Massive Graviton Geons:
self-gravitating massive gravitational waves

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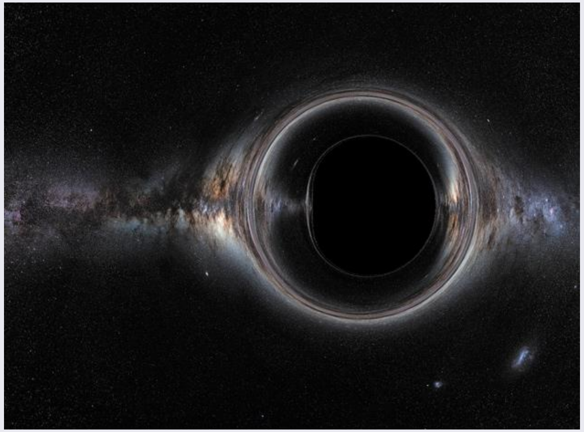
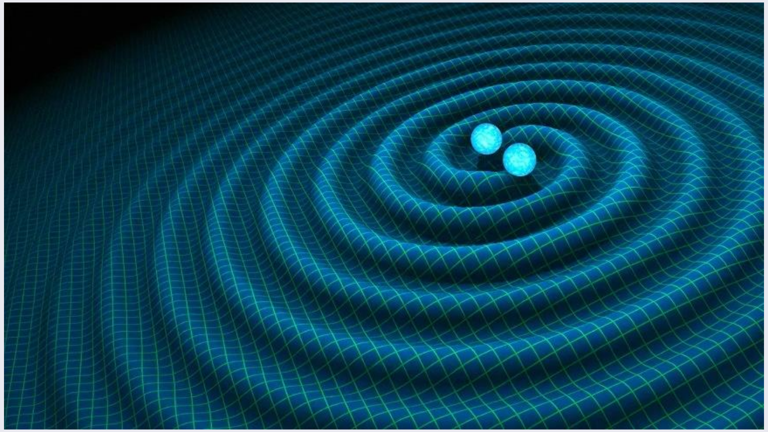
KA, K. Maeda, Y. Misonoh, and H. Okawa,
PRD 97, 044005 (2018), [arXiv: 1710.05606].

Introduction

Vacuum solutions to the Einstein equation?



Black Holes



Gravitational Waves

LIGO and Virgo observed both of them!

GW150914

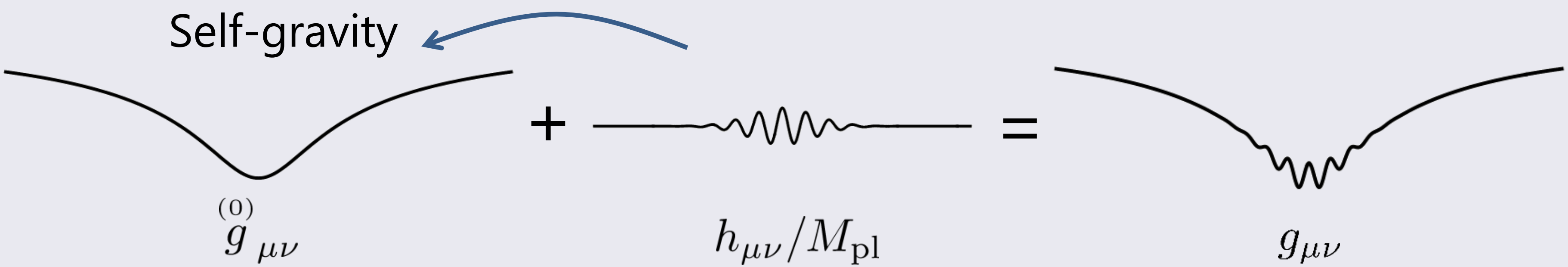
Initial mass: $65.3M_\odot = 36.2M_\odot + 29.1M_\odot$ → Final mass: $62.3M_\odot$

The energy is radiated by GWs!

GWs have their gravitational energy!

Due to the nonlinearities of the Einstein equation,
GWs (=perturbations) themselves change the background geometry.

Is it possible to realize self-gravitating gravitational waves?



