

SUPERSYMMETRIC BRANES IN EXPANDING UNIVERSE

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□ Our results:

- ✿ **The supersymmetry is preserved in dynamical M2-brane background.**
- ☞ **This is similar to the result which has been obtained in 10-dimensional IIB theory (with dynamical D3-brane).**

(H. Kodama & K. Uzawa, JHEP 0507 061 (2005))

[1] Introduction

- **The dynamics of geometry on the basis of unified theory.**
- **There is viable unified theory at present is supergravity and string theory.**
- **In order to study the cosmological evolution, we use the time-dependent solution in string theory.**

- ★ **Cosmological model, geometrical structure**
 - ★ **Black hole in expanding universe**
 - ◎ **Supersymmetry**
-

How to analyze the evolution of background geometry for the unified theory?

- We will discuss their dynamics in the classical framework of supergravity.



low energy effective theory of string

⇒ We can study the time evolution of metric, the number of preserved SUSY from the viewpoint of general relativity.

- **Static solution in higher-dimensional supergravity**
- **11-dimensional supergravity**
(Freund & Rubin, Phys. Lett. B **97** (1980) **233**)
(Duff & Nilsson, & Pope, Phys.Lett. **129B** (1983) **39**)
(Duff & Stelle, Phys.Lett. **253B** (1991) **113**)
(Horava & Witten, Nucl.Phys. **B475** (1996) **94**)
(K. Becker & M. Becker, Nucl.Phys. **B477** (1996) **155**)
- **10-dimensional supergravity**
(Candelas & Horowitz & Strominger & Witten, Nucl.Phys. **B258** (1985) **46**)
(Strominger, Nucl.Phys. **B274** (1986) **253**)
(Duff & Lu, Phys.Lett. **B273** (1991) **409**)
(Bergshoeff & de Roo & Eyras & Janssen & van der Schaar, Nucl.Phys. **B494** (1997) **119**)
(Bergshoeff & Lozano & Ortin, Nucl.Phys. **B518** (1998) **363**)



membrane objects (*p*-brane) in gravity theory

The name “*p*-brane” is generally used to indicate a classical solution which is extended in *p* directions.

(Horowitz & Strominger, Nucl.Phys. B360 (1991) 197–209)

Then, it has *p* spacelike translational Killing vectors in the context of a theory containing gravity

⌚ Time dependent solutions in gravity theory

- **4d theory**

(Kastor & Traschen, Phys.Rev. D47 (1993) 5370–5375)

- **Higher dim gravity**

(Maki & Shiraishi, Class.Quant.Grav. 10 (1993) 2171–2178)

- **String theory**

(Gibbons, Lu, Pope, Phys.Rev.Lett.94 (2005) 131602)

(Kodama: Uzawa, JHEP 0507 (2005) 061)

(Binetruy, Sasaki, Uzawa, Phys.Rev.D80 (2009) 026001)

(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

(Minamitsuji & Ohta & Uzawa, Phys.Rev. D81 (2010) 126005)

◆ **Dynamical branes are related to**

- ***brane collision***

(**Gibbons & Lu & Pope, Phys.Rev.Lett. **94** (2005) 131602**)

(**Maeda & Minamitsuji & Ohta & Uzawa, Phys. Rev. D**82** (2010) 046007**)

(**Uzawa, Phys.Rev. D**90** (2014) 025024**)

- ***cosmic Big-Bang of our universe***

(**Chen, et al., Nucl.Phys. B**732** (2006) 118–135**)

(**Minamitsuji & Ohta & Uzawa, Phys. Rev. D**82** (2010) 086002**)

- ***black hole in expanding universe***

(**Maeda & Ohta & Uzawa, JHEP **0906** (2009) 051**)

(**Maeda & Nozawa, Phys.Rev. D**81** (2010) 044017**)

