YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

Gravitational Wave Physics and Astronomy Genesis

Slow-roll violation in production of primordial black hole

Hayato Motohashi

Center for Gravitational Physics

Yukawa Institute for Theoretical Physics

HM & Wayne Hu, PRD 92, 043501 (2015), [arXiv:1503.04810]

PRD 96, 023502 (2017), [arXiv:1704.01128]

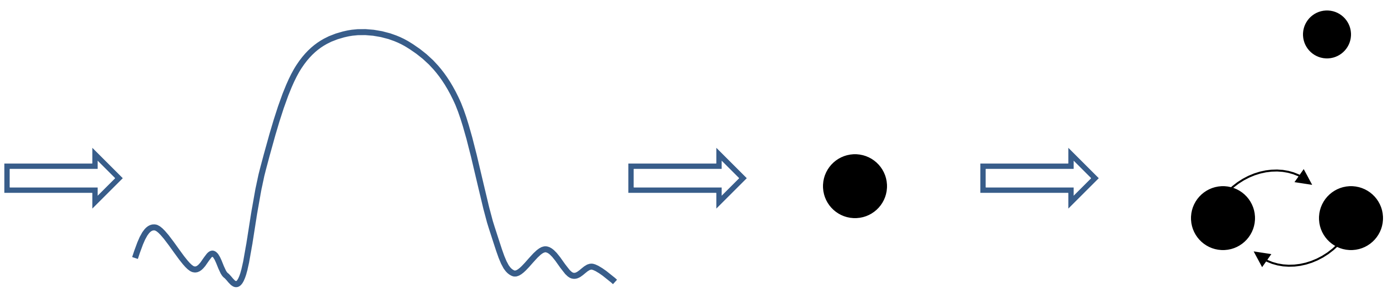
PRD 96, 063503 (2017), [arXiv:1706.06784]

2018.03.03-04 2nd workshop on gravity and cosmology by young researchers

Sasaki, Suyama, Tanaka, Yokoyama, 1603.08338
Kocsis, Suyama, Tanaka, Yokoyama, 1709.09007

This talk

Inflation



DM
GW source

References

Carr, Kohri, Sendouda, Yokoyama, 0912.5297
Carr, Kuhnel, Sandstad, 1607.06077
Sasaki, Suyama, Tanaka, Yokoyama, 1801.05235

Zel'dovich, Novikov (1966)
Hawking (1971)
Carr (1975)

- High density region collapses at horizon reentry
  if $\delta \equiv \delta\rho/\bar{\rho} > \delta_c \sim 0.3 - 0.7$

- PBH fraction at formation

$$\beta \equiv \frac{\rho_{\rm PBH}}{\rho_{\rm tot}} = 2\int_{\delta_c}^{\infty} P(\delta)d\delta = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_M}\right) \approx \sqrt{\frac{2}{\pi}}\frac{\sigma_M}{\delta_c} e^{-\frac{\delta_c^2}{2\sigma_M^2}}$$

- $\delta \propto \zeta$ holds at the horizon reenty

Musco, Miller, 1201.2379
Harada, Yoo, Kohri, 1309.4201
Young, Byrnes, Sasaki, 1405.7023

$$\Longrightarrow \quad \beta \approx \sqrt{\frac{2}{\pi}}\frac{\Delta_\zeta}{\zeta_c} e^{-\frac{\zeta_c^2}{2\Delta_\zeta^2}}, \qquad \zeta_c = \frac{9}{2\sqrt{2}}\delta_c = 0.95 - 2.2$$

Zel'dovich, Novikov (1966)
Hawking (1971)
Carr (1975)

- High density region collapses at horizon reentry

if $\delta \equiv \delta\rho/\bar{\rho} > \delta_c \sim 0.3 - 0.7$

Spherical collapse of closed universe

- PBH fraction at formation

$$\beta \equiv \frac{\rho_{\text{PBH}}}{\rho_{\text{tot}}} = 2\int_{\delta_c}^{\infty} P(\delta) d\delta = \text{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_M}\right) \approx \sqrt{\frac{2}{\pi}} \frac{\sigma_M}{\delta_c} e^{-\frac{\delta_c^2}{2\sigma_M^2}}$$

Gaussian $P(\delta) = \frac{1}{\sqrt{2\pi}\sigma_M} e^{-\frac{\delta^2}{2\sigma_M^2}}$

$\sigma_M \ll \delta_c$: Rare event

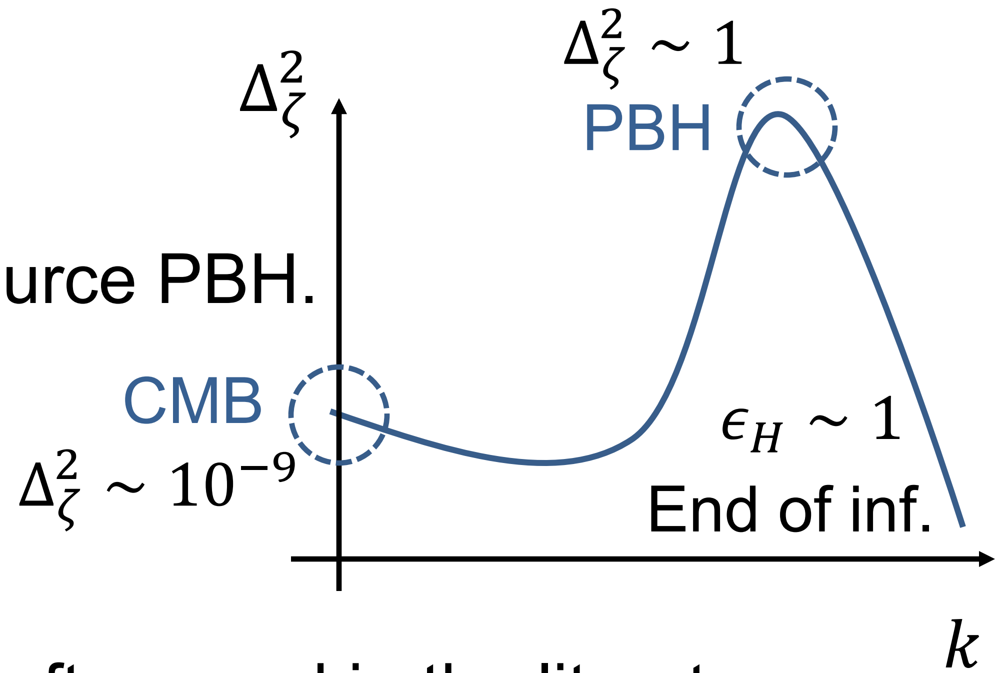
Nearly scale inv. Δ_ζ^2 for a few efolds around the reenty

- $\delta \propto \zeta$ holds at the horizon reenty

Musco, Miller, 1201.2379
Harada, Yoo, Kohri, 1309.4201
Young, Byrnes, Sasaki, 1405.7023

$$\Longrightarrow \quad \beta \approx \sqrt{\frac{2}{\pi}} \frac{\Delta_\zeta}{\zeta_c} e^{-\frac{\zeta_c^2}{2\Delta_\zeta^2}} \; , \qquad \zeta_c \overset{\text{RD}}{=} \frac{9}{2\sqrt{2}} \delta_c = 0.95 - 2.2$$

Large peak in Δ_ζ^2



A large peak of Δ_ζ^2 can source PBH.

