



Constraining inflation models from primordial perturbations

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1. Introduction

Inflation

Inflation- extremely rapid expansion of the early universe

- Solving problems of big-bang cosmology

 - Flatness problem

 - Horizon problem

 - Unwanted relics

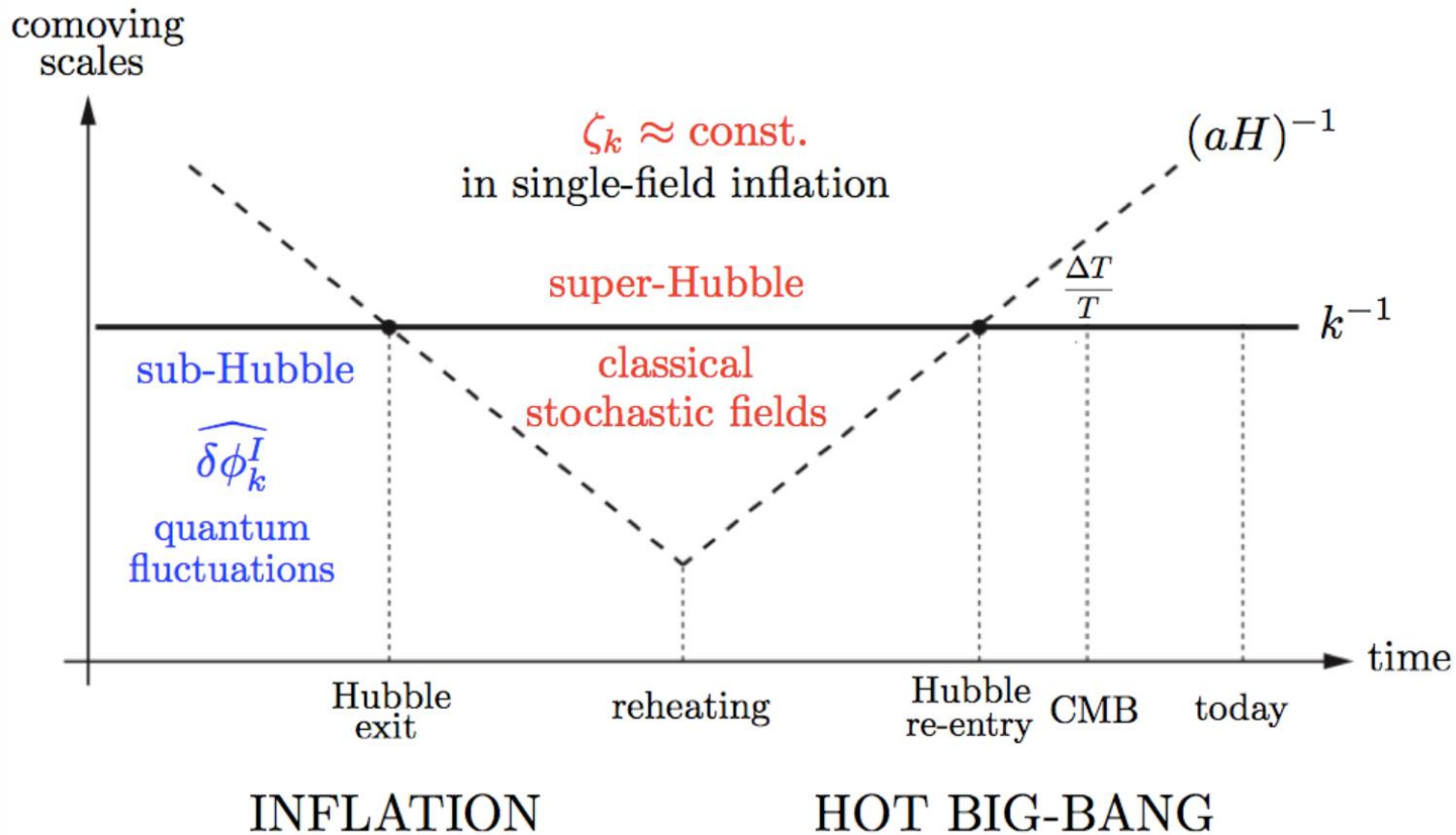
- Providing origin of the structures in the Universe

Almost scale invariant, adiabatic and Gaussian
primordial density fluctuations



Consistent with current observations (CMB, LSS, etc)

Generation of primordial perturbations



Sub-Hubble quantum fluctuations $\delta\phi_k$ $\longrightarrow$ Super-Hubble classical perturbations ζ_k

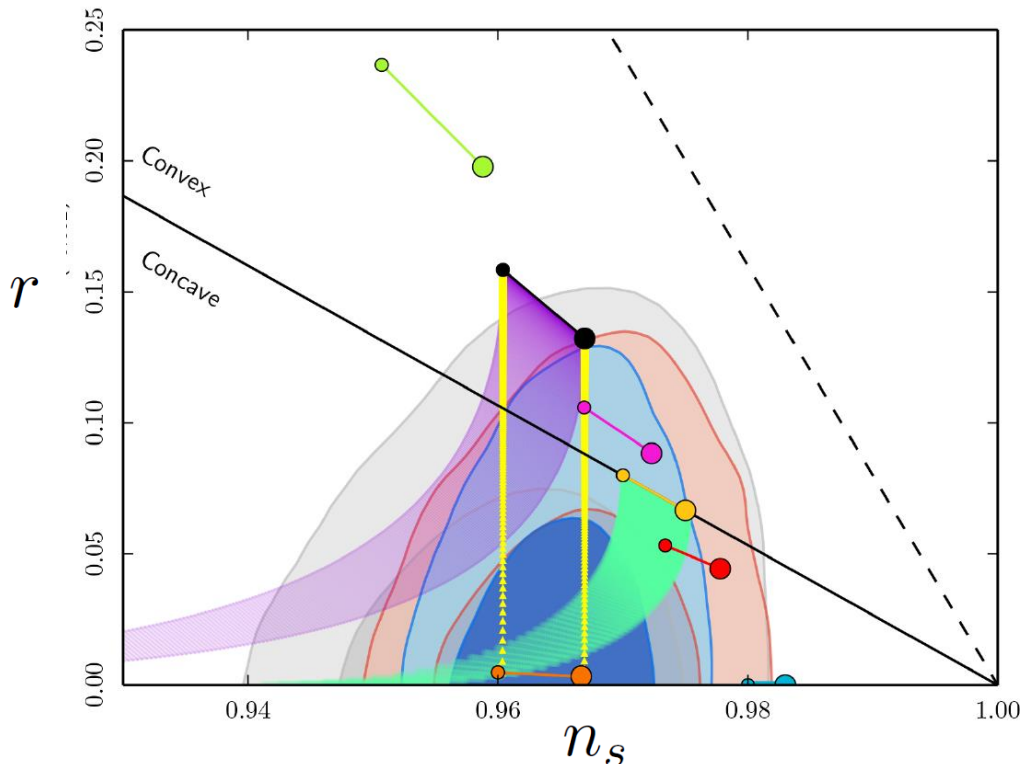
Constraints from primordial power spectrum

- Primordial power spectrum

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') P_\zeta(k)$$

well approximated by $\frac{k^3}{2\pi^2} P_\zeta(k) = A_s(k_\star) \left(\frac{k}{k_\star} \right)^{n_s-1}$ $k_\star = 0.05 \text{Mpc}^{-1}$

- Constraints from Planck Ade et al `16



For standard single-field
slow-roll inflation models

$$n_s - 1 \simeq -2\epsilon - \eta$$

$$r \equiv \frac{P_h}{P_\zeta} = 16\epsilon$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \quad \eta \equiv \frac{\ddot{\epsilon}}{H\epsilon}$$