Gravitational “Geons”

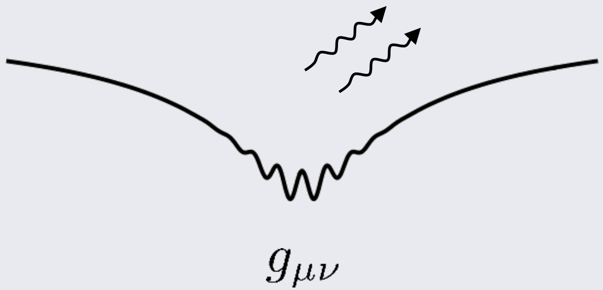
The original idea of “geon” is a gravitational electromagnetic entity.
= a realization of classical “body” by gravitational attraction.

Wheeler, 1955.

Gravitational Geons

Gravitational geons are singular-free time periodic vacuum solutions to GR.

Brill and Hartle, 1964, Anderson and Brill, 1997.



Gravitational geons

not stable and decay in time.

Gibbons and Stewart, 1984.

can be stable in asymptotically AdS?

e.g., Dias, Horowitz, Marolf and Santos, 2012.

This may not be the case in modified gravity.

Geons can be a proof of beyond GR? Geons can be dark matter?

We consider gravitational geons composed of massive graviton.

Massive gravitons?

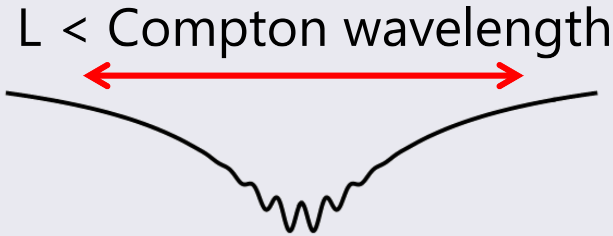
Massive modes as with other gauge theories? as KK modes?

It should break the gauge symmetry of graviton.
→ At least, we have to introduce two "metrics": $g_{\mu\nu}$ and $f_{\mu\nu}$.

If only one of them is dynamical: massive gravity (5 dof)
If both of them are dynamical: bigravity (2+5 dof)

We only consider bigravity theory.

In massive gravity, we may not find non-relativistic geons (not long-lived).



Localized scale $\simeq$ Compton wavelength

→ relativistic object

Massive gravitons?

Two dynamical tensors: $g_{\mu\nu}$ and $f_{\mu\nu}$ (Hassan and Rosen, 2011)

$$S = \frac{1}{2\kappa_g^2}\int d^4x\sqrt{-g}R(g) + \frac{1}{2\kappa_f^2}\int d^4x\sqrt{-f}\mathcal{R}(f) - \frac{m^2}{\kappa^2}\int d^4x\sqrt{-g}\sum_{i=0}^{4} b_i \mathscr{U}_i(g,f)$$

$$\mathscr{U}_n(g,f) = -\frac{1}{n!(4-n)!}\epsilon^{\cdots\cdot}\epsilon_{\cdots\cdot}(\gamma^\mu{}_\nu)^n \qquad \gamma^\mu{}_\alpha\gamma^\alpha{}_\nu = g^{\mu\alpha}f_{\alpha\nu} \qquad \kappa^2 = \kappa_g^2 + \kappa_f^2$$

Free parameters: $\kappa_g, \kappa_f, m, b_i \; (i = 0, 1, 2, 3, 4)$

Bigravity contains one massless graviton and one massive graviton.

We do not assume any particular value of the graviton mass.

We consider self-gravitating massive gravitational waves.

High frequency approximation

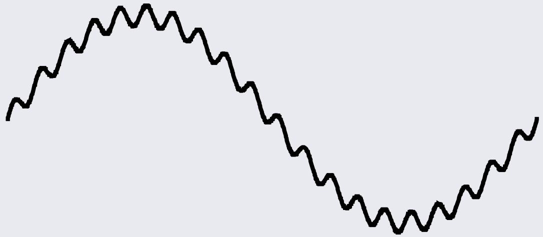
In general, there is no way to decompose
``background`` and ``perturbations`` if backreaction is included.

