qquad V(\Phi) = \frac{1}{4}(\Phi^2 - \sigma^2)^2$$

- initial data : Gaussian bubble

$$\Phi(t=0,r) = 2\sigma e^{-r^2/R_0^2} - \sigma$$

$$\Pi(t=0,r) = 0$$





What is Oscillon?

- Oscillon = extremely long-lived time dependent solution of scalar field
  - typical initial data
    - Gaussian bubble

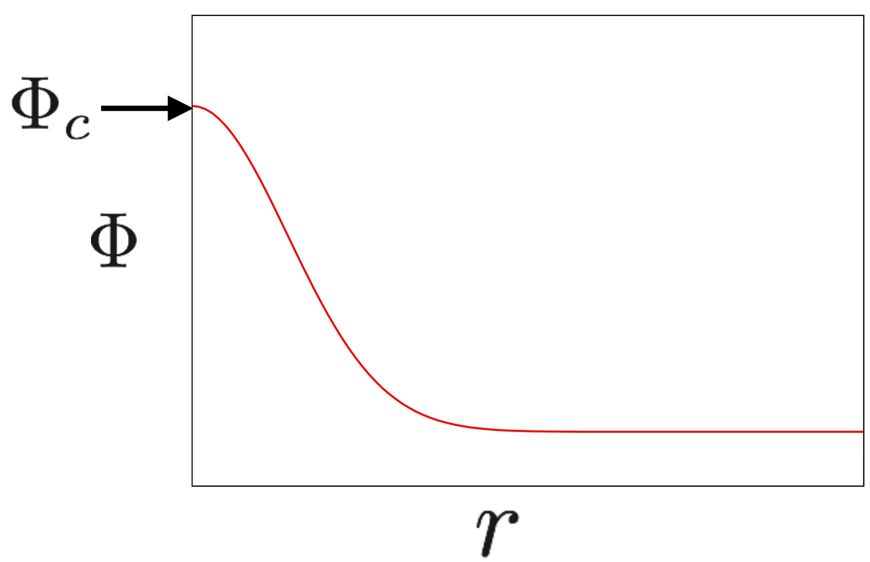
$$\Phi(t=0,r) = (\Phi_c + \sigma)e^{-r^2/R_0^2} - \sigma$$

    - tanh bubble

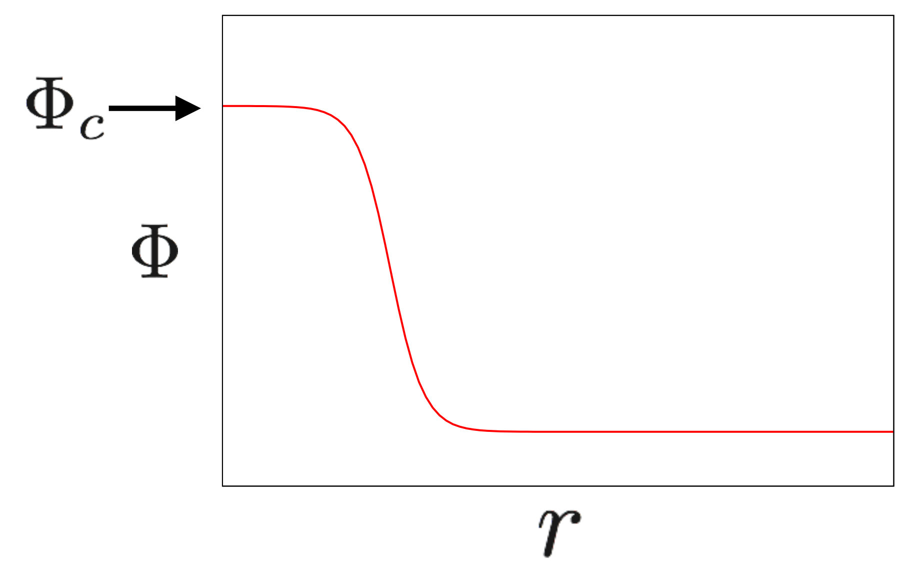
$$\Phi(t=0,r) = \frac{1}{2}[(-\sigma - \Phi_c)\tanh(r - R_0) - \sigma + \Phi_c]$$

Φ_c : value of the scalar field at the bubble's core

Gaussian bubble



tanh bubble



Life time vs bubble radius

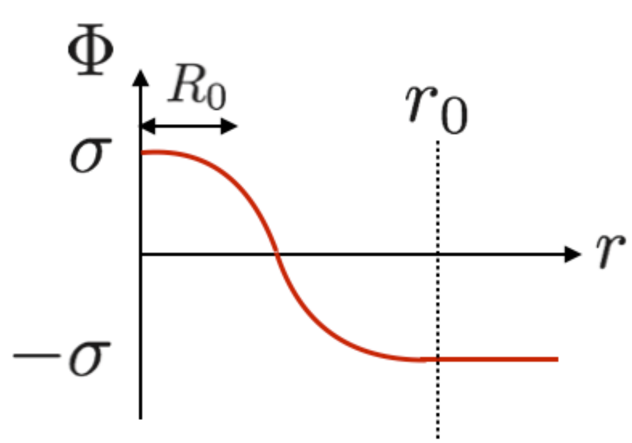
- Oscillon = extremely long-lived time dependent solution of scalar field
- Oscillon naturally appears during the collapse of bubble.
- Definition of life time : t_{f}

r_0 : sufficient large radius (fix)