Primordial bispectrum

- Primordial bispectrum

dimensionless function

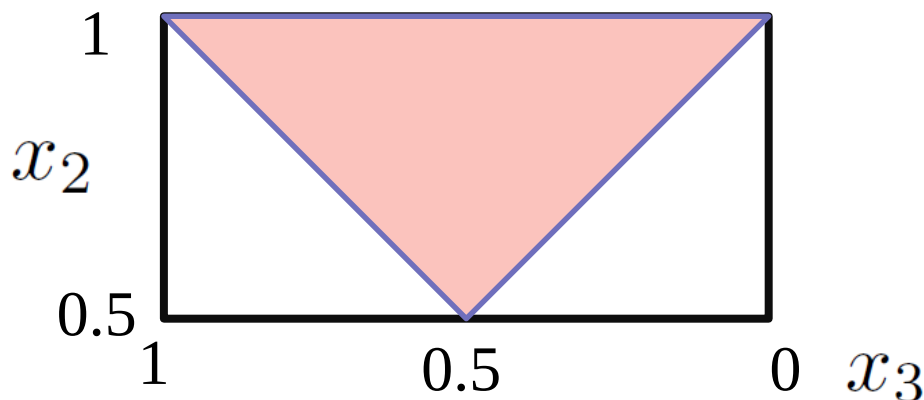
$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta(\sum_i \mathbf{k}_i) \left[(2\pi)^4 \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2} A_s^2 \right]$$

wavevectors $\mathbf{k}_i$ form a tetrahedron in Fourier space

S has information of amplitude as well as shape-dependence

- Shape-dependence of bispectrum

In most models, S depends only on $x_2 \equiv k_2/k_1$ and $x_3 \equiv k_3/k_1$



For $k_3 \leq k_2 \leq k_1$

Allowed region is

$$1 \geq x_2 \geq x_3$$

$$1 \leq x_2 + x_3$$



2. Primordial Non-Gaussianity (PNG)

δN -formalism

Sasaki, Stewart '96, Sasaki, Tanaka '98

• Curvature perturbation

$$\zeta = \Psi - \frac{\delta\rho}{3(\rho + P)}$$

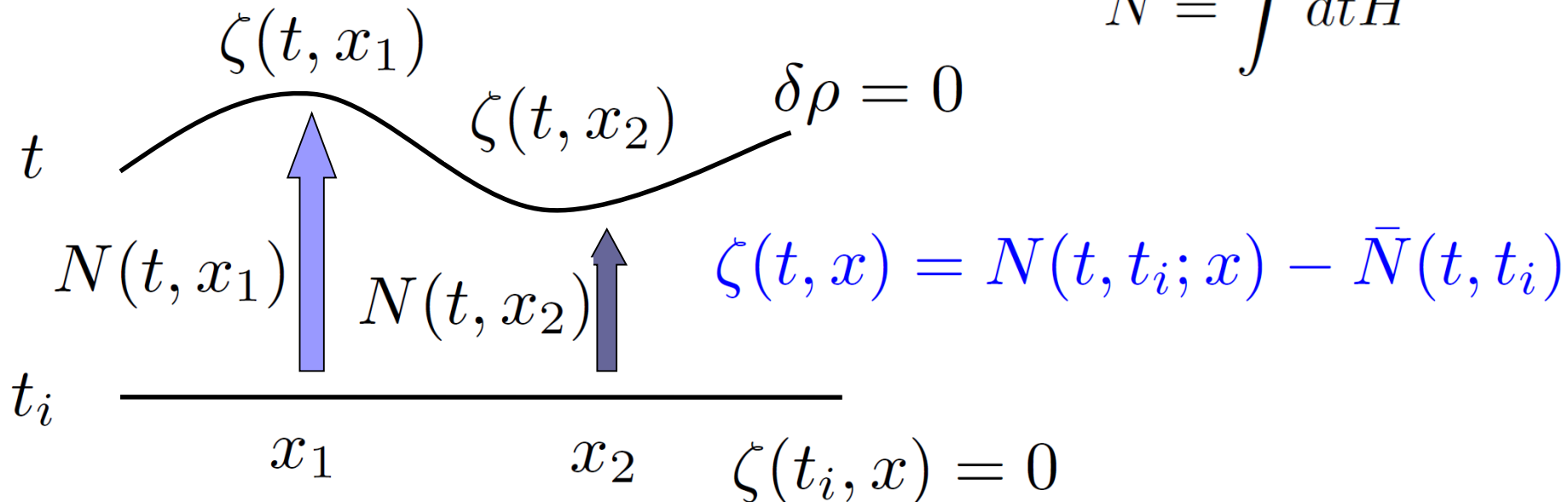
gauge invariant

$$^{(3)}R = \frac{4}{a^2} \nabla^2 \Psi$$

Curvature perturbations on superhorizon scales

= fluctuations in local e-folding number

$$N = \int dt H$$



Origins of primordial non-Gaussianity

Lyth, Rodriguez '05

• Assumption

$$\left\{ \begin{array}{l} N(t, t_i; x) = N(t, \phi^A(t_i, x)) \quad A = 1, 2, \dots \\ \text{ex.) attractor solutions of multi-field inflation} \\ \phi_i^A = \bar{\phi}_i^A + Q_i^A \end{array} \right.$$

→ $\zeta = N_A Q^A + \frac{1}{2} N_{AB} Q^A Q^B \quad N_A \equiv \left. \frac{\partial N}{\partial \phi_i^A} \right|_{\bar{\phi}_i^A} \quad N_{AB} \equiv \left. \frac{\partial^2 N}{\partial \phi_i^A \partial \phi_i^B} \right|_{\bar{\phi}_i^A}$

• Primordial bispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = N_A N_B N_C \langle Q_{\mathbf{k}_1}^A Q_{\mathbf{k}_2}^B Q_{\mathbf{k}_3}^C \rangle$$

Intrinsic bispectrum of field fluctuations

$$+ N_A N_B N_{CD} \left(\langle Q_{\mathbf{k}_1}^A Q_{\mathbf{k}'_1}^C \rangle \langle Q_{\mathbf{k}_2}^B Q_{\mathbf{k}'_2}^D \rangle + \text{perms.} \right)$$

Non-linear relation between Q_i^A and ζ

Multi-field models

Cases with effectively light canonical scalar fields

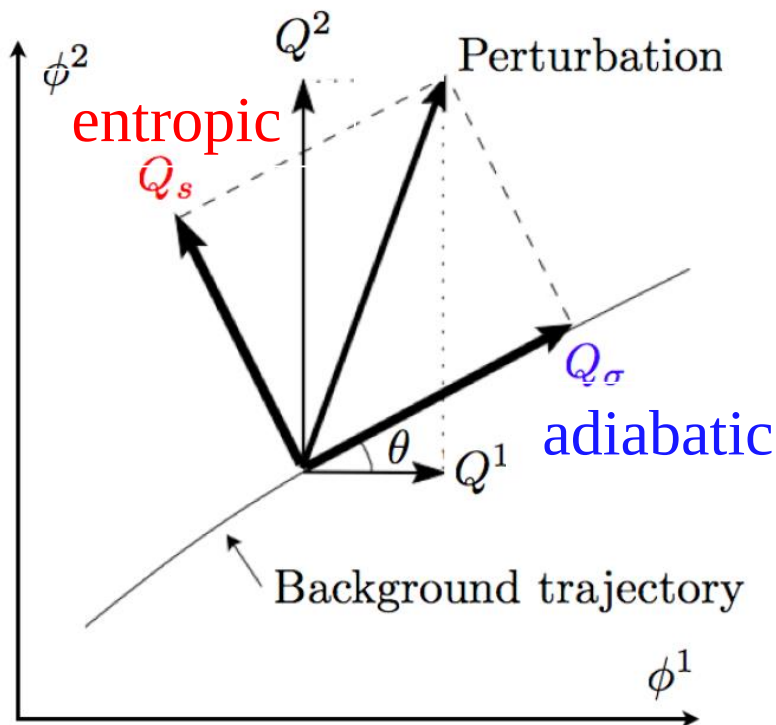
- Power spectra soon after Hubble crossing

