Stable cosmology in chameleon bigravity

第二回　若手による重力・宇宙論研究会　　2018年03月03日

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Based on

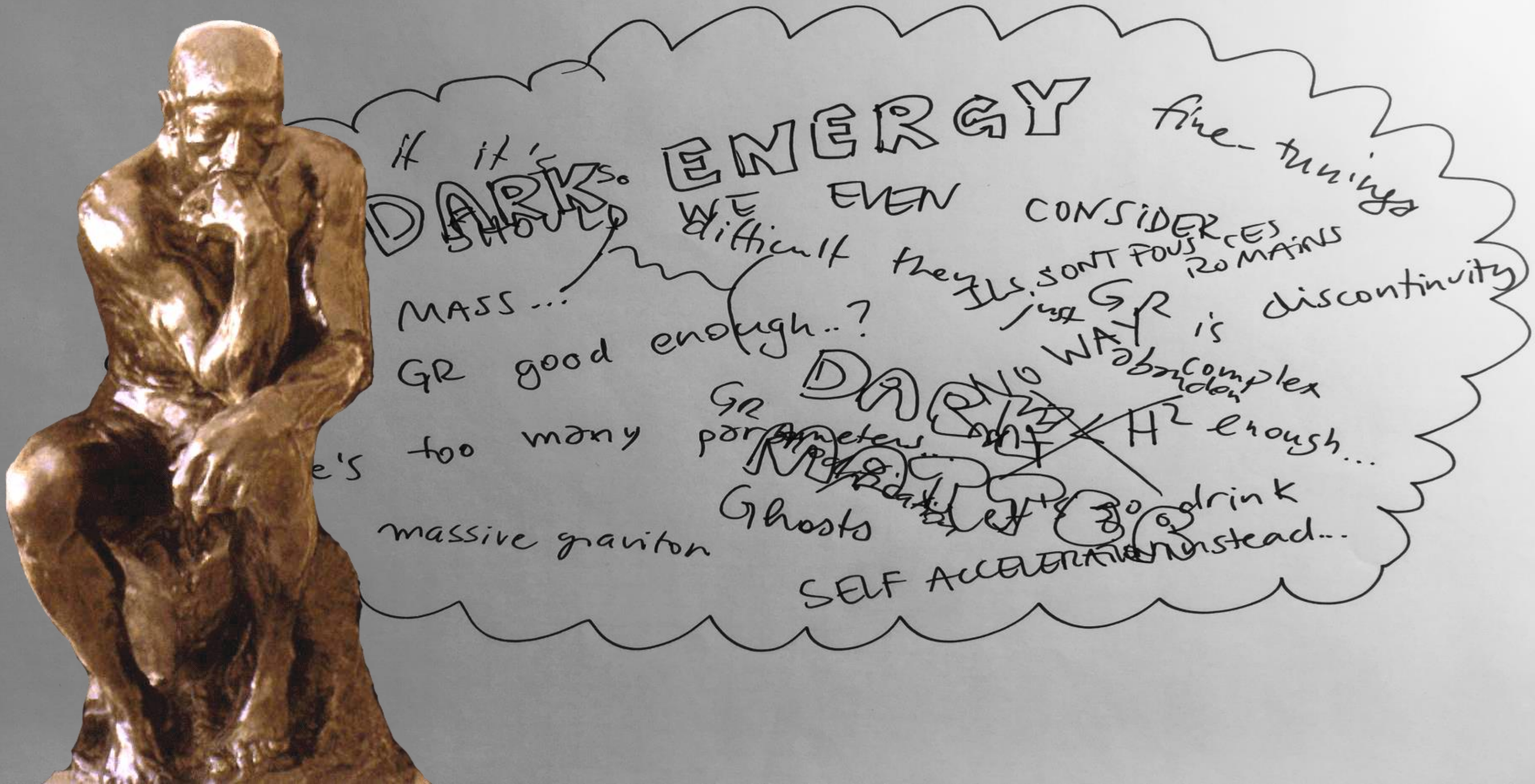
Stable cosmology in chameleon bigravity

arXiv 1711.04655, with

A. De Felice, S. Mukohyama, and Y. Watanabe

Outline

1. Introduction and motivations



Massive bigravity

Can we extend the general relativity by considering two interacting metrics $g_{\mu\nu}$ and $f_{\mu\nu}$?

The non linear theory is given by Hassan and Rosen, 1109.3515

$$S_{\mathrm{HR}} = S_{\mathrm{EH}}[g_{\mu\nu}] + S_{\mathrm{EH}}[f_{\mu\nu}] + S_m$$

with the dRGT interaction term (de Rham, Gabadadze, Tolley, 1011.1232)

$$S_m = M_g^2 m^2 \int d^4x \sqrt{-g}\beta_i e_i \left[s^\mu{}_\nu\right] \quad \text{with} \quad s^\mu{}_\nu \equiv \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu$$

Two branches of cosmological solutions:

- Self-accelerating (unstable)
- Normal branch (stable) (fine tunings needed…)

De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 1404.0008

Fine-tuning problems in bigravity

a. There are neg. norm states if $m_T^2 < \mathcal{O}(1)H^2$ (Higuchi bound) (Higuchi, 1989)

b. Fine-tuning needed to pass solar system tests with Vainshtein screening…

c. … and to have an interesting phenomenology (De Felice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka, 1404.0008)

Environment dependence

Can we make the graviton mass

- heavy enough in the early Universe ?
- heavy enough in astrophysical systems ?
- light enough in other settings ?

