

定常軸対称時空における円軌道の安定性

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Nonradial stability of marginal stable circular orbits in stationary axisymmetric spacetimes

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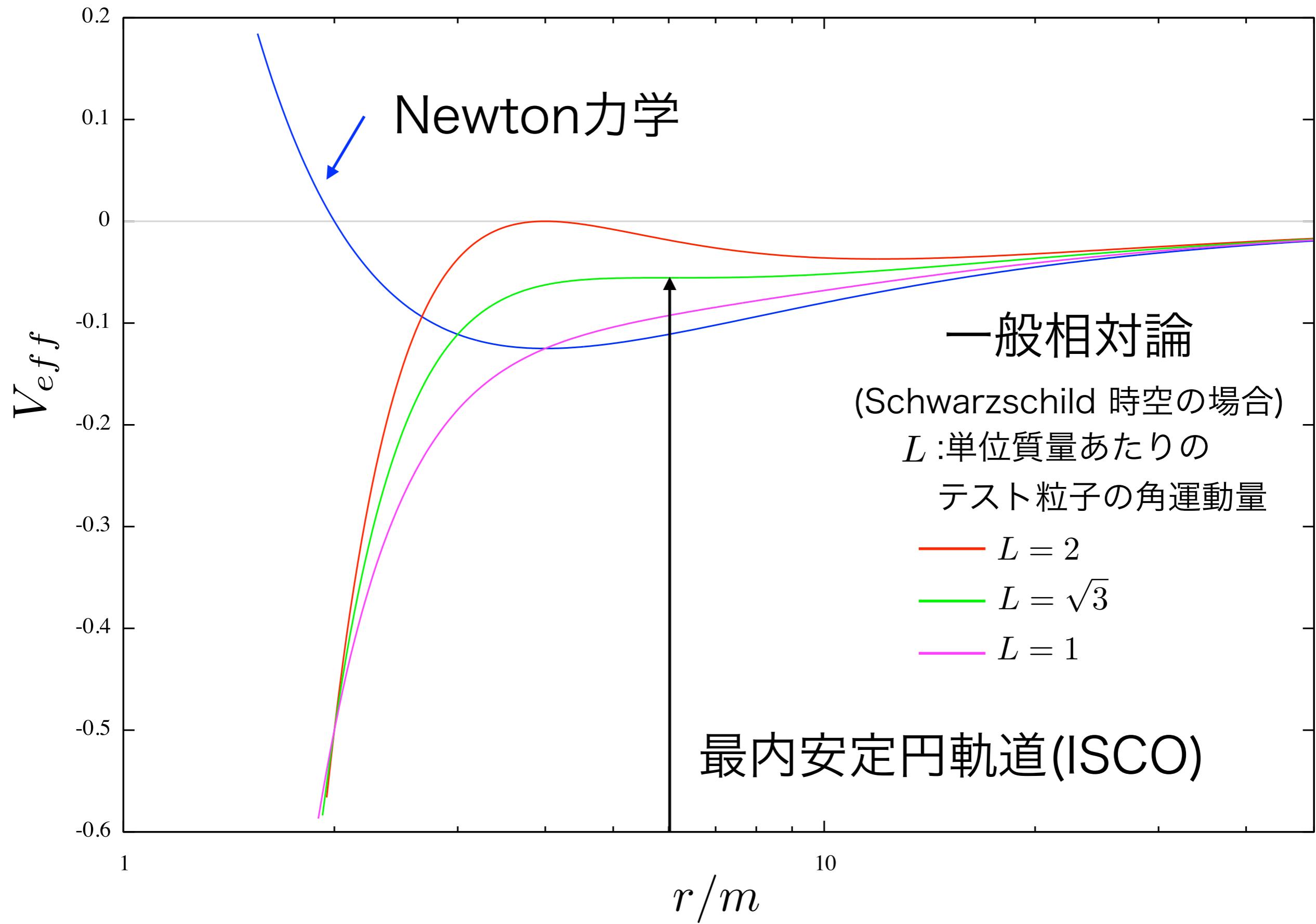
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We study linear nonradial perturbations and stability of a marginal stable circular orbit such as the innermost stable circular orbit of a test particle in stationary axisymmetric spacetimes which possess a reflection symmetry with respect to the equatorial plane. A zenithal stability criterion is obtained in terms of the metric components, the specific energy, and angular momentum of a test particle. The proposed approach is applied to the Kerr solution and Majumdar-Papapetrou solution to the Einstein equation. Moreover, we reexamine marginal stable circular orbits for a modified metric of a rapidly spinning black hole that has been recently proposed by Johannsen and Psaltis [Phys. Rev. D **83**, 124015 (2011)]. We show that, for the Johannsen and Psaltis model, circular orbits that are stable against radial perturbations for some parameter region become unstable against zenithal perturbations. This suggests that the last circular orbit for this model may be larger than the innermost stable circular orbit.