- **Time dependent Dp -brane solution**

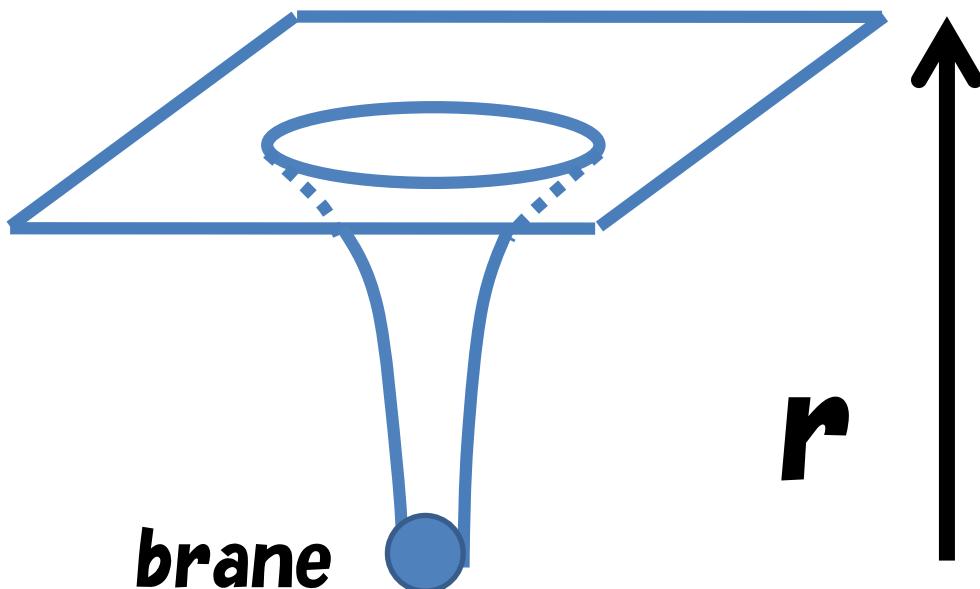
(G.W. Gibbons, H. Lu, C.N. Pope Phys.Rev.Lett.94 (2005) 131602)

(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

$$ds^2 = h^{-(D-p-3)/(D-2)} \eta_{\mu\nu} dx^\mu dx^\nu + h^{(p+1)/(D-2)} (dr^2 + r^2 d\Omega_{D-p-2}^2),$$

$$h(t, r) = c_1 t + c_2 + M r^{-D+p+3}, \quad F_{p+2} = d(h^{-1}) \wedge dt \wedge dx^1 \wedge \cdots \wedge dx^p,$$

$$e^\phi = h^{c/2}, \quad c^2 = 4 - 2(p+1)(D-p-3)(D-2)^{-1}$$



- **Asymptotically Time dependent vacuum spacetime**



- **Static spacetime at $r \rightarrow 0$**

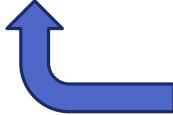
For trivial or vanishing dilaton, we find

(P. Binetruy, M. Sasaki, K. Uzawa, Phys.Rev. D 80 (2009) 026001)

$$ds^2 = h^{-(D-p-3)/(D-2)}(x, y)\eta_{\mu\nu}(X)dx^\mu dx^\nu + h^{(p+1)/(D-2)}(x, y)w_{ij}(Y)dy^i dy^j ,$$

$$R_{ij}(Y) = -\frac{\lambda(D-p-3)(p+1)}{2(D-2)} w_{ij}(Y)$$

$$h(x, y) = \frac{\lambda}{2} \boxed{x^\mu x_\mu} + c_\mu x^\mu + c + h_1(y)$$

 $-t^2 + (x^1)^2 + (x^2)^2 + \dots ,$

★ Cosmology:

(Binetruy, Sasaki, Uzawa, Phys.Rev.D80:026001,2009)

(Maeda, Ohta, Uzawa, JHEP 0906:051,2009)

We assume an isotropic and homogeneous three space in the four-dimensional spacetime.

Note that the time dependence in the metric comes from only one brane even if we consider several branes.

- Solutions in the original higher-dimensional theory (10D or 11D).

- For each case, the scale factor of 4-dimensional universe is given by $a(\tau) \propto \tau^\lambda$, where τ denotes the cosmic time.
- Since the three-dimensional spatial space of our universe stays in the transverse space to the brane, D-dimensional theory gives the fastest expansion of our universe.
- The power of the scale factor becomes $\lambda = (p+1)/(D+p-1) < 1$ for $D > 2$.
It is impossible to find the cosmological model that our universe is accelerating expansion .