Leading-order slow-roll

$$\Delta_\zeta^2 \approx \frac{H^2}{8\pi^2\epsilon_H}$$

Approximation $\epsilon_H \approx \epsilon_V$ is often used in the literature

$$\Delta_\zeta^2 \approx \frac{V}{24\pi^2\epsilon_V} \quad , \quad \frac{d\phi}{dN} = -\frac{V'}{V}$$

$$\epsilon_H \equiv -\dot{H}/H^2$$
$$\epsilon_V \equiv (V'/V)^2/2$$

Is $\epsilon_H \approx \epsilon_V$ valid?

$$\frac{\epsilon_V}{\epsilon_H} = \left(1 + \frac{1}{2(3-\epsilon_H)}\frac{d\ln\epsilon_H}{dN}\right)^2$$

Naively, a large $\frac{d\ln\epsilon_H}{dN}$ seems necessary for PBH.

Monochromatic mass function

- PBH mass = Horizon mass

Horizon reentry

$$M = M_H = \frac{4\pi\rho}{3H^3} = \frac{1}{2GH} \simeq 10^{24}\left(\frac{g_*}{g_{*0}}\right)^{\frac{1}{6}} a_H^2 M_\odot$$

$$\simeq 10^5 \left(\frac{t_H}{1\mathrm{s}}\right) M_\odot$$

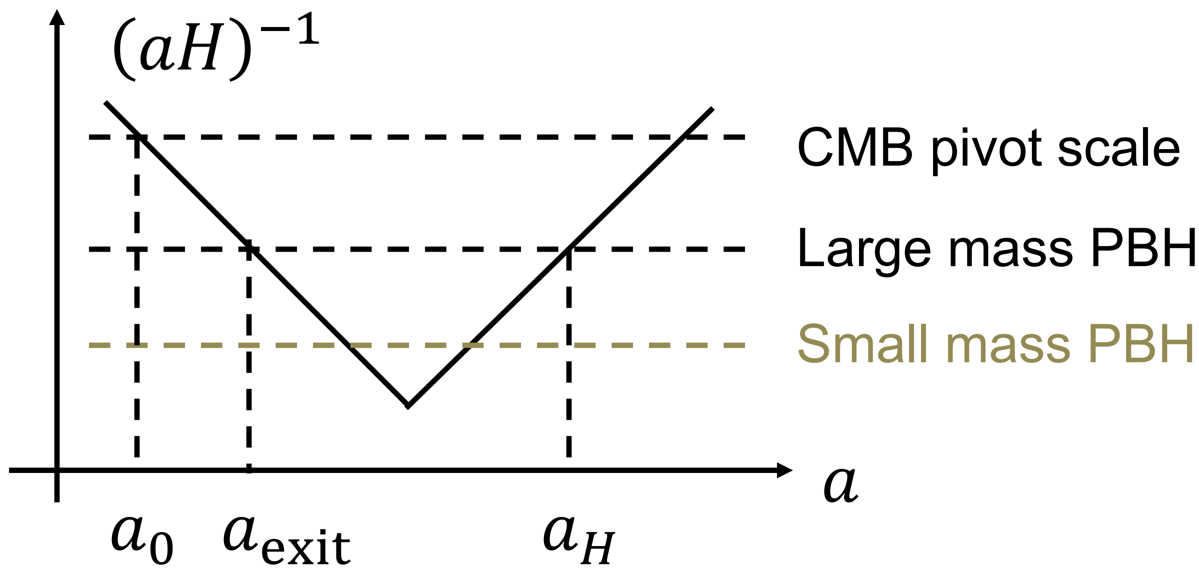
- PBH fraction at formation

RD: $H^2 = \frac{8\pi G}{3}\frac{\pi^2}{30} g_* T^4$

$$= \left(\frac{g_*}{g_{*0}}\right)^{-\frac{1}{3}} \Omega_r H_0^2 a^{-4}$$

$$\beta \equiv \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{tot}}} = \frac{\Omega_{\mathrm{PBH}} a_H^{-3}}{\Omega_{\mathrm{r}}\left(\frac{g_*}{g_{*0}}\right)^{-\frac{1}{3}} a_H^{-4}}$$

$$\approx 10^{-9}\left(\frac{g_*}{g_{*0}}\right)^{\frac{1}{4}}\left(\frac{\Omega_{\mathrm{PBH}} h^2}{0.12}\right)\left(\frac{M}{M_\odot}\right)^{1/2}$$



- efolds between CMB and PBH scales

$$\Delta N \equiv \ln\left(\frac{a_{\rm exit}H_{\rm inf}}{a_0 H_{\rm inf}}\right) = \ln\left(\frac{a_H H}{0.05\mathrm{Mpc}^{-1}}\right)$$

$$\approx 18 - \frac{1}{12}\ln\frac{g_*}{g_{*0}} - \frac{1}{2}\ln\frac{M}{M_\odot}$$

HM, Hu, 1706.06784

No go for slow-roll

$$\beta \approx \sqrt{\frac{2}{\pi}}\frac{\Delta_\zeta}{\zeta_c}e^{-\frac{\zeta_c^2}{2\Delta_\zeta^2}} \approx 10^{-9}\left(\frac{\Omega_{\rm PBH}h^2}{0.12}\right)\left(\frac{M}{M_\odot}\right)^{1/2} \qquad \zeta_c = 1.3$$

$$\Delta N \approx 18 - \frac{1}{2}\ln\frac{M}{M_\odot}$$

Given $(\Omega_{\rm PBH}, M)$

$\Longrightarrow \Delta_\zeta^2(k_{\rm PBH})$ and ΔN

SR: $\Delta_\zeta^2 \propto \epsilon_H^{-1} \Longrightarrow \Delta\ln\epsilon_H$

$\Delta_\zeta^2(k_{\rm CMB}) \approx 10^{-9}$

$$\left|\frac{\Delta\ln\epsilon_H}{\Delta N}\right|$$

HM, Hu, 1706.06784

No go for slow-roll

$M_{\rm min} \approx 10^{-21} M_\odot$: Smallest PBH mass that does not evaporate by matter-radiation equality barring merging and accretion $\Longrightarrow$ Lower bound

For $(\Omega_{\rm PBH}, M) = (\Omega_{\rm DM}, M_{\rm min})$

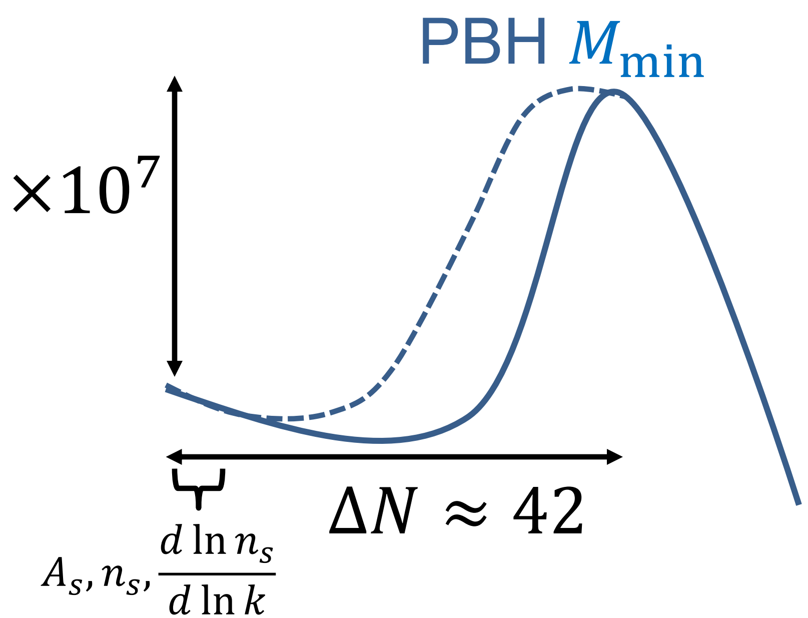
✓ $\Delta^2_\zeta(k_{\rm PBH}) \approx 0.02 \Longrightarrow \times 10^7$ from $\Delta^2_\zeta(k_{\rm CMB}) \approx 10^{-9}$

