Post-Newtonian parametrization of the Minimal Theory of Massive Gravity

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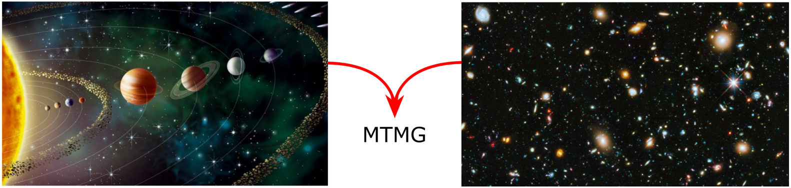
The second workshop on gravity and cosmology
by young researchers

Philosophy

- Construct a minimal theory of massive gravity, *ie.* propagating only two tensor *dof.*
  - ↪ test it at cosmological scales : *work in progress by A. De Felice, S. Mukohyama & M. Oliosi*,
  - ↪ test it in the Solar System : *this work, also in progress...*



NB: The latest realisation contains also a quasi-dilatonic scalar field, for the sake of viable self-accelerating cosmology[1].
But to begin with, let's do it in a more simple setup.

[1] A. De Felice, S. Mukohyama, M. Oliosi, arXiv:1709.03108 & 1701.01581.

Contents

Introduction : a very short story of massive gravity[5]

- 1939 : Fierz and Pauli built the first consistent linear theory of a massive graviton[2]
  - ↪ no Ostrogradski ghost, but vDVZ discontinuity,
  - ↪ and the "naive" non-linear completion has a Boulware-Deser ghost...
- 2000 : Dvali, Gabadadze and Porrati provide the first explicit model of a healthy massive graviton, arising from a 5D braneworld model[3]
  - ↪ the "degravitation" tackles the Old Cosmological Constant problem,
  - ↪ but the self-accelerating branch bears a ghost...
- 2010 : de Rham, Gabadadze and Tolley construct a 4D ghost-free massive gravity (dRGT gravity)[4]
  - ↪ can be extended to bi-gravity, multi-gravity...

[2]M. Fierz, W. Pauli, Proc. Roy. Soc. Lond., **A173**, 211–232 (1939).

[3]G. Dvali, G. Gabadadze, M. Porrati, arXiv:hep-th/0005016.

[4]C. de Rham, G. Gabadadze, A. J. Tolley, arXiv:1011.1232.

[5]C. de Rham, arXiv:1401.4173.

Motivations for a Minimal Theory Massive Gravity

- A graviton of mass $m \sim H_0 \sim 10^{-33}\,eV$ could naturally tackle the problem of late-time acceleration $\Rightarrow$ dRGT massive gravity,

$\rightsquigarrow$ but many *dof* : 2 tensors + 2 vectors + 1 scalar,

$\rightsquigarrow$ and no stable homogeneous and isotropic solutions[6]...

- Let's try to propagate only 2 tensor *dof* $\Rightarrow$ MTMG !

$\rightsquigarrow$ stable homogeneous and isotropic solutions exist[7],

$\rightsquigarrow$ but one has to pay a price : breaking of Lorentz invariance.

[6] A De Felice, A. E. Gumrukcuoglu, S. Mukohyama, arXiv:1206.2080.
[7] A De Felice, S. Mukohyama, arXiv:1512.04008.

Construction of MTMG : overview

Recipe

- start from dRGT gravity,

↪ break Lorentz invariance and add the Stueckelberg fields,

↪ perform an analysis *à la Dirac*,

↪ add constraints to kill enough degrees of freedom,

↪ your MTMG is ready, you can play with it !

Construction of MTMG : in more detail

- **dRGT gravity** : introducing a fiducial (*ie.* non-dynamical) metric f

$$S_{\mathrm{dRGT}} = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R[g] - \frac{M_{\mathrm{Pl}}^2 m^2}{2}\int \mathrm{d}^4x\sqrt{-g}\sum_{i=0}^{4} c_i\, E_i(X),$$

with E_n the 4D symmetric polynomials and $X^\mu{}_\nu \equiv \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu$.