Gravitons propagating on background

$$g_{\mu\nu} = g_{\mu\nu}^{\text{background}} + \delta g_{\mu\nu}^{\text{perturbation}}$$
$$f_{\mu\nu} = f_{\mu\nu}^{\text{background}} + \delta f_{\mu\nu}^{\text{perturbation}}$$

Backreaction from gravitons

However, they can be decomposed when perturbations are high-frequency.
(Isaacson, 1968)



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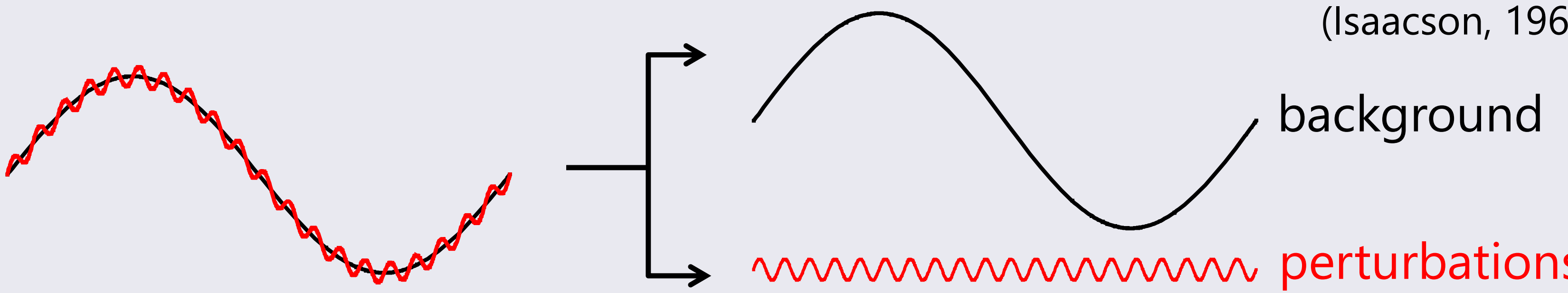


How to define energy of GW? (in GR)

The spacetime is decomposed into "background" and "perturbation".

$$g_{\mu\nu} = \overset{(0)}{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} \quad \text{with} \quad \partial \overset{(0)}{g}_{\mu\nu} \sim \frac{1}{L_B}\,, \quad \partial h_{\mu\nu} \sim \frac{h}{\lambda}\,, \quad h/M_{\rm pl} \ll 1$$

The high-frequency/momentum approximation ($\lambda \ll L_B$)

$$R_{\mu\nu} = \overset{(0)}{R}_{\mu\nu} + \delta \overset{(1)}{R}_{\mu\nu} + \delta \overset{(2)}{R}_{\mu\nu} + \cdots$$

$\overset{(0)}{R} \sim \partial^2 \overset{(0)}{g}$: only low-frequency part

$\delta \overset{(1)}{R} \sim \partial^2 h$: only high-frequency part

$\delta \overset{(2)}{R} \sim h\partial^2 h$: both low-frequency and high-frequency parts

$h \propto \sum e^{ikx} \to$ $h(k)h(k) \propto e^{2ikx}$: high-frequency part

$h(k)h(-k) \propto 1$: low-frequency part

How to define energy of GW? (in GR)

Einstein equation is decomposed into low- and high-frequency parts.

Low-frequency part: $\overset{(0)}{R}_{\mu\nu} = -\langle \delta \overset{(2)}{R}_{\mu\nu}(h) \rangle_{\rm low} \rightarrow \quad \frac{1}{L_B^2} = \frac{h^2/M_{\rm pl}^2}{\lambda^2}$

with $\lambda \ll L_B$

High-frequency part: $\delta \overset{(1)}{R}_{\mu\nu} = -\langle \delta \overset{(2)}{R}_{\mu\nu} \rangle_{\rm hight}$

$$\rightarrow \quad \frac{h}{\lambda^2} = \frac{h^2}{\lambda^2} \quad \rightarrow \quad \overset{(1)}{G}_{\mu\nu} = 0$$

The energy-momentum tensor is defined by nonlinear terms

$$\langle T_{\rm gw}^{\mu\nu} \rangle_{\rm low} = -\left(\overset{(0)}{g}{}^{\mu\alpha} \overset{(0)}{g}{}^{\nu\beta} - \frac{1}{2} \overset{(0)}{g}{}^{\mu\nu} \overset{(0)}{g}{}^{\alpha\beta} \right) \langle \delta \overset{(2)}{R}_{\alpha\beta} \rangle_{\rm low} + \cdots$$