✓ $k_{\rm PBH}$ exits the horizon $\Delta N \approx 42$ after $k_{\rm CMB} = 0.05 {\rm Mpc}^{-1}$

Slow-roll violation

$$\left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38$$

for any single-field canonical inflation



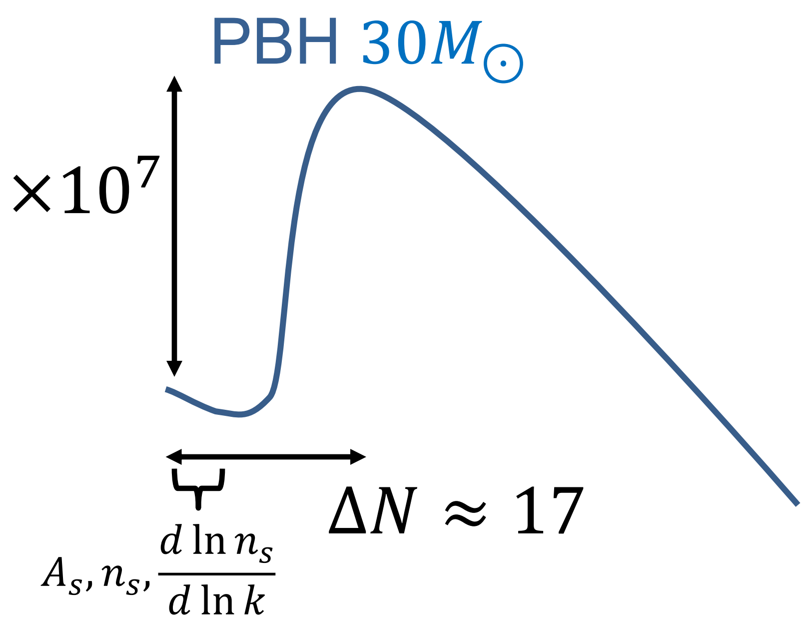
HM, Hu, 1706.06784

No go for slow-roll

$$(\Omega_{\mathrm{PBH}}, M) = \begin{cases} (\Omega_{\mathrm{DM}}, M_{\mathrm{min}}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38 \\ (10^{-3}\Omega_{\mathrm{DM}}, M_{\mathrm{min}}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.37 \\ (10^{-3}\Omega_{\mathrm{DM}}, 30 M_{\odot}) & \Rightarrow \left|\dfrac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.99 \end{cases}$$

PBH = LIGO event scenario

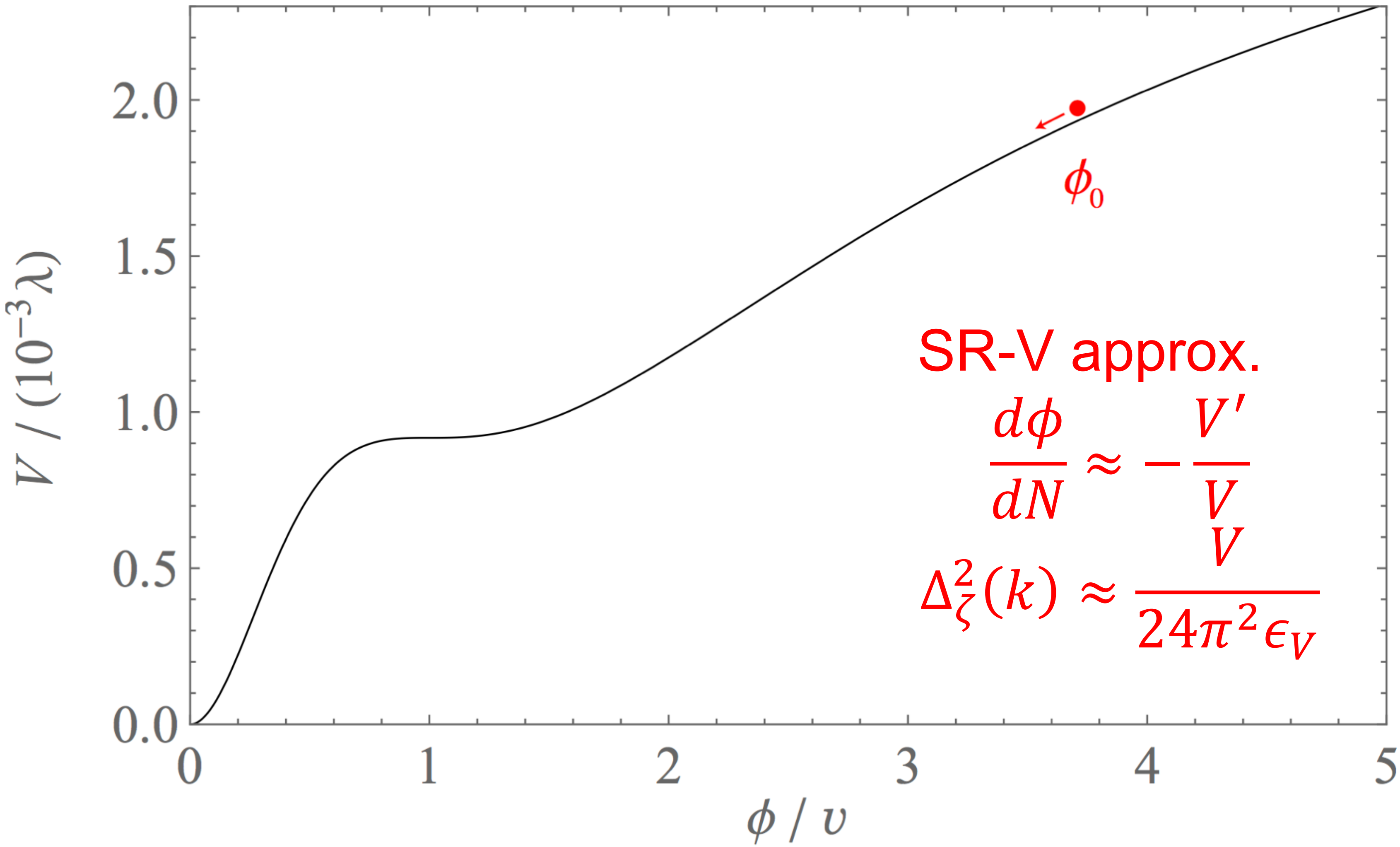
Sasaki, Suyama,
Tanaka, Yokoyama, 1603.08338