Sasaki, Stewart '96

$$\langle Q_{\mathbf{k}}^A Q_{\mathbf{k}'}^B \rangle_* = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{H_*^2}{2k^3} \delta^{AB}$$

- Super-Hubble evolution of ζ

Gordon, Wands, Bassett, Maartens '01



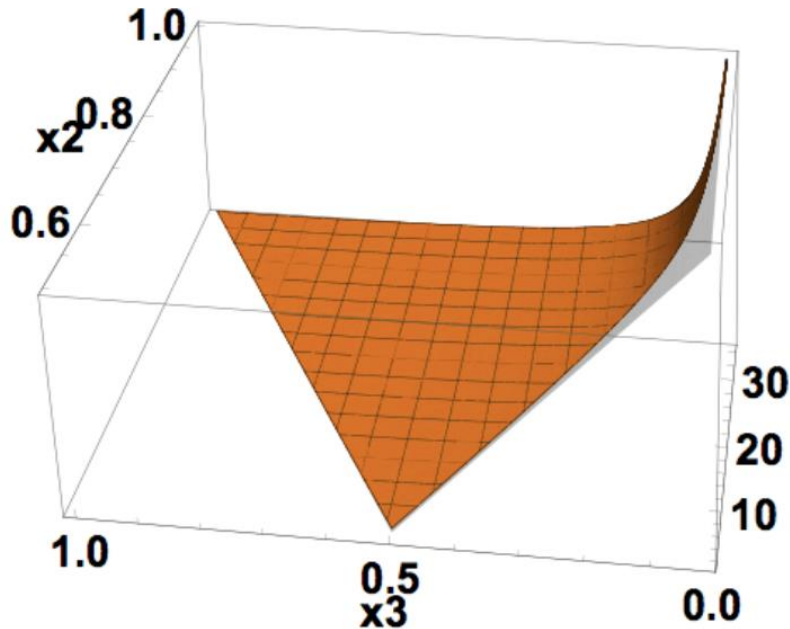
$$\zeta = - \frac{H}{\sqrt{(\dot{\phi}^1)^2 + (\dot{\phi}^2)^2}} Q_\sigma$$

$$\dot{\zeta} = -2 \frac{H\dot{\theta}}{\sqrt{(\dot{\phi}^1)^2 + (\dot{\phi}^2)^2}} Q_s$$

➡ Non-trivial N_{AB} from $\dot{\theta} \neq 0$

Local-type primordial non-Gaussianity

- Shape of local-type primordial bispectrum



$$S^{\text{local}} = \frac{3}{10} f_{\text{NL}}^{\text{local}} \left(\frac{k_1^2}{k_2 k_3} + 2 \text{ perm.} \right)$$

$$f_{\text{NL}}^{\text{local}} = \frac{5}{6} \frac{N_A N_B N^{AB}}{(N_C N^C)^2}$$

Maximum in the squeezed limit

$$k_3 \ll k_2 \simeq k_1$$

- Other models with non-linearities of the transfer mechanism
 - Modulated reheating (at the end of inflation)
 - Curvaton scenario (after the inflation)

The shape of the primordial bispectrum takes a universal form !!

Consistency relation

- Squeezed-limit bispectrum from single field inflation

Maldacena '03, Creminelli, Zaldarriaga '04

$$\lim_{k_3 \rightarrow 0} B_\zeta(k_1, k_2, k_3) = (1 - n_s(k_1)) P_\zeta(k_1) P_\zeta(k_3)$$

[Conservation of ζ_k on super-Hubble scales
→ k_3 mode locally acts as a background field

➡ Detection of $f_{\text{NL}}^{\text{local}}$ rule out most single field inflation models !!

- Loop hole

Chen, Firouzjahi, Namjoo, Sasaki '13

Based on non-attractor background solution, we can violate this, as the 'decaying' mode can become important on large scales

Local-type primordial trispectrum

- Primordial trispectrum

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv (2\pi)^3 \delta\left(\sum_{\mathbf{i}} \mathbf{k}_i\right) T_\zeta(\mathbf{k}_i)$$

wavevectors $\mathbf{k}_i$ form a tetrahedron in Fourier space

- Local-type primordial trispectrum

$$T_\zeta^{\text{local}}(\mathbf{k}_i) = \frac{54}{25} g_{\text{NL}}^{\text{local}} [P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perm.}] \\ + \tau_{\text{NL}}^{\text{local}} [P_\zeta(|\mathbf{k}_1 + \mathbf{k}_3|) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perm.}]$$

$$g_{\text{NL}}^{\text{local}} = \frac{25}{54} \frac{N_{ABC} N^A N^B N^C}{(N_D N^D)^3} \quad \text{can exist even if } f_{\text{NL}}^{\text{local}} = 0$$

$$\tau_{\text{NL}}^{\text{local}} = \frac{N_{AB} N^{AC} N^B N_C}{(N_D N^D)^3} \geq \left(\frac{6}{5} f_{\text{NL}}^{\text{local}}\right)^2 \quad \text{equality for single source}$$

Suyama and Yamaguchi '08

Intrinsic non-Gaussianity of field fluctuations

- Expansion of the action

$$g_{\mu\nu}(t, x) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, x) \quad \phi^A(t, x) = \bar{\phi}^A(t) + Q^A(t, x)$$

➡ $S = \bar{S} + S^{(2)}(\delta g_{\mu\nu}, Q^A) + \underline{S^{(3)}(\delta g_{\mu\nu}, Q^A) + \dots}$

interactions of the fields

- In-in formalism Calzetta and Hu '87, Weinberg '05

The expectation value of an observable $O(t)$

$$\langle in|O(t)|in\rangle = \langle 0| \left[\bar{T} \exp \left(i \int_{-\infty}^t H_I(t') dt' \right) \right] O^I(t) \left[T \exp \left(-i \int_{-\infty}^t H_I(t'') dt'' \right) \right] |0\rangle$$

I denotes the use of the interaction picture

At leading order ➡ $\langle O(t) \rangle = 2 \operatorname{Re} \left[-i \int_{-\infty}^t dt' \langle 0|O^I(t)H_I(t')|0\rangle \right]$

Single-field k-inflation

- Model

Armendariz-Picon et al. '99

$$\mathcal{L} = P(X, \phi) \quad \text{with} \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

cf. $\mathcal{L} = X - V(\phi)$ for a canonical scalar field

- Linear fluctuations (leading order in slow-varying)

Garriga, Mukhanov '99

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = \left(\frac{H^2}{8\pi^2 \epsilon \underline{c_s}} \right)_* \quad c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

sound speed

- Third-order action

Chen, Huang, Kachru, Shiu '07

$$S^{(3)} = \int dt d^3x a^3 \epsilon \left[\left(\frac{1}{c_s^2} - 1 \right) \left(\zeta \frac{(\partial\zeta)^2}{a^2} - \frac{3}{c_s^2} \zeta \dot{\zeta}^2 \right) + \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} \right) \frac{1}{H c_s^2} \dot{\zeta}^3 \right]$$

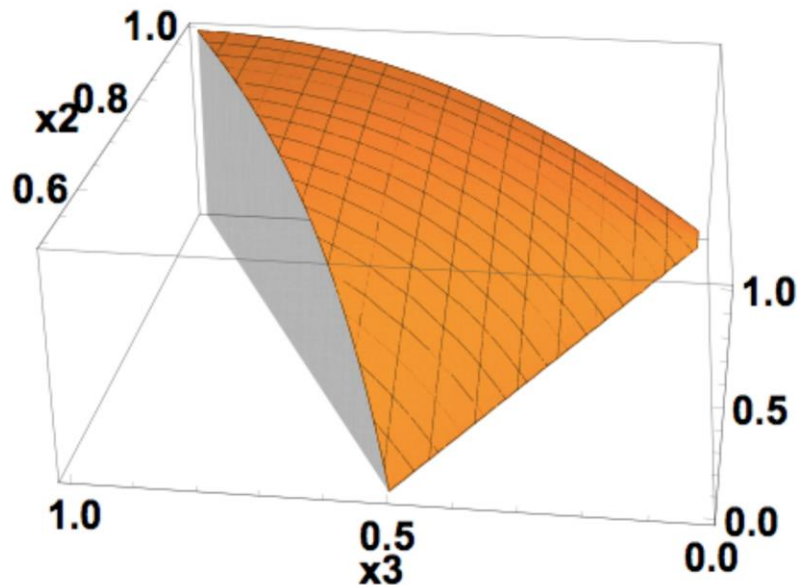
$$\Sigma = XP_{,X} + 2X^2P_{,XX} \quad \lambda = X^2P_{,XX} + \frac{2}{3}X^3P_{,XXX}$$

