Stable cosmology in chameleon bigravity
Based on

Stable cosmology in chameleon bigravity

arXiv 1711.04655, with
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Outline

1. Introduction and motivations
2. Description of the theory
3. Our goal: realistic background cosmology
4. The details
   i. Action
   ii. Scaling solutions
   iii. Stability
5. Numerics and results
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1. Introduction and motivations
Massive bigravity

Can we extend the general relativity by considering two interacting metrics $g_{\mu\nu}$ and $f_{\mu\nu}$?

The non linear theory is given by Hassan and Rosen, 1109.3515

$$S_{HR} = S_{EH}[g_{\mu\nu}] + S_{EH}[f_{\mu\nu}] + S_m$$

with the dRGT interaction term (de Rham, Gabadadze, Tolley, 1011.1232)

$$S_m = M_g^2 m^2 \int d^4x \sqrt{-g} \beta_i e_i \left[ s^\mu_{\nu} \right] \text{ with } s^\mu_{\nu} \equiv \left( \sqrt{g^{-1}f} \right)^\mu_{\nu}$$

Two branches of cosmological solutions:

- Self-accelerating (unstable)
- Normal branch (stable) (fine tunings needed...)

De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 1404.0008
Fine-tuning problems in bigravity

a. There are neg. norm states if $m_T^2 < O(1)H^2$ (Higuchi bound) (Higuchi, 1989)

b. Fine-tuning needed to pass solar system tests with Vainshtein screening...

c. ... and to have an interesting phenomenology (De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 1404.0008)
Environment dependence

Can we make the graviton mass
- heavy enough in the early Universe?
- heavy enough in astrophysical systems?
- light enough in other settings?

May be solved if the graviton mass scales as the energy density!

Use a messenger: **chameleon scalar field**

Khoury and Weltman, arXiv: 0309411
Chameleon mechanism

\[ \ddot{\phi} + 3H\dot{\phi} = -\alpha A(\phi)T + m^2 V(\phi) \]
Chameleon mechanism

Schematically

\[ m_T^2 \propto \rho \propto H^2 \]

In astrophysical setting:

Chameleon mechanism for both the scalar field and the graviton!

In cosmological setting:

Higuchi bound can be satisfied at all times!
2. Chameleon bigravity

De Felice, Uzan, Mukohyama, 1702.04490

- A theory of 2 gravitons and 1 scalar field
- Chameleon → Environment-dependent graviton mass
  Khoury and Weltman, astro-ph/0309300
- This extends **massive bigravity** and addresses the fine-tuning problems
- The theory becomes applicable to the early Universe
3. Goal of the work

- Show that the theory can accommodate a “realistic” background cosmology!

Does everything work as planned?
- Higuchi bound
- Stability
- Modes
- We do not cover before radiation domination
4. The details

Does this make sense...

????

Bigravity
The action

Chameleon bigravity side

\[ s_{\mu \nu} \equiv \left( \sqrt{g^{-1}f} \right)^{\mu \nu} \]

\[ S_{\text{EH}} = \frac{M_g^2}{2} \int \sqrt{-g} R[g] d^4x + \frac{\kappa M_g^2}{2} \int \sqrt{-f} R[f] d^4x, \]

\[ S_m = M_g^2 m^2 \int \sum_{i=0}^4 \beta_i(\phi) e_i[s] \sqrt{-g} d^4x, \]

\[ S_{\phi} = -\frac{1}{2} \int g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi \sqrt{-g} d^4x \]

Matter side

\[ S_{\text{mat}} = \int \mathcal{L}_{\text{mat}}(\psi, \tilde{g}_{\mu \nu}) d^4x \]

\[ \tilde{g}_{\mu \nu} \equiv A(\phi)^2 g_{\mu \nu} \]
Background cosmology

Exponential couplings

\[ A(\phi) = e^{\beta \phi / M_g}, \quad \beta_i(\phi) = -c_i e^{-\lambda \phi / M_g} \]

⇒ Existence of scaling solutions

Friedmann Ansätze

\[ ds^2_g = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad ds^2_f = \xi^2(t) \left[ -c^2(t) dt^2 + a^2(t) \delta_{ij} dx^i dx^j \right] \]

Friedmann equations

\[
\begin{align*}
3H^2 &= \frac{1}{M_g^2} \left[ \rho A^4 + \frac{1}{2} \dot{\phi}^2 \right] + m^2 R(\xi, \phi), \\
3H_j^2 &= \frac{m^2}{\kappa \xi^3} U, \dot{\xi}(\xi, \phi),
\end{align*}
\]