Case study 1: Inflection model

Garcia-Bellido, Morales, 1702.03901

Ezquiaga, Garcia-Bellido, Morales, 1705.04861



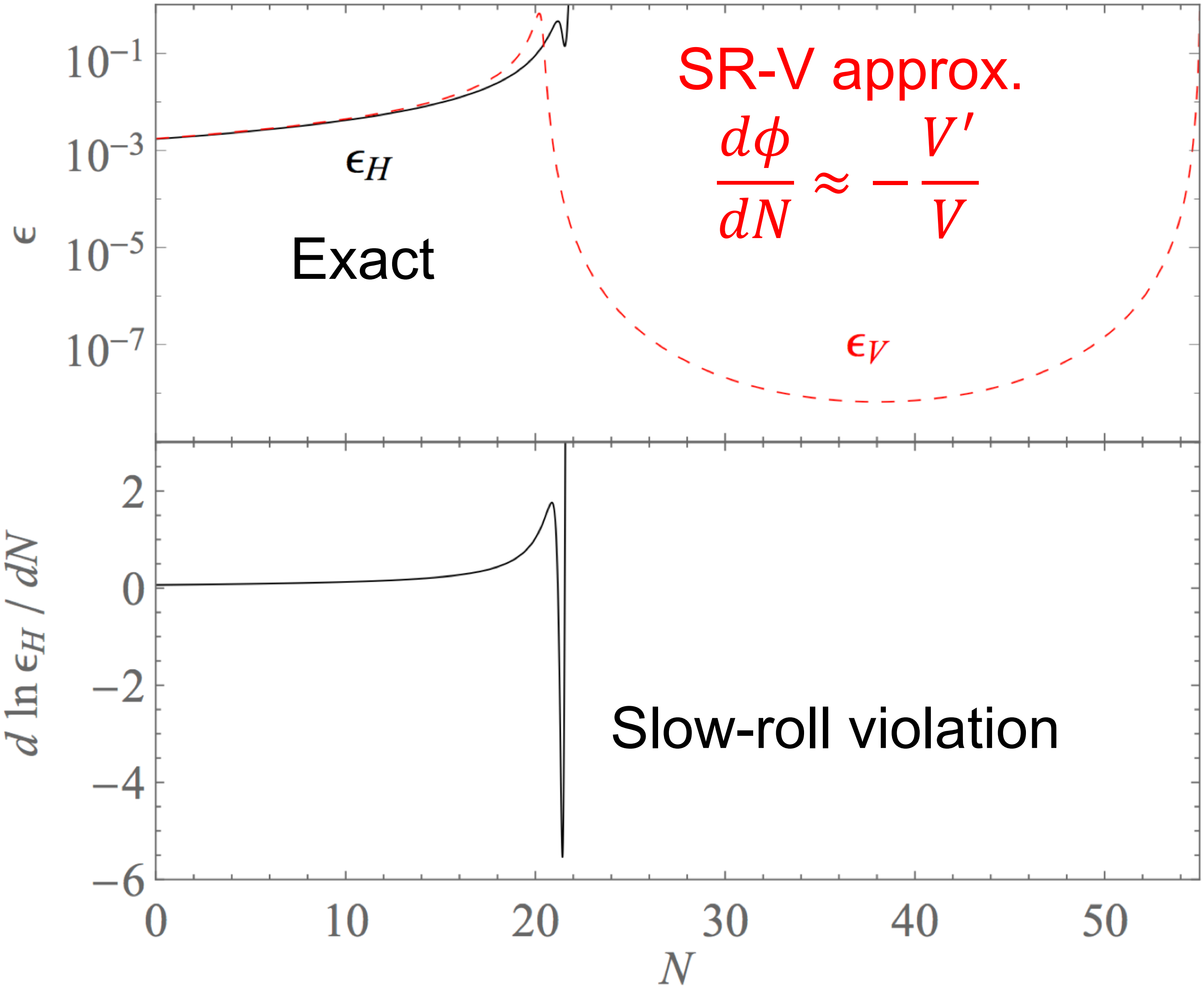
SR-V approx.