Equilateral-type primordial non-Gaussianity

- Shape of equilateral-type primordial bispectrum

Babich, Creminelli, Zaldarriaga '04

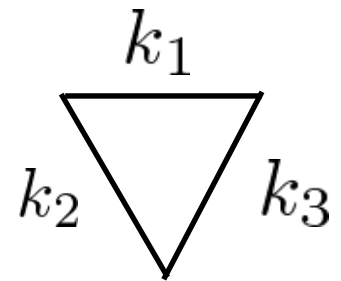
$$S^{k-\text{inf}} \simeq S^{\text{equil}} = \frac{9}{10} f_{\text{NL}}^{\text{equil}} \left[- \left(\frac{k_1^2}{k_2 k_3} + 2 \text{ perm.} \right) + \left(\frac{k_1}{k_2} + 5 \text{ perm.} \right) - 2 \right]$$



$$f_{\text{NL}}^{\text{equil}} = \mathcal{O} \left(\frac{1}{c_s^2}, \frac{\lambda}{\Sigma} \right)$$

Maximum in the equilateral limit

$$k_1 \sim k_2 \sim k_3$$



This shape emerges in more general higher-derivative scenarios

Ghost-inflation, Galileon-inflation, Horndeski theories,...

Equilateral-type primordial trispectrum

• Primordial trispectrum in k-inflation

Arroja, SM, Koyama, Tanaka '09

$$\begin{aligned} \frac{T_{\zeta}^{\dot{\sigma}^4}}{(2\pi^2\mathcal{P}_{\zeta})^3} &= \frac{221184}{25} \frac{g_{\text{NL}}^{\dot{\sigma}^4}}{(\sum k_i)^5 k_1 k_2 k_3 k_4} \\ \frac{T_{\zeta}^{\dot{\sigma}^2(\partial\sigma)^2}}{(2\pi^2\mathcal{P}_{\zeta})^3} &= -\frac{27648}{325} g_{\text{NL}}^{\dot{\sigma}^2(\partial\sigma)^2} \left[\frac{k_1^2 k_2^2 (\mathbf{k}_3 \cdot \mathbf{k}_4)}{(\sum k_i)^3 \Pi k_i^3} \left(1 + 3 \frac{(k_3 + k_4)}{\sum k_i} + 12 \frac{k_3 k_4}{(\sum k_i)^2} \right) + \text{perm.} \right] \\ \frac{T_{\zeta}^{(\partial\sigma)^4}}{(2\pi^2\mathcal{P}_{\zeta})^3} &= \frac{165888}{2575} g_{\text{NL}}^{(\partial\sigma)^4} \frac{[(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{k}_3 \cdot \mathbf{k}_4) + \text{perm.}]}{\sum k_i \Pi k_i} \\ &\quad \times \left(1 + \frac{\sum_{i<j} k_i k_j}{(\sum k_i)^2} + 3 \frac{\Pi k_i}{(\sum k_i)^3} \sum \frac{1}{k_i} + 12 \frac{\Pi k_i}{(\sum k_i)^4} \right) \\ g_{\text{NL}}^{\dot{\sigma}^4}, g_{\text{NL}}^{\dot{\sigma}^2(\partial\sigma)^2}, g_{\text{NL}}^{(\partial\sigma)^4} &= \mathcal{O} \left(\frac{1}{c_s^4} \right) \text{ written in terms of } P_{,X}, P_{,XX}, \dots \end{aligned}$$

These shapes also appear in effective field theory of inflation

Senatore, Zaldarriaga '11

Primordial non-Gaussianities in the CMB

- CMB angular bispectrum

$$\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle \quad \frac{\Delta T}{T}(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

The link between $a_{\ell m}$ and ζ is well known at linear order

But we need care about the nonlinear effects from GR itself

- Constraints from Planck

Ade et al `16

$$f_{\text{NL}}^{\text{local}} = 0.8 \pm 5.0$$

$$f_{\text{NL}}^{\text{equil}} = -4 \pm 43 \quad (68\% \text{ CL})$$

$$g_{\text{NL}}^{\text{local}} = (-9.0 \pm 7.7) \times 10^4$$

$$\tau_{\text{NL}}^{\text{local}} < 2800 \quad (95\% \text{ CL})$$

$$g_{\text{NL}}^{\dot{\sigma}^4} = (-0.2 \pm 1.7) \times 10^6$$

$$g_{\text{NL}}^{(\partial\sigma)^4} = (-0.1 \pm 3.8) \times 10^5$$

Perspectives

- The simplest single-field slow-roll models of inflation passed very stringent tests due to the lack of PNG

- The bounds on $f_{\text{NL}}^{\text{equil}}$ translate into a limit on c_s

$$c_s \geq 0.020 \quad (95\% \text{ CL})$$

Strong coupling scale in EFT $\Lambda_\star \sim \frac{1}{\sqrt{\zeta f_{\text{NL}}^{\text{equil}}}} H_{\text{inf}}$

Unless $f_{\text{NL}}^{\text{equil}} \leq 1$, new physics appears much below M_{Pl}

Baumann, Green `11

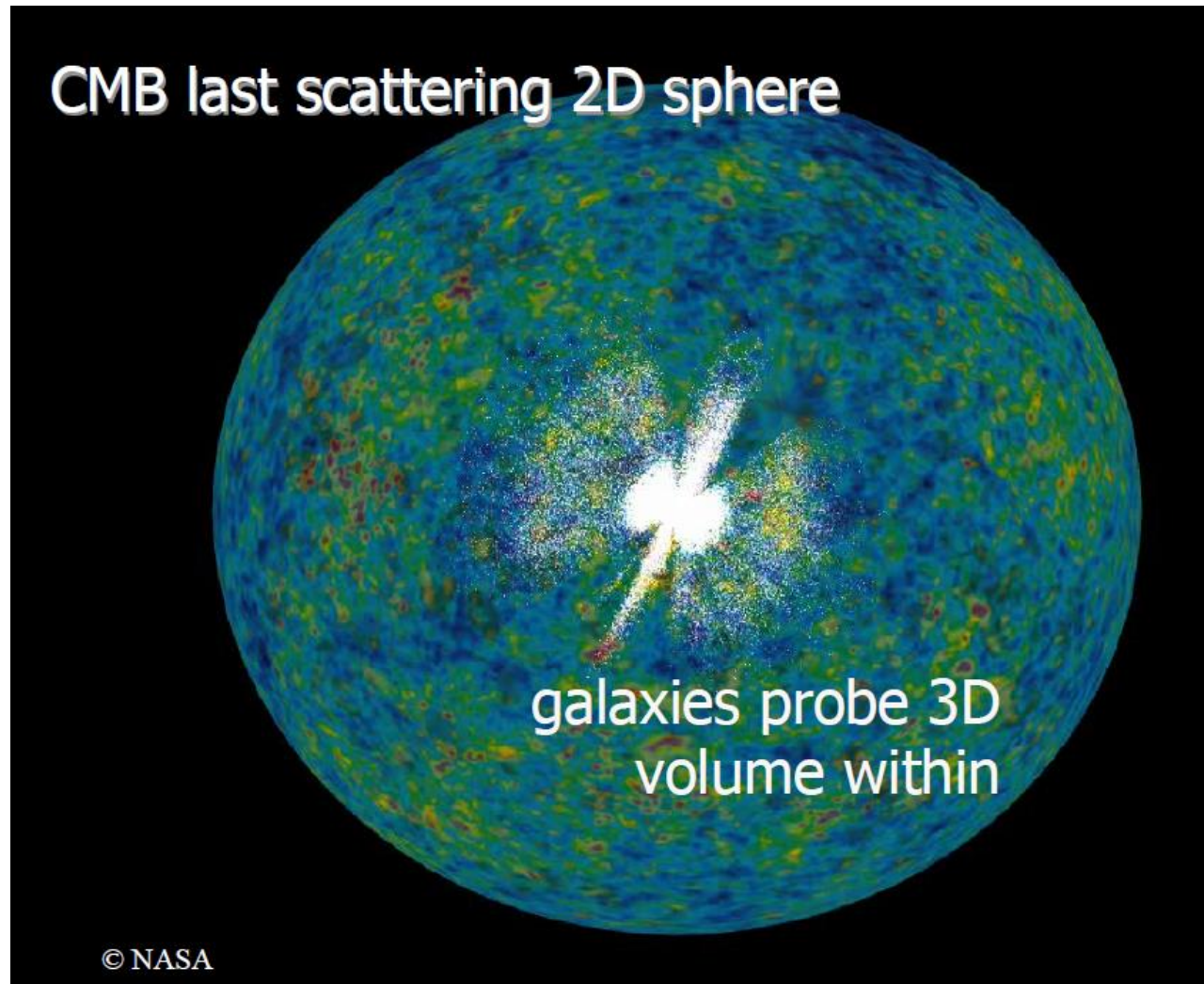
- $f_{\text{NL}}^{\text{local}}$ from multifield scenarios is very model-dependent
But large class of spectator models predicts $|f_{\text{NL}}^{\text{local}}| \geq \mathcal{O}(1)$

