ワームホールと重力レンズ及びその観測可能性

塚本直樹
Huazhong University of Science and Technology
華中科技大学

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NT and Y. Gong, arXiv:1612.08250(accepted in PRD).
NT, Phys. Rev. D **94**,124001 (2016).
NT. and T. Harada, Phys. Rev. D **95**, 024030 (2017).

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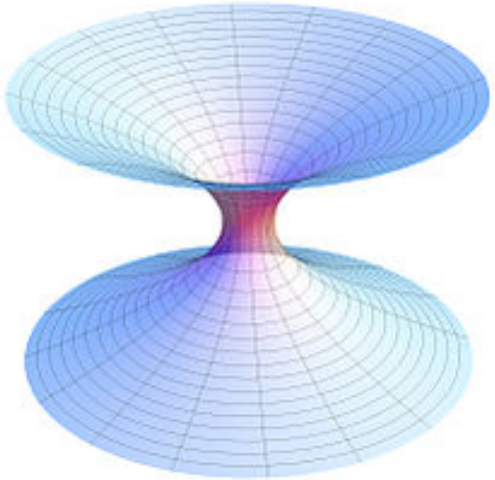
Morris-Thorne Wormhole Morris and Thorne (1988).

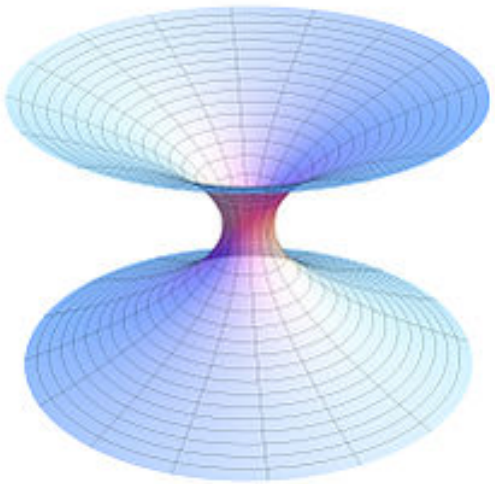
- A static and spherically symmetric WH.
- Two infinities are linked by a throat.
- If we assume general relativity without Λ, exotic matters ($w \equiv p/\rho_e < -1$) are needed to support the throat because of the violation of the null energy condition ($T_{\mu\nu}k^\mu k^\nu \geq 0$).

  (p: pressure, ρ_e: energy density, k^μ:null vector.)



- Cosmological observations (CMB+BAO) imply that our universe may be filled with exotic matter like a phantom scalar field ($w = -1.13 + 0.13 - 0.10$, Planck Collaboration, arXiv:1303.5076).
- If we add some terms to the action, we can make wormhole solutions without violation of the null energy condition.

- **Does our universe have throats?** WH formation is still an open question.
- we will concentrate on methods to find wormholes with gravitational lenses.
- From gravitational lens observations, we gave **the upper bound of number density** n of WH with a throat radius a
- $n \leq 10^{-9}\mathrm{AU}^{-3}$ with $a \sim 1\mathrm{cm}$:
  Fermi Gamma-ray Burst Monitor (Yoo, Harada, NT, 2013)
- $n \leq 10^{-4}h^3\mathrm{Mpc}^{-3}$ with $a \sim 10^{1-4}\mathrm{pc}$:
  Sloan Digital Sky Survey Quasar Lens Search(Takahashi and Asada, 2013).



Ellis wormhole Ellis 1973 and Bronikov 1973.

- The earliest and simplest example of Morris-Thorne class.

By solving the Einstein equations and the wave equation with respect to a phantom scalar field $\chi(r)$

$$R_{\mu\nu} - \frac{1}{2}R^{\lambda}{}_{\lambda}g_{\mu\nu} = -2\left(\chi(r)_{;\mu}\chi(r)_{;\nu} - \frac{1}{2}\chi(r)^{;\lambda}\chi(r)_{;\lambda}g_{\mu\nu}\right), \quad \chi(r)^{;\mu}{}_{;\mu} = 0$$

with the boundary condition $\lim_{r\to\infty}\chi(r) = 0$, we obtain a static and spherical wormhole solution as

$$\begin{aligned} ds^2 &= \frac{r^2+a^2-m^2}{\left|r^2+a^2-m^2\right|}\left\{-e^{-\frac{2m\chi(r)}{a}}dt^2 + e^{\frac{2m\chi(r)}{a}}\left[dr^2 + \left(r^2+a^2-m^2\right)d\Omega^2\right]\right\}, \\ \chi(r) &= \frac{a}{\sqrt{a^2-m^2}}\left[\frac{\pi}{2} - \arctan\frac{r}{\sqrt{a^2-m^2}}\right]. \end{aligned}$$

- As $r \to \infty$, it is asymptotic to Schwarzschild spacetime with the mass m.
  Its gravitational lens effects under the weak-field approximation are same as Schwarzschild lens.

- As $r \to -\infty$,

$$ds^2 = -\left(1 - \frac{2me^{\frac{m\pi}{a}}}{r}\right) d\left(e^{-\frac{m\pi}{a}} t\right)^2 + \frac{dr^2}{1 - \frac{2me^{\frac{m\pi}{a}}}{r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

  It is asymptotic to Schwarzschild spacetime with a negative mass $-me^{\frac{m\pi}{a}}$. **This is an example of so-called natural wormhole.**

- For $m = 0$, it becomes the **so-called Ellis wormhole**.

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2\theta d\phi^2),$$

Ellis wormhole Ellis 1973, Bronnikov 1973.

$$ds^2 = -dt + dr^2 + (r^2 + a^2)\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

- A throat and a photon sphere are at $r = 0$.
- The upper bound of number density n obtained as
  $n \leq 10^{-4} h^3 \mathrm{Mpc}^{-3}$ with $a \sim 10^{1-4}$pc from Sloan Digital Sky Survey Quasar Lens Search. (Takahashi and Asada, 2013)
  $n \leq 10^{-9} \mathrm{AU}^{-3}$ with $a \sim 1$cm from lensing effect of the gamma-ray bursts. (Yoo, Harada and NT, 2013)