- **Stueckelberg fields** : four scalar fields, ϕ^a, that play the role of Goldstone bosons for diffeomorphisms

$$g^{\mu\nu} \mapsto g^{\alpha\beta}\partial_\alpha\phi^\mu\partial_\beta\phi^\nu.$$

- **Breaking of Lorentz invariance** : ADM-decomposing the two metrics, $g_{\mu\nu} \mapsto (N, N_i, \gamma_{ij})$ and $f_{\mu\nu} \mapsto (M, M_i, \tilde{\gamma}_{ij})$, and setting a Minkowskian fiducial metric $M = 1, M_i = 0, \tilde{\gamma}_{ij} = \delta_{ij}$.

Construction of MTMG : in more detail

- **dRGT gravity** :

$$S_{\mathrm{dRGT}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R[g] - \frac{M_{\mathrm{Pl}}^2 m^2}{2} \int \mathrm{d}^4x \sqrt{-g} \sum_{i=0}^{4} c_i \, E_{4-i}(X),$$

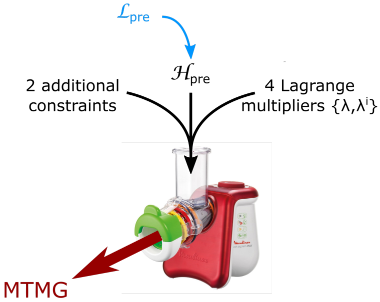
with E_n the 4D symmetric polynomials and $X^{\mu}{}_{\nu} \equiv \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$.

$\Rightarrow$ **precursor theory** :

$$S_{\mathrm{pre}} = S_{\mathrm{GR}} - \frac{M_{\mathrm{Pl}}^2 m^2}{2} \int \mathrm{d}^4x \sqrt{\gamma} \sum_{i=0}^{4} c_i \left[N \, e_{4-i}\left(\mathfrak{K}\right) + e_{3-i}\left(\mathfrak{K}\right) \right],$$

with e_n the 3D symmetric polynomials and $\mathfrak{K}^p{}_q \equiv \left(\sqrt{\gamma^{-1}\tilde{\gamma}}\right)^{p}{}_{q}$.

Construction of MTMG : in more detail



$$\mathcal{E} \equiv \frac{1}{N}\sum_{i=0}^{3} c_i e_{3-i}(\mathfrak{K}),$$

$$\tilde{\mathcal{E}} \equiv \frac{1}{N}\sum_{i=1}^{4} c_i e_{4-i}(\mathfrak{K}),$$

$$\tilde{\mathcal{F}}^p{}_q \equiv \frac{\delta\tilde{\mathcal{E}}}{\delta\mathfrak{K}^p{}_q}.$$

$$S_{\rm MTMG} = S_{\rm GR} - \frac{M_{\rm Pl}^2 m^2}{2}\int \mathrm{d}^4x\, N\sqrt{\gamma}\,\mathcal{W},$$

with

$$\mathcal{W} \equiv \mathcal{E} + N\tilde{\mathcal{E}} + \tilde{\mathcal{F}}^p{}_q\left(\mathcal{D}_p\lambda^q + \lambda K^{qr}\gamma_{rp}\right) - \frac{m^2\lambda^2}{4}\left(\left[\tilde{\mathcal{F}}^2\right] - \frac{1}{2}\left[\tilde{\mathcal{F}}\right]^2\right).$$

Current state of the art

- MTMG by itself cannot account for a stable self-accelerating de Sitter solution...

$\Rightarrow$ one has to add a quasi-dilatonic field with a global symetry

$$\sigma \to \sigma + \sigma_0, \qquad \phi^i \to \phi^i e^{\sigma_0/M_{\text{Pl}}}, \qquad \phi^0 \to \phi^0 e^{(1+\alpha)\sigma_0/M_{\text{Pl}}},$$

to allow it[8].