Suyama, Takahashi, Yamaguchi, Yokoyama `13



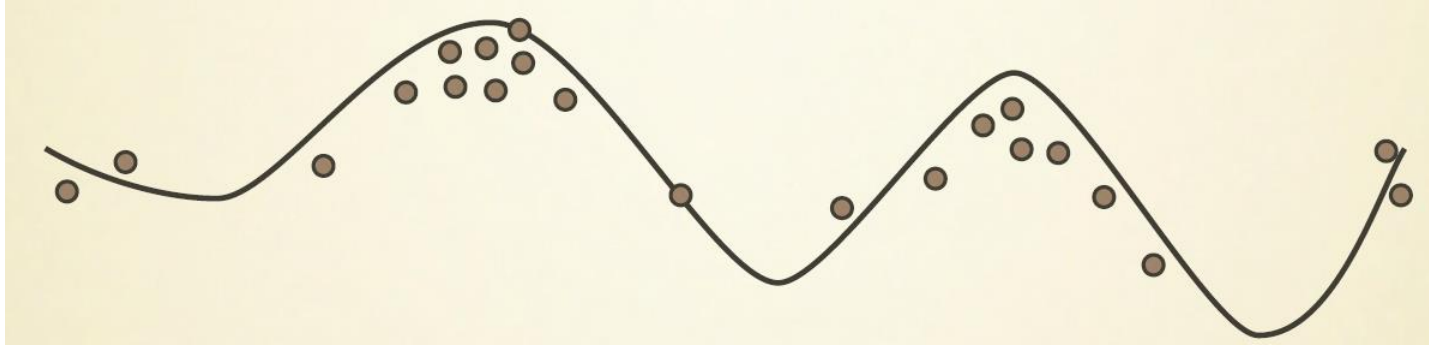
3. Constraints on Primordial NG from LSS

CMB vs LSS



LSS can give more stringent constraint !!

Bias between mass and astrophysical objects



Distribution of astrophysical objects on large scales:

$$\delta_n(x, t) \equiv \frac{\delta n}{n}$$

biased tracer of underlying matter (dark matter + baryons):

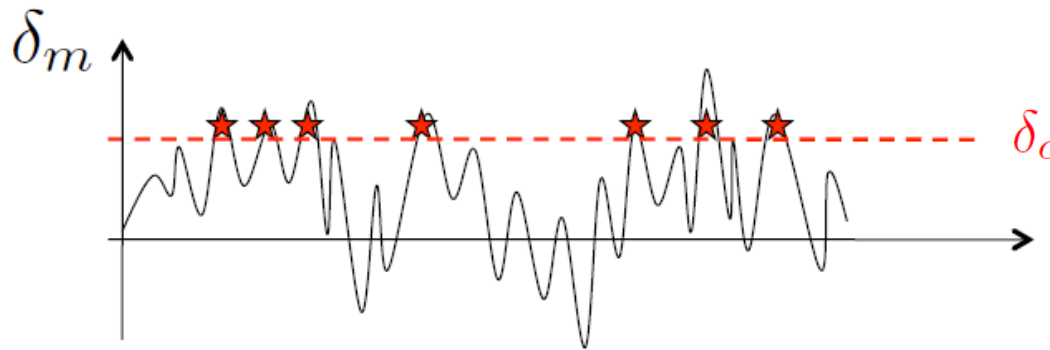
$$\delta_m(x, t) \equiv \frac{\delta \rho_m}{\rho_m}$$

There should be a relation between the two

↑
formation process of the astrophysical objects

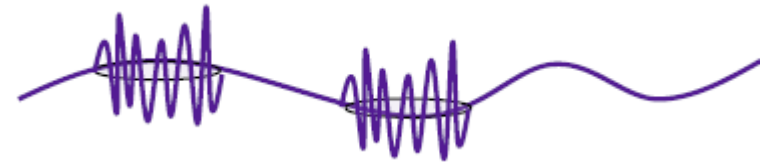
Simple picture of bias

Small scale density peaks exceeding threshold collapse under their own gravity and form virialized objects



- Peak-background split

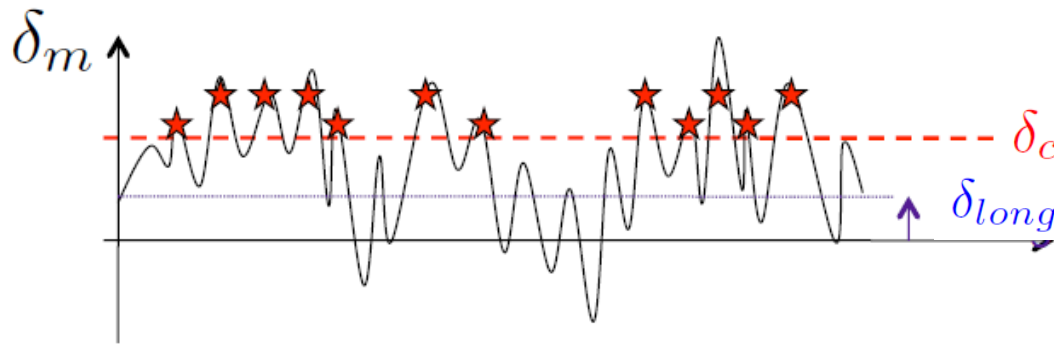
$$\delta_m = \delta_{short} + \delta_{long}$$



Large-scale fluctuations δ_{long} raise local background density,

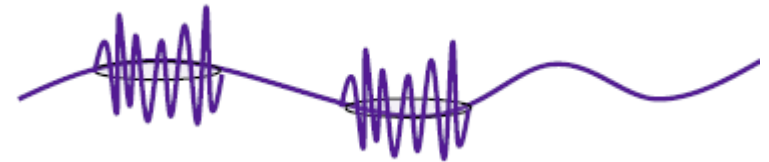
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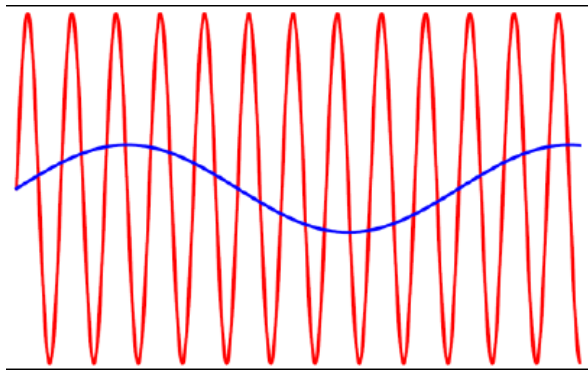


Large-scale fluctuations δ_{long} raise local background density, which lowers effective threshold for collapse and enhances number of peaks above threshold !!