Non-local operation, e.g., spatial average or time average

Graviton $T^{\mu\nu}$ in Bigravity

Assuming $|\partial^2 g_{\mu\nu}| \ll m^2$ (no Vainshtein effect) and taking Isaacson average,

we find the Einstein and Klein-Gordon equations

$$G^{\mu\nu}[\overset{(0)}{g}] \simeq \frac{1}{M_{\rm pl}^2}(\langle T^{\mu\nu}_{\rm gw}\rangle_{\rm low} + \langle T^{\mu\nu}_{G}\rangle_{\rm low})$$

$$\Box h_{\mu\nu} \simeq 0\,, \quad (\Box - m^2)\varphi_{\mu\nu} \simeq 0$$ + TT conditions

where $T^{\mu\nu}_{\rm gw} \sim (\partial h_{\mu\nu})^2\,, \quad T^{\mu\nu}_{G} \sim (\partial \varphi_{\mu\nu})^2 + m^2\varphi^2_{\mu\nu}$

$$M_{\rm pl} = \frac{\kappa}{\kappa_g \kappa_f}\,, M_G = \frac{\kappa}{\kappa_g^2}$$

The metrics are given by

$$g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} + \frac{\varphi_{\mu\nu}}{M_G}\,, \quad f_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} - \frac{\varphi_{\mu\nu}}{\alpha M_G}\,,$$

$$(\alpha = M_{\rm pl}^2/M_G^2)$$

We shall ignore the massless gravitational waves $h_{\mu\nu}$.

Newtonian limit of bigravity

We then assume that the massive gravitons are non-relativistic.

$$\overset{(0)}{g}_{\mu\nu}dx^\mu dx^\nu = -(1+2\Phi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j$$

$$\varphi_{\mu\nu} = \begin{pmatrix} \psi_{00} & \psi_{0i} \\ * & \frac{\psi_{\rm tr}}{3}\delta_{ij} + \psi_{ij} \end{pmatrix} e^{-imt} + \text{c.c.}\,,$$

↑ traceless, $\psi^i{}_i = 0$

where $\Phi, \psi_{..}$ are slowly varying functions.

The transverse-traceless condition leads to $|\psi_{00}|, |\psi_{\rm tr}| \ll |\psi_{0i}| \ll |\psi_{ij}|$

Finally, we obtain the Poisson-Schrodinger equations

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \quad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

Scale invariance

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \qquad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

Note that the equations are invariant under

$$\Phi \to \lambda^2\Phi\,, \quad \psi_{ij} \to \lambda^2\psi_{ij}\,, \quad |x^i| \to \lambda^{-1}|x^i|\,, \quad t \to \lambda^{-2}t$$

The mass of the localized ψ_{ij}: $\quad M \to \lambda M$

$$M := \int d^3x\, \frac{m^2}{4}\psi_{ij}^*\psi^{ij}$$

Increasing mass → small radius (compact object)

Newtonian approximation is valid as long as $R \ll m^{-1}$.

$$R_{\rm min} \sim m^{-1}\,, \quad M_{\rm max} \sim (Gm)^{-1} \sim 1M_\odot\left(\frac{10^{-10}{\rm eV}}{m}\right)$$

Self-gravitating bound state

The bound state of the Poisson-Schrodinger eqs. with intrinsic spin.

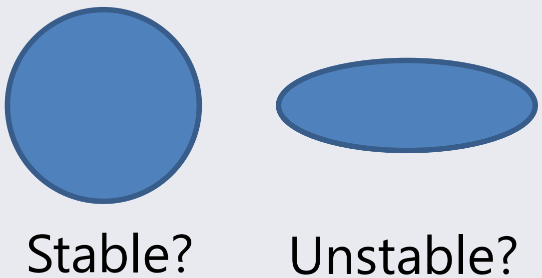
$$\psi_{ij}(t,\mathbf{x}) = \psi_{ij}(\mathbf{x})e^{-iEt}\,, \quad i\frac{\partial}{\partial t} \to E$$

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \quad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,, \qquad \text{Spin-2}$$

Cf. $$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*\psi\,, \quad i\frac{\partial}{\partial t}\psi = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi\,, \qquad \text{Spin-0}$$

Only difference is the intrinsic spin

ψ_{ij} : symmetric traceless tensor $\quad \psi$: scalar



What is the most stable configuration?

Angular momentum of bound state

Maybe… spherically symmetric configuration (monopole)?

However, it is NOT because of the intrinsic spin!

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \qquad E\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

The most stable = The lowest energy eigenvalue

$$\Delta = \frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - \frac{\ell(\ell+1)}{r^2}$$

= The lowest angular momentum

There are total angular momentum j and orbital angular momentum ℓ.