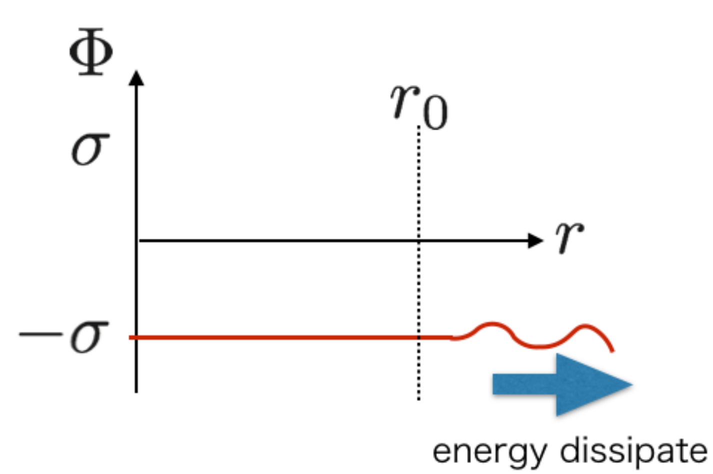
$$E(t;r_0) = \int_0^{r_0} \left\{ \dot{\Phi}^2 + (\nabla\Phi)^2 + V(\Phi) \right\} 4\pi r^2 dr$$