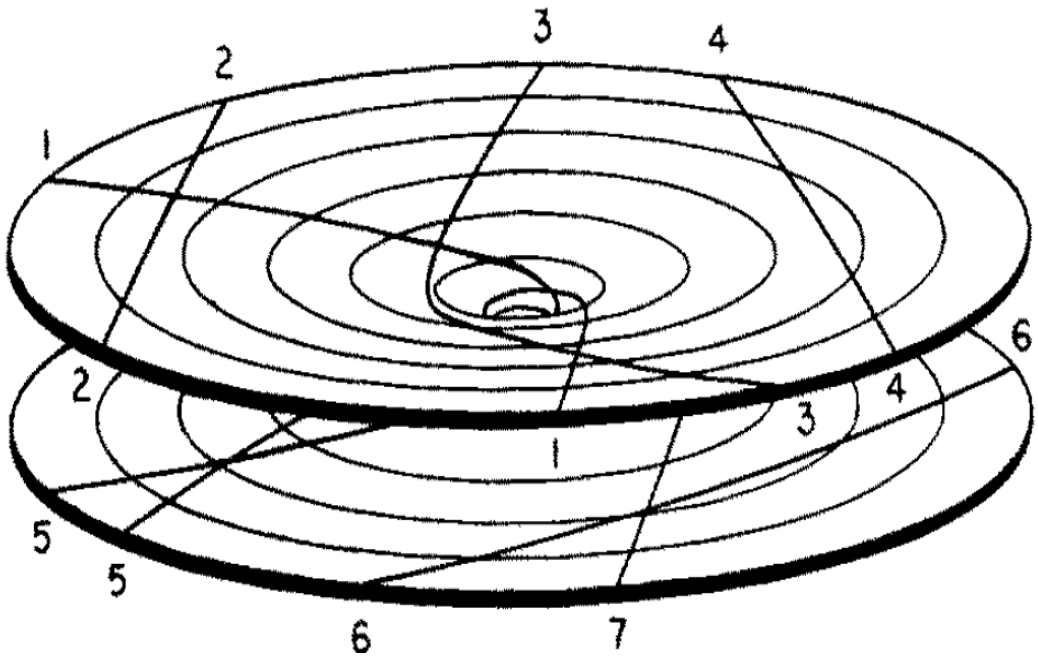
- We often see the wormhole metric without referring Ellis and Bronnikov papers because of Morris and Thorne. James, Tunzelmann, Franklin, and Thorne (2015) say "Fifteen years later, Morris and Thorne wrote down this same metric, among others, and being unaware of Ellis's paper, failed to attribute it to him, for which they apologize. Regretably, it is sometimes called the Morris-Thorne wormhole metric."

The Ellis wormhole is unstable but we may find stable wormholes with the Ellis wormhole metric.

- Some known wormhole solutions have the Ellis wormhole metric as the simplest case.
- Stability of a wormhole depends on the matter.
- Shatskii-Novikov-Kardashev wormhole has the Ellis wormhole metric but with a perfect fluid with negative density and a source-free radial electric or magnetic field is linearly stable under both spherically symmetric perturbations and axial perturbations. (Bronnikov et al. 2013)
  $T^{\nu}_{\mu} = q^2/(8\pi r^4)\mathrm{diag}(1,1,-1,-1) - 2q^2/(8\pi r^4)\mathrm{diag}(1,0,0,0)$.
- It seems to be the first example of stable wormhole without thin shells in General Relativity.

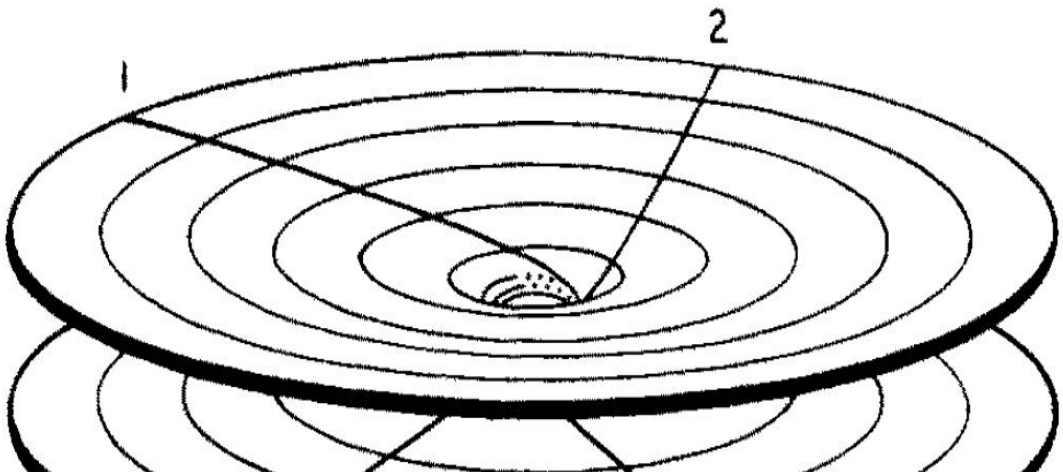
Large impact parameter case. Ellis 1973

- Usual lens configuration and retro lens configuration. (We will consider retro lens later.)



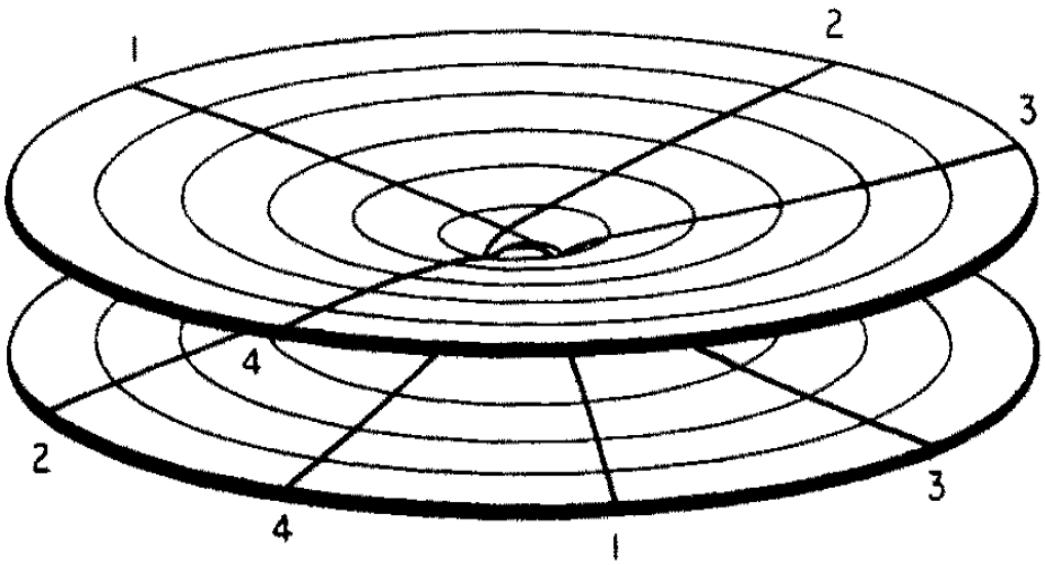
Critical impact parameter case. Ellis 1973