Angular momentum of bound state

There are total angular momentum and orbital angular momentum.

$$[\hat{\mathbf{L}}_I, \hat{\mathbf{L}}_J] = i \sum_K \epsilon_{IJK} \hat{\mathbf{L}}_K \,,$$

$$[\hat{\mathbf{J}}_I, \hat{\mathbf{J}}_J] = i \sum_K \epsilon_{IJK} \hat{\mathbf{J}}_K \,,$$

$$[\hat{\mathbf{J}}_I, \hat{\mathbf{L}}_J] = i \sum_K \epsilon_{IJK} \hat{\mathbf{L}}_K \,, \qquad \hat{\mathbf{J}}_I = \hat{\mathbf{L}}_I + \hat{\mathbf{S}}_I$$

We consider the angular momentum eigenstate.

$$\hat{\mathbf{L}}^2 \psi_{ij} = \ell(\ell+1)\psi_{ij} \,, \quad \hat{\mathbf{J}}^2 \psi_{ij} = j(j+1)\psi_{ij} \,, \quad \hat{\mathbf{J}}_z \psi_{ij} = j_z \psi_{ij} \,,$$

The Laplace operator is given by

$$\Delta = \frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr} - \frac{\hat{\mathbf{L}}^2}{r^2}$$

Self-gravitating bound state

Spin-2 case $j = \ell + s \ (s = 0, \pm 1, \pm 2)$

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*_{ij}\psi^{ij}\,, \qquad E\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

The monopole configuration $j = 0 \ \rightarrow \ell = 2 \ (s = -2)$

The quadrupole configuration $j = 2 \ \rightarrow \ell = 0 \ (s = +2)$ **Lowest energy**

Spin-0 case $j = \ell$

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi^*\psi\,, \qquad E\psi = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi\,,$$

The monopole configuration $j = 0 \ \rightarrow \ell = 0 \ (s = 0)$ **Lowest energy**

The lowest energy state in massive graviton geons must be quadrupole!

Monopole geon and Quadrupole geon

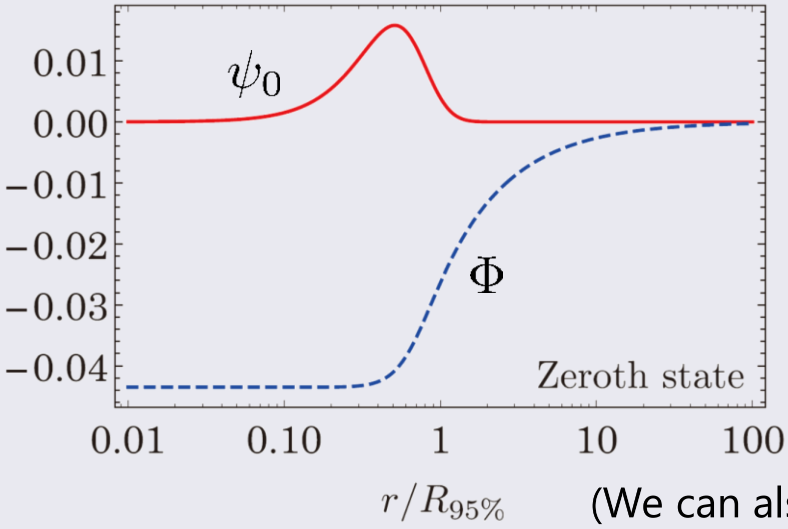
The monopole configuration

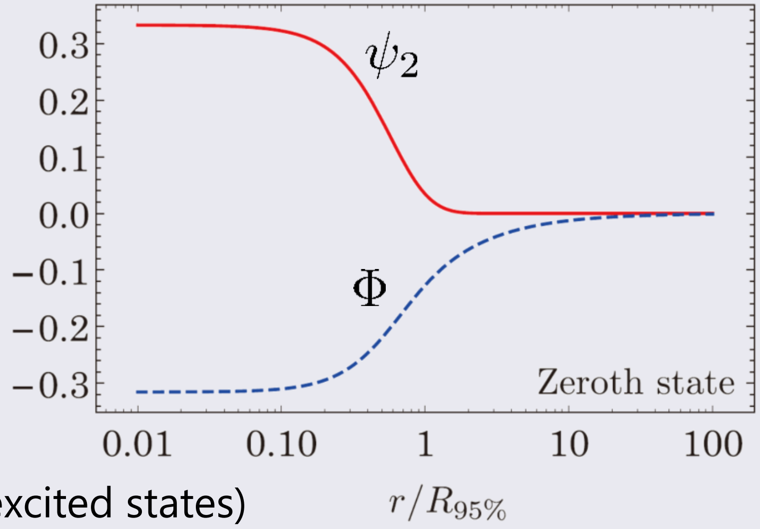
$$\psi_{ij} = \sqrt{16\pi}\psi_0(r)e^{-iEt}(T^{-2}_{0,0})_{ij} \, ,$$