● The characteristics of M-brane :

- Classical solution of 11-dim SUGRA
- Static limit of M-brane : Black brane
- M-brane on time-dependent background
⇒ Black hole in expanding universe

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)

★Outline my talk

- * *Property of static M-brane*
- * *Preserved supersymmetry*
- * *Singularities*
- * *Summary and comments*

[2] Property of static M-brane

(Duff & Stelle, Phys.Lett. **B253** (1991) 113–118)

(Gibbons & Townsend, Phys. Rev. Lett., **71**, 3754 (1993))

(Güven, Phys.Lett. **B276** (1992) 49–55)

◆ Background

(1) The *background has gravity, 4-form field strength, gravitino.*
⇒ *11-dimensional supergravity*

(2) Classical solutions.
ex) M2-brane, M5-brane, M-wave

Static M2-brane solution :

$$ds^2 = \left(1 + \frac{M}{r^6} \right)^{-2/3} \eta_{\mu\nu}(X) dx^\mu dx^\nu + \left(1 + \frac{M}{r^6} \right)^{1/3} (dr^2 + r^2 d\Omega_{(7)})$$

(1+2)-dim worldvolume spacetime

8-dim transverse space to brane

For static background, $\text{AdS}_4 \times S^7$, the background has the full supersymmetry.
(Freund & Rubin, Phys. Lett. B 97 (1980) 233)



The dynamical D3-brane solution preserves $\frac{1}{4}$ SUSY in the conifold background.

(H. Kodama & K. Uzawa, JHEP 0507 061 (2005))

★ Question

Do supersymmetries preserve in the dynamical M-brane background?

How to obtain the SUSY solutions ...

Dynamical case :

- ***pp-wave***

(Matthias Blau, et al.,
JHEP 0201 (2002) 047)

(M. Sakaguchi, K. Yoshida,
JHEP 0311 (2003) 030)

- ***D3-brane***

(H. Kodama & K. Uzawa,
JHEP 0507 061 (2005))



[3] Preserved supersymmetry (11d SUGRA)

The 11-dimensional action is invariant under local SUSY transformations :

$e^A{}_M$: graviton Ψ_M : gravitino,
 A_{MNP} : 3-form gauge potential

$$\delta e^A{}_M = \bar{\varepsilon} \Gamma^A \Psi_M ,$$

$$\delta A_{MNP} = -3 \bar{\varepsilon} \Gamma_{[MN} \Psi_{P]} ,$$

$$\begin{aligned}\delta \Psi_M &= D_M \varepsilon \\ &= \left[\nabla_M + \frac{1}{12 \cdot 4!} (\Gamma_M F_{MNPQ} \Gamma^{MNPQ} - 12 F_{MNPQ} \Gamma^{NPQ}) \right] \varepsilon\end{aligned}$$

★**Supersymmetry in dynamical M2-brane**

- The only fermionic field is the gravitino Ψ_M .
- Supersymmetric configuration is a nontrivial solution to the Killing spinor equation :

$$\delta \Psi_M = 0 ,$$

$$\Rightarrow \left[\nabla_M + \frac{1}{12 \cdot 4!} \left(\Gamma_M F_{MNPQ} \Gamma^{MNPQ} - 12 F_{MNPQ} \Gamma^{NPQ} \right) \right] \varepsilon = 0$$

★ Ansatz for fields

- **11-dim metric** **(1+2)-dim worldvolume spacetime**

$$ds^2 = g_{MN} dx^M dx^N = h^{-2/3}(x, r) \eta_{\mu\nu}(X) dx^\mu dx^\nu$$

$$+ h^{1/3}(x, r) (dr^2 + r^2 u_{ab}(Z) dz^a dz^a)$$



8-dim transverse space to brane

- **4-form field strength & gravitino**

$$F_{r\mu\nu\rho} = \pm h^{-2} \partial_r h \varepsilon_{\mu\nu\rho}, \quad \Psi_M = 0$$