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研究背景



研究背景

marginal stable circular orbit (MSCO) :

ISCOの様な安定円軌道と不安円軌道の境界となる円軌道

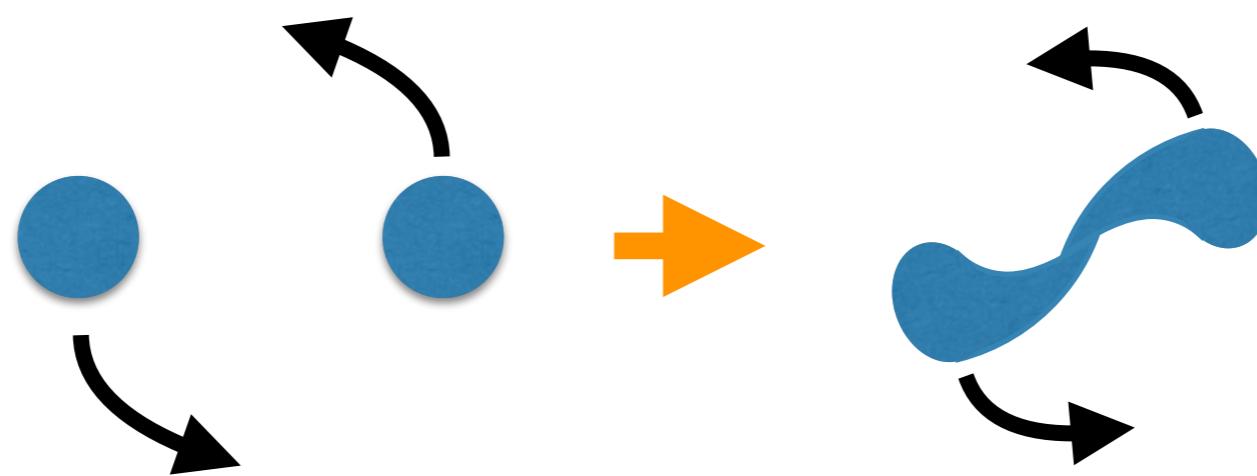
inspiralling phaseからmerging phaseになるLast circular orbit
はISCOと考えられている。

[L. Blanchet, Living Rev. Relativ. 9, 4 (2006);

S. Isoyama, L. Barack, S. R. Dolan, A. Le Tiec, H. Nakano, A. G. Shah, T. Tanaka, and N. Warburton, Phys. Rev. Lett. 113, 161101 (2014).]

