無衝突粒子系のスピンドル重力崩壊

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Introduction

What We Do

◎Spindle collapse with many collisionless particles

◎The system treated here

- Axi-symmetric on average but not exactly axi-sym.
  because of the random distribution of particles
- The same reference continuum as in Shapiro and Teukolsky(1991)

◎What we focus on

- Singularity formation
- Black hole formation
- Comparison with Sphapiro-Tekolsky(ST)

◎What we do not(cannot) address

- Generality of the results
- Event horizon
- Strength of the singularity

Non-spherical Collapse

◎Cosmic Censorship Conjecture(CCC)[Penrose1969]

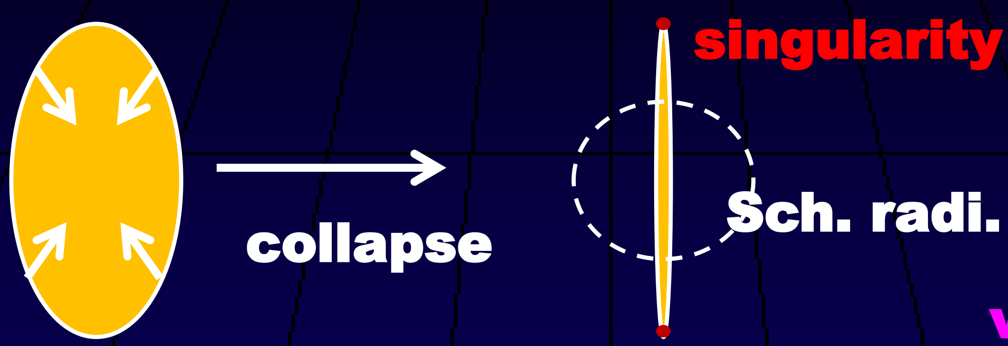
・“For spacetimes which contain physically reasonable matter fields and develop from generic nonsingular initial data, singularity should be clothed by a black hole horizon”

◎Hoop Conjecture[Thorne(1972)]

- “Black holes with horizons form when and only when a mass M gets compacted into a region whose circumference in every direction is C$\lesssim$4πM”

◎If hoop conjecture is correct

- Aspherical collapse might lead to naked singularity



violation of CCC?

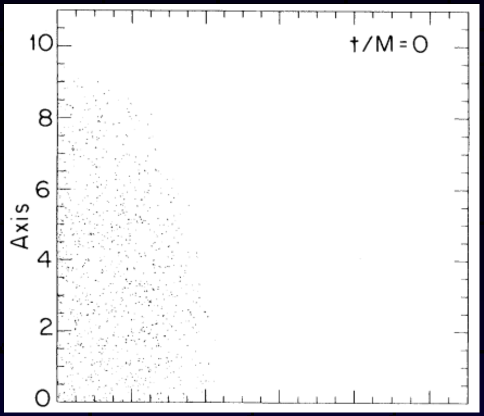
Shapiro and Teukolsky

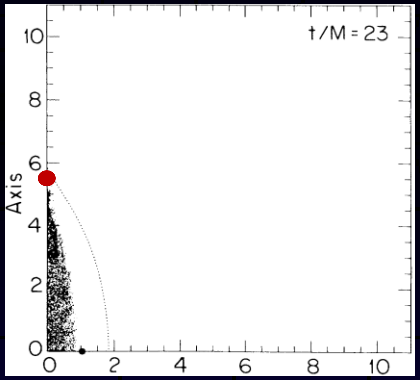
◎Axial sym. gravitational collapse

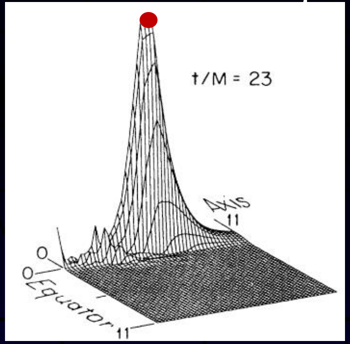
- Exactly axi-symmetric(2+1 simulation)
- Collisionless ring sources

Kretschmann curvature invariant

$$K_{\rm inv} = R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho}$$







- No horizon
- The Larger value of max $K_{\rm inv}$ for a finer resolution
- The calculation breaks down because of the “singularity”
- The position of max $K_{\rm inv}$ is outside the matter distribution

Singularity?

◎What do we expect from the singularity? The end?

- Extremely high curvature → Quantum gravity...?
- Unknown high energy particle physics might take place

Naked "singularity" is
a window into a new physics beyond our knowledge!

◎How to numerically investigate the singularity?

- We cannot predict the causal future of the singularity in principle.
  How to discuss whether it is naked or not without analyticity?
- We are not really interested in the naked singularity
  but the naked very high curvature region
- In the simulation, the singularity is automatically smoothed out
  due to finite resolution→the system can be practically analyzed

Table of Contents

Simulation Method

Previous Works and Ours

◎Simulation with collisionless particles

- Axisymmetric collapse[Shapiro-Teukolsky(1991)]
- Full 3D with BSSN[Shibata(1999)]
- Higher dim. spacetime axisymmetric[Yamada-Shinkai(2011)]

◎Our work

- Basically follow [Shibata(1999)]
- Simulate a similar situation as [Shapiro-Teukolsky(1991)]
- Compare the results with [Shapiro-Teukolsky(1991)]

Outline of the Simulation

◎2nd order leap frog with BSSN (with time filtering)

◎Maximal slice condition for α(lapse)

◎Flow of evolution

repeat

1. Evolve geometrical variables except for α(lapse)
2. Evolve particle variables solving geodesic eqs.
   *2nd order interpolation for geometry at particle position
3. Set energy momentum tensor
   *No α-dependence in our expression
4. Clean the Hamiltonian constraint
5. Set α by solving the elliptic eq. of the maximal slice condition

Geometrical Variables

◎**Metric**

$$\mathrm{d}s^2 = -\alpha^2\,\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i \mathrm{d}t)(\mathrm{d}x^j + \beta^j \mathrm{d}t)$$

$$\gamma_{ij} = \mathrm{e}^{4\psi}\,\tilde{\gamma}_{ij} \text{ with } \det\tilde{\gamma} = 1$$

◎**Projection tensor**

$$\gamma_\mu{}^\nu = n_\mu n^\nu + g_\mu{}^\nu \text{ with unit normal } \; n_\mu \coloneqq -\alpha(\mathrm{d}t)_\mu$$

◎**Extrinsic curvature**

$$K_{ij} = -\gamma_i{}^\mu \gamma_j{}^\nu \nabla_\mu n_\nu = \mathrm{e}^{4\psi}\,\tilde{A}_{ij} + \frac{1}{3} K \gamma_{ij}$$

◎**Equations based on BSSN scheme to be solved**

Stress-energy Tensor

◎For a point particle system

$$E = n_\mu n_\nu T^{\mu\nu} = \sum_p m_p \Gamma_p \frac{\delta^3(\vec{x}-\vec{x}_p)}{\sqrt{\gamma}}$$

$$J^i = -n_\nu \gamma^i{}_\mu T^{\mu\nu} = \sum_p m_p \Gamma_p V_p^i \frac{\delta^3(\vec{x}-\vec{x}_p)}{\sqrt{\gamma}}$$

$$S^{ij} = \gamma^i{}_\mu \gamma^j{}_\nu T^{\mu\nu} = \sum_p m_p \Gamma_p V_p^i V_p^j \frac{\delta^3(\vec{x}-\vec{x}_p)}{\sqrt{\gamma}}$$

with particle 4-velocity

$$u_p^\mu = \Gamma_p (n^\mu + V_p^\mu)$$

◎No α-dependence

◎Smoothing

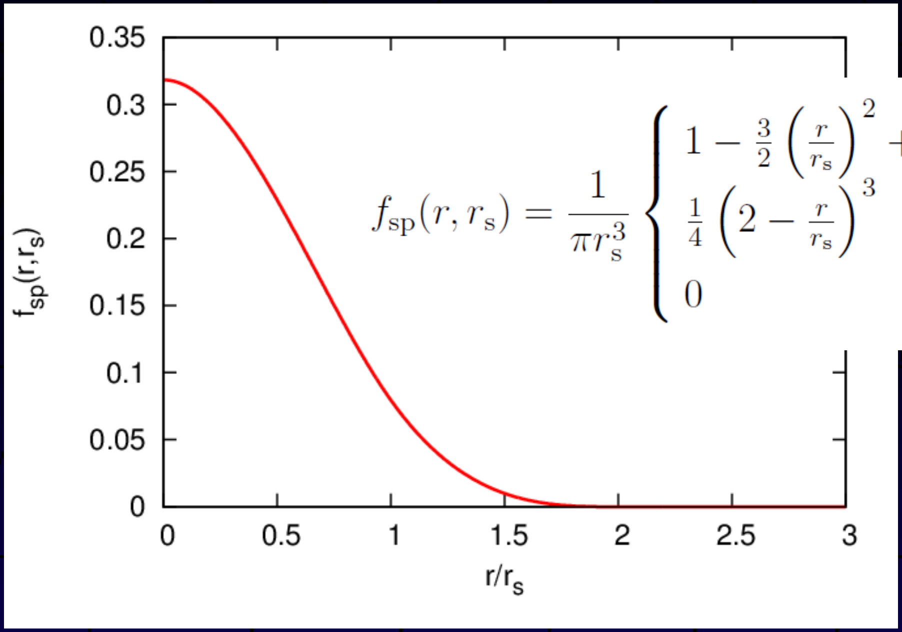
- $\delta^3(\vec{x}-\vec{x}_a) \to f_{\rm sp}(|\vec{x}-\vec{x}_a|, r_s)$