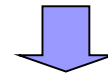
Linear bias from Gaussian fluctuations

Form of the enhancement depends on the statistical property

- Gaussian fluctuations of mass density



long and short wavelength modes are decoupled and evolve independently



no scale dependence in the bias

- Linear bias

$$\delta_n = b\delta_m \quad (b = 1 + b_L)$$

with $b \rightarrow b_G = \text{constant}$
on large scales

← Widely applied form as this is derived from the assumption

$$\delta_n(x) = f(\delta_m(x), (\nabla \delta_m(x))^2, \dots)$$

local function of δ_m

Influence of local-type PNG on bias

Dalal et al, '08, (See also Slosar et al, '08)

- local model of non-Gaussianity

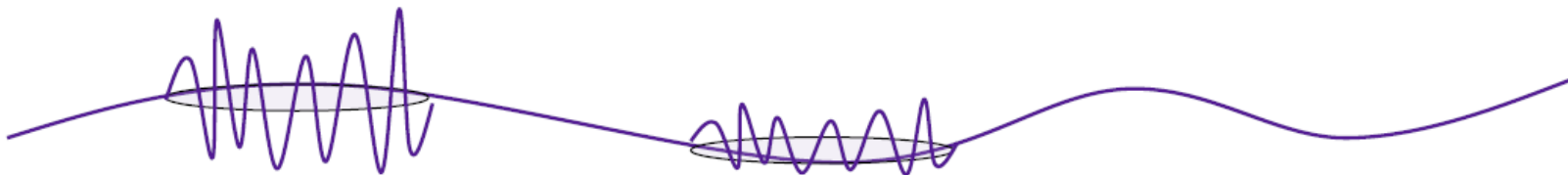
$$\Phi(x) = \phi_G(x) + f_{\text{NL}}^{\text{local}}(\phi_G^2(x) - \langle \phi_G^2 \rangle)$$

peak-background split of Gaussian potential fluctuations:

$$\phi_G(x) = \phi_s(x) + \phi_l(x)$$

$$\begin{aligned} \Rightarrow \Phi(x) = & (1 + \underline{f_{\text{NL}}^{\text{local}} \phi_l(x)}) \phi_s(x) + \phi_l(x) \\ & + f_{\text{NL}}^{\text{local}}(\phi_s^2(x) + \phi_l^2(x) - \langle \phi_s^2(x) \rangle - \langle \phi_l^2(x) \rangle) \end{aligned}$$

long wavelength modes add to local background density $+\delta_{\text{long}}$
and **modulate amplitude on small scales** $\times f_{\text{NL}} \phi_l$



Scale-dependent bias from local-type PNG

Dalal et al, '08, (See also Slosar et al, '08)

- Scale-dependence

large-scale modes enhance power on small scales $\propto f_{\text{NL}}\phi_l$

potential is related with the density via Poisson equation

$$\nabla^2 \Phi = 4\pi G_N \delta \rho_m \quad \Rightarrow \quad \phi_l = \frac{3}{2} \left(\frac{aH}{k} \right)^2$$

$$\Rightarrow \quad \delta b \propto \left(\frac{aH}{k} \right)^2 (b_G - 1) \quad \text{diverges on large scales}$$

“universal” mass functions

$$n(M) = n(M, \nu) = M^{-2} \nu f(\nu) \frac{d \ln \nu}{d \ln M} \quad \nu = \delta_c^2 / \sigma^2(M)$$

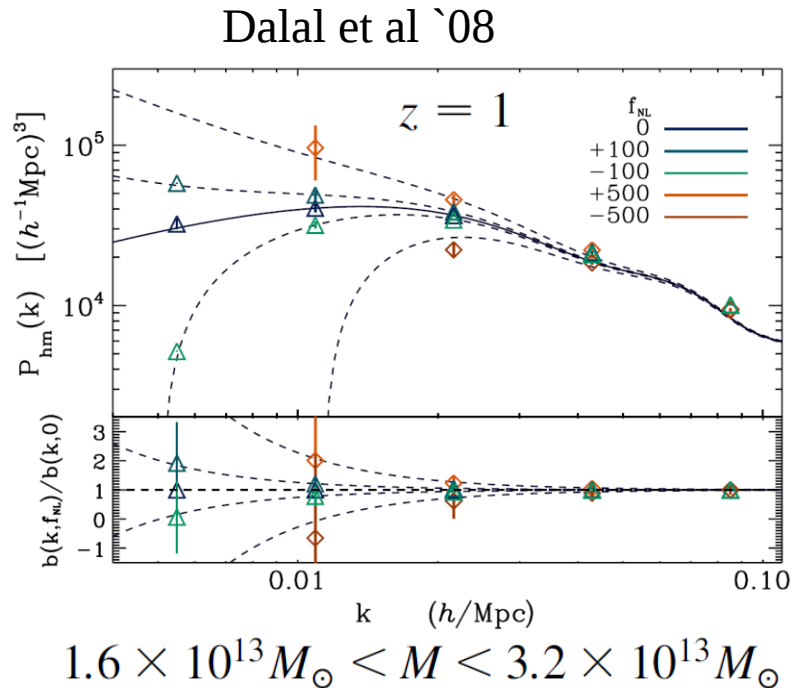
$$\Delta b = f_{\text{NL}}^{\text{local}} (b_G - 1) \frac{3\delta_c \Omega_m H_0^2}{k^2 T(k) D(a)}$$

$f(\nu)$: fraction of mass collapsing into halos

$T(k)$: transfer function

$D(a)$: growth factor

Constraints on local-type PNG



- Current constraints:

Leistedt, Peiris, Roth '14

$$-49 < f_{\text{NL}}^{\text{local}} < 31$$

$$-2.7 \times 10^5 < g_{\text{NL}}^{\text{local}} < 1.9 \times 10^5$$

(At 95 % CL)

- Forecast constraints:

Yamauchi et al '14

SKA (Square Km Array)

$$\Delta b = f_{\text{NL}}^{\text{local}} (b_{\text{G}} - 1) \frac{3\delta_c \Omega_m H_0^2}{k^2 T(k) D(a)}$$

$$\Delta f_{\text{NL}}^{\text{local}} \simeq 0.1$$

Integrated Perturbation Theory

Matsubara '12, '13, Bernardeau et al '08

- Multi-point propagator of biased objects

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta \delta_L(\mathbf{k}_1) \delta \delta_L(\mathbf{k}_2) \cdots \delta \delta_L(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \cdots + \mathbf{k}_n) \underline{\Gamma_X^{(n)}(\mathbf{k}_1, \mathbf{k}_2, \cdots, \mathbf{k}_n)}$$