LIGO観測データでは、フィッティング模型にISCO。

→今後の重力波観測で、ISCO半径近傍の観測される可能性がある。



Introduction

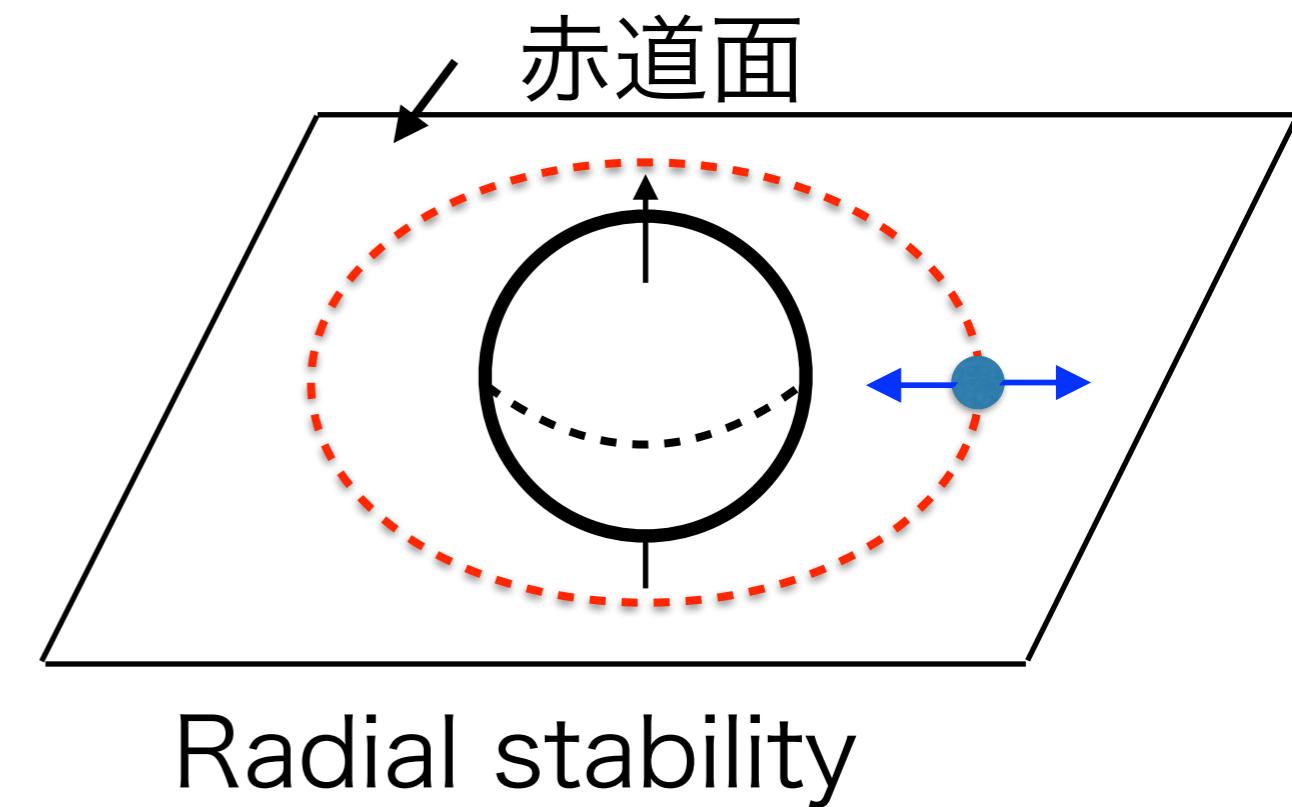
定常・軸対称時空のMSCO方程式は導出済み

[M. Stute & M. Camenzind, MNRAS **336**, 831 (2002)]

線素 : $ds^2 = f(dt - wd\phi)^2 - f^{-1}[e^{2\gamma}(d\rho^2 + dz^2) + \rho^2 d\phi^2]$

MSCO方程式 :

$$\begin{aligned} & \left[w_{,\rho} w_{,\rho\rho} f^5 \rho (2f - f_{,\rho} \rho) + w_{,\rho}^2 f^4 [2f^2 + (-f_{,\rho}^2 + f_{,\rho\rho} f) \rho^2] \right. \\ & + w_{,\rho} f^2 \sqrt{w_{,\rho}^2 f^4 + f_{,\rho} \rho (2f - f_{,\rho} \rho)} \\ & \times [2f^2 + 2f_{,\rho}^2 \rho^2 - f \rho (4f_{,\rho} + f_{,\rho\rho} \rho)] \\ & + \rho (2f - f_{,\rho} \rho) \{ 3f_{,\rho} f^2 - 4f_{,\rho}^2 f \rho + f_{,\rho}^3 \rho^2 \} \\ & \left. + f^2 [f_{,\rho\rho} \rho - w_{,\rho\rho} f \sqrt{w_{,\rho}^2 f^4 + f_{,\rho} \rho (2f - f_{,\rho} \rho)}] \} \right] / \\ & \left[f^2 \rho^2 \{ w_{,\rho}^2 f^4 + 3f_{,\rho} f \rho - f_{,\rho}^2 \rho^2 \} \right. \\ & \left. - f^2 [2 + w_{,\rho} \sqrt{w_{,\rho}^2 f^4 + f_{,\rho} \rho (2f - f_{,\rho} \rho)}] \} \right] = 0. \end{aligned} \quad (40)$$