May be solved if the graviton mass scales as the energy density !

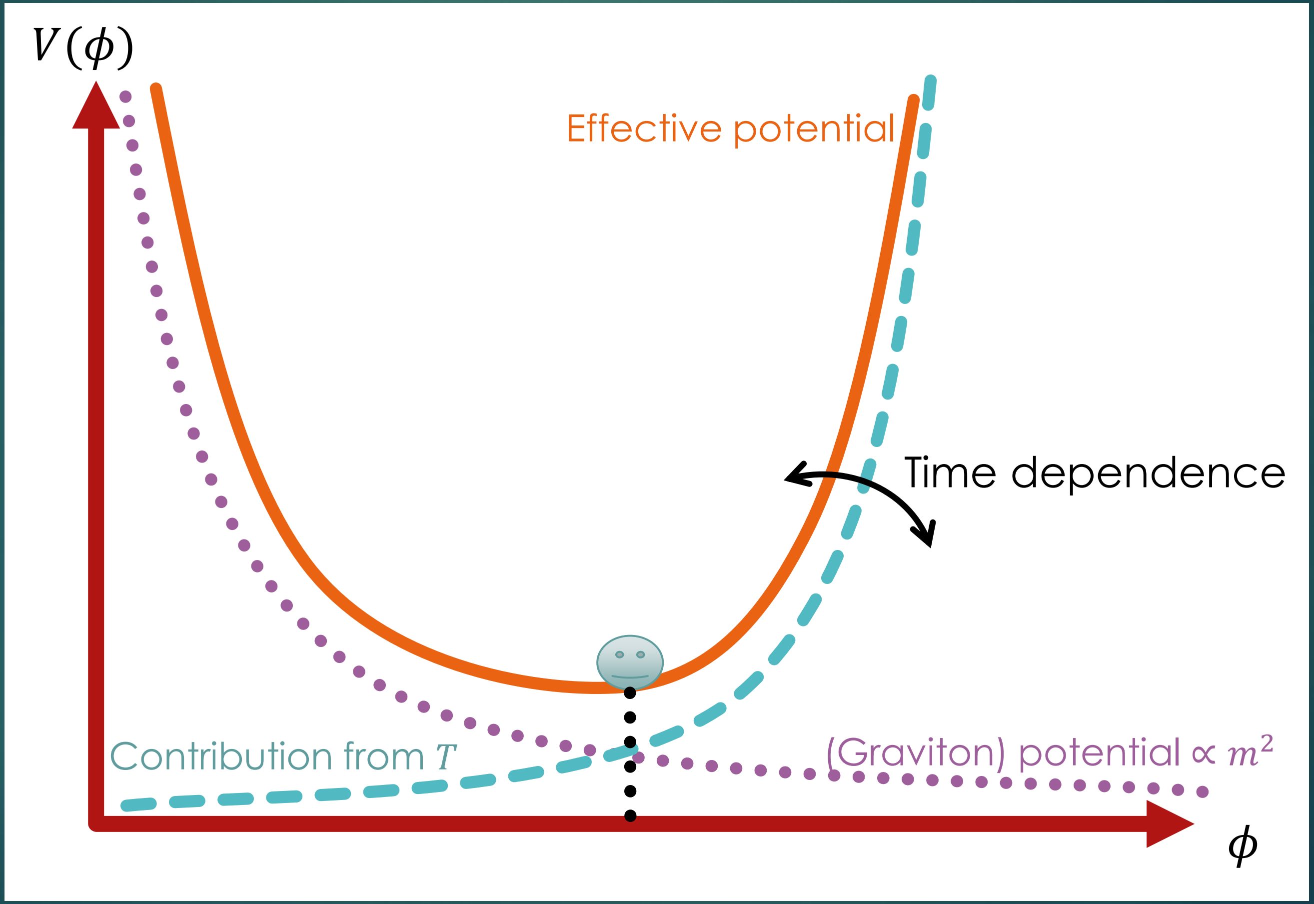
Use a messenger : **chameleon scalar field**

Khoury and Weltman, arXiv: 0309411

Chameleon boy (c) DC

Chameleon mechanism

$$\ddot{\phi} + 3H\phi = -\alpha A(\phi)T + m^2 V(\phi)$$



Chameleon mechanism

Schematically

$$m_T^2 \propto \rho \propto H^2$$

In astrophysical setting :

Chameleon mechanism for both the scalar field and the graviton !

In cosmological setting :

Higuchi bound can be satisfied at all times !