11-dimensional gamma matrices satisfying

$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2g^{MN},$$

$$\Gamma^\mu = h^{1/3} \gamma^\mu, \quad \Gamma^r = h^{-1/6} \gamma^r, \quad \Gamma^a = r^{-1} h^{-1/6} \gamma^a$$

and we define

$$\gamma_{(3)} = \gamma_0 \gamma_1 \gamma_2$$

★ Killing spinor equation

$$\bar{\nabla}_\mu \varepsilon = \left[\partial_\mu + \frac{1}{6} \boxed{\partial_\nu \ln h \gamma^\nu}_\mu - \frac{1}{6} h^{-3/2} \partial_r h \gamma^\mu \gamma^r (1 \pm \gamma_{(3)}) \right] \varepsilon,$$

$$\bar{\nabla}_r \varepsilon = \left[\partial_r - \frac{1}{12} h^{-1/2} \boxed{\partial_\nu h \gamma^\nu} \gamma^r + \frac{1}{6} \chi h^{-1} \partial_r h \gamma_{(3)} \right] \varepsilon,$$

$$\bar{\nabla}_a \varepsilon = \left[{}^Z \nabla_a - \frac{r}{12} h^{-1/2} \boxed{\partial_\nu h \gamma^\nu} \gamma_a - \frac{r}{12} h^{-1} \partial_r h \gamma^r \gamma_a (1 \pm \gamma_{(3)}) \right] \varepsilon$$

- If the function $h(x, r)$ is included in the spinor $\varepsilon = h^{1/6} \varepsilon_0$ (ε_0 : constant Killing spinor), we find ...

- **Solution for dynamical background**

$$\partial_\mu h \gamma^\mu \varepsilon = 0, \quad (1 \pm \gamma_{(3)}) \varepsilon = 0$$

- (i) **Induced effective mass for the spinor field**

$$\sim h^{-1} \partial_\mu h$$

- (ii) **This mass scale diverges at the naked singularity where the function h vanishes.**

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- (iii) ***the degree of supersymmetry breaking increases as the universe approaches the singularity.***
 - (iv) ***In the region with a large warp factor, the SUSY breaking becomes negligible.***

[4] Singularity in dynamical brane

(Binetruy, Sasaki, Uzawa, Phys.Rev.D80 026001 (2009))

- ☀ It is of great significance to understand the cosmological backgrounds profoundly.
- ☠ There is a naked singularity in the dynamical brane background due to ...
 - (i) the divergence of non-trivial dilaton
(This also appears in the static brane).
 - (ii) the time-dependence in the theory.

✿ M2-brane

(Duff & Stelle, Phys.Lett. **B253** (1991) 113–118)
(Güven, Phys.Lett. **B276** (1992) 49–55)

✎ matter (bosonic) :
gravity, 4-form field strength

	M2										t, x^1, x^2
	0	1	2	3	4	5	6	7	8	9	10
M2	o	o	o								
x^M	t	x^1	x^2	r	z^1	z^2	z^3	z^4	z^5	z^6	z^7

◆ Dynamical M2-brane background

(Binetruy & Sasaki & Uzawa, Phys.Rev. D80 (2009) 026001)

(Maeda & Ohta & Uzawa, JHEP 0906 (2009) 051)

(1+2)-dim worldvolume spacetime

$$ds^2 = \left(cu + \frac{M}{r^6} \right)^{-2/3} (-2du\,dv + dy^2)$$

$$+ \left(cu + \frac{M}{r^6} \right)^{1/3} [dr^2 + r^2 d\Omega_{(7)}^2] ,$$

$$u = \frac{1}{\sqrt{2}} (t - x) , \quad v = \frac{1}{\sqrt{2}} (t + x) ,$$

8-dim transverse space to brane

∞ Dynamics of geometry

- Geodesic equation :

$$\frac{d^2 r}{ds^2} + \Gamma^r_{MN} \frac{dx^M}{ds} \frac{dx^N}{ds} = 0$$

- ⊗ We can set a regular and smooth initial data for the M2-brane.

- **Solution of geodesic equation ($M=0$)**

$$u(s) = s_1(s - s_0)^3,$$

$$r(s) = r_0 \ln(s - s_0) + r_1$$

r_i , s_i ($i=0,1$) are constants.

**The singularity in the dynamical background
is not spacelike but null for $M=0$.**

**For $M \neq 0$, null geodesics reach the timelike
singularity in finite affine parameter.**

(K. Maeda & K. Uzawa, Phys.Rev.D93, 044003 (2016))

[5] Summary and comments

- (1) The dynamical M2-brane background preserved the supersymmetry.
- (2) The solutions of field equations cannot give a homogeneous expansion unless supersymmetries are completely broken.
- (3) For dynamical M2-brane, we can set smooth initial data evolving into a curvature singularity.