Spline Kernel

◎Smoothing

- $\delta^3(\vec{x}-\vec{x}_a) \to f_{\rm sp}(|\vec{x}-\vec{x}_a|, r_s)$

- r_s **gives typical size of each particle**



$$f_{\rm sp}(r, r_{\rm s}) = \frac{1}{\pi r_{\rm s}^3}\begin{cases} 1-\frac{3}{2}\left(\frac{r}{r_{\rm s}}\right)^2+\frac{3}{4}\left(\frac{r}{r_{\rm s}}\right)^3 & \text{for } 0 \le \frac{r}{r_{\rm s}} \le 1 \\ \frac{1}{4}\left(2-\frac{r}{r_{\rm s}}\right)^3 & \text{for } 1 \le \frac{r}{r_{\rm s}} \le 2 \\ 0 & \text{for } 1 < \frac{r}{r_{\rm s}} \end{cases}$$

◎Specific form of the kernel is not essential

Geodesic Equation

◎3+1 decomposition of geodesic equations

[Vincent et.al(1208.3927)]

$$\frac{\mathrm{d}\tau_p}{\mathrm{d}t} = \alpha/\Gamma_p$$

$$\frac{\mathrm{d}x_p^i}{\mathrm{d}t} = -\beta^i + \alpha V^i$$

$$\frac{d\Gamma_p}{\mathrm{d}t} = \Gamma_p V_p^i (\alpha K_{ij} V_p^j - \partial_i \alpha)$$

$$\frac{\mathrm{d}V_p^i}{\mathrm{d}t} = \alpha V_p^j \left[V_p^i \left(\partial_j \ln\alpha - K_{jk} V_p^k \right) + 2K^i{}_j - V_p^k \Gamma^i_{jk} \right] - \gamma^{ij}\partial_j\alpha - V_p^j \partial_j \beta^i$$

with 2nd order interpolation for geometry at particle position

Outline of the Simulation

◎2nd order leap frog with BSSN (with time filtering)

◎Maximal slice condition for α(lapse)

◎Flow of evolution

repeat

1. Evolve geometrical variables except for α(lapse)
2. Evolve particle variables solving geodesic eqs.
   *2nd order interpolation for geometry at particle position
3. Set energy momentum tensor
   *No α-dependence in our expression
4. Clean the Hamiltonian constraint
5. Set α by solving the elliptic eq. of the maximal slice condition

Constraint Cleaning

◎Hamiltonian constraint

$$\tilde{D}_i\tilde{D}^i\psi = -\tilde{D}_i\psi\tilde{D}^i\psi + \frac{1}{8}\tilde{R} - \mathrm{e}^{4\psi}\left(\frac{1}{8}\tilde{A}_{ij}\tilde{A}^{ij} + 2\pi E\right)$$

◎Cleaning

- Perform a few iteration steps to solve it(SOR method)

Others

◎BSSN with 2nd order finite differences

◎Maximal slice: $K = 0 \Rightarrow$ elliptic eq. for α

◎Numerical region: $0 \leq X, Y, Z \leq L$ (X, Y, Z:Cartesian)

◎Kreiss-Oligar dissipation term

Initial Data Construction

Initial Data

◎Assumptions

- Conformally flat: $dl^2 = \Psi^4 \delta_{ij} dx^i dx^j$

- Momentarily static: $K_{ij} = 0$

◎Momentum constraint

- Trivially satisfied by $J^i = 0 \;\Leftarrow\; V_p^i = 0, \Gamma_p = 1$

◎Hamiltonian constraint

$$\Delta \Psi = -2\pi E \Psi^5 = -2m \sum_p f_{sp}(|\vec{x} - \vec{x}_p|, r_s)/\Psi \qquad \text{with } \Psi = \mathrm{e}^{\psi}$$

- It can be numerically solved for given particle distribution

Reference Continuum

◎The same reference continuum as ST

[Nakamura et. al(PRD38,2972)]

◎Energy density $\bar{E}$ and the conformal factor $\bar{\Psi}$

- Assumption: $\frac{1}{2}\bar{E}\bar{\Psi}^5 = E_{\mathrm{N}} = \frac{3M_{\mathrm{N}}}{4\pi a^2 b} \quad \text{for } \frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} \leq 1$

$$= 0 \quad \text{for } \frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} > 1$$

- for $\Phi := 1 - \bar{\Psi}$

Hamiltonian constraint $\Rightarrow \Delta\Phi = 4\pi E_{\mathrm{N}}$

$$\Phi = -\frac{3M_{\mathrm{N}}}{2be}\beta - \frac{3M_{\mathrm{N}}}{4b^3e^3}(\beta - \sinh\beta\cosh\beta)R^2 - \frac{3M_{\mathrm{N}}}{2b^3e^3}(\tanh\beta - \beta)z^2$$

where $\sinh\beta = \frac{be}{a}$, $e = \sqrt{1 - a^2/b^2}$, $R = \sqrt{x^2 + y^2}$

◎The continuum initial data set is analytically given

Continuum to Particles

◎Mass of the continuum

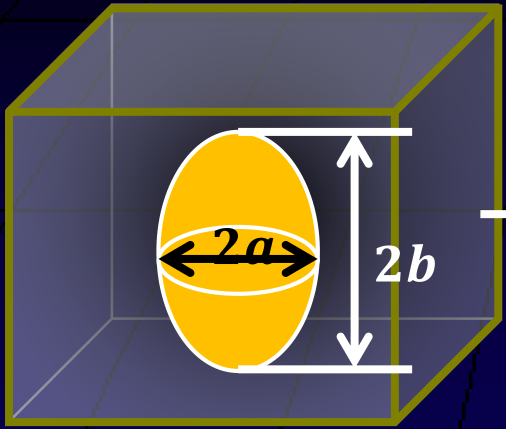
$$\lim_{r\to\infty}\bar{\Psi} = 1 - \lim_{r\to\infty}\Phi = 1 + \frac{M_\mathrm{N}}{r} \Rightarrow \text{ total mass: } M = 2M_\mathrm{N}$$

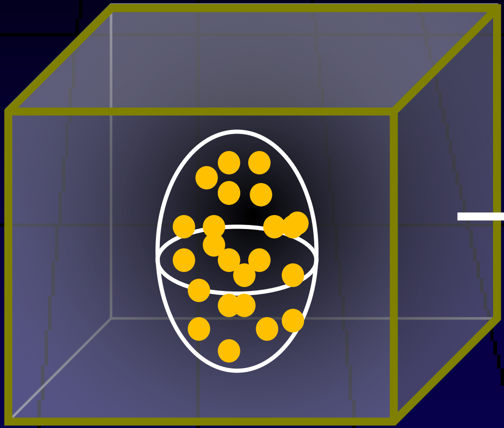
$$\text{rest mass: } M_0 = \int \bar{E}\bar{\Psi}^6 \mathrm{d}^3x = 2M_\mathrm{N} + \frac{6}{5}\frac{M_N^2}{be}\ln\frac{1+e}{1-e}$$

◎Particle distribution

- Number of particles ΔN in a grid box ΔV

$$\Delta N = \frac{\bar{E}\bar{\Psi}^6\Delta V}{m} = \frac{E_N\bar{\Psi}\Delta V}{m} \qquad \text{with } m = \frac{M_0}{N}$$





Numerically Solve Hamiltonian constraint for Ψ

Results(1)
Comparison with ST

Convergence Check

◎**Numerical domain: an octant region with reflection sym.**

$$0 < x, y, z < L \text{ with } L/M = 20$$

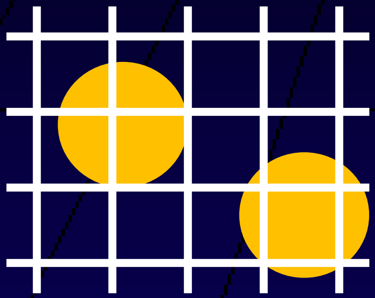
◎**Parameters for the spheroid(the same as ST)**

$$b/M = 10, e = 0.9$$

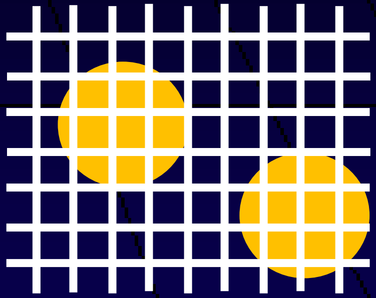
◎**Numerical parameters for convergence check**