The quadrupole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_2(r)e^{-iEt}\sum_{j_z} a_{j_z}(T^{+2}_{2,j_z})_{ij} \, ,$$

$$\tilde{E} = \frac{E}{(GM)^2m^3} = -0.027 \quad > \quad \tilde{E} = \frac{E}{(GM)^2m^3} = -0.16$$





(We can also find excited states)

Monopole geon and Quadrupole geon

The monopole configuration

$$\psi_{ij} = \sqrt{16\pi}\psi_0(r)e^{-iEt}(T^{-2}_{0,0})_{ij}\ ,$$

$$\tilde{E} = \frac{E}{(GM)^2m^3} = -0.027$$

$>$

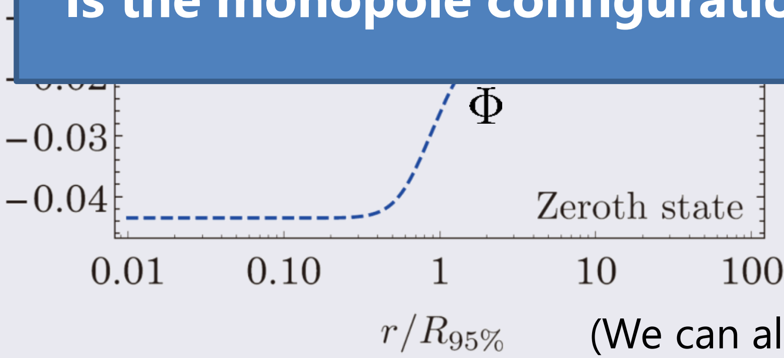
The quadrupole configuration

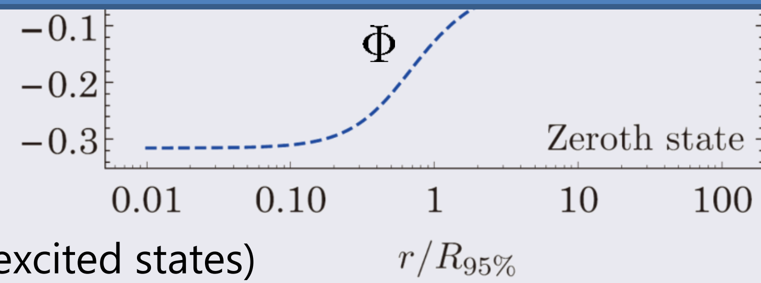
$$\psi_{ij} = \sqrt{16\pi}\psi_2(r)e^{-iEt}\sum_{j_z} a_{j_z}(T^{+2}_{2,j_z})_{ij}\ ,$$

$$\tilde{E} = \frac{E}{(GM)^2m^3} = -0.16$$

The lower energy state must be more stable than the higher state.

Is the monopole configuration unstable???





(We can also find excited states)

Stability of monopole geon

We thus study the perturbations around the monopole configuration.

We assume the perturbations do not spoil the Newtonian approx.

$$\Delta\Phi = \frac{m^2}{8M_{\rm pl}^2}\psi_{ij}^*\psi^{ij}\,, \quad i\frac{\partial}{\partial t}\psi_{ij} = \left(-\frac{\Delta}{2m} + m\Phi\right)\psi_{ij}\,,$$

We consider

$$\Phi = \Phi_0(r) + \delta\Phi(t,\mathbf{x})\,, \quad \psi_{ij} = \psi_{0,ij} + \delta\psi_{ij}(t,\mathbf{x})$$

$$\psi_{0,ij} = \sqrt{16\pi}\psi_0(r)e^{-iEt}(T_{0,0}^{-2})_{ij}\,,$$

Background spherical symmetry

→ perturbations can be expanded in terms of spherical harmonics.

Instability of monopole geon

The system is reduced into the eigenvalue problem after the Fourier transformation in the time domain.