$\Rightarrow$ there exists a de Sitter solution, which is an attractor, and stable !

$\Rightarrow$ the speed of the tensor modes coincides with the speed of light
$\hookrightarrow$ OK with GW170817 + GRB170817A,

$\Rightarrow$ no ghost when adding matter.

[8] A. De Felice, S. Mukohyama, M. Oliosi, arXiv:1709.03108 & 1701.01581.

PN parametrisation : the idea

The Post-Newtonian (PN) parametrisation of a theory is a setup dedicated to test its weak-field, slow-motion limit, *ie* the regime in which

$$\frac{Gm}{rc^2} \sim \frac{v^2}{c^2} \ll 1.$$

The basic idea is to expand any quantity appearing in the EOM in powers of c, to solve order by order and to compare with reality.
For a complete description, see [1].

Beware ! In GR the first corrections appear at c^{-2}
$\hookrightarrow$ the n$^{\text{th}}$ PN order is $\mathcal{O}(c^{-2n})$.

[1] C. M. Will, *Theory and Experiment in Gravitational Physics*, (1993), Cambridge University Press

The PPN metric

For a perfect fluid,

$$T^{\mu\nu} = [\rho(1+\Pi)+p]\,u^\mu u^\nu + p g^{\mu\nu},$$

Will and Nordtvedt (1972) found the most generic expansion of the metric at relevant PN order

$$\begin{aligned} g_{00} &= -1 + 2U - 2\beta U^2 - (\zeta_1 - 2\xi)\mathcal{A} - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\ &\quad + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4, \\ g_{0i} &= -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i, \\ g_{ij} &= (1 + 2\gamma U)\delta_{ij}, \end{aligned}$$

where

$$U(x,t) = \int \mathrm{d}^3x' \frac{\rho(x',t)}{|x - x'|}, \qquad \text{so} \quad \nabla^2 U = -4\pi\rho,$$

and the other potentials depends on v, Π, p,...

The PPN metric

$$g_{00} = -1 + 2U - 2\beta U^2 - (\zeta_1 - 2\xi)\mathcal{A} - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4,$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i,$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij}.$$

- 10 parameters are introduced in front of the potentials
  $\Rightarrow$ each parameter has a physical interpretation :

β	: amount of non-linearities *wrt.* GR,	(1 in GR)
γ	: amount of light deflection *wrt.* GR,	(1 in GR)
ξ	: preferred-location effects,	(0 in GR)
α_n	: preferred-frame effects,	(0 in GR)
α_3, ζ_n	: violation of conservation of total momentum.	(0 in GR)

PN parametrisation : the tests

The best apparatus for studying slow-motion, weak-field conditions is around us : the Solar system.

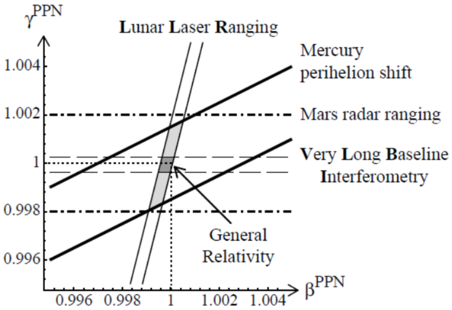


Figure from [2].

One can also obtain the PN parameters values from binary pulsars.

[2] G. Esposito-Farese, arXiv:gr-qc/9903058.

PN parametrisation in a nutshell

Extremely simple and intuitive path to follow

- ↪ write down all the EOM of your theory,
- ↪ expand all possible quantities in terms of $1/c$,
- ↪ solve your EOM order by order,
- ↪ compare the resulting metric with the PPN metric to read the values of the coefficients,
- ↪ cross fingers and compare those values with the current experimental bounds.

$\Rightarrow$ But in practice, a little bit more tricky...

So let's do it !

Equations of motion

- MTMG : 1 metric and 8 scalar fields $(\phi^\mu, \lambda, \lambda^i)$.