Number of particles $N = 125000$

Particle size $r_s = 2L/75$

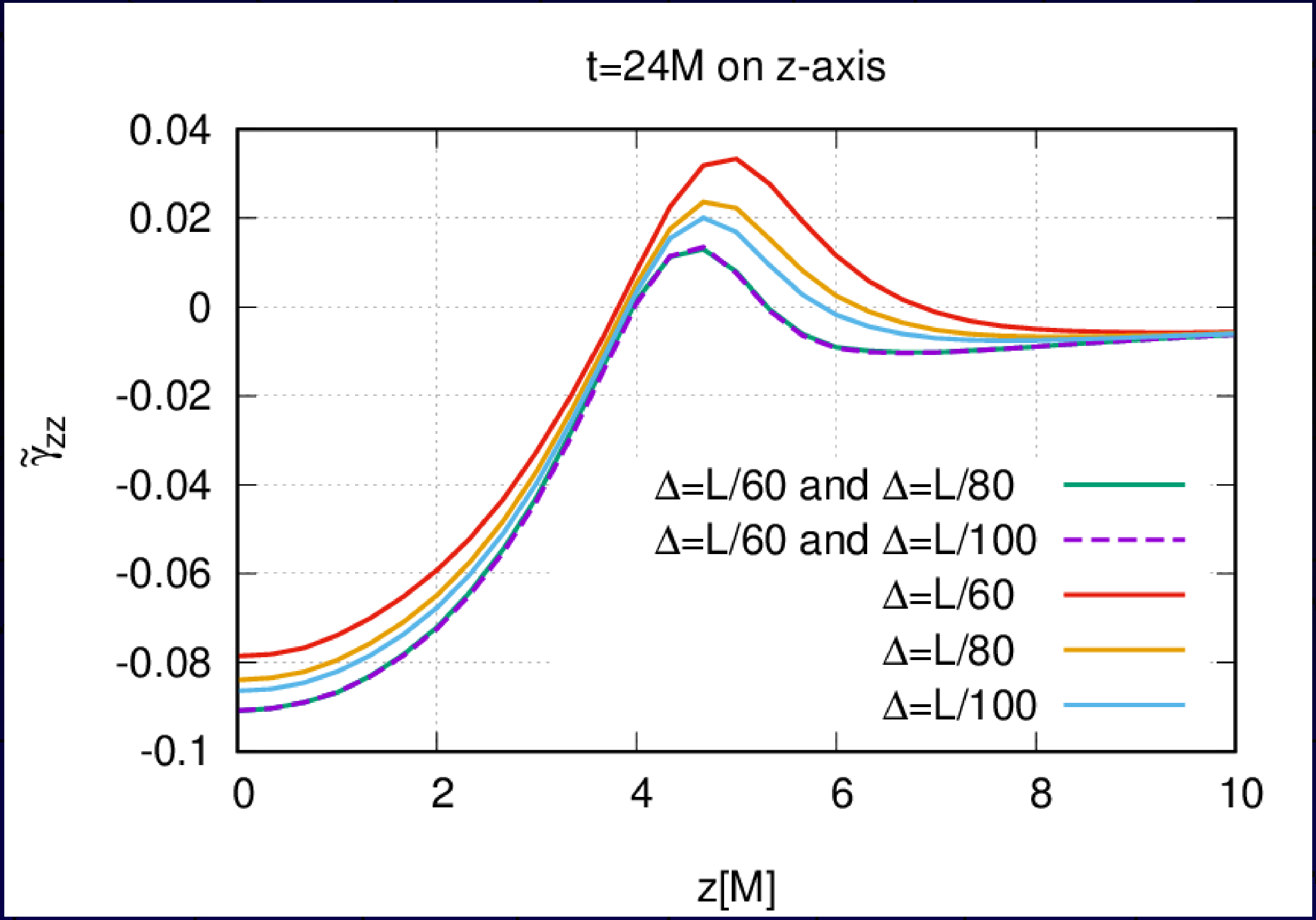


finer resolution



Convergence Check

◎Clear 2nd order convergence



Resolution Dependence

◎If we fix the particle size, the resolution for the geometry is limited by the particle size

◎Numerical parameters for main calculations

Finest: grid interval $\Delta = L/120$, $N = 10^6, r_s = L/75$

Others: $N \propto \Delta^{-3}, r_s \propto \Delta$

Parameters

◎Numerical domain: an octant region with reflection sym.

$0 < x, y, z < L$ **with** $L/M = 20$

◎Parameters for the spheroid(the same as ST)