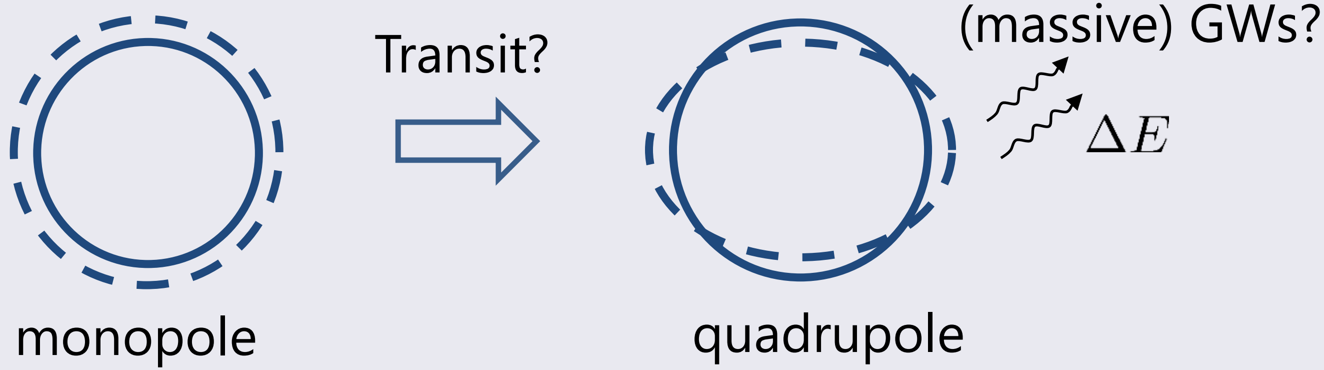
$$\delta\Phi(t,\mathbf{x}) \to \delta\Phi(r) Y_{j,j_z} e^{-i\omega t}, \cdots$$

	$j=1$	$j=2$	$j=3$
$\tilde{\omega}_0$	0.00000	0.00000	0.004440
$\tilde{\omega}_1$	0.004674	$0.0005155i$	0.004918
$\tilde{\omega}_2$	0.00622	0.008190	0.005600
$\tilde{\omega}_3$	0.01078	0.008469	0.01133
$\tilde{\omega}_4$	0.01132	0.008660	0.01189
$\tilde{\omega}_5$	0.01551	0.01070	0.01346
$\tilde{\omega}_6$	0.01581	0.01358	0.01559

The monopole geon is unstable against quadrupole mode perturbations.

Stability of geons

The unstable perturbations may be the transition mode.



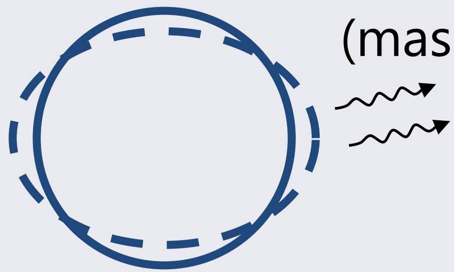
$$\tilde{E} = \frac{E}{(GM)^2 m^3} = -0.027 \quad > \quad \tilde{E} = \frac{E}{(GM)^2 m^3} = -0.16$$

The monopole may transit to the quadrupole by releasing binding energy.

Stability of geons

GWs could be emitted due to non-spherically symmetric oscillations.

quadrupole



(massless and/or massive) GWs?

But, the emission is small because of the large hierarchy between the time and the length scales.

Anisotropic pressure $\sim T^{\rm TT}_{G,ij}(\mathbf{x})e^{-2imt}$, $\quad \partial_k T^{\rm TT}_{G,ij}(\mathbf{x}) \ll m T^{\rm TT}_{G,ij}(\mathbf{x})$

(GWs are emitted if $\omega^2 = k^2$ or $\omega^2 = k^2 + m^2$)

$$|h^{TT}_{ij}| \propto \int dr' r' \psi^2(r') \sin[2mr']$$

→ The non-relativistic quadrupole geon is an (approximately) stable object.

Production of geons

Coherent massive GW

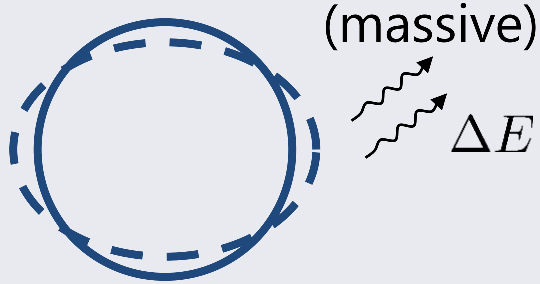
Jeans instability
(KA and Maeda, '18)

Excited states?

Transit?