δ_X : number density field of the biased objects

Gravitational evolution

Lagrangian bias,

δ_L : linear density field which is related with

primordial curvature perturbation Φ through

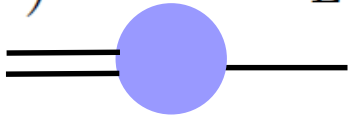
$$\delta_L(k) = \mathcal{M}(k) \Phi(k); \quad \mathcal{M}(k) = \frac{2}{3} \frac{D(z)}{D(z_*)(1+z_*)} \frac{k^2 T(k)}{H_0^2 \Omega_{m0}}$$

z_* : arbitrary redshift at the matter-dom. era

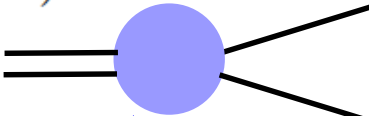
Amplitude is fixed by σ_8

Diagrammatic approach

- Multi-point propagator of biased objects

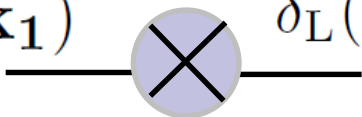


$$\Gamma_X^{(1)}(\mathbf{k}_1) \delta^{(3)}(\mathbf{k}_1 - \mathbf{k})$$

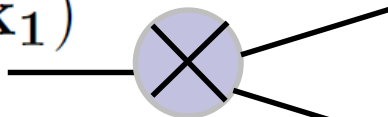


$$\Gamma_X^{(2)}(\mathbf{k}_1, \mathbf{k}_2) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k})$$

- Spectra of linear density field



$$\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) P_L(k_1)$$



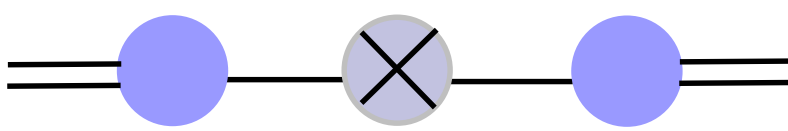
$$\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_L(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$



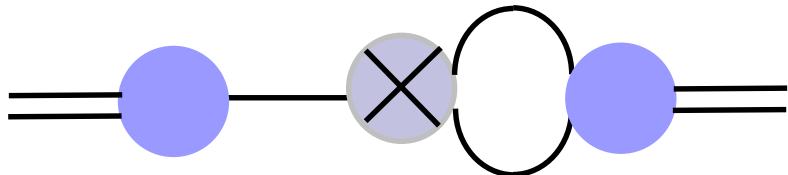
spectra of biased objects (Halo/Galaxy) systematically !!

Halo/Galaxy power spectrum with PNG

- Diagrams for the power spectrum of the biased objects

P_0

 $\Rightarrow \propto \mathcal{M}(k)^2 P_\Phi(k) \propto \underline{k}$

large scale limit
 $k \ll \underline{p}$ typical scale of the biased objects

P_{bis}


large scale limit $\Rightarrow$

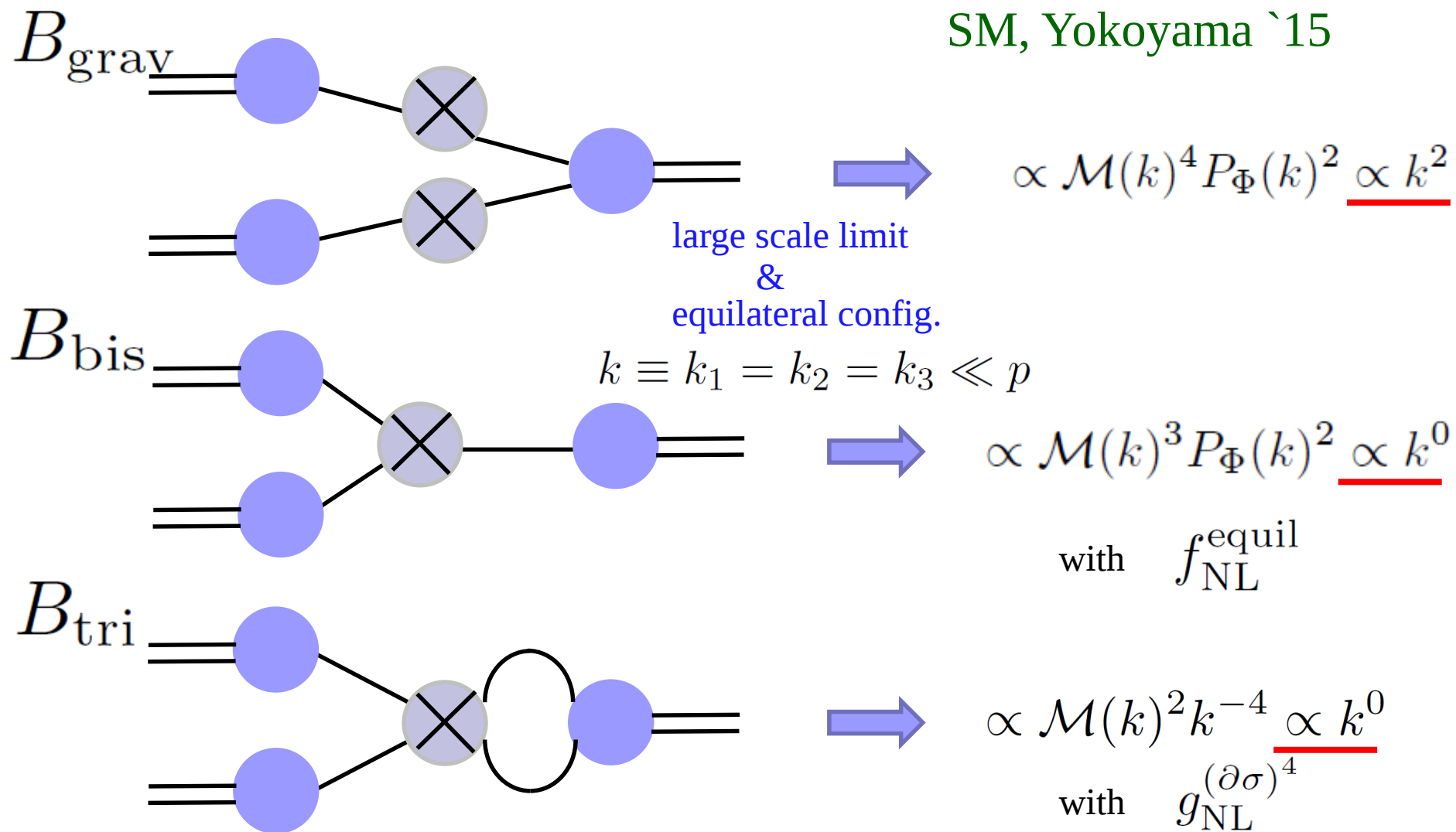
$$\left[\begin{array}{ll} \propto \mathcal{M}(k) k^{-3} \underline{\propto k^{-1}} & \text{with } f_{\text{NL}}^{\text{local}} \\ \propto \mathcal{M}(k) k^{-1} \underline{\propto k} & \text{with } f_{\text{NL}}^{\text{equil}} \end{array} \right.$$