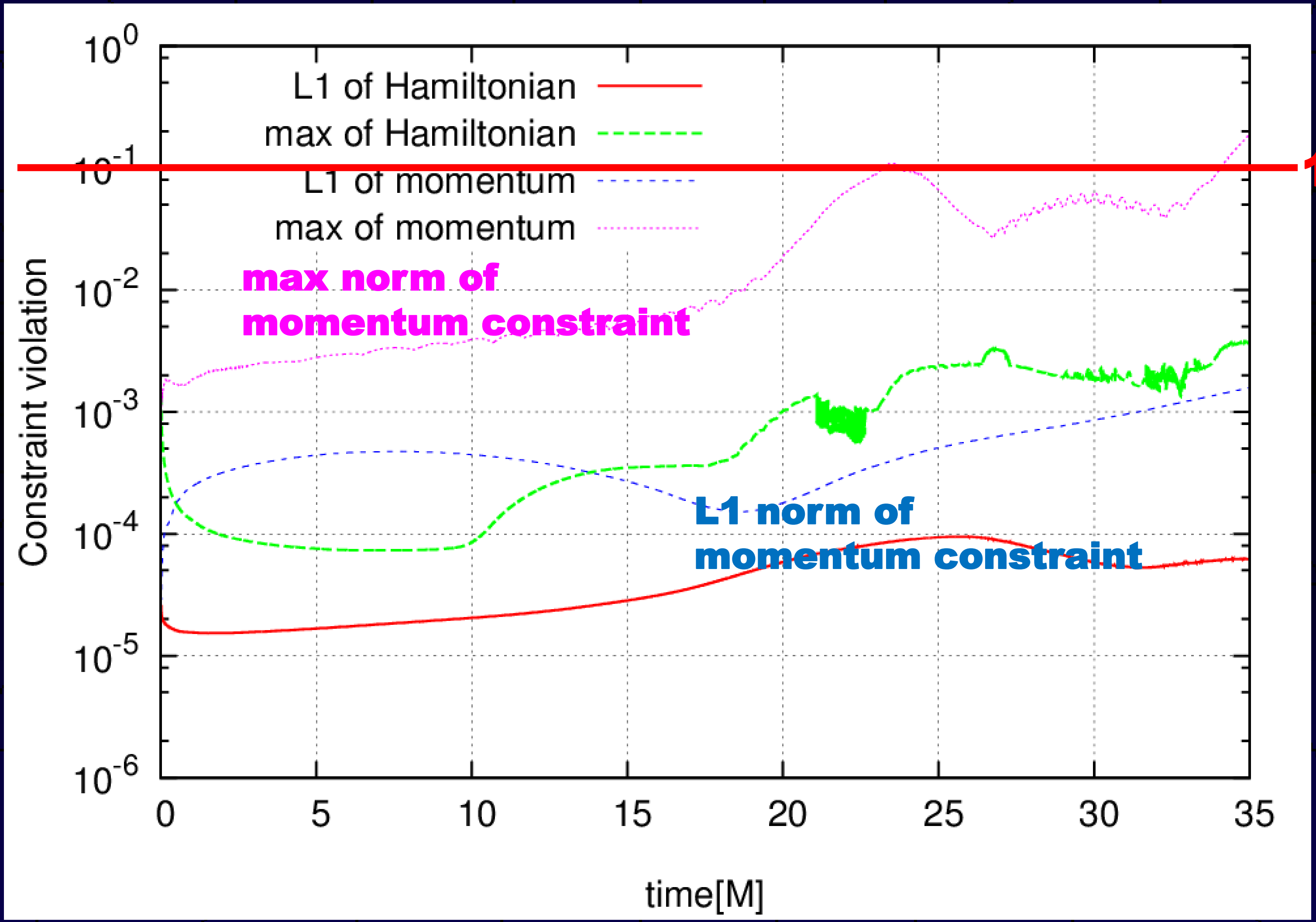
$b/M = 10, e = 0.9$

◎Numerical parameters

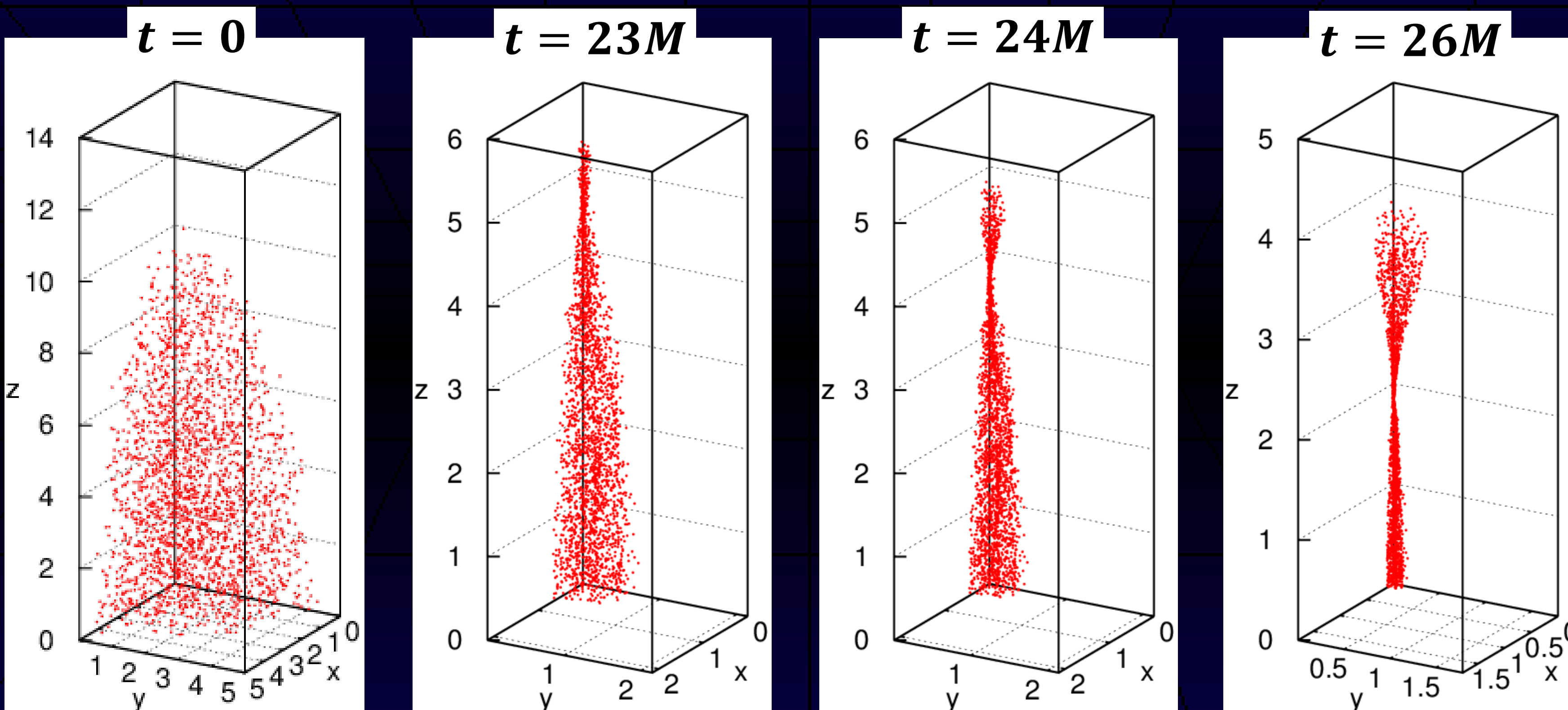
Finest: grid interval $\Delta = L/120$**,** $N = 10^6, r_s = L/75$