$$\frac{d\phi}{dN} \approx -\frac{V'}{V}$$

$$\Delta_\zeta^2(k) \approx \frac{V}{24\pi^2\epsilon_V}$$

HM, Hu, 1706.06784

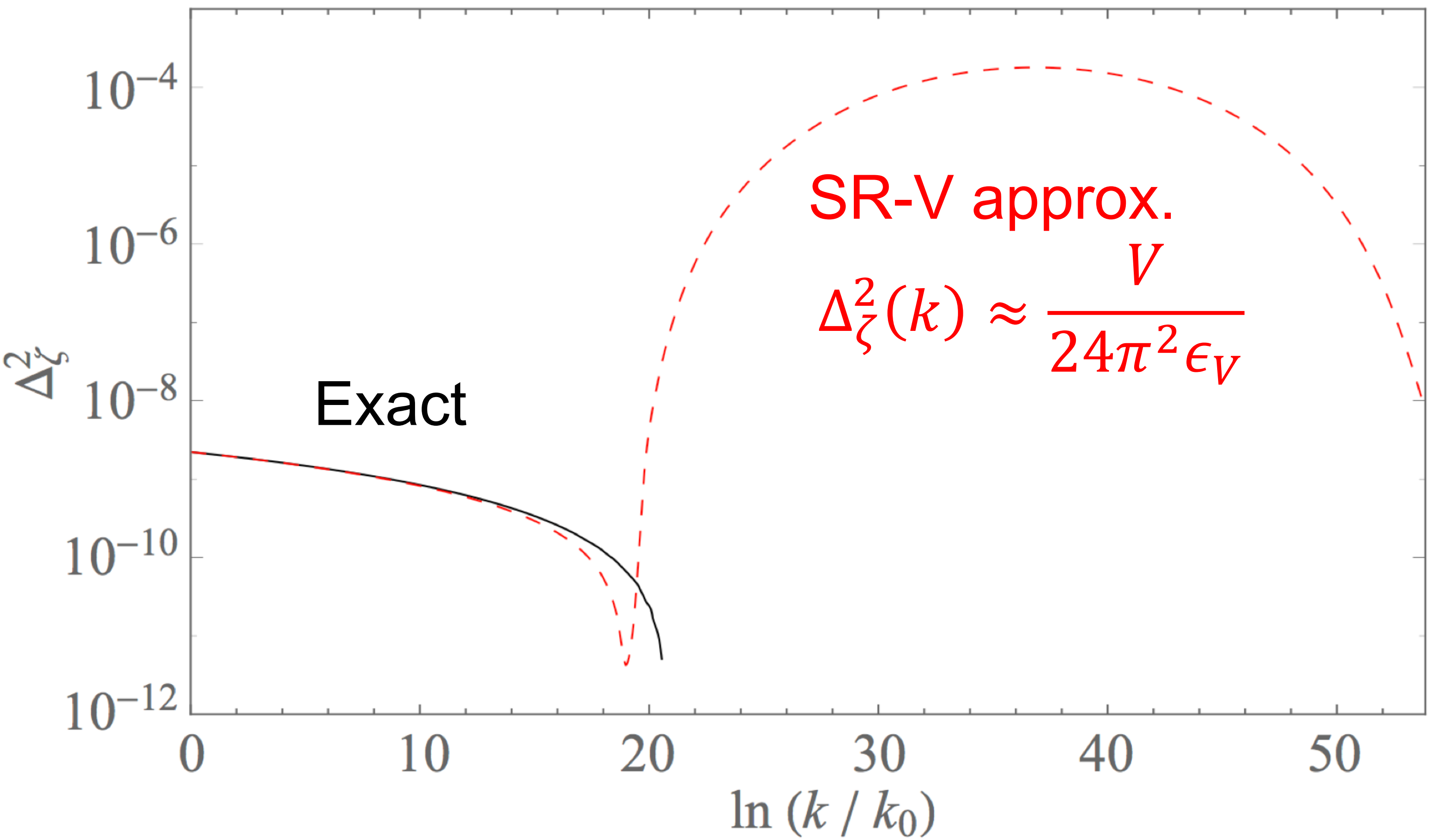
Case study 1: Inflection model



HM, Hu, 1706.06784

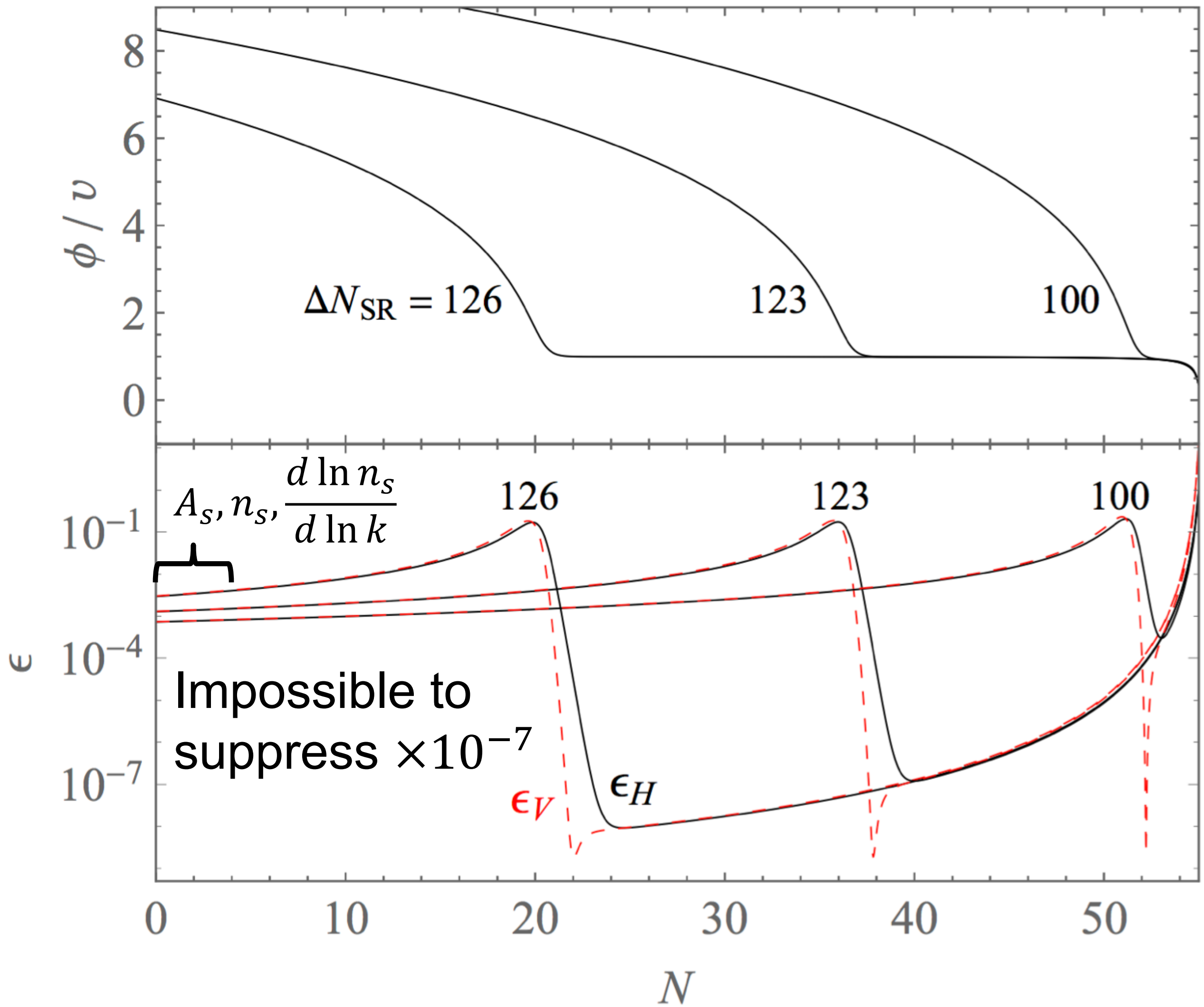
Case study 1: Inflection model

See also Germani, Prokopec, 1706.04226



HM, Hu, 1706.06784

Case study 1: Inflection model



See also Ballesteros, Taoso, 1709.05565 for different inflection potential for $\times 10^{-7}$ suppression of ϵ_H.

HM, Hu, 1503.04810, 1704.01128

Improve approximation

Large SR violation $\left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38$

- SR-V : $\epsilon_H \approx \epsilon_V$ $\Longrightarrow$ Particularly bad
- Standard SR : $\Delta_\zeta^2 \approx \left.\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right|_{ks=1}$ $\Longrightarrow$ Not good
- Optimized SR : $\Delta_\zeta^2 \approx \left.\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right|_{ks=x_1}$ $\Longrightarrow$ Works well

- Minimize truncation error by optimization $ks = x_1$
- Apply for Horndeski, GLPV, subclass of DHOST

$$L_{s2} = \frac{a^3 b_s \epsilon_H}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2}\zeta^2\right)$$

$$L_{t2} = \frac{a^3 b_t}{4c_t^2}\left(\dot{\gamma}_{+,\times}^2 - \frac{c_t^2 k^2}{a^2}\gamma_{+,\times}^2\right)$$

Unitary gauge L_2

Kobayashi et al, 1105.5723

Gleyzes et al, 1304.4840, 1411.3712

Kase et al, 1409.1984

Langlois et al, 1703.03797

Unitary gauge vs comoving gauge

For canonical inflation $\delta G_i^0 = \delta {T^{\phi}}_i^0 \propto \delta\phi$

$\Longrightarrow$ unitary gauge ($\delta\phi = 0$) = comoving gauge ($\delta {T^{\phi}}_i^0 = 0$)

HM, Hu, 1704.01128

Unitary gauge vs comoving gauge

For noncanonical inflation $\delta G_i^0 = \delta T^{\phi}{}_i^0 \not\propto \delta\phi$

$\Longrightarrow$ unitary gauge ($\delta\phi = 0$) $\neq$ comoving gauge ($\delta G_i^0 = 0$)

$$\zeta_{\text{comov}} = \zeta_{\text{uni}} - \frac{\Delta}{H\epsilon_H} \qquad \Delta \equiv H\delta N_{\text{uni}} - \dot{\zeta}_{\text{uni}}$$

$$\delta N_{\text{comov}} = \delta N_{\text{uni}} - \frac{d}{dt}\left(\frac{\Delta}{H^2\epsilon_H}\right)$$