enhancement
 no enhancement

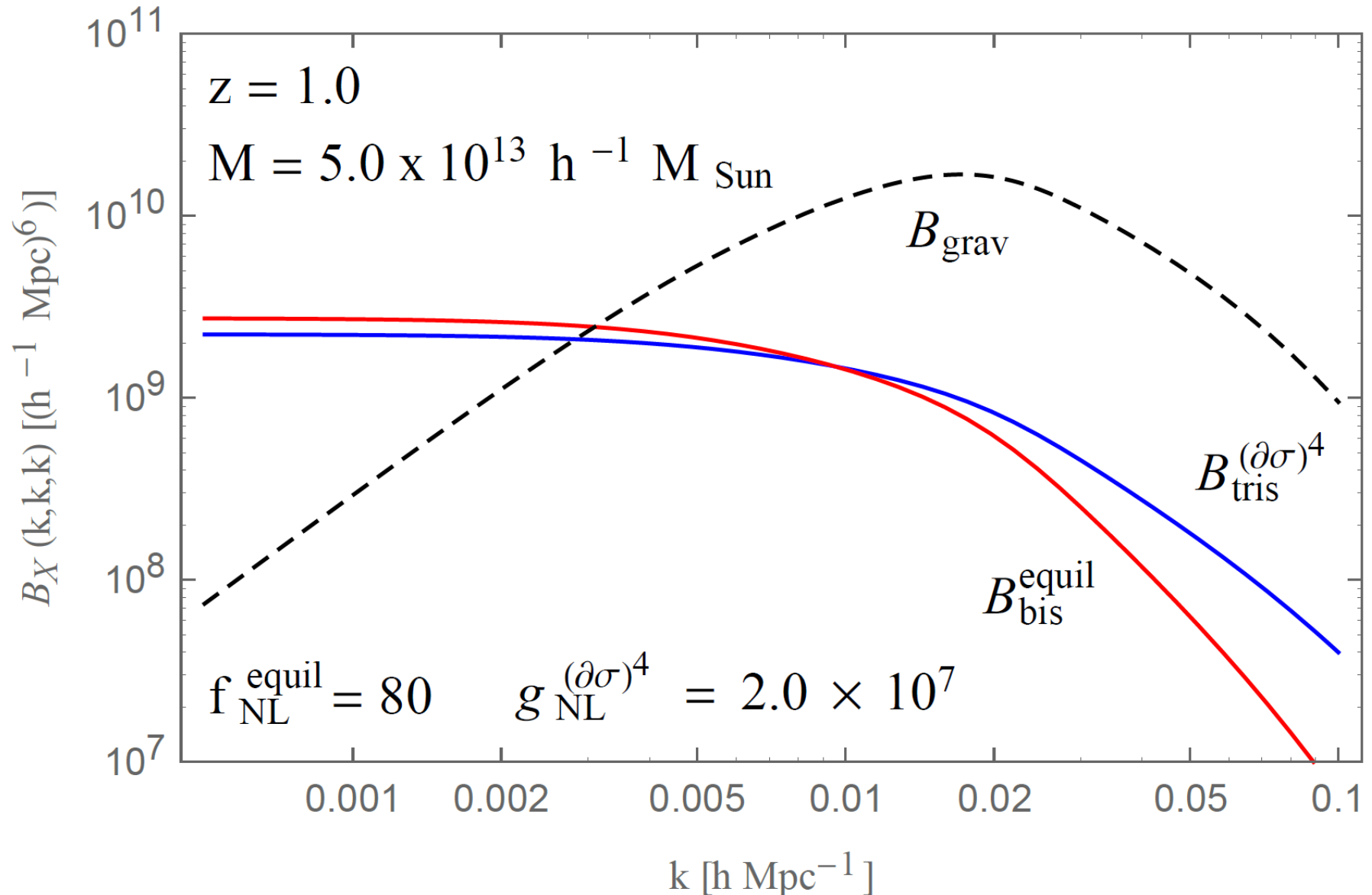
Halo/galaxy bispectrum with PNG

- Diagrams for the bispectrum of the biased objects

SM, Yokoyama '15



Scale-dependence of halo/galaxy bispectrum



Forecast constraints

Hashimoto, SM, Yokoyama '16

Can LSS obtain more severer constraints than CMB ?

- Ongoing/future surveys on LSS



HSC
deep



DES
wide



LSST
deep & wide

	f_{sky}	z_m	$\bar{n}_s$ [arcmin $^{-2}$]
HSC [21]	0.0375 (1,500 deg 2)	1.0	35
DES [22]	0.125 (5,000 deg 2)	0.5	12
LSST [23]	0.5 (20,000 deg 2)	1.5	100

sky coverage

mean source redshift

mean number density of source

Strategy

From integrated perturbation theory

Three-dimensional spectra: $B_{XYZ}(k_1, k_2, k_3)$, $X = h$ or m

project on celestial sphere

$$\Delta_h^{(2)}(\boldsymbol{\theta}) = \int_0^\infty dz \, \underline{W_h(z)} \delta_h^{(3)}(\chi(z)\boldsymbol{\theta}, z), \quad \text{halo}$$

$$\kappa(\boldsymbol{\theta}) = \int_0^\infty dz \, \underline{W_\kappa(z)} \delta_m^{(3)}(\chi(z)\boldsymbol{\theta}, z) \quad \text{matter (lensing)}$$

Angular spectra: $B_{abc}(\ell_1, \ell_2, \ell_3)$, $a = h$ or κ

Fisher analysis $\text{Cov}[B_{abc}(\ell_i, \ell_j, \ell_k), B_{a'b'c'}(\ell_l, \ell_m, \ell_n)]$

$$F_{\alpha\beta} = \sum_{\ell_i=\ell_{\min}}^{\ell_{\max}} \frac{\partial \mathbf{B}_i(\mathbf{p})}{\partial p_\alpha} \underline{(\text{Cov}^B)^{-1}_{ij}} \frac{\partial \mathbf{B}_j(\mathbf{p})}{\partial p_\beta} \bigg|_{\mathbf{p}=\mathbf{p}_0}, \quad \mathbf{B}_i = \begin{pmatrix} (B_{hhh})_i \\ (B_{hh\kappa})_i \\ (B_{h\kappa\kappa})_i \end{pmatrix},$$

weight function and covariance matrix depend on surveys

Expected constraints on $f_{\text{NL}}^{\text{equil}}$, $g_{\text{NL}}^{\text{equil}}$ $\leftarrow (1/F_{\alpha\alpha})^{1/2}, ([F]_{\alpha\alpha}^{-1})^{1/2}$

Constraints on equilateral-type PNG

Survey		B_{hhh}	$B_{hhh} + B_{hh\kappa} + B_{h\kappa\kappa}$
HSC	$\sigma(f_{\text{NL}}^{\text{equil}})$	$3.2 \times 10^3 (2.9 \times 10^3)$	$2.3 \times 10^3 (2.1 \times 10^3)$
	$\sigma(g_{\text{NL}}^{(\partial\sigma)^4})$	$3.2 \times 10^8 (2.9 \times 10^8)$	$2.9 \times 10^8 (2.7 \times 10^8)$
DES	$\sigma(f_{\text{NL}}^{\text{equil}})$	$1.6 \times 10^3 (1.6 \times 10^3)$	$1.1 \times 10^3 (1.1 \times 10^3)$
	$\sigma(g_{\text{NL}}^{(\partial\sigma)^4})$	$1.6 \times 10^9 (1.7 \times 10^9)$	$8.2 \times 10^8 (7.7 \times 10^8)$
LSST	$\sigma(f_{\text{NL}}^{\text{equil}})$	$9.2 \times 10^2 (8.0 \times 10^2)$	$7.0 \times 10^2 (6.4 \times 10^2)$
	$\sigma(g_{\text{NL}}^{(\partial\sigma)^4})$	$5.3 \times 10^7 (4.6 \times 10^7)$	$4.9 \times 10^7 (4.4 \times 10^7)$

cf. $\sigma \left(f_{\text{NL}}^{\text{equil}} \right) = 43$, $\sigma \left(g_{\text{NL}}^{(\partial\sigma)^4} \right) = 1.3 \times 10^6$

from Planck

Summary

- PNG has information on various types of nonlinearity of inflation models and is helpful to distinguish between them
- Currently, from CMB , no significant PNG is observed and the simplest single-field slow-roll inflation models are consistent
- From scale-dependent bias, the future/ongoing projects on LSS can constrain $f_{\text{NL}}^{\text{local}}$ more and we can expect $\Delta f_{\text{NL}}^{\text{local}} \simeq 0.1$
- From halo/galaxy bispectrum, we can constrain $f_{\text{NL}}^{\text{equil}}$, but constraints from LSS are looser than that from CBM

Discussions

- Multitracer technique (Yamauchi et al `16) is helpful and gets $\Delta f_{\text{NL}}^{\text{equil}} \sim 20$ for SKA, slightly better than Planck
- We can constrain $f_{\text{NL}}^{\text{equil}}$ more from information of small scales if we specify nonlinear and nonlocal bias (Gleyzes et al `16)
- PNG is also generated by models with Exited initial states, resonance and feature models, models with gauge fields,...
- Primordial tensor perturbations and small scale fluctuations can also constrain inflation models significantly