Others: $N \propto \Delta^{-3}, r_s \propto \Delta$

Constraint Violation

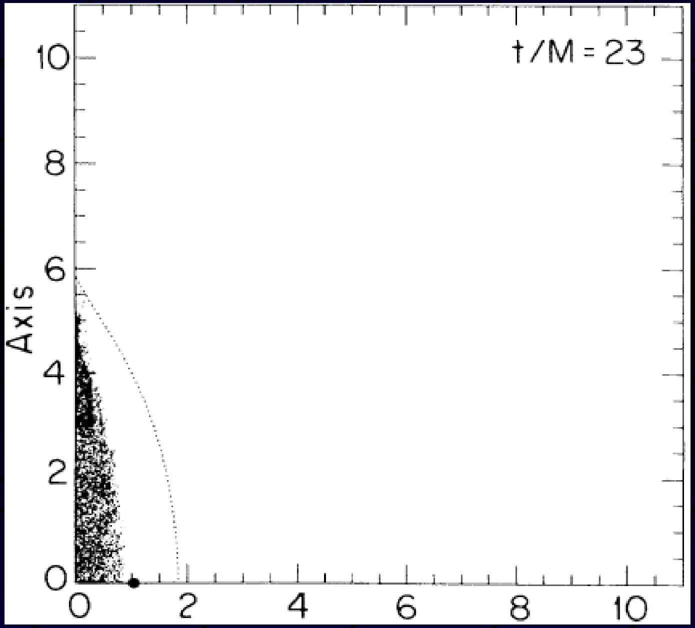


Snapshots: particles

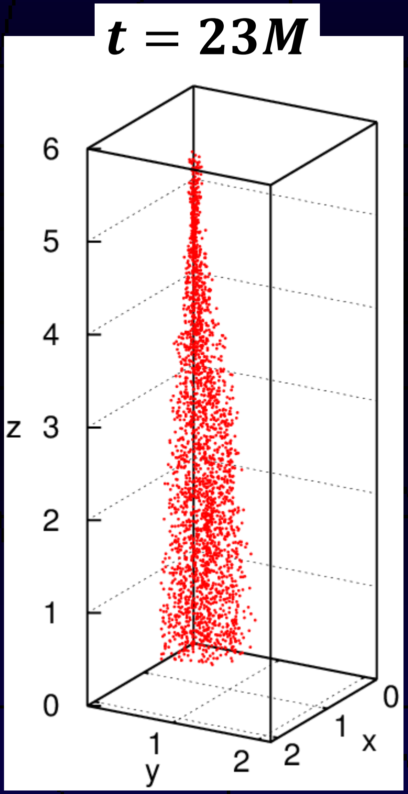


Apparent Shape at t=23M

Shapiro-Teukolsky



Our simulation

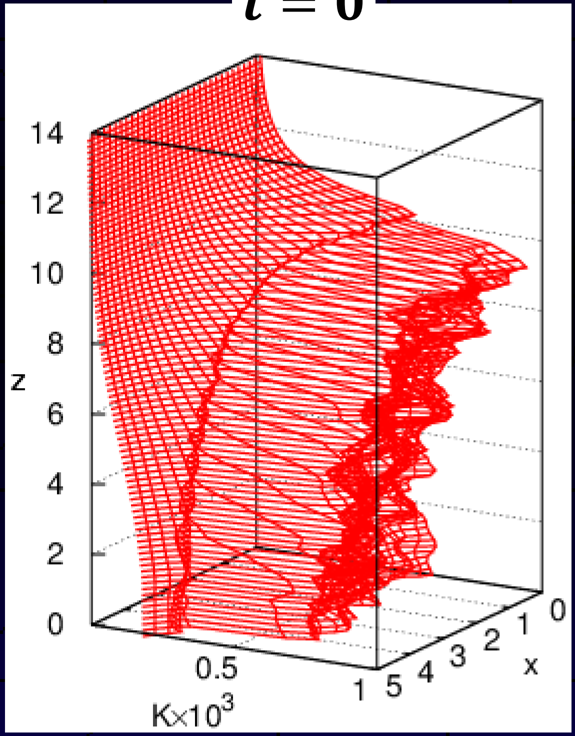


***Note: shift gauge condition is different from each other**

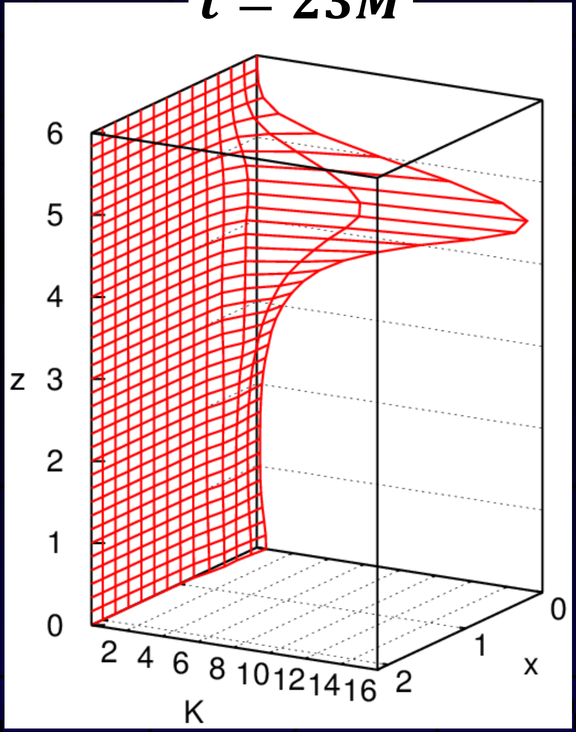
Snapshots: Kretschmann

◎On y=0 plane ◎Peak on z-axis

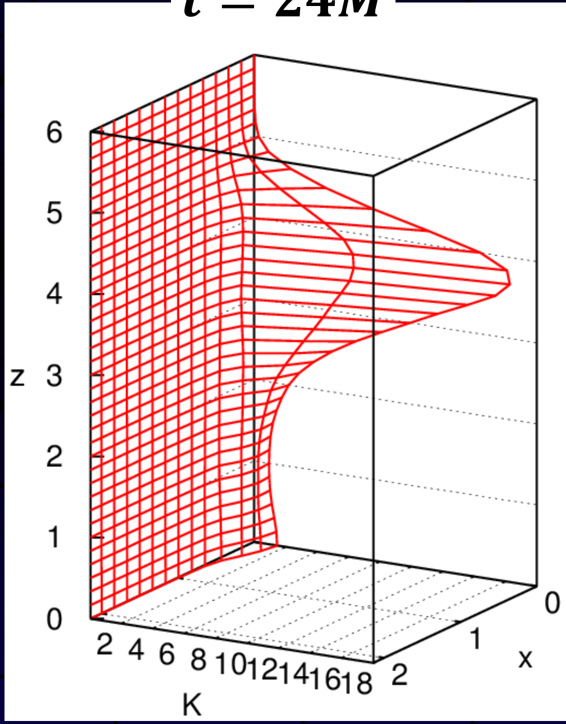
$t = 0$



$t = 23M$



$t = 24M$



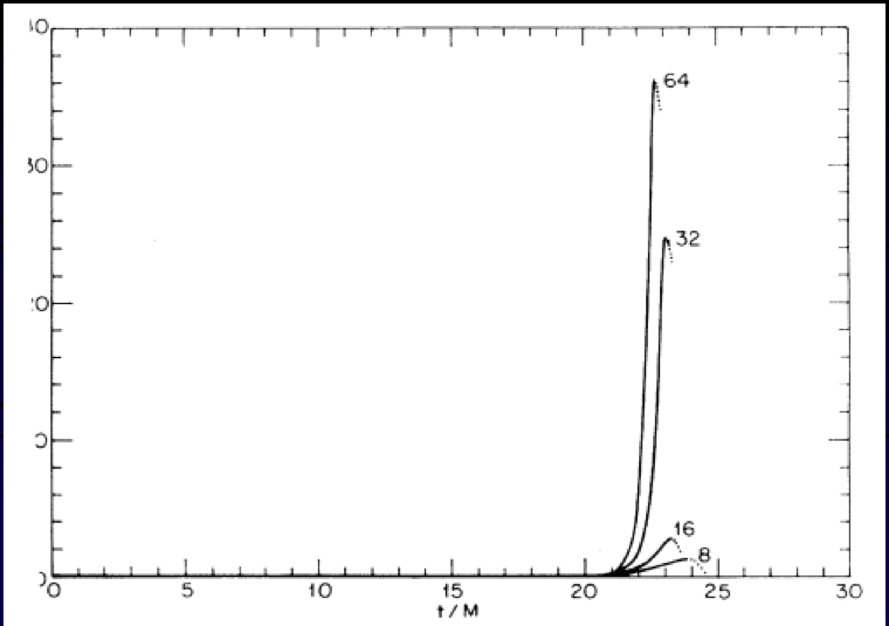
Evolution of K_{peak}

◎K_{peak} : peak value of Kretschmann inv. at each time

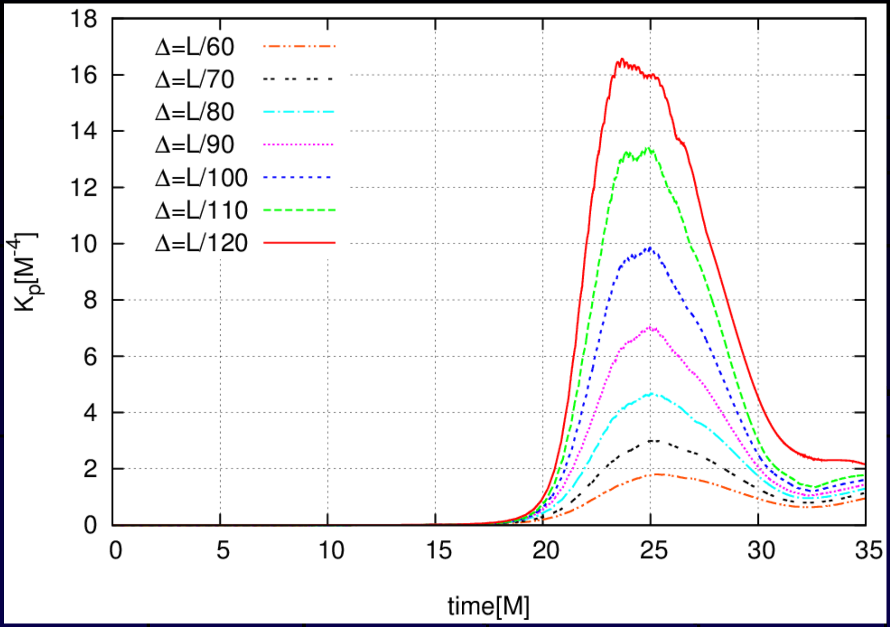
◎Value of K_{peak} starts to increase around t~20M

◎The faster growth for the finer resolution.

Shapiro-Teukolsky



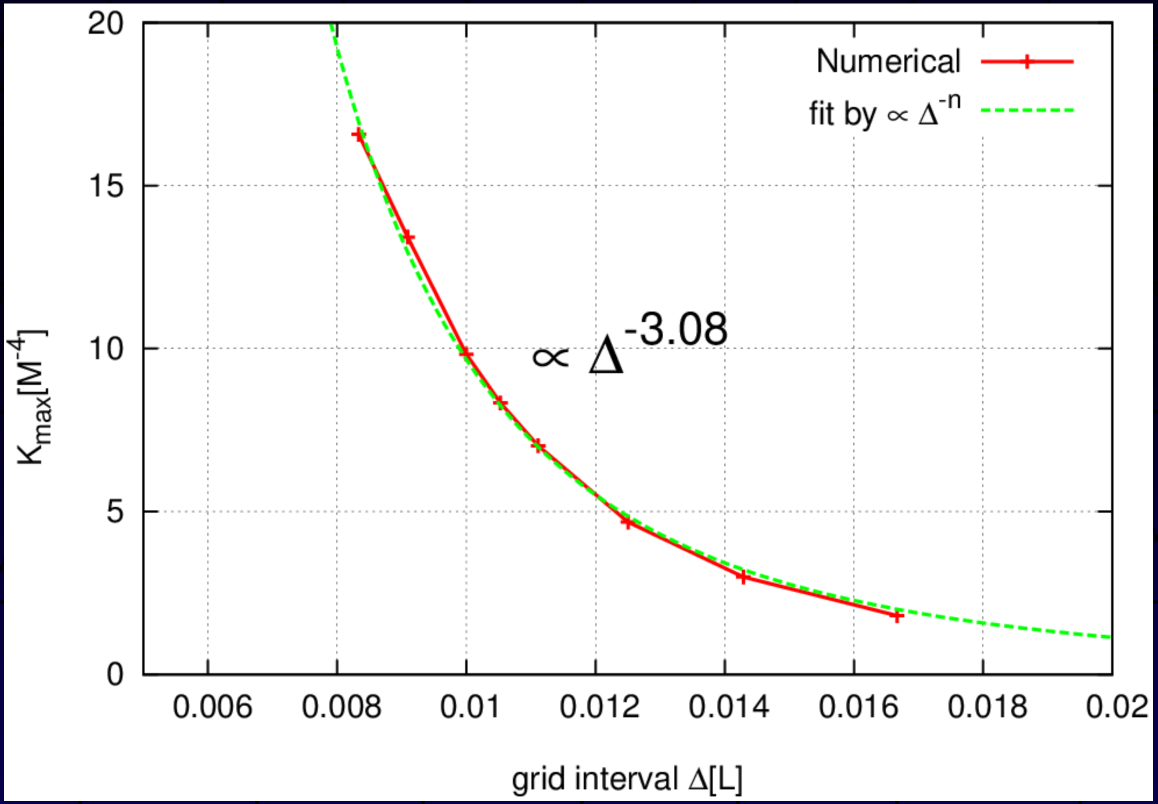
Our simulation



Resolution Dependence

◎K_{max} : maximum value of K_{peak} for one realization

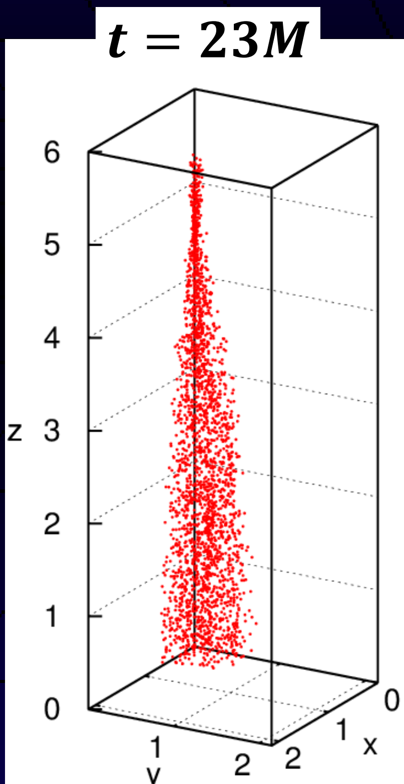
◎The larger value of K_{max} for the finer resolution similarly to ST