For theories with 2nd-order EOM for scalar perturbation

Einstein eq $\supset$ constraint eq $\delta N_{\text{uni}} \propto \dot{\zeta}_{\text{uni}} \Longrightarrow \Delta = \Gamma\dot{\zeta}_{\text{uni}}$

$$|\zeta_{\text{comov}} - \zeta_{\text{uni}}| = \left|\frac{d\ln\zeta_{\text{uni}}}{dN}\right|\left|\frac{\Gamma}{\epsilon_H}\right|$$

$$\zeta_{\text{comov}} \approx \zeta_{\text{uni}} \text{ if } \begin{cases} \Gamma \approx 0 \text{ (canonical case: } \Gamma = 0) \\ \zeta_{\text{uni}} \approx \text{const.} \end{cases}$$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

Stewart, astro-ph/0110322

1. Write down the formal solution of Mukhanov-Sasaki equation by using Green function (Generalized SR)
2. First order iteration

$$\ln \Delta^2(k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

Window function

Source function

$x \equiv ks$

Sound horizon

- Function of H, ϵ_H etc
- Information of model

$$W(x) = \frac{3\sin 2x}{2x^3} - \frac{3\cos 2x}{x^2} - \frac{3\sin 2x}{2x}$$

$$G(\ln x) = -2\ln f + \frac{2}{3}(\ln f)'$$

$$\approx \begin{cases} \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1} \\ \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1} \end{cases}$$

$$\epsilon_H \equiv -\dot{H}/H^2$$

$$\delta_1 \equiv d\ln\epsilon_H/dN/2 - \epsilon_H$$

$$\sigma_{i1} \equiv d\ln c_i/dN$$

$$\xi_{i1} \equiv d\ln b_i/dN$$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

1. Write down the formal solution of Mukhanov-Sasaki equation by using Green function (Generalized SR)
2. First order iteration

$$\ln \Delta^2(k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

$x \equiv ks$

↑

Sound horizon

3. Taylor expand $G(\ln x)$ around the evaluation point $\ln x_f$

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

4. Truncate at p and optimize $\ln x_f$ so that $q_{p+1}(\ln x_f) = 0$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$$x \equiv ks$$

- Standard SR

$\ln \Delta^2 = G(0)$: correction $= O(q_1(0)G'(0))$

$\ln x_f = 0$: horizon exit

$q_1(0) \approx 1.06$

$1/\Delta N$

$\simeq 0.35$ for $\Delta N \sim 3$

$$G(\ln x) = -2\ln f + \frac{2}{3}(\ln f)'$$

$$\approx \begin{cases} \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1} \\ \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1} \end{cases}$$