$$\frac{E(t_{\mathrm{f}};r_0)}{E(t=0;r_0)} < \text{(some small value)}$$

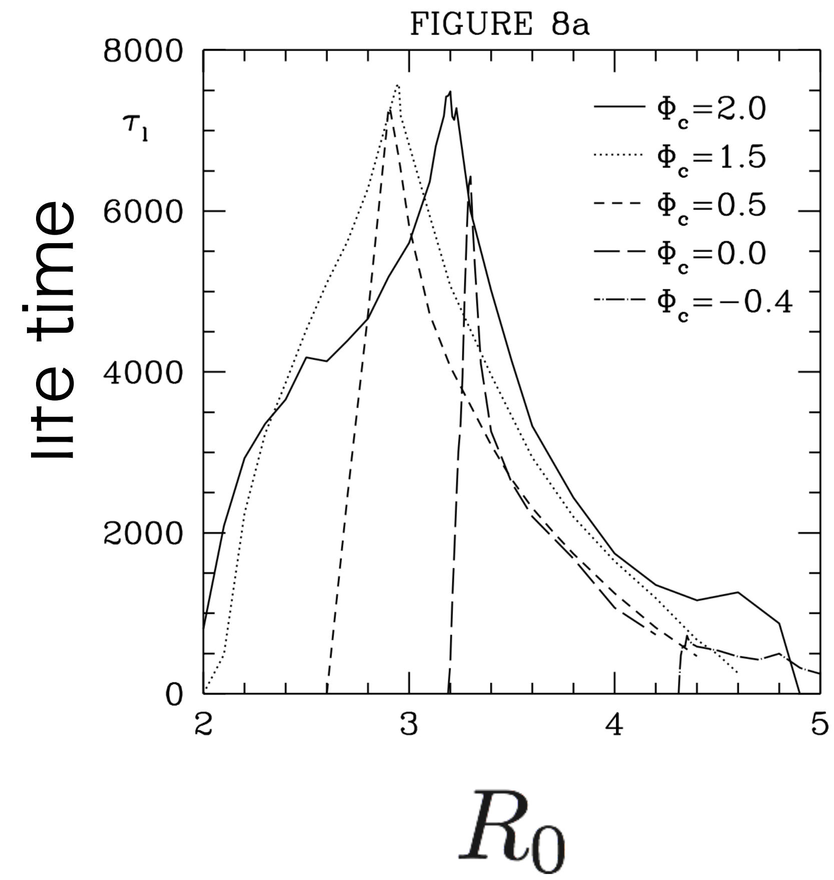
$t = 0$



$t = t_{\mathrm{f}}$

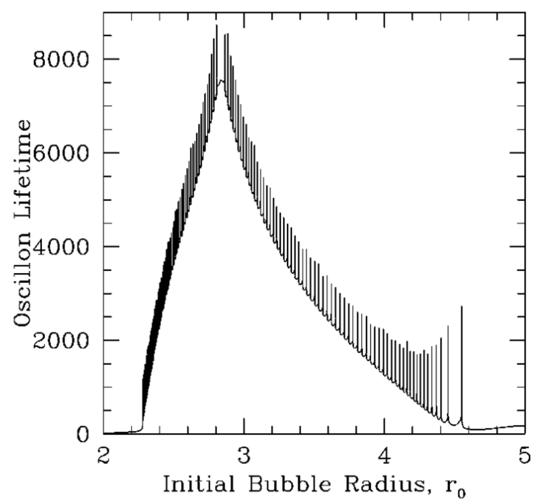


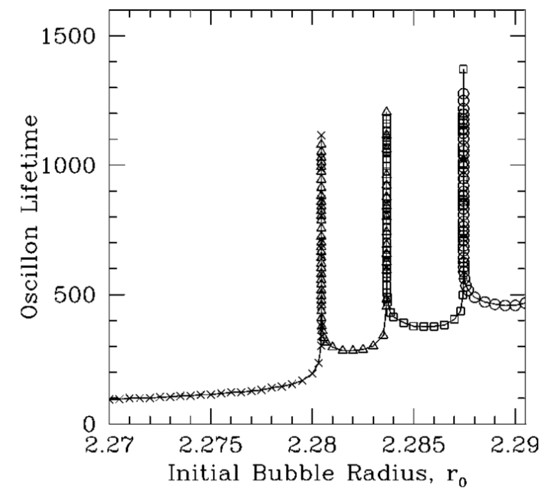
Gaussian bubbles case



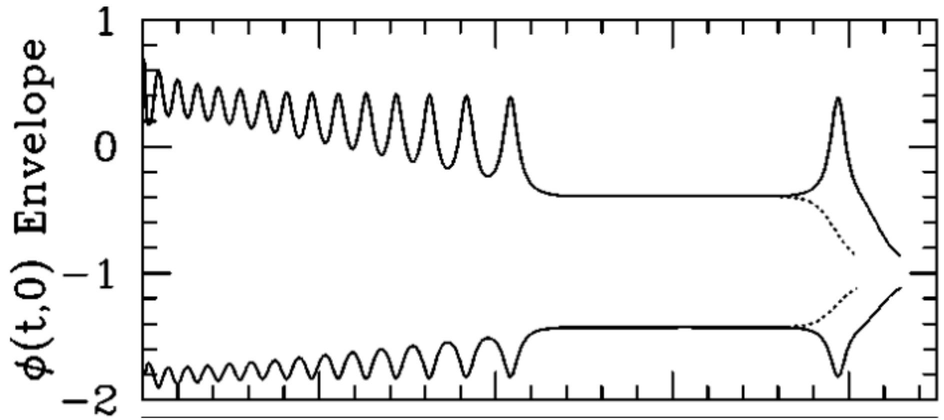
Fine structure of Oscillon

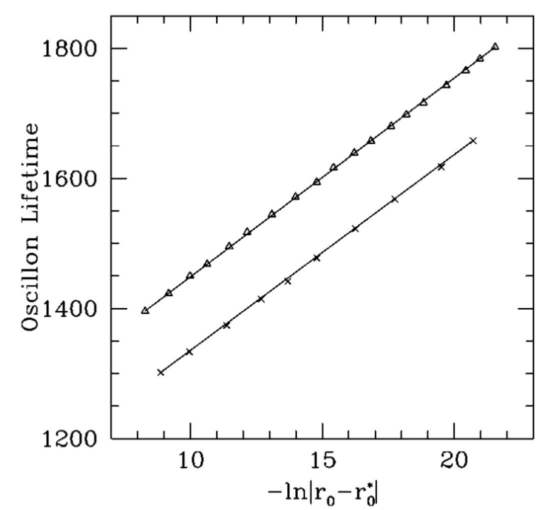
- Fine structure of lifetime (Honda et al 2002)
  - When we tune the initial parameter, resonance structure of lifetime appears.





  - Critical behavior appears around the fine structure. (critical solution exist)





$$t_f \sim \gamma \ln |R_0 - R_*|$$

What we want to do?

- We want to construct Oscillon with gravity.
  - Dose it exist (not only in week gravity case but also strong gravity case) ?
  - its property ?
  - It may be one of the final state or intermediate state of gravitational collapse.
- What is interesting ?
  - Can Oscillon collapse to black hole ?
  - If critical behavior appears, there are special solutions associated with critical behavior.

➡We need to develop the numerical code for spherically symmetric spacetime.

System and initial data

- EOM

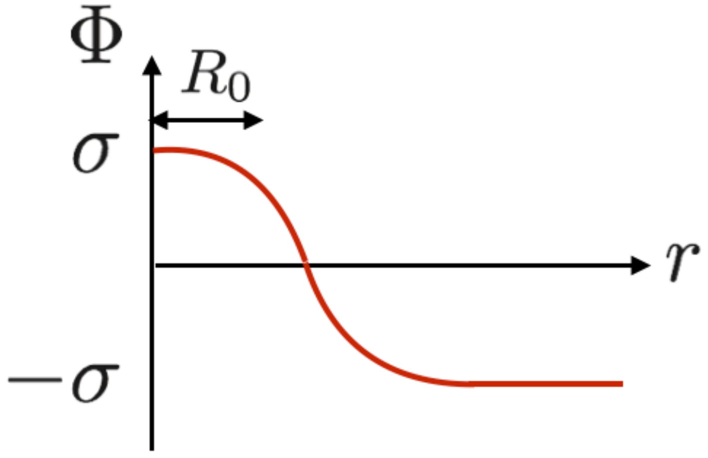
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla^2\Phi = V'(\Phi) \qquad V(\Phi) = \frac{1}{4}(\Phi^2 - 1)^2 \qquad G : \text{Newton constant}$$

- initial data
  - momentary static bubble

$$\begin{cases} \Phi(t=0,r) = -1 + 2e^{-r^2/R_0^2} \\ \Pi(t=0,r) = 0 \end{cases}$$

$$\begin{cases} \tilde{\gamma}_{ij}dx^i dx^j = dr^2 + r^2 d^2\Omega \\ K_{ij} = 0 \end{cases}$$



▸ ϕ is given from the solution of Hamiltonian constraint.