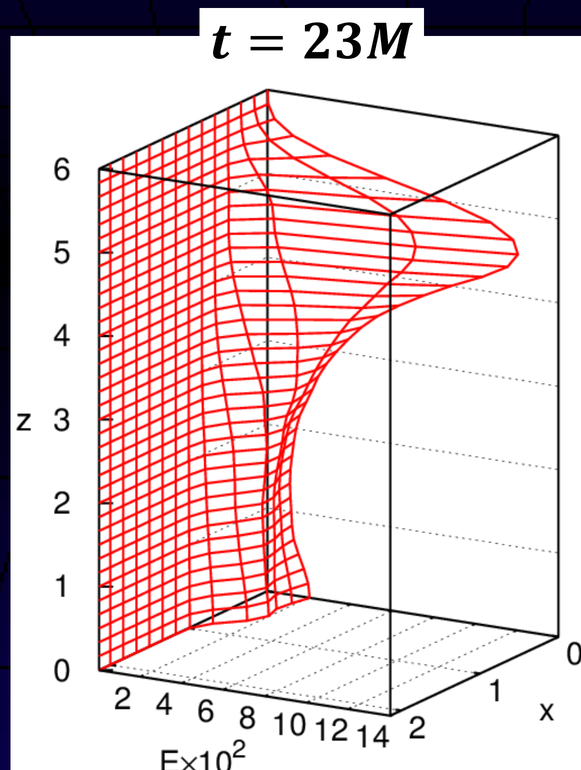
Peak Position

◎Shape of Kretschmann traces the density distribution

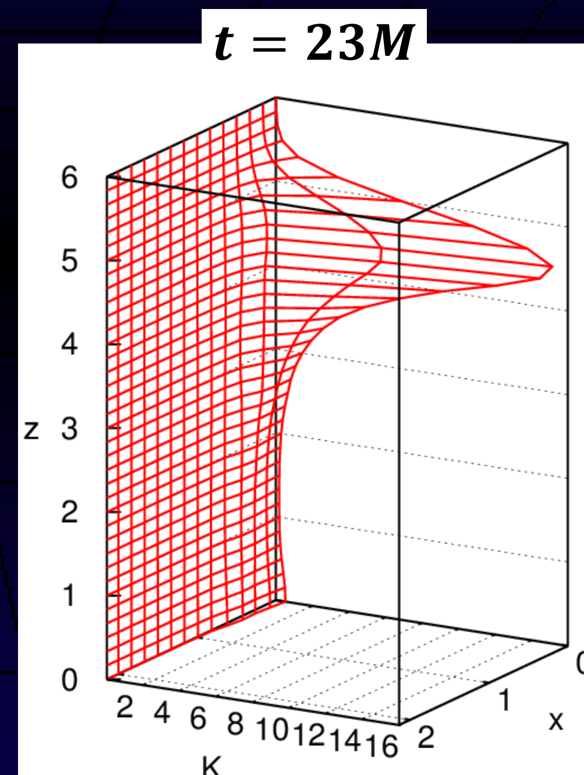
◎Peak position is inside the matter contrary to ST



particles



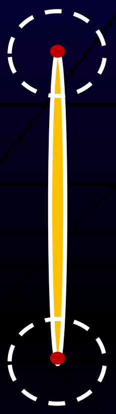
density



Kretschmann

No Horizon?

◎We searched for a horizon enclosing the origin
but could not find it→no horizon?

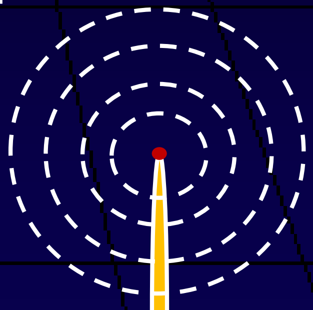


◎What about small horizon just encloses the top?

◎To address this possibility,
we plot the value of the expansion

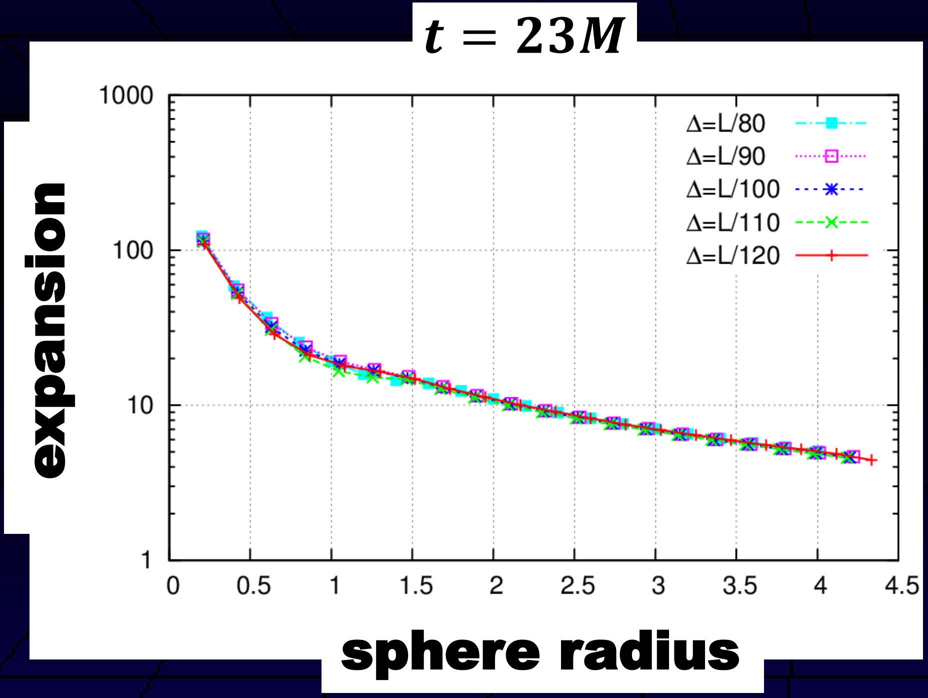
$$\Theta = D_i s^i + K_{ij} s^i s^j - K$$

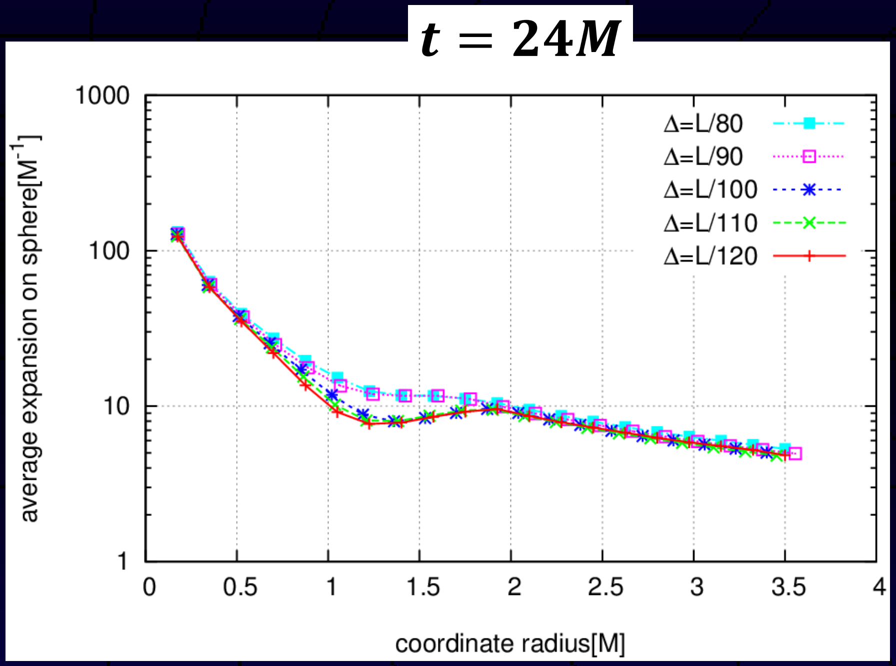
on spheres centered at the peak of Kretschmann inv.
instead of using our apparent horizon finder
which cannot find a small horizon



Expansion

◎Average expansion on a sphere centered at the top as a function of the radius





◎No trapped region(at least within our resolution)