- The throat is the photon sphere.
- 喉に光が巻き付く

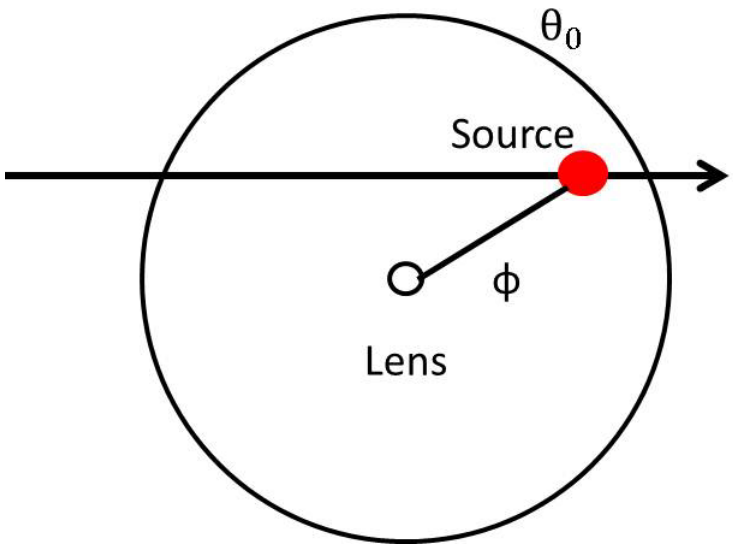


Small impact parameter case. Ellis 1973

- 喉に落ちる



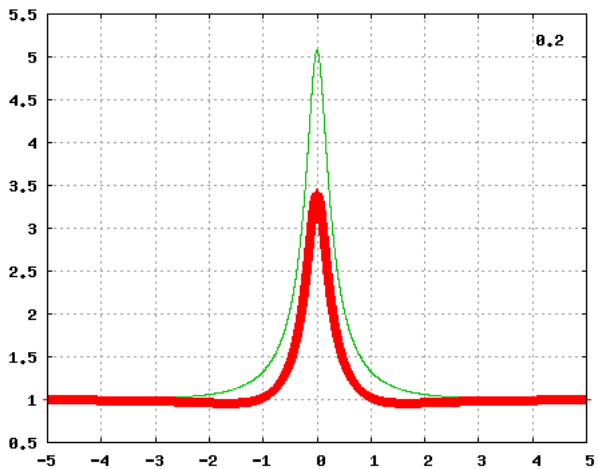
Microlens with large impact parameter. Abe, 2010.

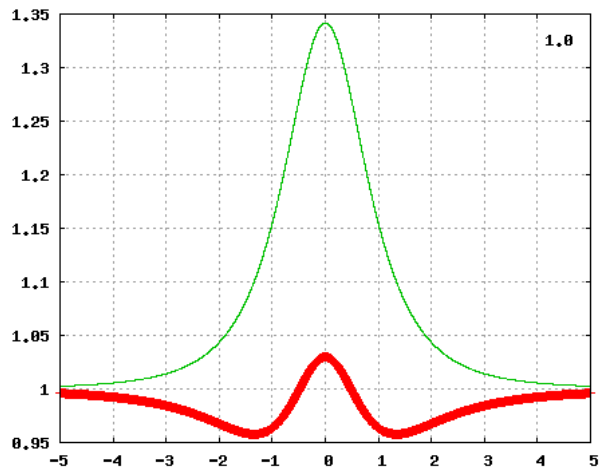


- A source go across near a lens object on the lens plane.
- The light curves have characteristic shapes.

Green :Mass lens

Red :Ellis WH





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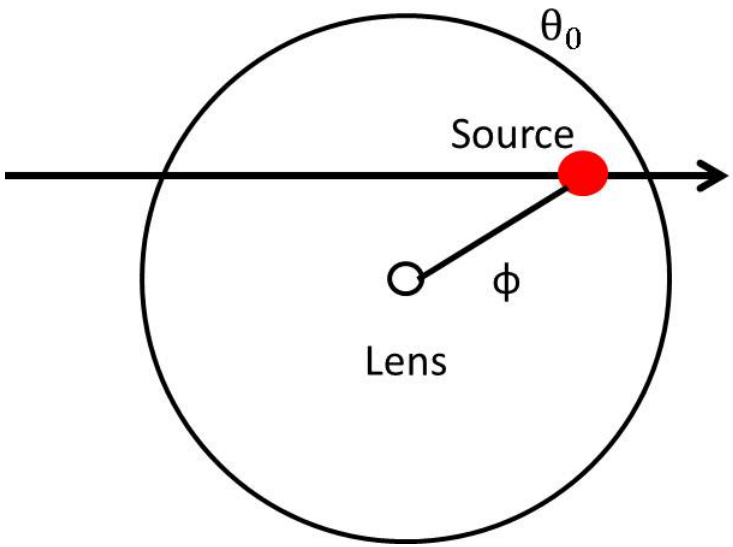
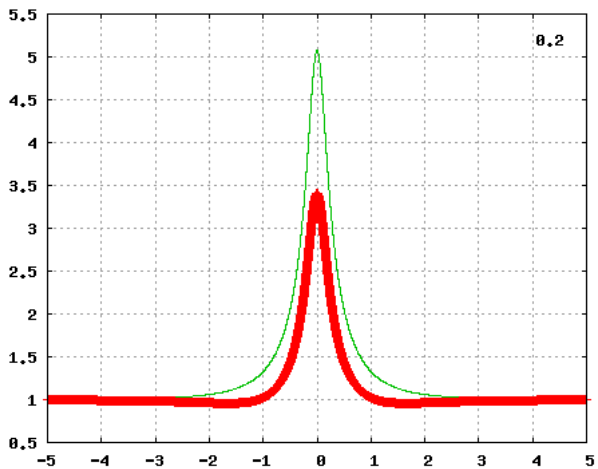
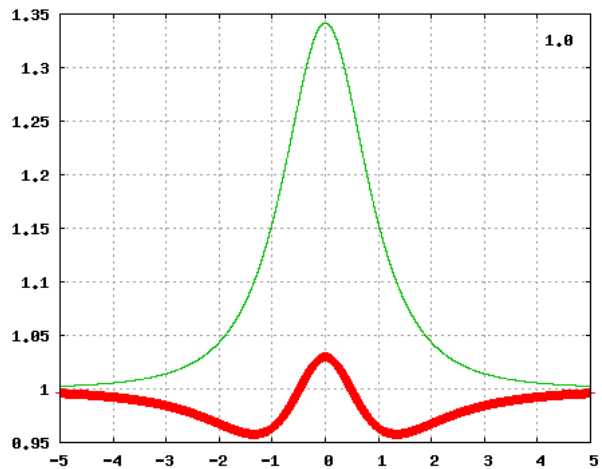


- Ellis WH in or near our galaxy can be found with its characteristic demagnification.
- Light curve for Mass lens is always magnified.