↪ Einstein's equations

$$R_{\mu\nu} + m^2 \mathcal{M}_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right),$$

↪ EOM of the Stueckelberg fields : conservation laws traducing the Bianchi identities

$$\nabla_\mu \mathcal{J}^\mu_\nu = 0,$$

↪ EOM of the auxiliary fields

$$\frac{\lambda m^2}{2}\left(\left[\tilde{\mathcal{F}}^2\right] - \frac{1}{2}\left[\tilde{\mathcal{F}}\right]^2\right) - \tilde{\mathcal{F}}^p{}_q K^{qr} \gamma_{pr} = 0,$$

$$\mathcal{D}_p \tilde{\mathcal{F}}^p{}_i + \frac{\tilde{\mathcal{F}}^p{}_i}{\sqrt{\det \gamma^{-1}}} \nabla_\mu \left[\sqrt{\det \gamma^{-1}}\,(g^{\mu\nu} + n^\mu n^\nu)\gamma_{ps}\partial_\nu \phi^s\right] = 0.$$

Expansion of the quantities

- for the metric : perturbation around a spatially flat background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu},$$

with $\bar{g}_{\mu\nu} = \text{diag}[-n_0, 1, 1, 1,]$, $h_{00} \sim h_{ij} \sim 1\text{PN}$ and $h_{0i} \sim 1.5\text{PN}$,

- for the matter sector, let's assume a perfect fluid ansatz

$$T^{\mu\nu} = [\rho\,(1+\Pi) + p]\, u^\mu u^\nu + p g^{\mu\nu},$$

with $\Pi/\rho \sim p/\rho \sim 1\text{PN}$,

- for the Stueckelberg fields, let's take

$$\phi^\mu = x^\mu + \varphi^\mu.$$

No assumption *a priori* on the PN order of the auxiliary fields $\varphi^\mu, \lambda, \lambda^i$,
$\hookrightarrow$ determination *via* the EOM.

Solving the auxiliary EOM

- At lowest order, the auxiliary EOM yield "gauge conditions" relating $\partial_\mu h_{\nu\sigma}$, $\partial_\mu \varphi^\nu$, $\partial_\mu \lambda$ and $\partial_\mu \lambda^i$, slightly broken by terms of order $m^2\lambda$.

$\hookrightarrow$ *eg.* the EOM of λ is

$$(c_1+2c_2+c_3)\left[\partial_k h_{0k} - \frac{1}{2}\partial_0 h_{kk} + n_0 \partial_k^2 \varphi^0 - \frac{3\sqrt{n_0}}{4}(c_1+2c_2+c_3)m^2\lambda\right] = 0,$$

and indicates two branches. But the branch $c_1 + 2c_2 + c_3 = 0$ is forbidden by the cosmology !

$\hookrightarrow$ or the EOM of ϕ^i

$$(c_1 + c_2)\left[\partial_i h_{00} + \sqrt{n_0}\left(\partial_k^2 \lambda^i - \partial_i \partial_k \lambda^k\right)\right] = 0.$$

Solving Einstein's equations

$$R_{\mu\nu} + m^2 \mathcal{M}_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)$$

- $R_{\mu\nu}$ and $T_{\mu\nu}$ are both 1PN quantities, but $\mathcal{M}_{\mu\nu}$ has a 0PN part.

↪ $m^2 \mathcal{M}^{0PN}_{\mu\nu}$ plays the role of an effective cosmological constant that has to be set to 0 in order for the PN setup to work (spatially flat space).

↪ induces one additional constraint on the c_n parameters

$$c_1 + 3c_2 + 3c_3 + c_4 = 0,$$

which fixes c_4, the "bare cosmological constant" of the theory as

$$S_{\rm MTMG} \supset -\frac{M_{\rm Pl}^2 m^2}{2} \int \mathrm{d}^4x \sqrt{-g}\, c_4.$$

To be continued