Our numerical code

- Our numerical code
  - It is written in C++.
  - GBSSN formulation - spherically symmetric case
  - free evolution
  - time integration : iterative Crank Nicolson shceme
  - spatial derivative : central difference
  - totally second order accuracy
  - We add 2nd order numerical dissipation term in each time evolution equation.
  - We use inhomogeneous grid
  - parallel computation by using Open MP

Kodama mass

- Kodama mass = locally conserved energy in the spherically symmetric system
  - definition
    - Kodama vector

$$K^A = \epsilon^{AB}\partial_B R$$

(vector on 2-dim manifold charted by t and r)

(extend)

➡ K^μ (vector on 4-din manifold)

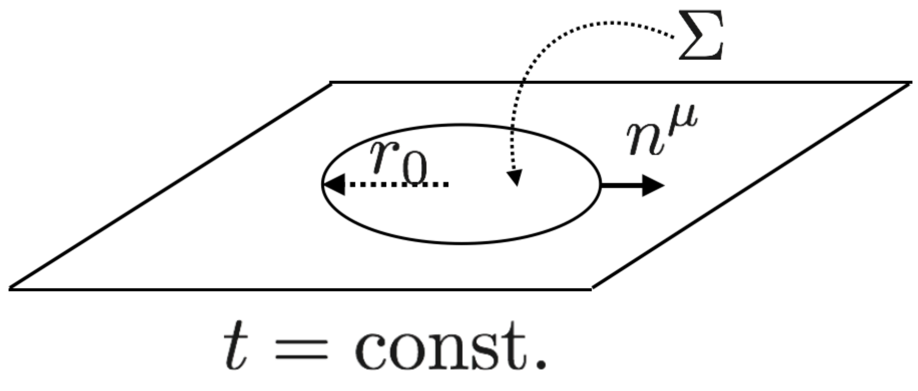
    - Kodama mass

$$S^\mu = T^\mu{}_\nu K^\nu$$

$$M(t;r_0) = \int_\Sigma S^t \alpha\sqrt{\gamma}dx^3$$

$$ds^2 = \underset{(t,r)}{\mathcal{G}_{AB}} dx^A dx^B + R^2 d^2\Omega$$

$$\epsilon_{AB} = \sqrt{-\mathcal{G}}\varepsilon_{AB} \qquad \varepsilon_{tr} = 1$$



Kodama mass

- proof

$$\epsilon_{AB} = \sqrt{-\mathcal{G}}\varepsilon_{AB}$$
$$\varepsilon_{tr} = 1$$
$$\epsilon_{tr} = \alpha a$$

$$\begin{aligned} ds^2 &= \mathcal{G}_{AB}dx^A dx^B + r^2 d^2\Omega \\ &= -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d^2\Omega \end{aligned}$$

- component of $K^A = \epsilon^{AB}\partial_B r$

$$K^t = \epsilon^{tr}\partial_r r = -\frac{1}{\alpha a},\ K^r = 0$$

- component of $S^\mu = T^\mu_{\ \nu}K^\nu$

$$\begin{cases} S^t = \dfrac{1}{\alpha^3 a}T_{tt} = \dfrac{1}{\alpha^3 a}G_{tt} = \dfrac{1}{\alpha^3 a}\left(\dfrac{2\alpha^2\partial_r a}{ra^3} + \dfrac{\alpha^2}{r^2} - \dfrac{\alpha^2}{r^2 a^2}\right) \\ S^r = \dfrac{1}{\alpha a^3}T_{rt} = \dfrac{1}{\alpha a^3}G_{rt} = -\dfrac{2\partial_t a}{r\alpha a^4} \end{cases}$$

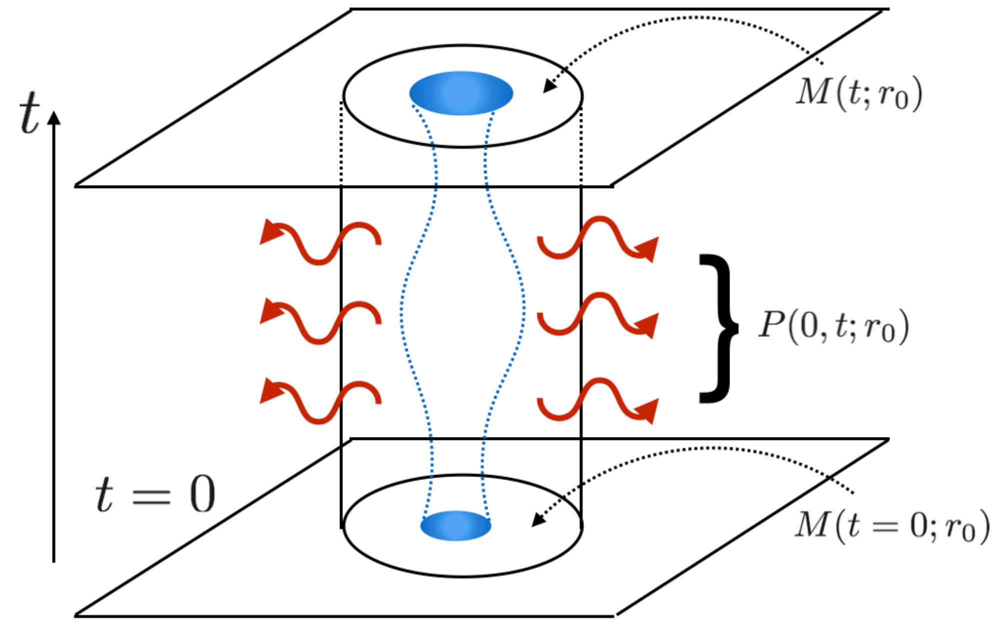
➡ $\partial_\mu(\sqrt{-g}S^\mu) = 0$

- conservation law of Kodama mass

$$\nabla_\mu S^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}S^\mu) = 0$$