Green :Mass lens

Red :Ellis WH

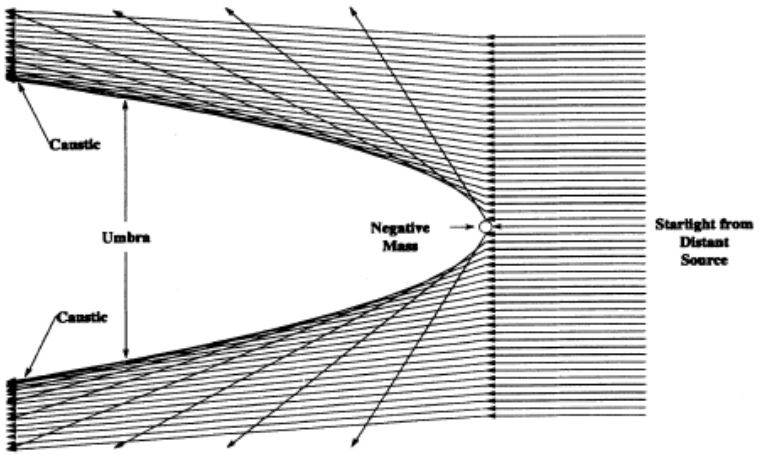


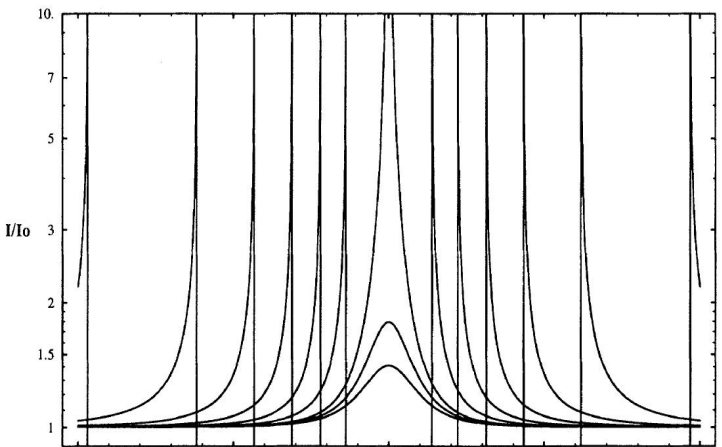


負の質量のワームホールのマイクロレンズ Cramer et al. (1995)

number density $n \leq 10^{-8}(10^{-4})h^3\mathrm{Mpc}^{-3}$ with $|M| > 10^{15}(10^{12})M_\odot$

SDSS Quasar Lens Search, Takahashi and Asada, 2013.

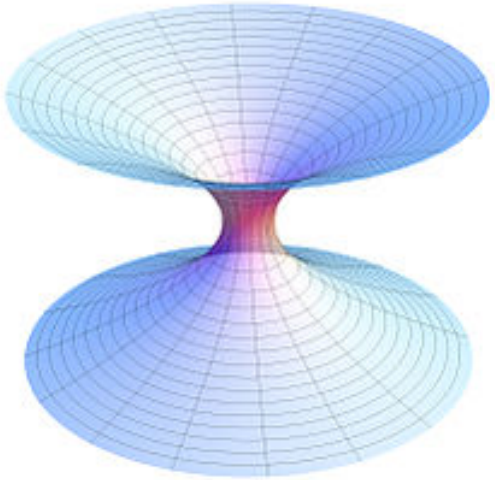




喉を通る光の光度曲線

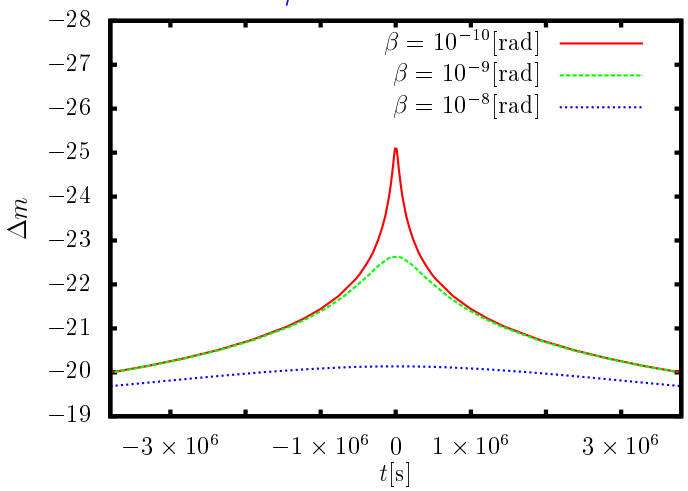
NT and Harada 2017.

- 弱重力場の観測では、質量が負、ゼロ、正の何かがあると言えるだけで、ワームホールだとはいえない。
- ワームホールの特徴に関わる観測量を考えれば、ワームホールと他のレンズを区別できるかもしれない。
- ワームホールの特徴って？　photon sphereを持つという意味で、強い重力場を持つけれども、イベントホライズンはない、喉がある、測地的完備、、、
- 喉を通る光の光度曲線by using the exact lens equation (See Perlick 2004).
- 近似されたレンズ方程式は知られていないので、　Perlick 2004のexact lens equationを数値的に解く。
- 簡単なEllis wormholeについて解くけれども、正の質量の通行可能なワームホールにも簡単に適応可能。

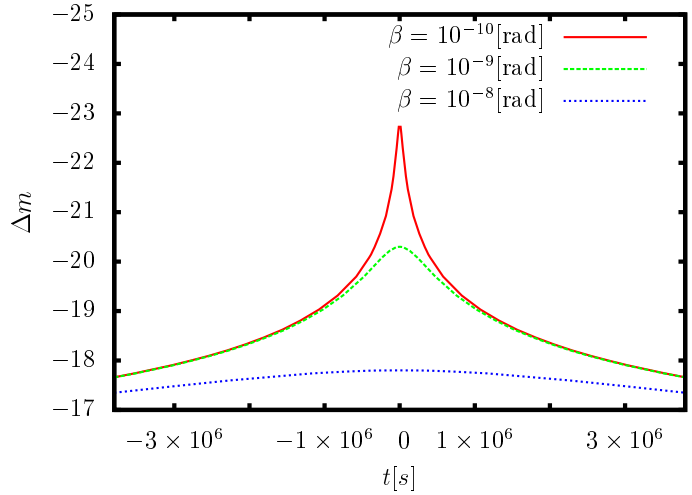


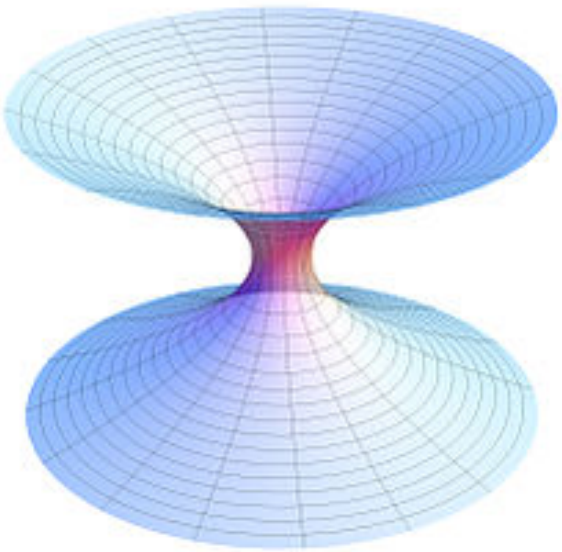
喉を通る光による光度曲線NT and Harada 2017.