HM, Hu, 1503.04810, 1704.01128

Optimized slow-roll approximation

$$\ln \Delta^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$x \equiv ks$

- Standard SR

$\ln \Delta^2 = G(0)$: correction $= O(q_1(0)G'(0))$

$\ln x_f = 0$: horizon exit

$q_1(0) \approx 1.06$

$1/\Delta N$

$\simeq 0.35$ for $\Delta N \sim 3$

- Optimized SR

$\ln \Delta^2 = G(\ln x_1)$: correction $= O(q_2(\ln x_1)G''(\ln x_1))$

$\ln x_f = \ln x_1 \approx 1.06$ with $q_1(\ln x_1) = 0$

$\sim$1 efold before horizon exit

$q_2(\ln x_1) \approx -0.36$

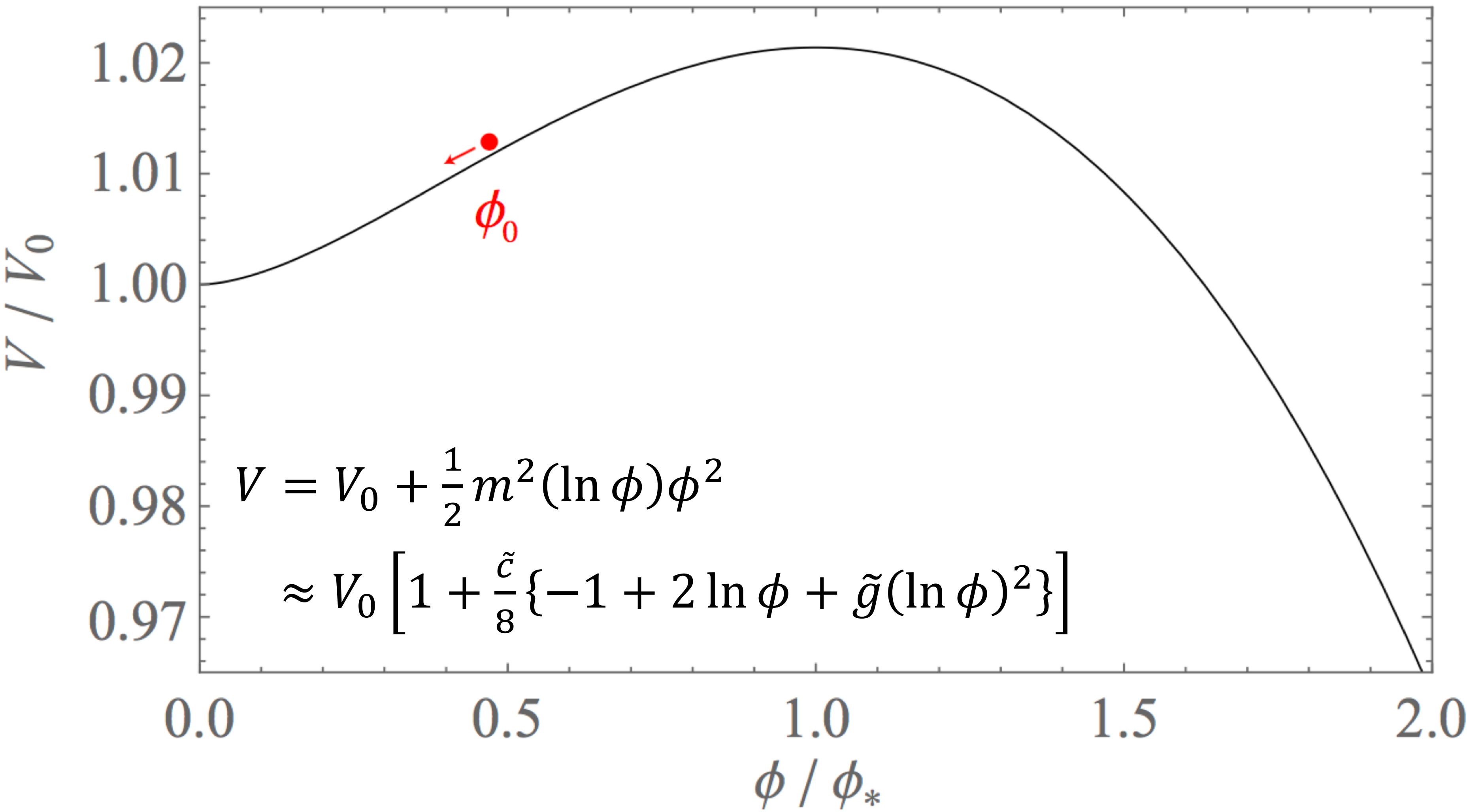
$1/\Delta N^2$

$\simeq 0.04$ for $\Delta N \sim 3$

HM, Hu, 1706.06784

Case study 2: Running mass model

Drees, Erfani,1102.2340

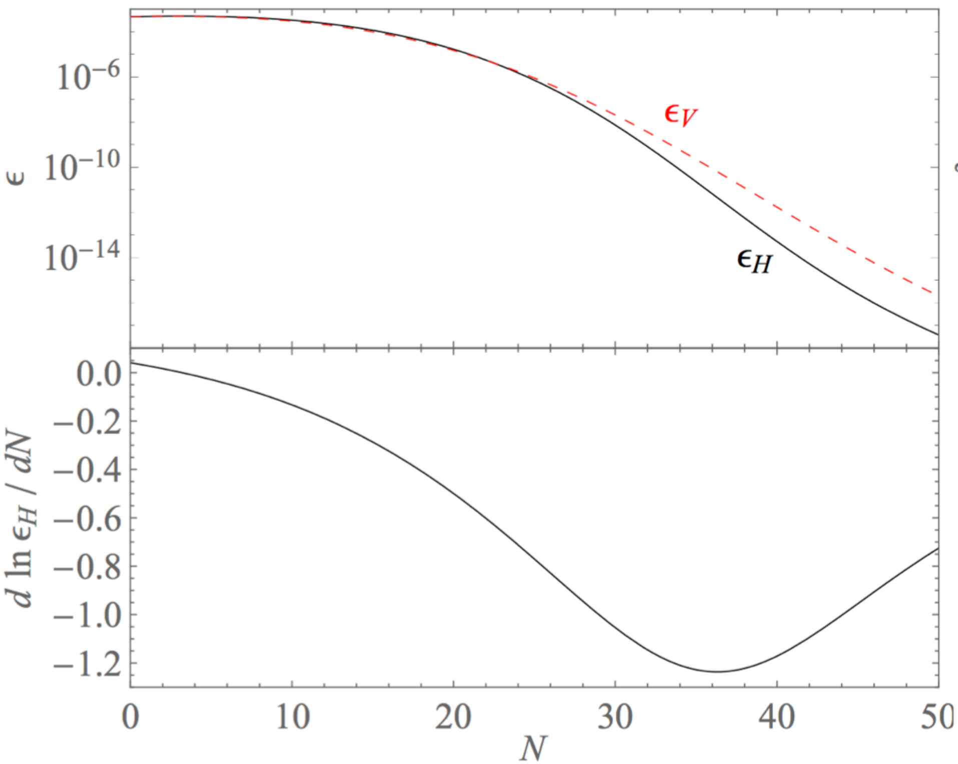


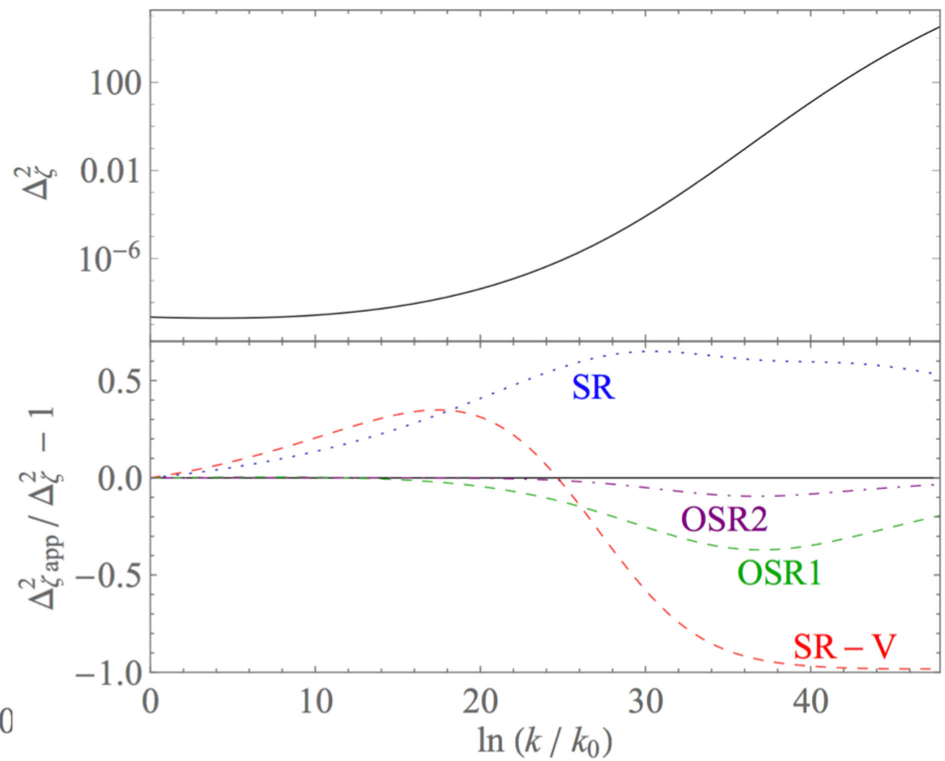
$$V = V_0 + \frac{1}{2} m^2 (\ln\phi)\phi^2$$

$$\approx V_0 \left[1 + \frac{\tilde{c}}{8}\{-1 + 2\ln\phi + \tilde{g}(\ln\phi)^2\}\right]$$