(massive) GWs?

quadrupole



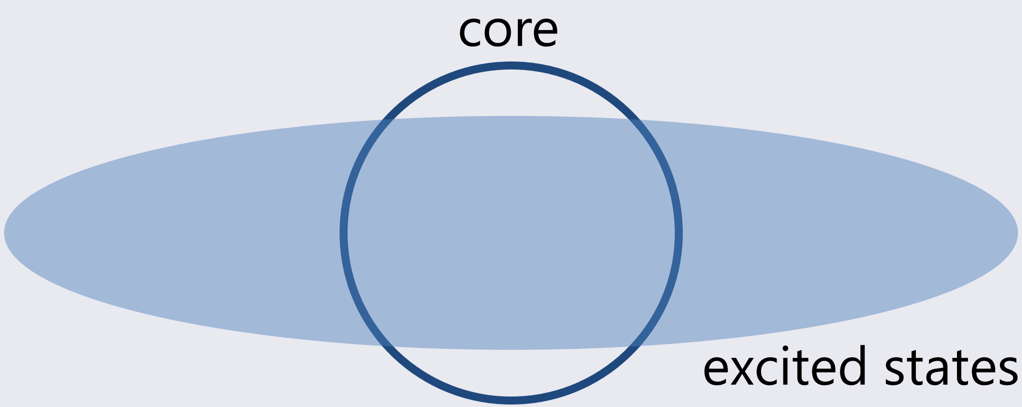
If the graviton mass is quite light, the scenario should be more complicated.

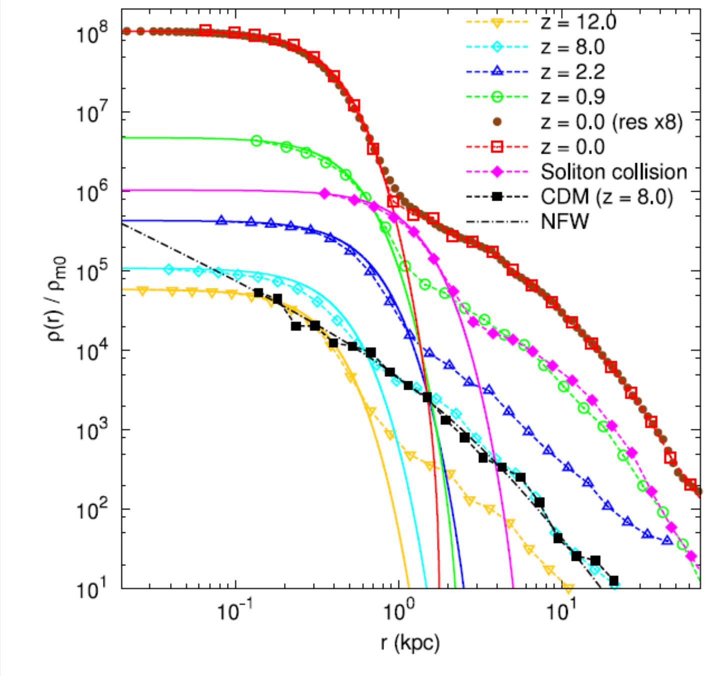
Geons as field dark matter

If a mass is $\sim 10^{-22}$eV, massive graviton can be a fuzzy dark matter.

Ultralight axion: spin-0 DM
Massive graviton: spin-2 DM

In FDM, the central part of DM halos is given by the "soliton" (=geon).





From Schive et al, 2014

Geons as field dark matter

Although the field configuration is not spherically symmetric, the energy distribution is spherically symmetric.

ψ_{ij} : not spherical $\qquad$ $\psi^*_{ij}\psi^{ij}$: spherical

and the energy distribution is exactly the same as that of spin-0 case.

Spin-2 FDM could shear successes of spin-0 FDM.

Is there any differences?

Spin-0: isotropic oscillation, Spin-2: anisotropic oscillation

GWs could (not?) be emitted during the formation of DM halos?

DM is not new "particle" but spacetime itself $g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{\varphi_{\mu\nu}}{M_G}$

Summary

Massive graviton geons = self-gravitating massive GWs

New vacuum solutions to bigravity theory.

The ground state must be non-spherical.

Spin-0: ground state = monopole ⇒ $\ell = j = 0$

Spin-2: ground state = quadrupole ⇒ $\ell = 0, \ j = 2$

Ultralight massive graviton can be FDM as well.

Note that DM is not new "particle" but spacetime itself

$$g_{\mu\nu} \simeq \overset{(0)}{g}_{\mu\nu} + \frac{\varphi_{\mu\nu}}{M_G},$$

Possible prospects: Hairy BHs?, Geon as BE condensate? etc...