$\phi \sim \pi$



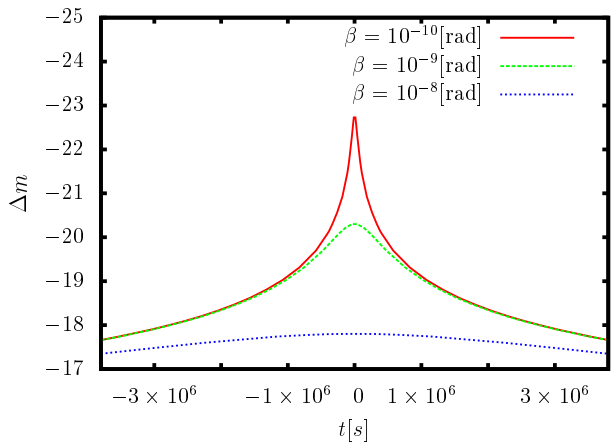
$\phi \sim 0$

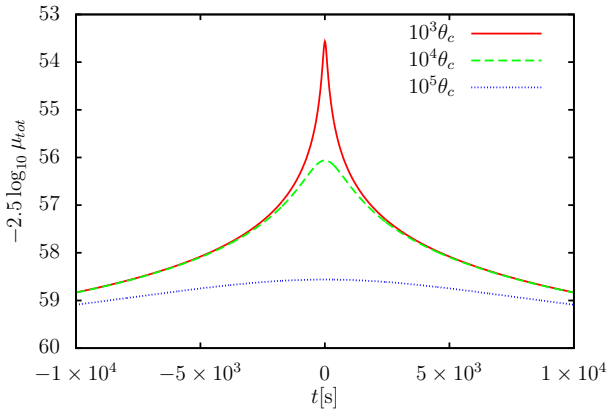


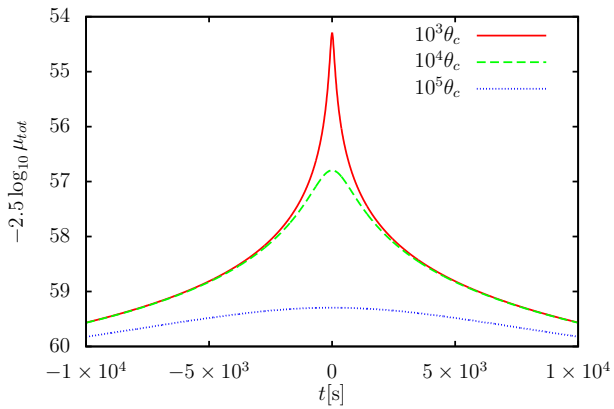


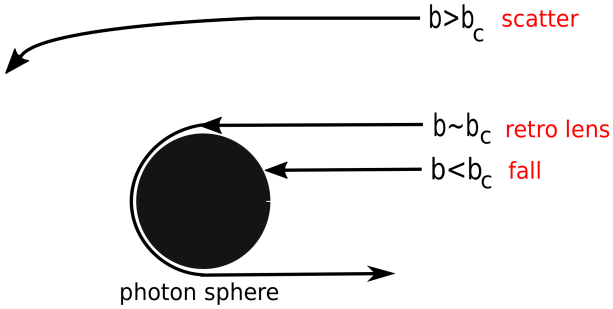
喉を通らないけれども**photon sphere**に反射された光の光度曲線（レトロレンズ）と比較

ワームホール、喉を通る　　ブラックホール　　ワームホール、喉を通らない









喉を通る光の内、レンズ効果で増光されて光度曲線に寄与するものは、**photon sphere**に巻き付きた光である。形が似ているので、区別するのは難しそう。

エリスワームホールの曲がり角α

$$ds^2 = -dt + dr^2 + (r^2 + a^2)\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

$$\alpha = 2K(a/b) - \pi, \text{ Chetouani and Clement(1984)}$$

where $K(k)$ is the complete elliptic integral of the first kind,

$$K(k) \equiv \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}. \tag{2}$$

Using *Higher Transcendental Functions* edited by A. Erdelyi, Vol. II, α in the strong deflection limit $b \to b_c \equiv a$ is given by

$$\alpha(b) = -\log\left(\frac{b}{b_c} - 1\right) + 3\log 2 - \pi + O((b-b_c)\log(b-b_c)). \text{ NT PRD(2016)}$$

Nandi et al. (2006)の結果と合わない。

$$\alpha(b) = -\log\left(\frac{1}{4} - \sqrt{\frac{b^2}{b_c^2} - 1}\right) + 2\log 2 - \pi + O((b-b_c)).$$

Strong deflection limit $b \to b_c$ Bozza 2002.

Deflection angle α is given by a following form:

$$\alpha(b) = -\bar{a} \log\left(\frac{b}{b_c} - 1\right) + \bar{b} + O\ (b - b_c)$$

- For Schwarzschild BH, $\bar{a} = 1$, $\bar{b} = \log[216(7 - 4\sqrt{3})] - \pi$. Darwin (1959)

- Bozza claimed that $\bar{b}$ of Reissner-Nordström BH cannot be obtained analytically.

Strong deflection limit $b \to b_c$ Bozza 2002. NT 2016.

Deflection angle α is given by a following form:

$$\alpha(b) = -\bar{a}\log\left(\frac{b}{b_c} - 1\right) + \bar{b} + O((b - b_c)\log(b - b_c)).$$

- For Schwarzschild BH, $\bar{a} = 1$, $\bar{b} = \log[216(7 - 4\sqrt{3})] - \pi$. Darwin (1959)
  $-5\sqrt{3}/162(b - b_c)\log(b - b_c)$. Iyer and Petters (2007)
- Bozza claimed that $\bar{b}$ of Reissner-Nordström BH cannot be obtained analytically. However, it can be obtained in a straightforward way. NT and Gong (2016)