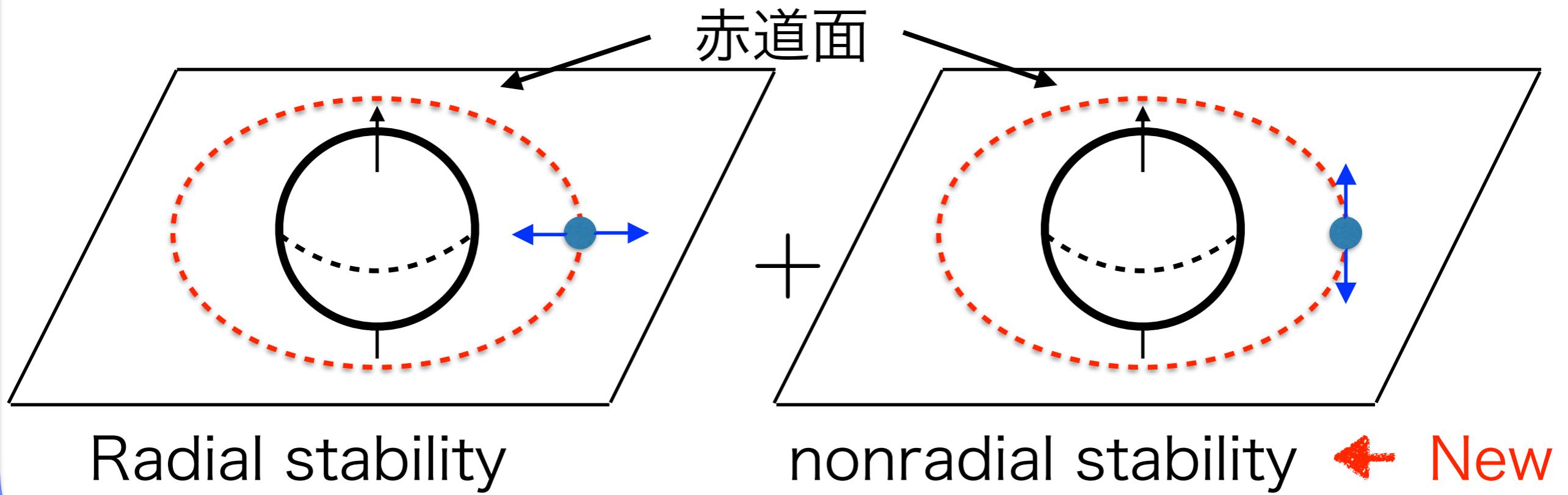


- nonradial stabilityについて議論していない

Marginal stable circular orbits

T. Ono et al., PRD 94, 064042 (2016)

- 定常軸対称時空におけるテスト粒子の円軌道の radial perturbation だけでなく nonradial perturbation に対する安定性
- 新たに、Nonradial stability criterion を定式化した。



Marginal stable circular orbits

定常軸対称時空の計量 [T. Lewis, Proc. Roy. Soc. A, **136**, 176 (1932)]

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -A(y^p, y^q) dt^2 - 2H(y^p, y^q) dt d\phi \\ &\quad + F(y^p, y^q)(\gamma_{pq} dy^p dy^q) + D(y^p, y^q) d\phi^2 \end{aligned}$$

ここでは極座標を採用する。

$$\begin{aligned} ds^2 &= -A(r, \theta) dt^2 - 2H(r, \theta) dt d\phi + B(r, \theta) dr^2 \\ &\quad + C(r, \theta) d\theta^2 + D(r, \theta) d\phi^2 \end{aligned}$$

仮定

計量 $g_{\mu\nu}$ は $\frac{\partial g_{\mu\nu}}{\partial \theta} \Big|_{\theta=\pi/2} = 0$ を満たす。

(例: $\theta = \frac{\pi}{2}$ で反転対称な時空)

Marginal stable circular orbits

反変計量

$$g^{\mu\nu} = \begin{pmatrix} -\frac{D}{AD+H^2} & 0 & 0 & -\frac{H}{AD+H^2} \\ 0 & \frac{1}{B} & 0 & 0 \\ 0 & 0 & \frac{1}{C} & 0 \\ -\frac{H}{AD+H^2} & 0 & 0 & \frac{A}{AD+H^2} \end{pmatrix} = \begin{pmatrix} -\tilde{A} & 0 & 0 & -\tilde{H} \\ 0 & \tilde{B} & 0 & 0 \\ 0 & 0 & \tilde{C} & 0 \\ -\tilde{H} & 0 & 0 & \tilde{D} \end{pmatrix}$$

テスト粒子のラグランジアン

$$\mathcal{L} = -At^2 - 2Ht\dot{\phi} + Br^2 + C\dot{\theta}^2 + D\dot{\phi}^2$$

保存量

$$\varepsilon \equiv \frac{1}{2} \frac{\partial \mathcal{L}}{\partial \dot{t}} = -At - H\dot{\phi}, \quad l \equiv \frac{1}{2} \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -H\dot{t} + D\dot{\phi}$$

Marginal stable circular orbits (radial stability)

測地線方程式の r 成分

$$\begin{aligned} \ddot{r} + \frac{1}{2B(r, \theta)} & \left\{ \frac{\partial B(r, \theta)}{\partial r} \dot{r}^2 - \frac{\partial C(r, \theta)}{\partial r} \dot{\theta}^2 + 2 \frac{\partial B(r, \theta)}{\partial \theta} \dot{r} \dot{\theta} \right\} \\ = -\frac{1}{2B(r, \theta)} & \left\{ \frac{\partial A(r, \theta)}{\partial r} \dot{t}^2 + 2 \frac{\partial H(r, \theta)}{\partial r} \dot{t} \dot{\phi} - \frac{\partial D(r, \theta)}{\partial r} \dot{\phi}^2 \right\} \end{aligned}$$

$\dot{t} = -\tilde{A}\varepsilon - \tilde{H}l$, $\dot{\phi} = -\tilde{H}\varepsilon + \tilde{D}l$ を代入

$$2B\ddot{r} + \frac{\partial B}{\partial r} \dot{r}^2 - \frac{\partial C}{\partial r} \dot{\theta}^2 + 2 \frac{\partial B}{\partial \theta} \dot{r} \dot{\theta} = \frac{\partial \tilde{A}}{\partial r} \varepsilon^2 + 2 \frac{\partial \tilde{H}}{\partial r} \varepsilon l - \frac{\partial \tilde{D}}{\partial r} l^2$$

Marginal stable circular orbits (radial stability)