Results(2)
Spindle Collapse with a Horizon

Parameters

◎We keep the shape and increase the mass

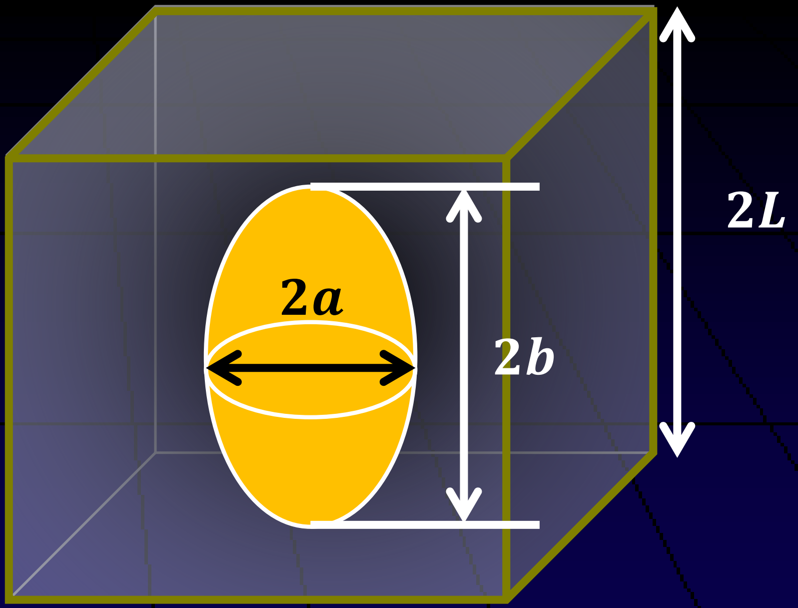
$$L/M = 20 \longrightarrow L/M = 13/2$$

$$e = 0.9 \longrightarrow e = 0.9$$

$$b/M = 10 \longrightarrow b/M = 13/4$$

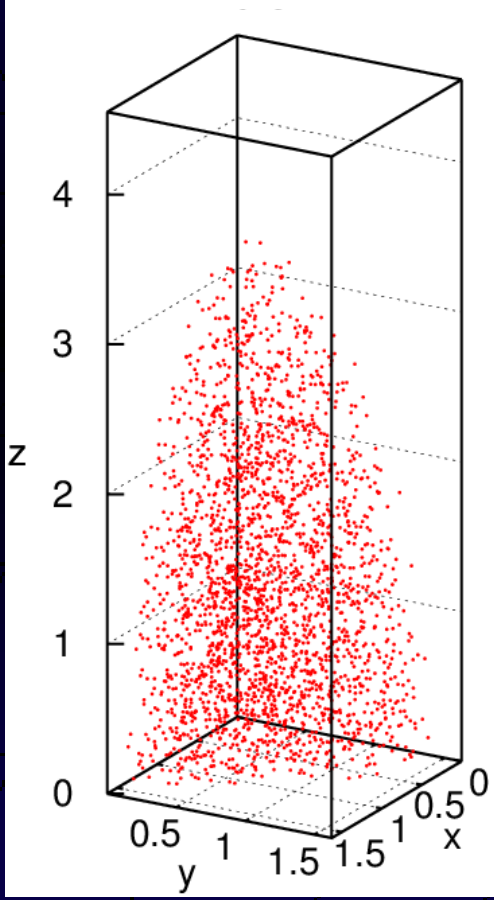
\# of particles $N = 10^6$

Particle size $r_s = L/75$

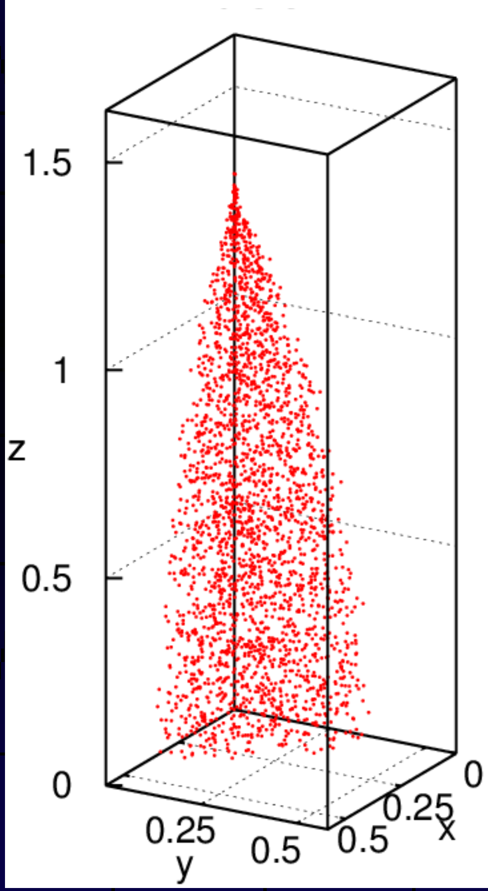


Snapshots: particles

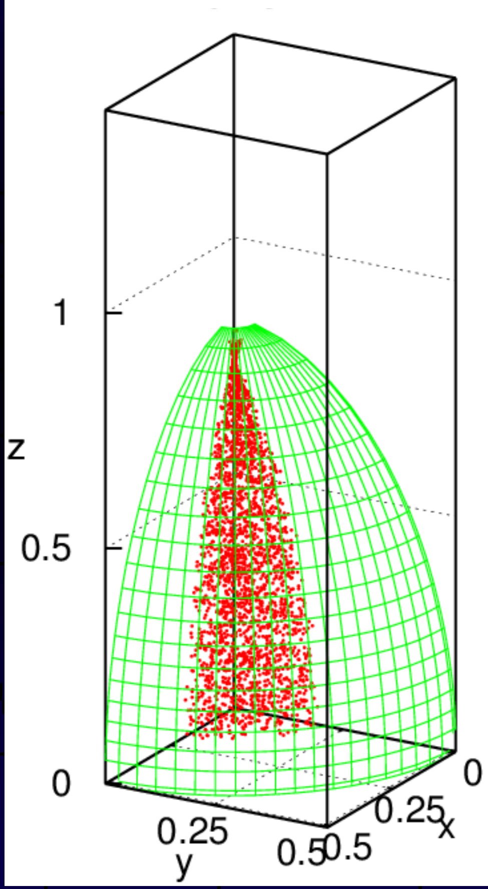
$t = 0$



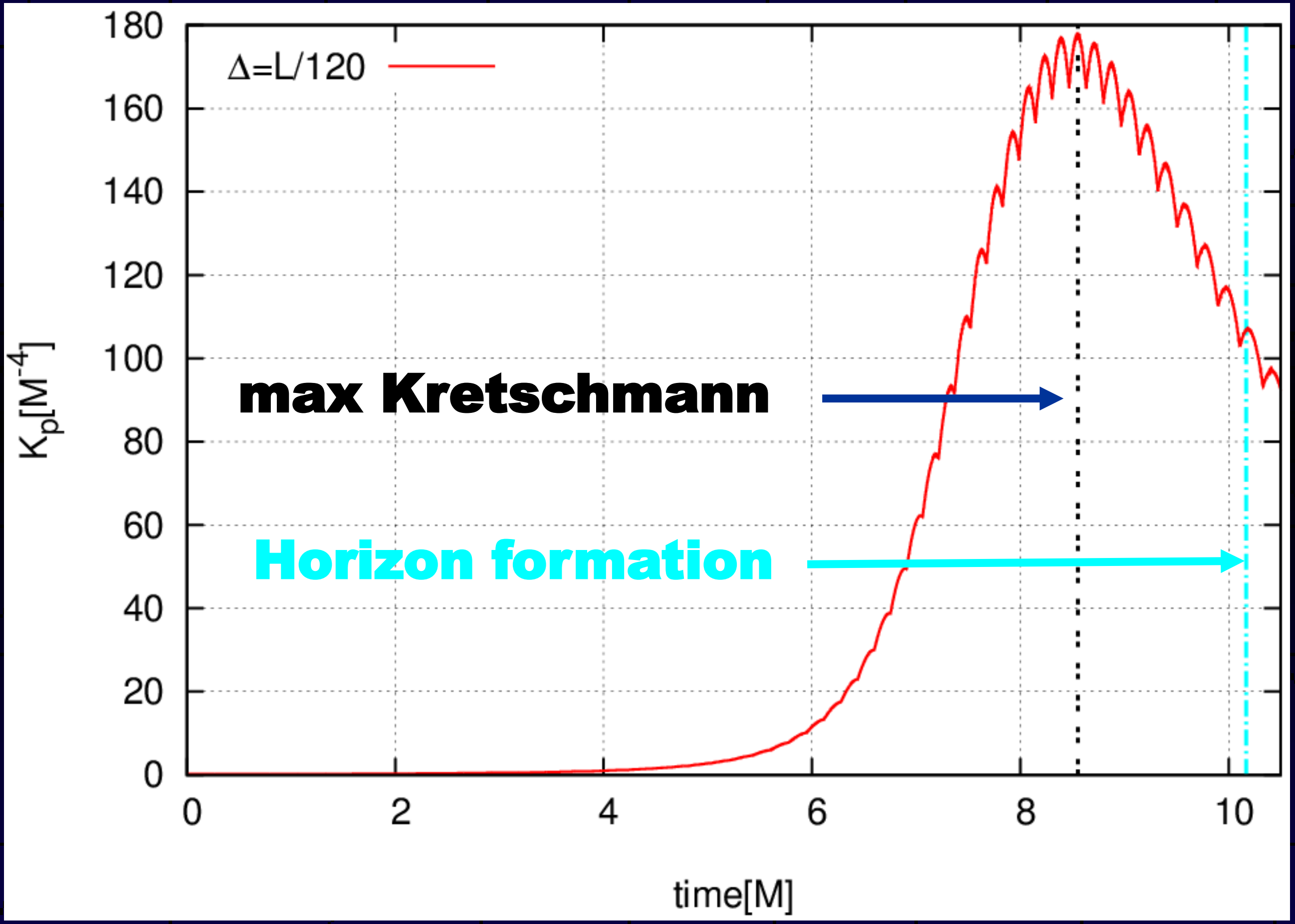
$t = 8.6M$



$t = 10.2M$

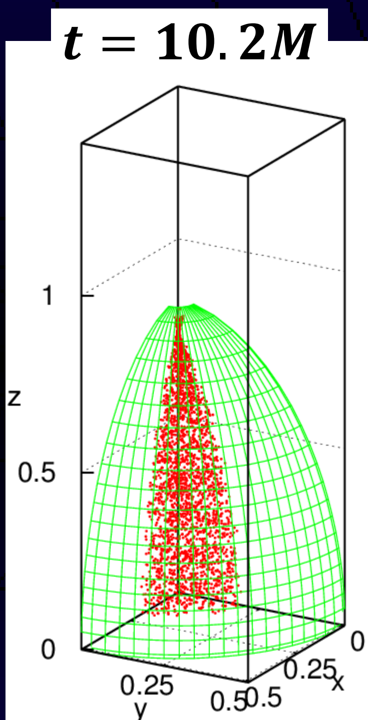


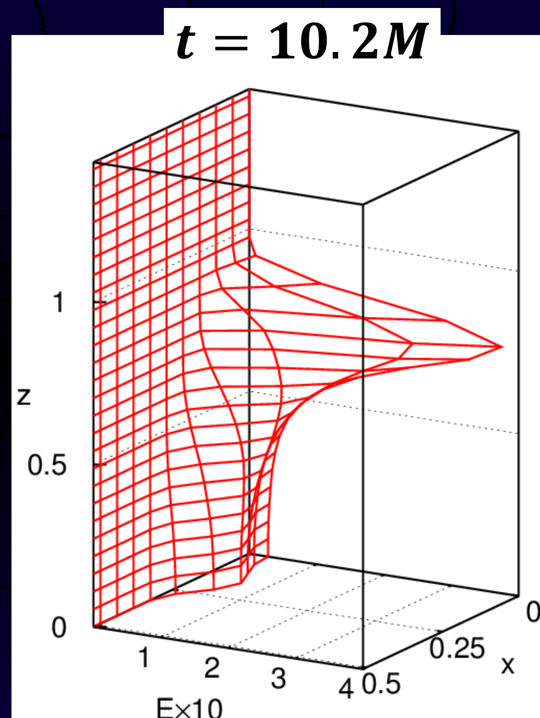
Evolution of K_{peak}

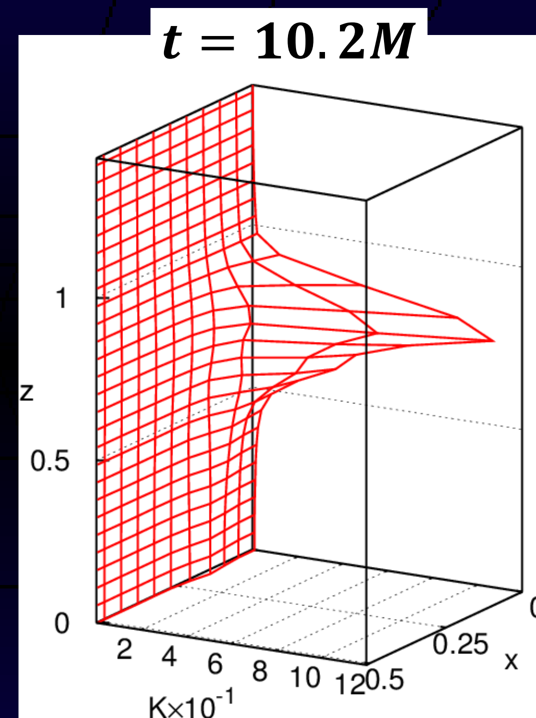


◎Horizon formation after the max Kretschmann inv.

At Horizon Formation Time

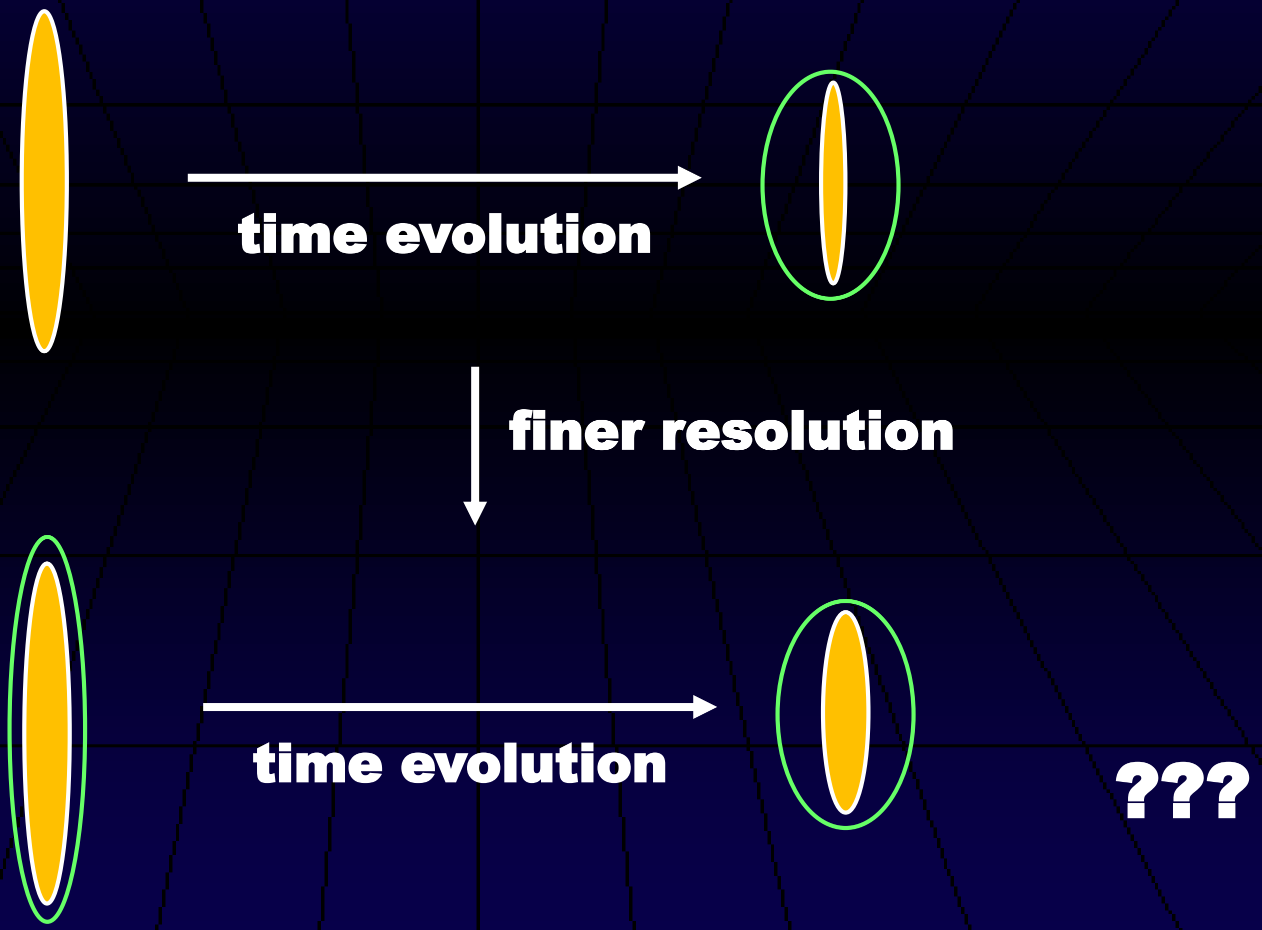




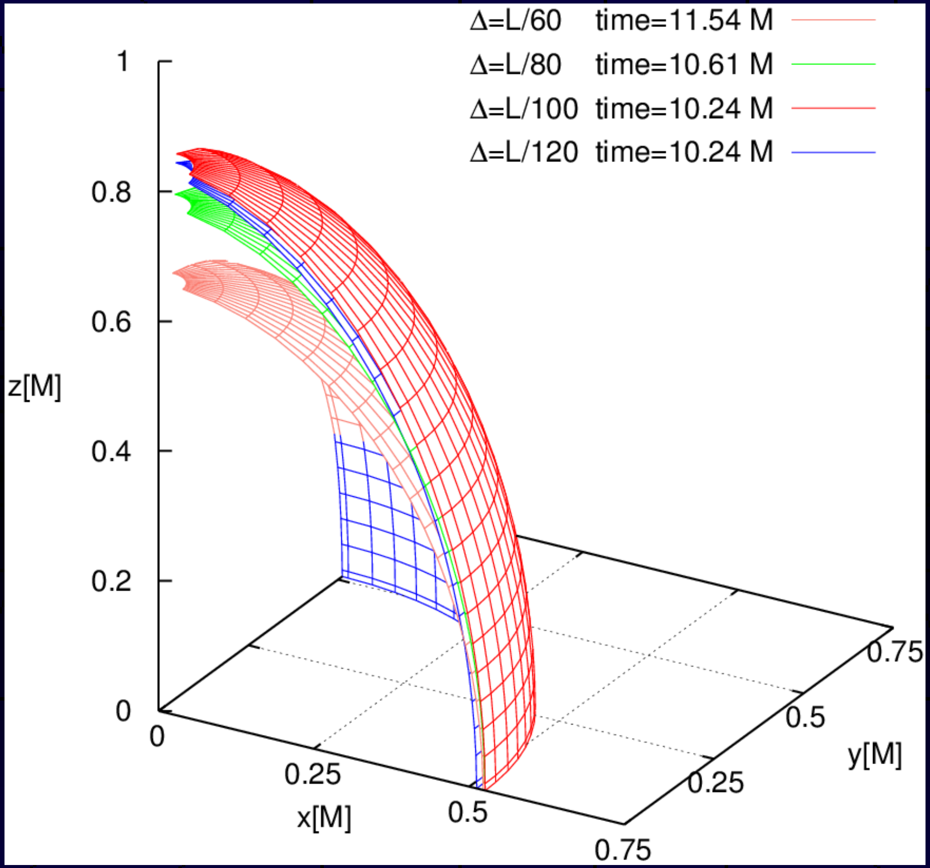


Elongated Horizon?

◎Elongate horizon for finer resolution?



Resolution Dependence



◎**Convergence of the formation time and the shape**

⇒**No horizon when** $K_{\mathrm{peak}} = K_{\mathrm{max}}$ **even for finer resolution**

Summary
—Comparison with ST—

Comparison with ST

◎Shapiro-Tekolsky

- Axi-symmetric
- Collisionless ring sources

◎Our setting

- not exactly in our case
- particles

◎How the results changes

- No horizon at the time of max. Kretschmann→Same
- The larger value of max. Kretschmann for the finer resolution
  → Same in our case(support naked singularity formation)
- The calculation breaks down because of the "singularity" in ST
  → Does not crash and finally collapses to BH for some cases
- The position of max K-inv. is outside the matter distribution in ST
  → Inside the matter distribution, mainly from Ricci part
  The reason for this discrepancy is not clear.
  Is ST type singularity unstable without exact symmetry?

Open Questions

◎Event horizon

- The singularity could be covered by the global event horizon

◎How general? Other initial data?

- Effects of velocity dispersion?

◎What is the reason for the discrepancy with ST?

- Is the vacuum singularity formation with axi-sym. unstable under the general non-symmetric perturbation?

◎Character of the singularity

- Is the singularity weaker than the shell focusing singularity? Is this Spacelike?

Thank you
for your attention!