1st Einstein equations

\[
\begin{align*}
2 \dot{H} &= -\frac{1}{M_g^2} \left[ (\rho + P) A^4 + \dot{\phi}^2 \right] + m^2 \xi (c - 1) J(\xi, \phi), \\
2 \dot{H}_f &= m^2 \frac{1 - c}{\kappa \xi^2} J(\xi, \phi),
\end{align*}
\]

Scalar equations

\[ \ddot{\phi} + 3H \dot{\phi} = -\alpha A^4 (\rho - 3P) + M_g^2 m^2 Q, \dot{\phi}(\xi, \phi), \]

\[
\begin{align*}
R &= U - \xi U, \xi / 4, \\
J &= R, \xi / 3, \\
Q &= (c - 1) R - cU, \\
U &= - (\beta_4 \xi^4 + 4 \beta_3 \xi^3 + 6 \beta_2 \xi^2 + 4 \beta_1 \xi + \beta_0),
\end{align*}
\]
Scaling solutions

- Exact **radiation** dominated and \( \Lambda \)-dominated solutions

\[
\frac{d\varphi}{dN_e} = \frac{n}{\lambda}, \quad \frac{1}{h} \frac{dh}{dN_e} = -\frac{2}{n}, \quad \xi = \text{const}, \quad c = \text{const}
\]

\( \varphi \equiv \phi/M_g, \quad h \equiv \frac{H}{m} \)

\( N_e = \ln \left( \frac{a(t)}{a_i} \right) \)

- **Dust**-dominated, under condition

\[
\beta \left( \lambda^2 - \frac{3c}{c + \kappa \xi^2} \right) = 0
\]

- When \( \beta \ll 1 \) yields an approximate scaling solution.
Scaling solutions

The scaling solutions under homogeneous perturbations yield

\[
\begin{align*}
\ln h &= \ln h_0 - \frac{n}{2} N_e + \epsilon h^{(1)}, \\
\varphi &= \frac{n N_e}{\lambda} (1 + \epsilon \varphi^{(1)}), \\
\xi &= \bar{\xi} + \epsilon \xi^{(1)}, \\
c &= c^{(0)} + \epsilon c^{(1)},
\end{align*}
\]

yield

\[
\begin{align*}
\varphi^{(1)'}'' + \left(1 + \frac{2}{N_e}\right) \varphi^{(1)'} + A_r \varphi^{(1)} &= 0, \\
\varphi^{(1)'''} + \left(\frac{3}{2} + \frac{2}{N_e}\right) \varphi^{(1)'} + A_m \varphi^{(1)} &= 0
\end{align*}
\]

where

\[
A_i > 0
\]
Inhomogeneous perturbations

ADM splitting

\[ ds_g^2 = -N^2 dt^2 + \gamma_{ij} (N^i dt + dx^i) (N^j dt + dx^j), \quad ds_f^2 = -\tilde{N}^2 dt^2 + \tilde{\gamma}_{ij} (\tilde{N}^i dt + dx^i) (\tilde{N}^j dt + dx^j) \]

Perturbations

\[ \phi = \bar{\phi} + \delta \phi, \quad \psi_\alpha = \bar{\psi}_\alpha + \delta \psi_\alpha \]

\[ N = N(1 + \Phi), \quad N_i = N_i + \delta N_i, \quad \gamma_{ij} = a^2 \delta_{ij} + \delta \gamma_{ij}, \]

\[ \tilde{N} = \tilde{N}(1 + \tilde{\Phi}), \quad \tilde{N}_i = \tilde{N}_i + \delta \tilde{N}_i, \quad \tilde{\gamma}_{ij} = \tilde{a}^2 \delta_{ij} + \delta \tilde{\gamma}_{ij} \]

Decomposition in SO(3) representations

\[ \delta N_i = Na (\partial_i B + B_i) \]

\[ \delta \gamma_{ij} = a^2 \left[ 2\delta_{ij} \bar{\Psi} + \left( \partial_i \partial_j - \frac{\delta_{ij}}{3} \Delta \right) E + \partial_i \partial_j (E_j) + h_{ij} \right] \]

\[ \delta \tilde{N}_i = \tilde{N}\tilde{a} (\partial_i \tilde{B} + \tilde{B}_i) \]

\[ \delta \tilde{\gamma}_{ij} = \tilde{a}^2 \left[ 2\delta_{ij} \bar{\tilde{\Psi}} + \left( \partial_i \partial_j - \frac{\delta_{ij}}{3} \Delta \right) \tilde{E} + \partial_i \partial_j (\tilde{E}_j) + \tilde{h}_{ij} \right] \]
Inhomogeneous perturbations