測地線方程式の r 成分に摂動 : $r = r_c + \delta r, \theta = \frac{\pi}{2} + \delta\theta$

Taylor展開する。

$$\begin{aligned}
 & 2 \left(B|_c + \left. \frac{\partial B}{\partial r} \right|_c \delta r + \left. \frac{\partial B}{\partial \theta} \right|_c \delta \theta \right) \ddot{\delta r} + \left(\left. \frac{\partial B}{\partial r} \right|_c + \left. \frac{\partial^2 B}{\partial r^2} \right|_c \delta r + \left. \frac{\partial^2 B}{\partial r \partial \theta} \right|_c \delta \theta \right) \dot{\delta r}^2 \\
 & - \left(\left. \frac{\partial C}{\partial r} \right|_c + \left. \frac{\partial^2 C}{\partial r^2} \right|_c \delta r + \left. \frac{\partial^2 C}{\partial r \partial \theta} \right|_c \delta \theta \right) \dot{\delta \theta}^2 + 2 \left(\left. \frac{\partial B}{\partial \theta} \right|_c + \left. \frac{\partial^2 B}{\partial r \partial \theta} \right|_c \delta r + \left. \frac{\partial^2 B}{\partial \theta^2} \right|_c \delta \theta \right) \dot{\delta r} \dot{\delta \theta} \\
 & = \left(\left. \frac{\partial \tilde{A}}{\partial r} \right|_c + \left. \frac{\partial^2 \tilde{A}}{\partial r^2} \right|_c \delta r + \left. \frac{\partial^2 \tilde{A}}{\partial r \partial \theta} \right|_c \delta \theta \right) \varepsilon^2 + 2 \left(\left. \frac{\partial \tilde{H}}{\partial r} \right|_c + \left. \frac{\partial^2 \tilde{H}}{\partial r^2} \right|_c \delta r + \left. \frac{\partial^2 \tilde{H}}{\partial r \partial \theta} \right|_c \delta \theta \right) \varepsilon l \\
 & - \left(\left. \frac{\partial \tilde{D}}{\partial r} \right|_c + \left. \frac{\partial^2 \tilde{D}}{\partial r^2} \right|_c \delta r + \left. \frac{\partial^2 \tilde{D}}{\partial r \partial \theta} \right|_c \delta \theta \right) l^2
 \end{aligned}$$

$$\delta r \text{ の } 0 \text{ 次} \quad \left. \frac{\partial \tilde{A}}{\partial r} \right|_c \varepsilon^2 + 2 \left. \frac{\partial \tilde{H}}{\partial r} \right|_c \varepsilon l - \left. \frac{\partial \tilde{D}}{\partial r} \right|_c l^2 = 0$$

$$\delta r \text{ の } 1 \text{ 次} \quad \left. \frac{\partial^2 \tilde{A}}{\partial r^2} \right|_c \varepsilon^2 + 2 \left. \frac{\partial^2 \tilde{H}}{\partial r^2} \right|_c \varepsilon l - \left. \frac{\partial^2 \tilde{D}}{\partial r^2} \right|_c l^2 = 0$$

Marginal stable circular orbits (radial stability)

δr の 0 次と 1 次の式をまとめる

MSCO 方程式：

$$\left[\frac{d\tilde{A}}{dr} \frac{d^2\tilde{D}}{dr^2} - \frac{d\tilde{D}}{dr} \frac{d^2\tilde{A}}{dr^2} \right]^2 - 4 \left[\frac{d\tilde{A}}{dr} \frac{d^2\tilde{H}}{dr^2} - \frac{d\tilde{H}}{dr} \frac{d^2\tilde{A}}{dr^2} \right] \left[\frac{d\tilde{D}}{dr} \frac{d^2\tilde{H}}{dr^2} - \frac{d\tilde{H}}{dr} \frac{d^2\tilde{D}}{dr^2} \right] = 0$$

M. Stute and M. Camenzind, Mon. Not. R. Soc. **336**, 831
(2002). の結果と同等

Marginal stable circular orbits (nonradial stability)

測地線方程式の θ 成分を Taylor 展開する。

$\delta\theta$ の 1 次

$$2C \left(r_c, \frac{\pi}{2} \right) \frac{\delta\ddot{\theta}}{\delta\theta} = \frac{\partial^2 \tilde{A}}{\partial\theta^2} \Bigg|_c \varepsilon^2 + 2 \frac{\partial^2 \tilde{H}}{\partial\theta^2} \Bigg|_c \varepsilon l - \frac{\partial^2 \tilde{D}}{\partial\theta^2} \Bigg|_c l^2$$

$$\frac{\delta\ddot{\theta}}{\delta\theta} < 0 \Leftrightarrow \frac{\partial^2 \tilde{A}}{\partial\theta^2} \Bigg|_c \varepsilon^2 + 2 \frac{\partial^2 \tilde{H}}{\partial\theta^2} \Bigg|_c \varepsilon l - \frac{\partial^2 \tilde{D}}{\partial\theta^2} \Bigg|_c l^2 < 0 \quad \text{stable}$$

$$\frac{\delta\ddot{\theta}}{\delta\theta} > 0 \Leftrightarrow \frac{\partial^2 \tilde{A}}{\partial\theta^2} \Bigg|_c \varepsilon^2 + 2 \frac{\partial^2 \tilde{H}}{\partial\theta^2} \Bigg|_c \varepsilon l - \frac{\partial^2 \tilde{D}}{\partial\theta^2} \Bigg|_c l^2 > 0 \quad \text{unstable}$$