$$\Rightarrow \quad \boxed{M(t=0;r_0) + P(t=0,t;r_0) = M(t;r_0)}$$

$$\begin{cases} M(t;r_0) = \int_\Sigma S^t \alpha \sqrt{\gamma} dx^3 \\ P(t=0,t;r_0) = \int_0^t dt \int_{\partial\Sigma} d^2x \; n_\mu S^\mu \end{cases}$$



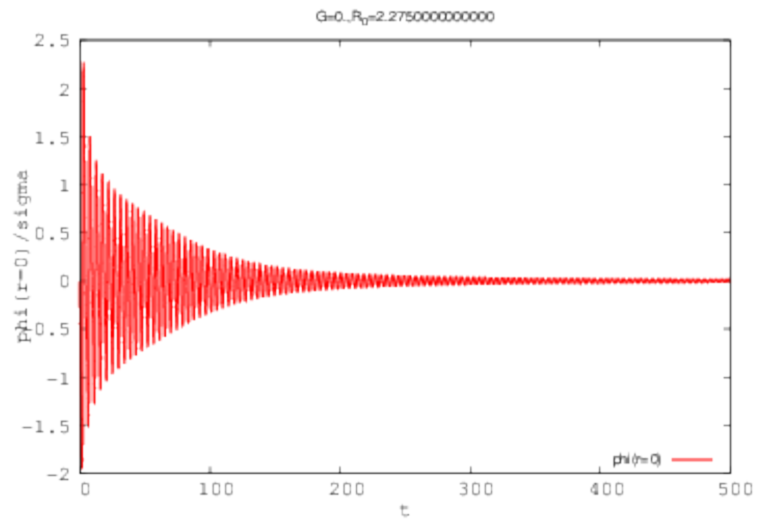
- We define the lifetime of Oscillon from Kodama mass in the case of Oscillon with gravity.

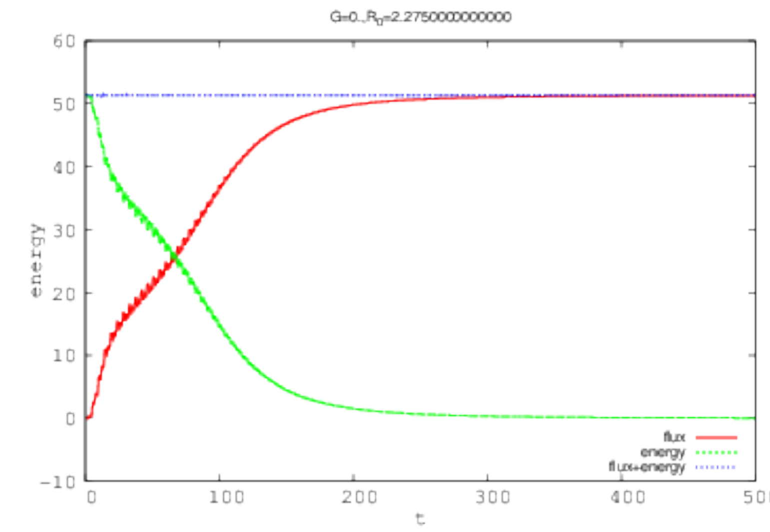
Oscillon with gravity (preliminary result)