**2x2 tensor modes**
- \( c_{T1} = 1, c_{T2} = c \)
- 2 massive modes
  \[ m_{T}^2 = m^2 \Gamma \frac{c + \kappa \xi^2}{\kappa \xi} \]
- & 2 massless modes

Non trivial no-ghost condition:
\[ c > 0 \]

**1x2 vector modes**
- \( c_{V} = m^2 \Gamma \frac{c+1}{2\xi J} \)
- 2 massive modes
  \[ m_{V}^2 = m_{T}^2 \]

Non trivial no-ghost condition:
\[ J > 0 \]

Non trivial no-gradient instability condition:
\[ \Gamma > 0 \]

**2 scalar modes**
- massive modes
- non trivial sound speeds

Non trivial no-ghost condition (large expression)
Non trivial no-gradient instability condition (large expression)

+ matter modes
5. Numerics
Equations for numerics

Set of equations to integrate

\[
\begin{aligned}
    h' &= h'(h, \xi, \varphi, \varphi'), \\
    \varphi'' &= \varphi''(h, \xi, \varphi, \varphi'), \\
    \xi' &= \xi'(h, \xi, \varphi, \varphi'),
\end{aligned}
\]

Initial conditions: quasi-radiation dominated scaling solution

\[
    h_i' \approx -2h_i, \quad \varphi_i' \approx \varphi_{sc} = \frac{4}{\lambda}, \quad \varphi_i'' \approx 0, \quad \xi_i' \approx 0
\]

Parameters for numerics

New choice of parameters so that $J > 0$ is always satisfied

\[ c_3 c_1 - c_2^2 = A, \quad c_1 + 2c_2 + c_3 = B \]

Finally we chose the parameters

\[ c_{in} = \frac{101}{100}, \quad c_{V,in}^2 = 1, \quad A = 1, \quad B = 1, \]
\[ \Omega_{\Lambda i}^{EF} = 1 \times 10^{-30}, \quad \Omega_{d i}^{EF} = 1 \times 10^{-5}, \quad \Omega_{k i}^{EF} = \frac{3}{200}, \quad \Omega_{V i}^{EF} = \frac{1}{200}, \]
\[ \beta = 1 \times 10^{-2}, \quad \lambda = \frac{40}{3} \]

NB : these are non unique...
Numerical results

Evolution as planned!
- Radiation – dust – $\Lambda$ domination
- Stable scaling solutions
- Small numerical errors

What about the Higuchi bound and the sound-speeds?
Numerical results

Again just as planned!

- $m_T^2 \gg H^2$ at all times
- Positive sound-speeds, close to 1
- No-ghost conditions are satisfied

Promising!

Proof of existence for a stable cosmology in chameleon bigravity!
Summary

i. **Chameleon bigravity** solves the fine-tuning problems of bigravity and extends its reach

ii. Scaling solutions were described

iii. Stability conditions under homogeneous and inhomogeneous perturbations were found

iv. The model propagates 2x2 tensor, 1x2 vector, 2 scalar + matter modes

v. Numerical integration and example background cosmology were achieved
Future outlook

A promising model, with avenues for further study!

_E.g._ constraints from:

i. More precise background cosmology

ii. Evolution of perturbations

iii. Solar-system tests

iv. GW wave-forms modified due to graviton oscillations
Merci beaucoup !
Back-up 1: density dependence

Compare the density at late times and cosmological distances $\rho_\infty$ with the local density $m_{loc}$

$$\frac{m_{T,loc}^2}{m_{T,\infty}^2} = \left( \frac{\rho_{loc}}{\rho_\infty} \right)^{\frac{\lambda}{\lambda + 4\beta}}$$

If $\beta$ is small enough...
Back-up 2: other graphs

- $-\frac{\dot{H}}{H^2}$
- $\kappa$
- $\varphi$
- $\ddot{\varphi}$

number of e-folds $N_e$
Backup 3: Higuchi condition and strong coupling scale

Graviton mass $m_T^2$

Strong coupling

$\Lambda_3 \sim 3 \sqrt{m_T^2 M_P}$

Higuchi bound

Cosm. density $\rho$