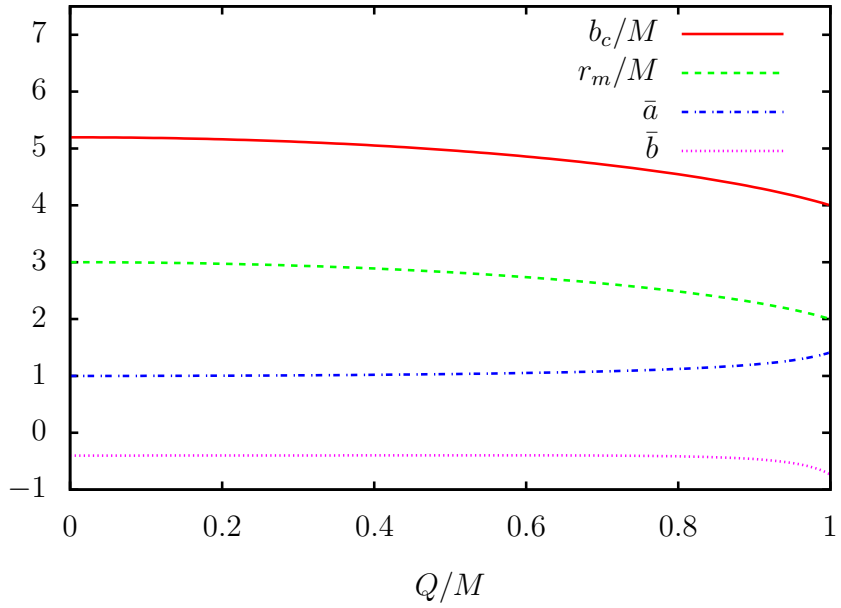
Bozza's formalism does not work in ultrastatic spacetimes (g_{tt} =const).
Ellis wormholeの場合もBozzaの方法を少し変えると計算でき、完全楕円積分と公式集を使った場合と一致した。

$\bar{a}$ and $\bar{b}$ of RN BH NT and Gong (2016)

$$\bar{b} = 2\bar{a}\log\frac{2^{\frac{3}{2}}(3Mr_m - 4Q^2)^{\frac{3}{2}}\left(2\sqrt{Mr_m - Q^2} - \sqrt{3Mr_m - 4Q^2}\right)}{Mr_m(Mr_m - Q^2)} - \pi.$$

$$\bar{a} = r_m/\sqrt{3Mr_m - 4Q^2}\ .\ \ r_m = \left(3M + \sqrt{9M^2 - 8Q^2}\right)/2.$$

$$b_c = r_m^2/\left(\sqrt{r_m^2 - 2Mr_m + Q^2}\right).$$



Eiroa et al. (2002) と Bozza (2002) の数値計算の表と一致する。

Deflection angnle in the strong deflection limit $b \to b_c$ or $r_0 \to r_m$ NT (2016)

$$\alpha(b) = -\bar{a}\log\left(\frac{b}{b_c} - 1\right) + \bar{b} + O((b - b_c)\log(b - b_c)),$$

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2, \lim_{r\to\infty} A(r), B(r) \to 1, C(r) \to r^2.$$

$$\alpha(r_0) = I(r_0) - \pi, \qquad I(r_0) \equiv 2\int_{r_0}^{\infty} \sqrt{\frac{B}{\left(\frac{A_0 C}{A C_0} - 1\right) C}} dr.$$

$$I(r_0) = I_D(r_0) + I_R(r_0)$$

$$\begin{aligned} I_D(b) &= -\frac{r_m}{\sqrt{c_2(r_m)}}\log\left(\frac{b}{b_c} - 1\right) + \frac{r_m}{\sqrt{c_2(r_m)}}\log r_m^2\left(\frac{C_m''}{C_m} - \frac{A_m''}{A_m}\right) \\ &\quad + O((b - b_c)\log(b - b_c)). \end{aligned}$$

$$c_2(r_m) = \frac{C_m r_m^2}{2B_m}\left(\frac{C_m''}{C_m} - \frac{A_m''}{A_m}\right).$$

Deflection angnle in the strong deflection limit $b \to b_c$ or $r_0 \to r_m$ NT (2016)

$$\alpha(b) = -\bar{a}\log\left(\frac{b}{b_c} - 1\right) + \bar{b} + O((b - b_c)\log(b - b_c))$$

$$\bar{a} = \sqrt{\frac{2B_m A_m}{C_m'' A_m - C_m A_m''}}$$

$$\bar{b} = \bar{a}\log\left[r_m^2\left(\frac{C_m''}{C_m} - \frac{A_m''}{A_m}\right)\right] + I_R(r_m) - \pi,$$

Bozza (2002)と比べて、**ultrastatic spaticetime**で破綻しない。$I_R(r_m)$の積分が一般的に単純。**Reissner-Nordström BH**の場合も解析的に求まる。

Deflection angnle in the strong deflection limit $b \to b_c$ or $r_0 \to r_m$ NT (2016)

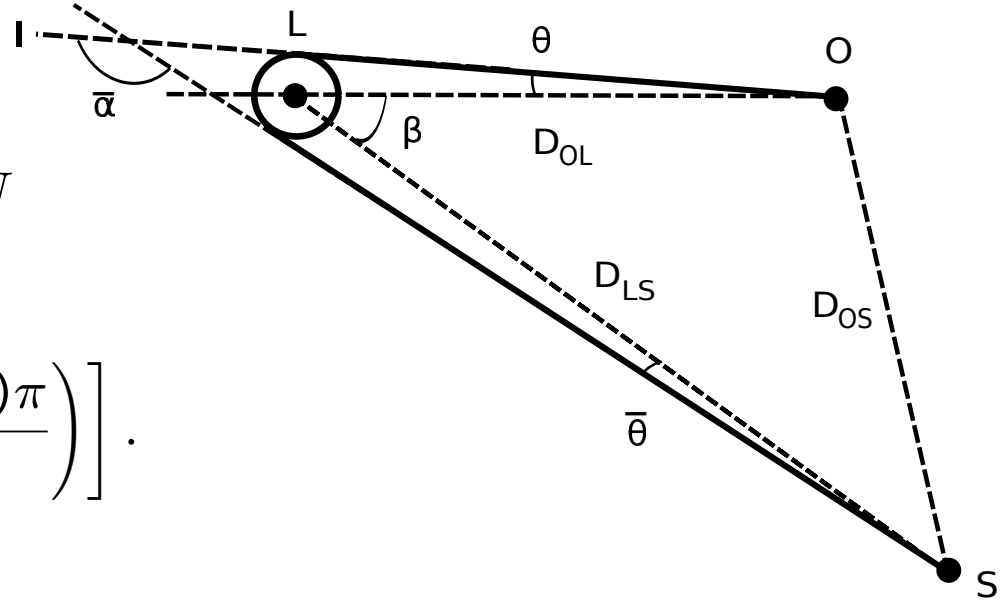
$$\alpha(b) = -\bar{a}\log\left(\frac{b}{b_c} - 1\right) + \bar{b} + O((b - b_c)\log(b - b_c))$$

$$\bar{a} = \sqrt{\frac{2B_m A_m}{C_m'' A_m - C_m A_m''}}$$

$$\bar{b} = \bar{a}\log\left[r_m^2\left(\frac{C_m''}{C_m} - \frac{A_m''}{A_m}\right)\right] + I_R(r_m) - \pi,$$