- Some examples of numerical simulation

$\Phi(t, r=0)$ Kodama mass

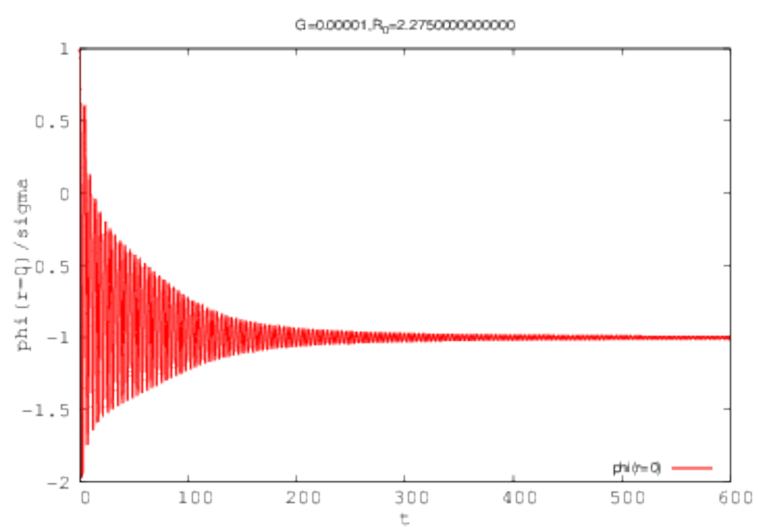
$G = 0$
$R_0 = 2.2750$

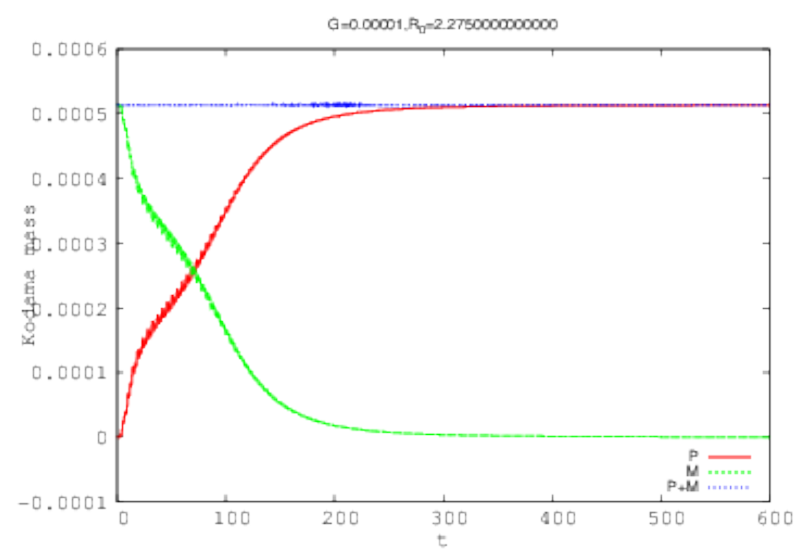




no Oscillon

$G = 10^{-5}$
$R_0 = 2.2750$



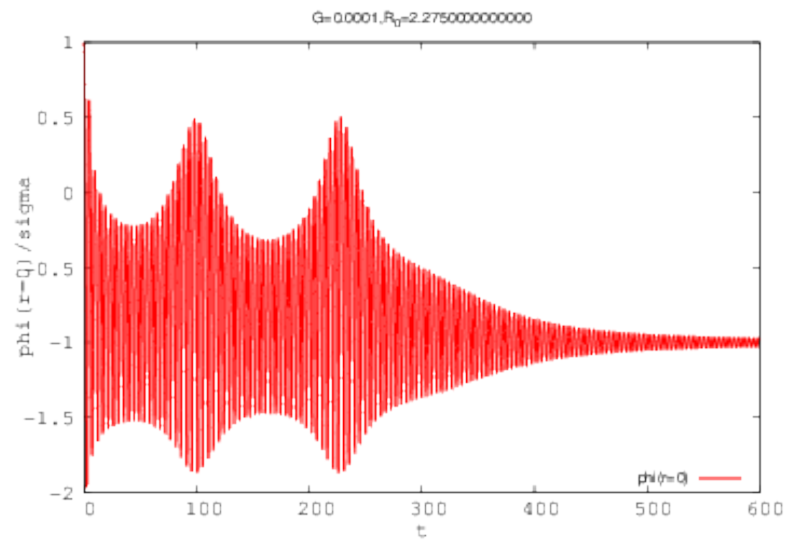


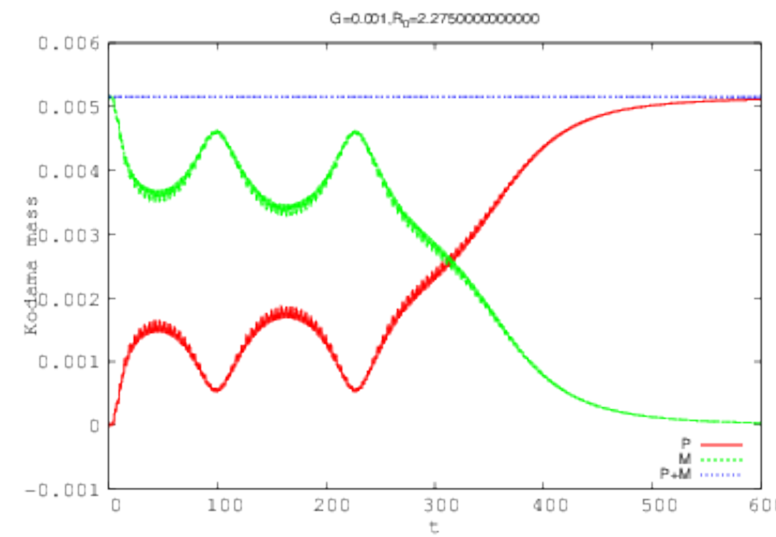
no Oscillon

Oscillon with gravity (preliminary result)