HM, Hu, 1706.06784

Case study 2: Running mass model





Slow-roll violation

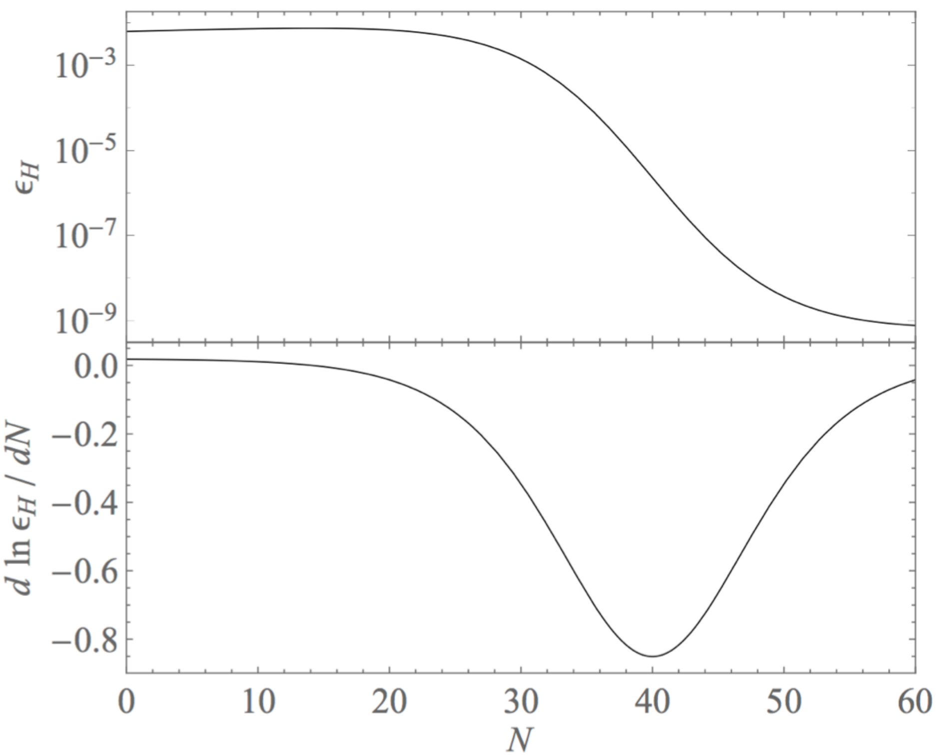
OSR still works well

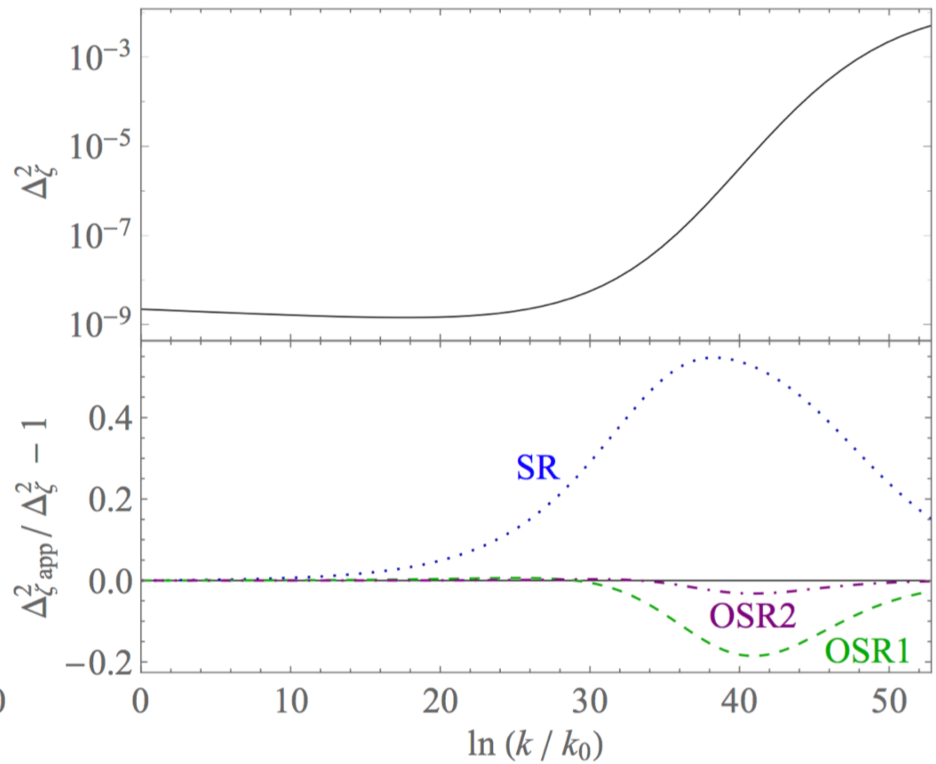
HM, Hu, 1706.06784

Case study 3: Slow roll step model

Parametrize $\ln \epsilon_H$ directly

$$\ln \epsilon_H = C_1 + C_2 N - C_3 \left[1 + \tanh\left(\frac{N - N_s}{d}\right)\right]$$



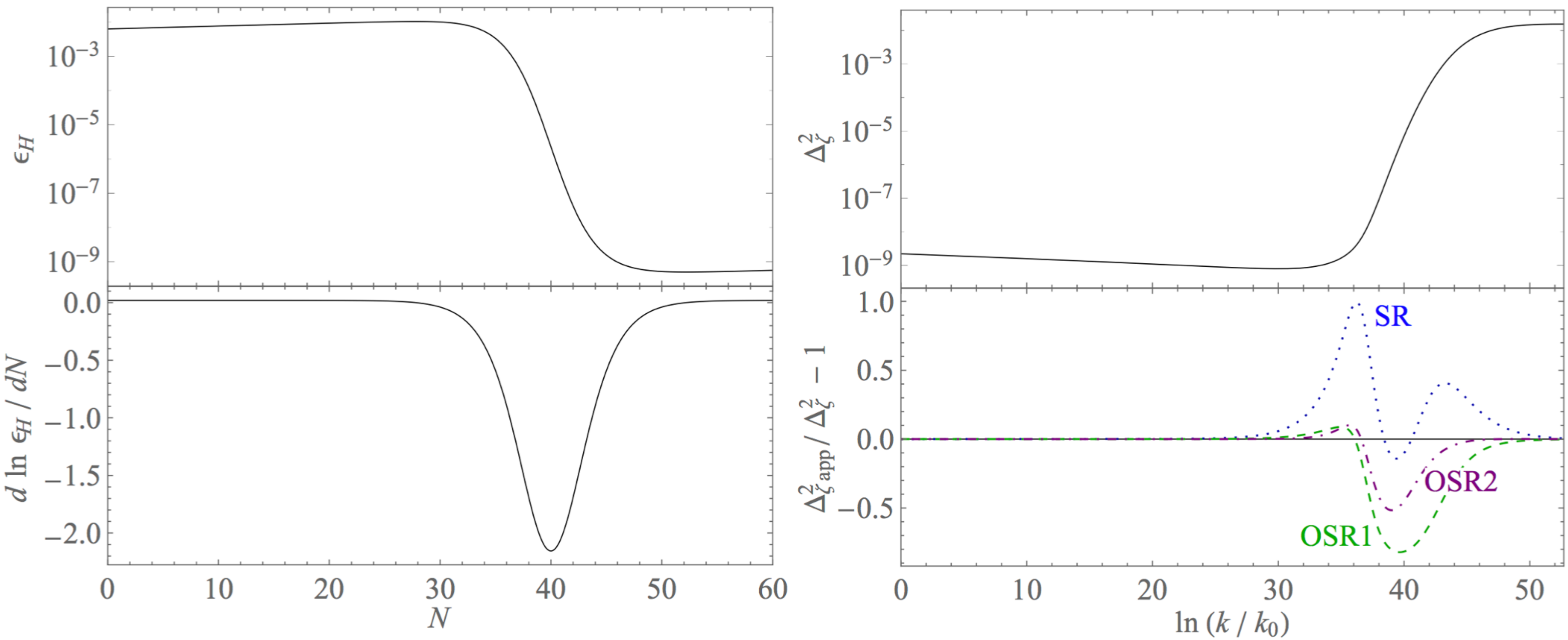


HM, Hu, 1706.06784

Case study 3: Slow roll step model

Parametrize $\ln \epsilon_H$ directly

$$\ln \epsilon_H = C_1 + C_2 N - C_3 \left[1 + \tanh\left(\frac{N - N_s}{d}\right)\right]$$



For $\Delta N < 10$, all approximations do not work.

Summary

- PBH production requires

  slow-roll violation $\left|\frac{\Delta \ln \epsilon_H}{\Delta N}\right| > 0.38$

$\Rightarrow$ Previous slow-roll analyses need reconsideration

  - Inflection model: No sufficient peak in Δ_ζ^2
  - Running mass model: Shift PBH mass scale

- Improved approximation: Optimized slow roll

  - Slow-roll step model:

    OSR remains a good description for models

    with 10^7 amplification of Δ_ζ^2 in $\Delta N > 10$.

- Applies for Horndeski, GLPV, subclass of DHOST.

  Unitary gauge = comoving gauge if ζ_{uni} ≈const.