Bozza (2002)と比べて、**ultrastatic spaticetime**で破綻しない。$I_R(r_m)$の積分が一般的に単純。**Reissner-Nordström BH**の場合も解析的に求まる。ある時空が与えられたときに、**Bozza (2002)**や**NT (2016)**の結果を使うのではなく、自分で考えて、上手く計算することが大切です。

レトロレンズ Holz and Wheeler 2002, Bozza and Mancini 2004.



$$\beta = \pi - \bar{\alpha}(\theta) + \theta + \bar{\theta}, \qquad \bar{\alpha} = \alpha - 2\pi N$$

N は光の巻き数。$\beta \sim 0$ の時、

$$\theta_N = \pm \frac{b_c}{D_{OL}} \left[1 + \exp\left(\frac{\bar{b} - (1+2N)\pi}{\bar{a}} \right) \right].$$

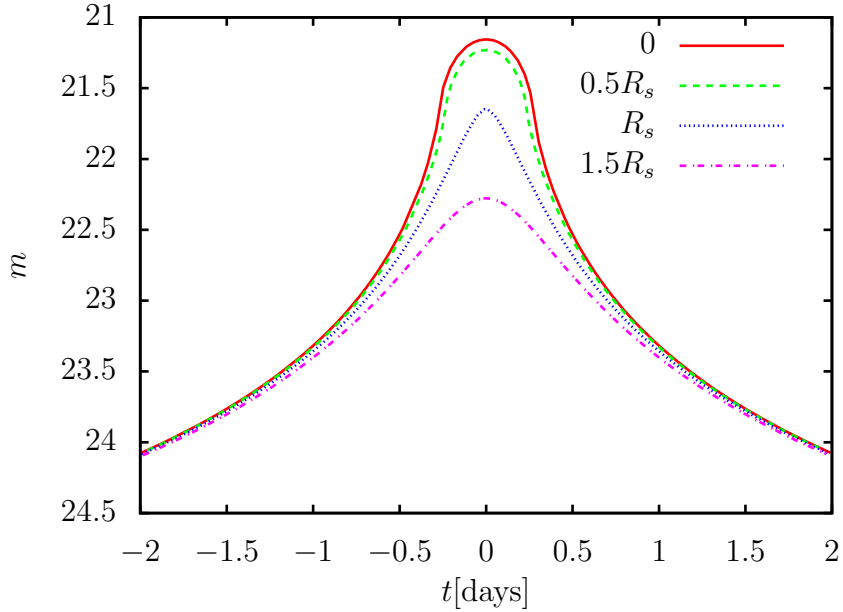
The total magnification is

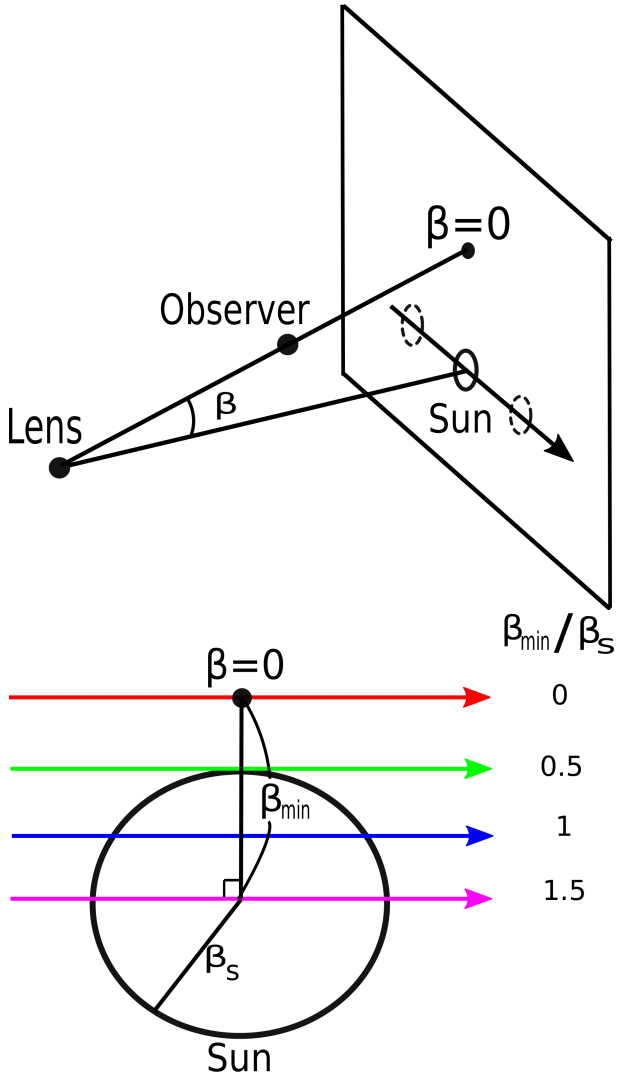
$$\mu_{totN} = 2 \frac{D_{OS}^2}{D_{OL}^2} \frac{b_c^2 e^{(\bar{b}-(1+2N)\pi)/\bar{a}} \left[1 + e^{(\bar{b}-(1+2N)\pi)/\bar{a}} \right]}{\pi \bar{a} R_s^2} \left| \int_{Disk} \frac{\beta' d\beta' d\phi}{\sin(\pi - \beta')} \right|,$$

where R_s is the radius of Sun and we have regarded Sun as an uniform-luminous disk.