$\Phi(t, r=0)$ | Kodama mass

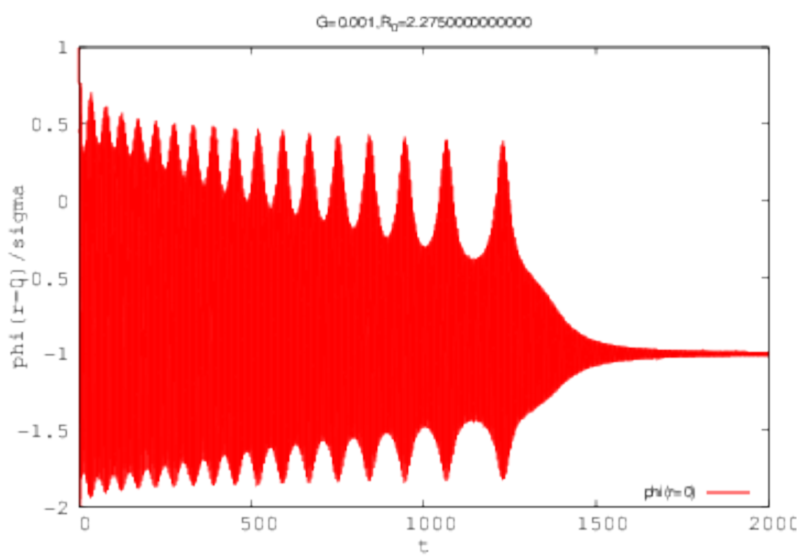
$G = 10^{-4}$
$R_0 = 2.2750$

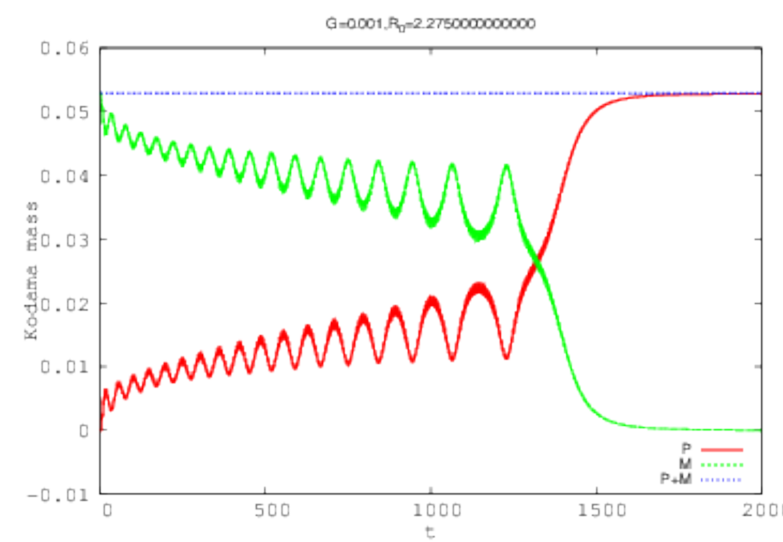




Oscillon

$G = 10^{-3}$
$R_0 = 2.2750$



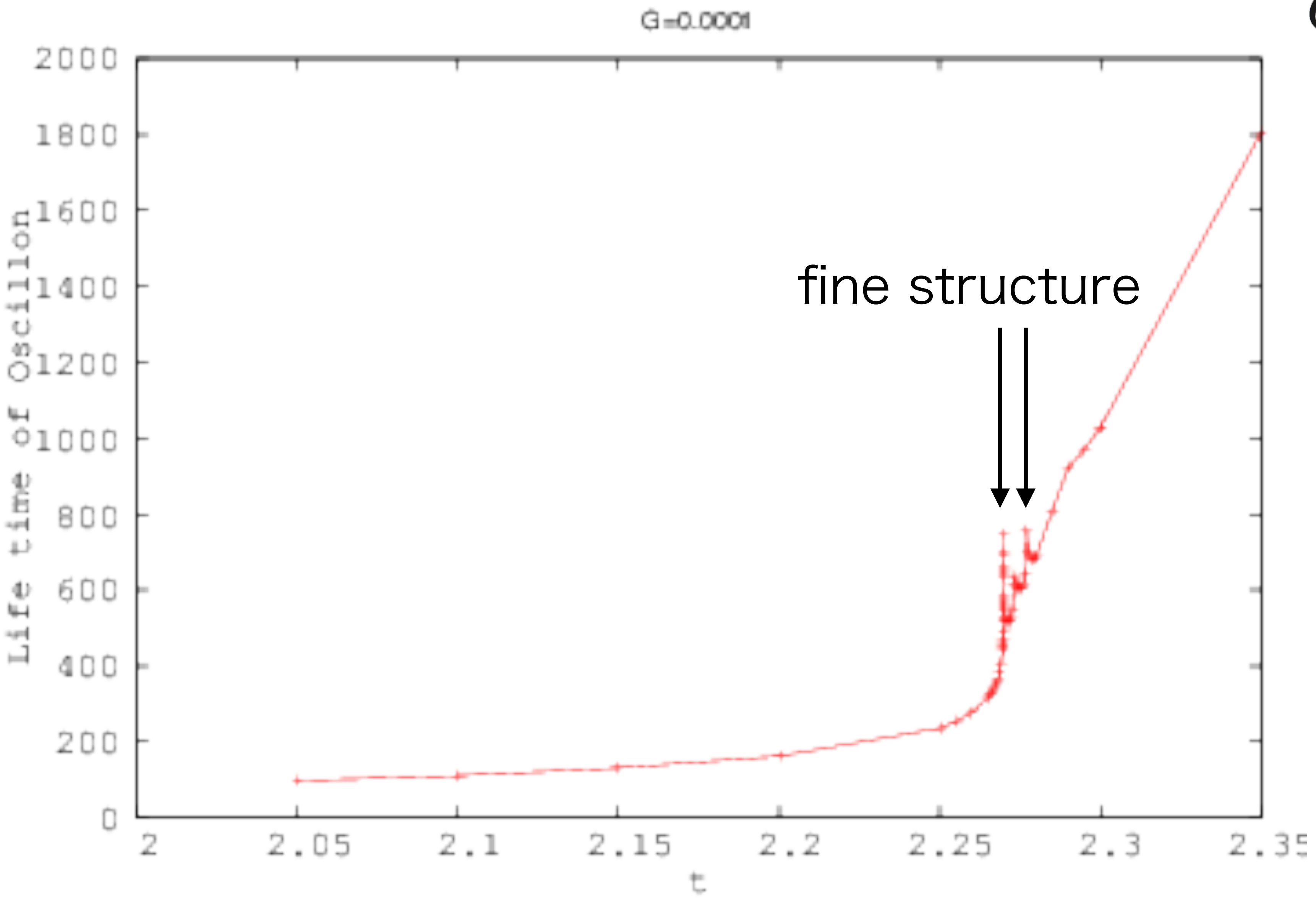


Oscillon

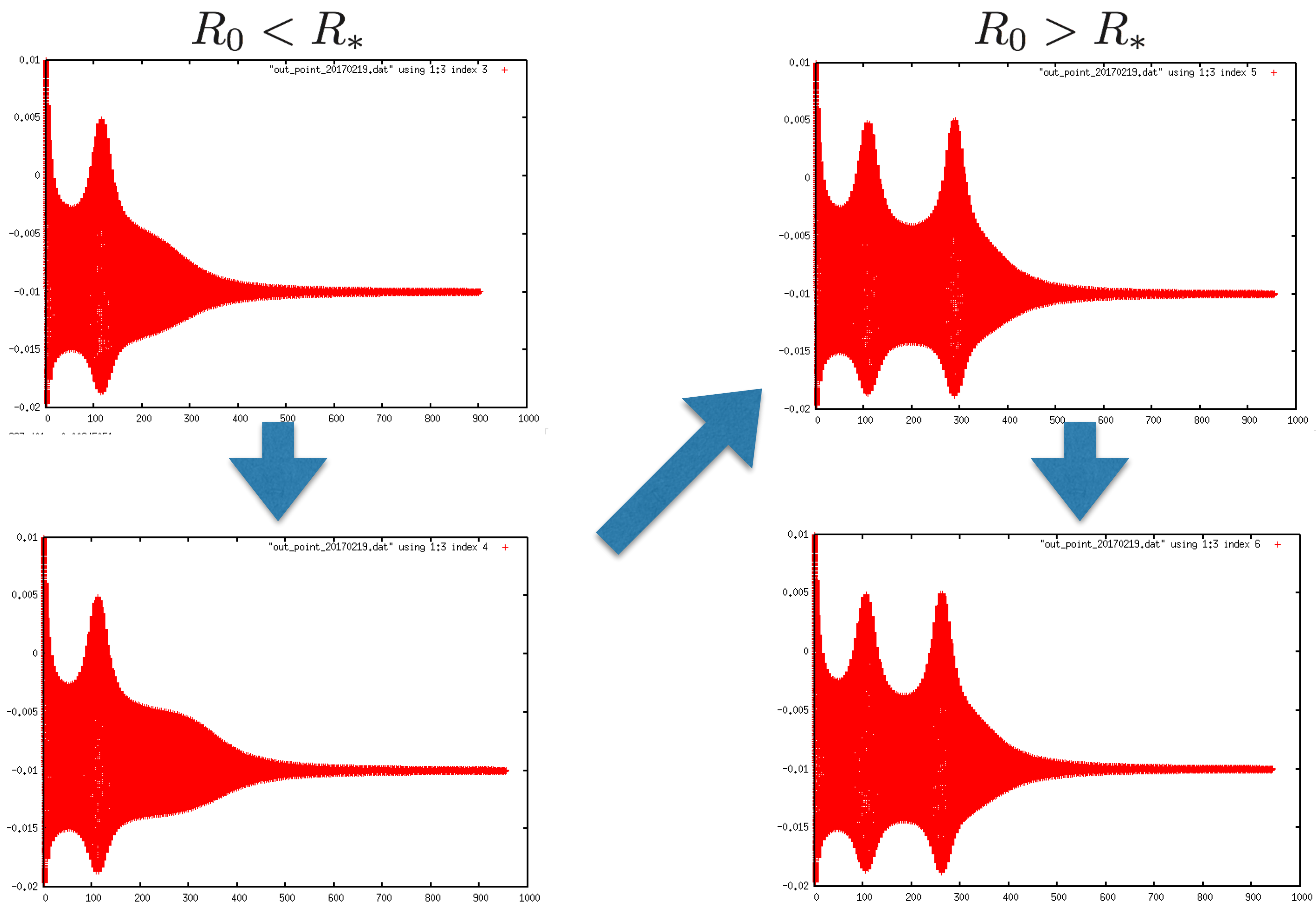
Life time of Oscillon with gravity (preliminary result)

- initial bubble radius vs life time of Oscillon with gravity

$G = 10^{-4}$



Life time of Oscillon with gravity (preliminary result)



Summary and future work

- Summary
  - Oscillon = long lifetime localized solution of scalar filed on Minkwski background
  - It is the product of nonlinear effect of scalar field.
  - We construct the Oscillon with gravity.
  - We get sign of its fine structure in the life time.
- Future work
  - Can Oscillon collapse to black hole ?
  - Can we check the critical behavior ?
  - What is the properties of the critical solution ?