Johannsen and Psaltis's model

Johannsen and Psaltis's modified Kerr 計量

[Johannsen T. and Psaltis D., Phys. Rev. D, **83** (2011) 124015.]

$$ds^2 = -[1 + h(r, \theta)] \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr \sin^2 \theta}{\Sigma} \times [1 + h(r, \theta)] dt d\phi \\ + \frac{\Sigma[1 + h(r, \theta)]}{\Delta + a^2 \sin^2 \theta h(r, \theta)} dr^2 + \Sigma d\theta^2 + \left[\sin^2 \theta \left(r^2 + a^2 + \frac{2a^2 Mr \sin^2 \theta}{\Sigma} \right) + h(r, \theta) \frac{a^2(\Sigma + 2Mr) \sin^4 \theta}{\Sigma} \right] d\phi^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 - 2Mr + a^2,$$

$$h(r, \theta) = \epsilon_3 \frac{M^3 r}{\Sigma^2}$$

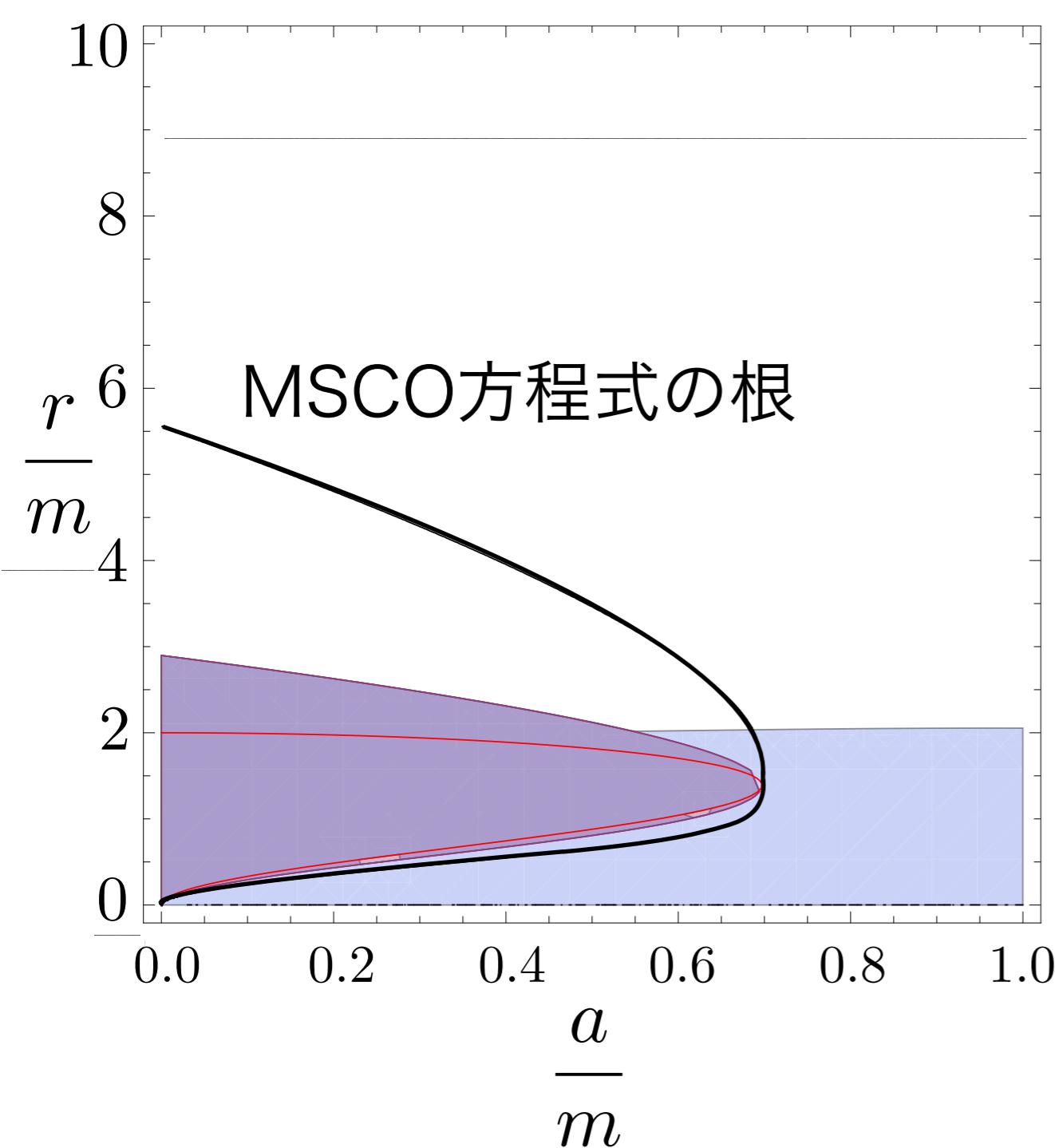
ϵ_3 はこれまでの一般相対性理論の観測からは制限されないパラメータ

ブラックホール近傍で Kerr とは異なる効果
→ ISCO に大きく影響

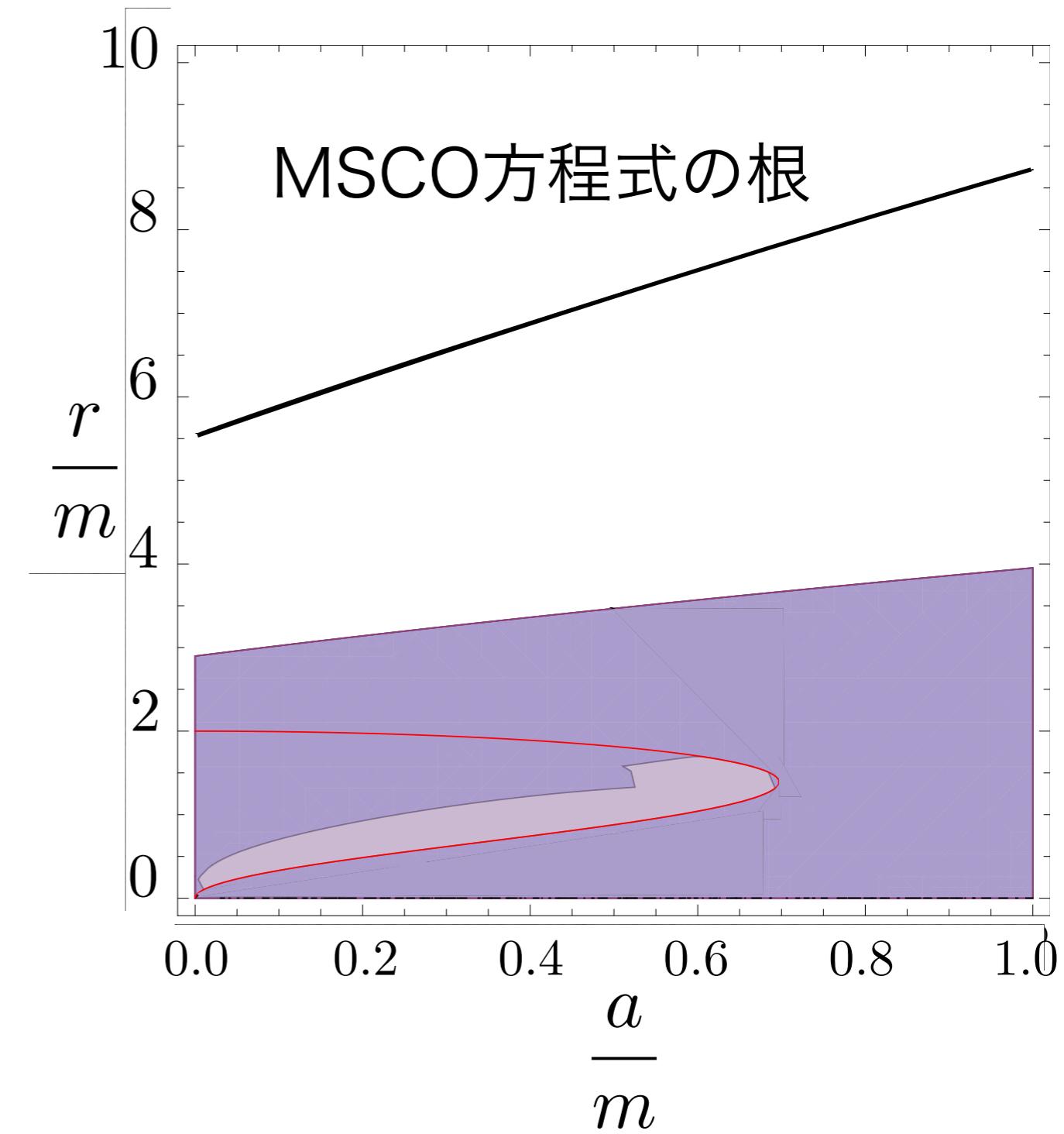
Johannsen and Psaltis's model

$\epsilon_3 = 2$ の場合

prograde の場合



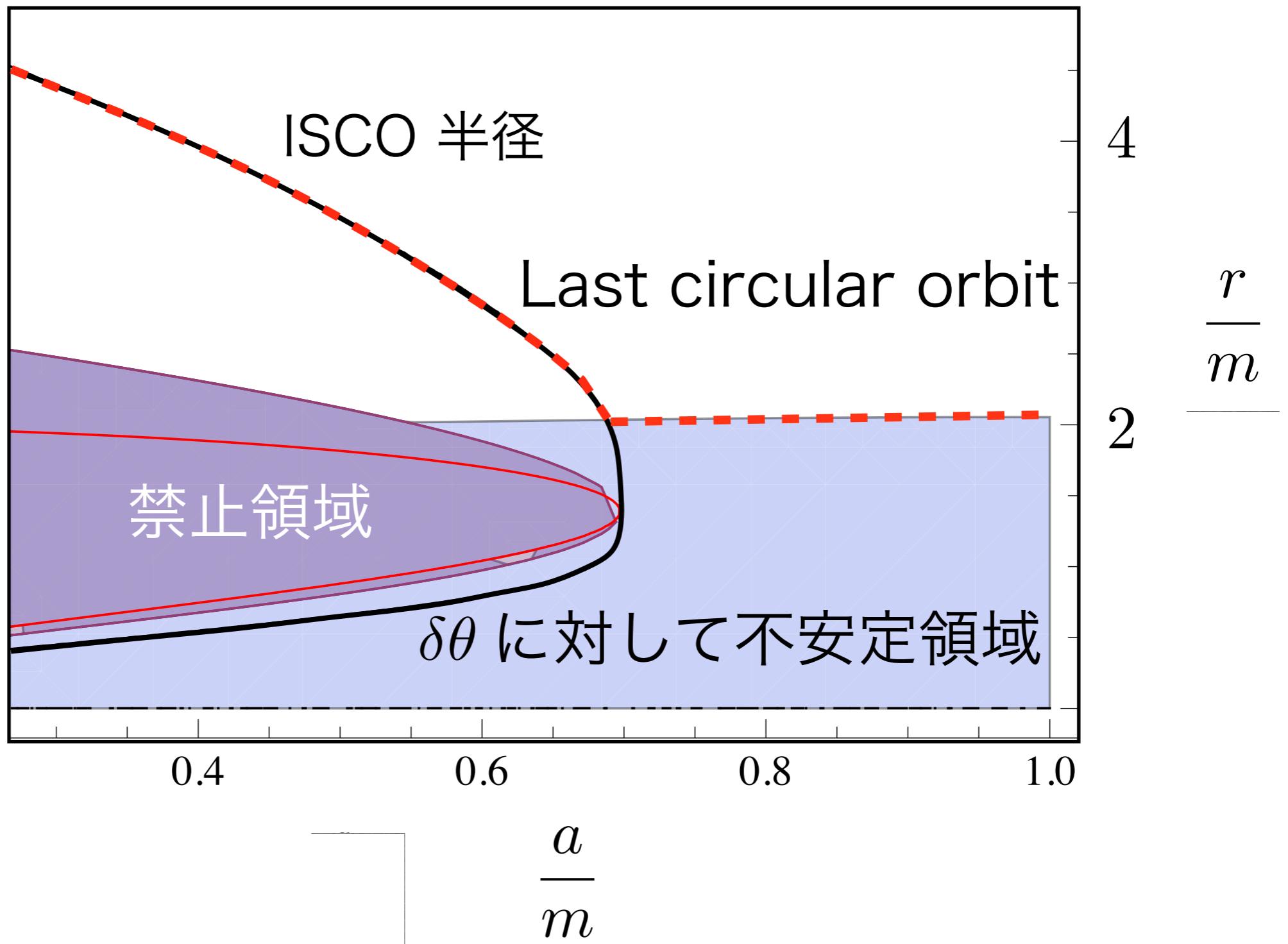
retrograde の場合



Johannsen and Psaltis's model

$\epsilon_3 = 2$ case

prograde の場合

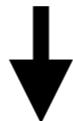


まとめ

先行研究では議論されていなかった定常軸対称時空におけるテスト粒子のMSCOの nonradial stability について議論し、安定性の条件を定式化した。

$$\left. \frac{\partial^2 \tilde{A}}{\partial \theta^2} \right|_c \varepsilon^2 + 2 \left. \frac{\partial^2 \tilde{H}}{\partial \theta^2} \right|_c \varepsilon l - \left. \frac{\partial^2 \tilde{D}}{\partial \theta^2} \right|_c l^2 < 0$$

Kerr時空とJohannsen&Psaltisのモデルにおけるテスト粒子のMSCOの nonradial stabilityを考察した。



Johannsen&Psaltisのモデルにおいて、
Last circular orbitはISCOよりも大きくなる場合がある。

今後の課題

Kerr-de Sitter時空、Kerr-Newman時空の
MSCO方程式、nonradial stabilityの計算

Nonradial perturbationに対して不安定な円軌道が
その後どのような軌道になるか？

軌道面を赤道面に限定しない場合への拡張。