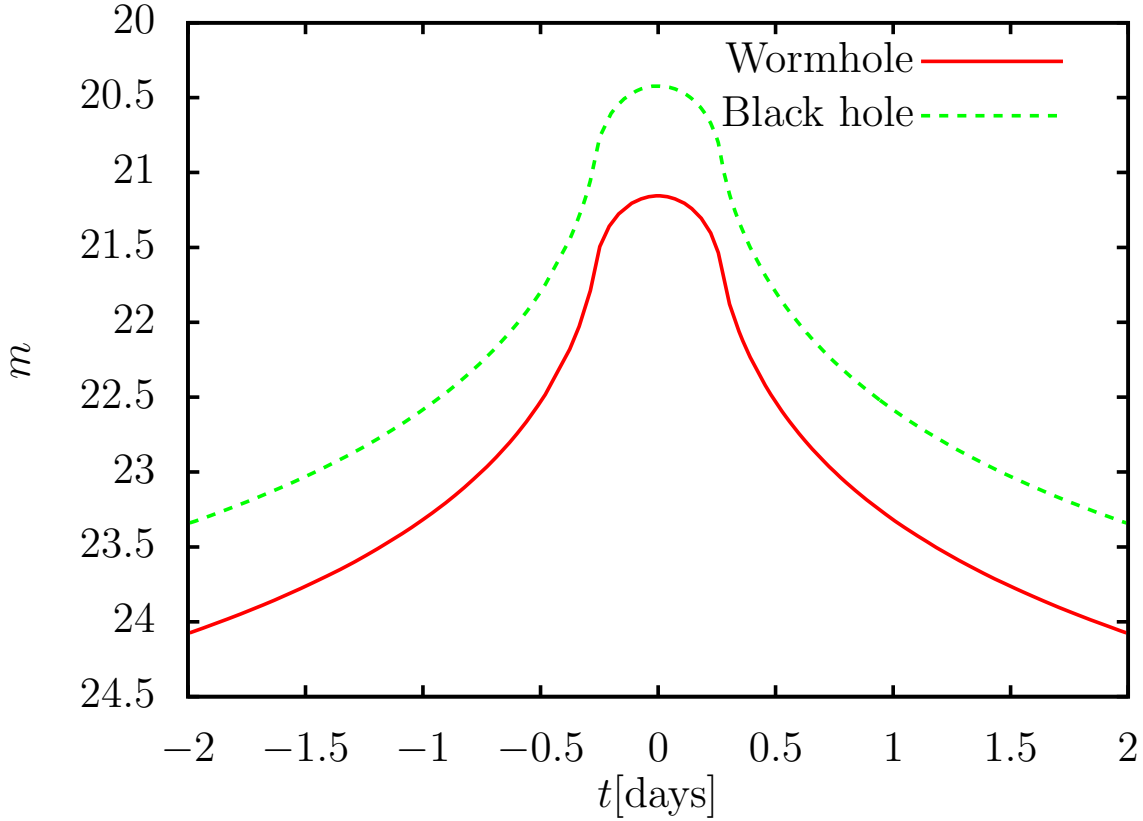
From the shape of the peak, we can estimate β_{min}.

- $a = 10^6$km at $D_{OL} = 1$pc
- the orbital velocity of Sun.





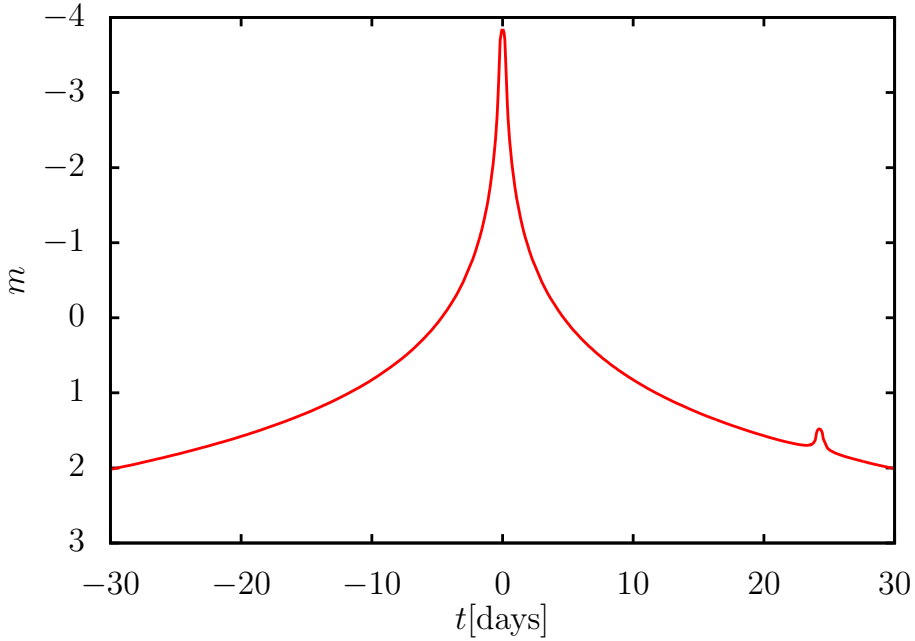
光源が広がっていても、**photon sphrere**に散乱される光の光度曲線の形はやっぱり似ている



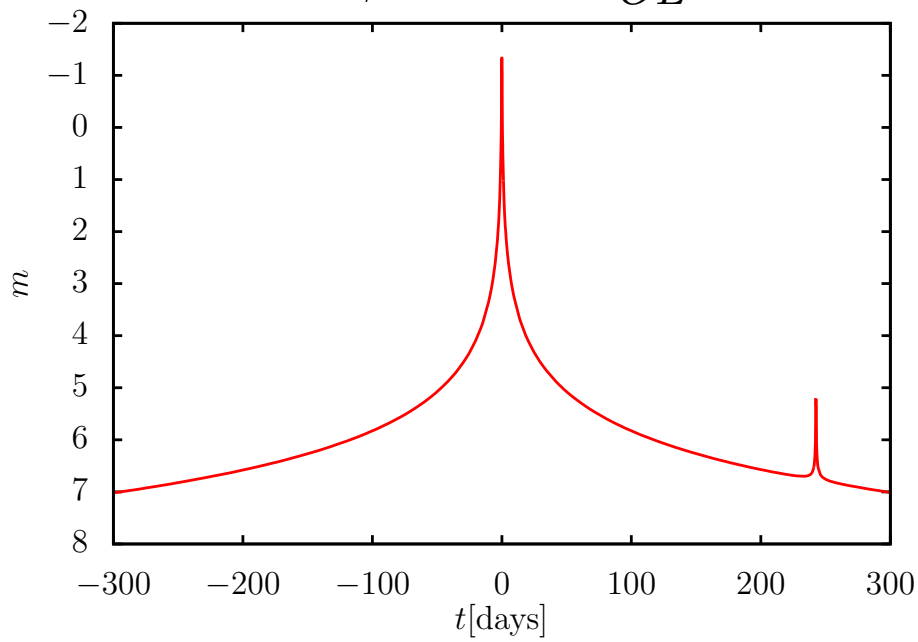
$b_c = 10^6$km and $\beta_m = 0$ at $D_{OL} = 1$pc.

光度曲線の二つ目のピーク

$a = 10^{11}$km, $\beta_m = 0$, $D_{OL} = 1$pc.



$a = 10^{12}$km, $\beta_m = 0$,$D_{OL} = 10$pc.



- Unknown parameters are β_{min}, D_{OL}, and a
- ピークの形、$m_{N=0}$, $m_{N=1}$, $\Delta t = 2\pi a$,

議論とまとめ

- 重力レンズ効果でワームホールを探すことができる。
- 観測データから個数密度への制限をつけられる。例えば、マイクロレンズでの原始ブラックホールへの制限をワームホールへの制限に焼き直せる。
- ワームホールの喉に注目するとワームホールと他の天体と区別できるかもしれない。
- モリス・ソーンのワームホールの喉はphoton sphere
- photon sphereに散乱された光の性質は、似ているようだ。
- photon sphereに散乱された光の重力レンズ効果の詳細を調べることで、何か手がかりが得られるかもしれない。