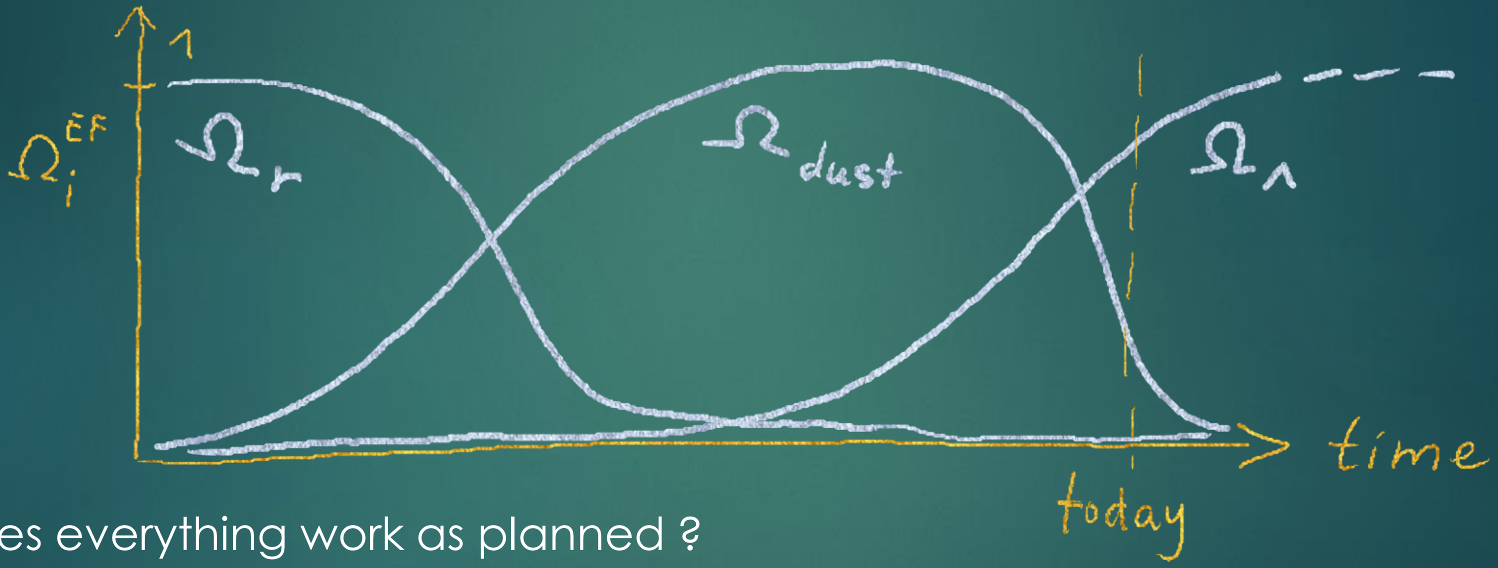
2. Chameleon bigravity

De Felice, Uzan, Mukohyama, 1702.04490

- A theory of 2 gravitons and 1 scalar field
- Chameleon → Environment-dependent graviton mass Khoury and Weltman, astro-ph/0309300
- This extends **massive bigravity** and addresses the fine-tuning problems
- The theory becomes applicable to the early Universe

3. Goal of the work

- Show that the theory can accommodate a "realistic" background cosmology !



Does everything work as planned ?

- Higuchi bound
- Stability
- Modes
- We do not cover before radiation domination

4. The details

The action

Chameleon bigravity side

$$s^{\mu}{}_{\nu} \equiv \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$$

$$S_{\rm EH} = \frac{M_g^2}{2}\int R[g]\sqrt{-g}d^4x + \frac{\kappa M_g^2}{2}\int R[f]\sqrt{-f}d^4x\,,$$

$$S_m = M_g^2 m^2 \int \sum_{i=0}^{4} \beta_i(\phi) e_i[s]\sqrt{-g}d^4x\,,$$

$$S_\phi = -\frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\sqrt{-g}d^4x$$

Matter side

$$S_{\rm mat} = \int \mathcal{L}_{\rm mat}(\psi, \tilde{g}_{\mu\nu})d^4x$$

$$\tilde{g}_{\mu\nu} \equiv A(\phi)^2 g_{\mu\nu}$$

Background cosmology

Exponential couplings $A(\phi) = e^{\beta\phi/M_g}\,, \quad \beta_i(\phi) = -c_i e^{-\lambda\phi/M_g}$

➔ Existence of scaling solutions

Friedmann Ansätze

$$ds_g^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j\,, \quad ds_f^2 = \xi^2(t)\left[-c^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j\right]$$

Friedmann equations

$$3H^2 = \frac{1}{M_g^2}\left[\rho A^4 + \frac{1}{2}\dot{\phi}^2\right] + m^2 R(\xi,\phi)\,,$$

$$3H_f^2 = \frac{m^2}{4\kappa\xi^3}U_{,\xi}(\xi,\phi)\,,$$

1st Einstein equations

$$2\dot{H} = -\frac{1}{M_g^2}\left[(\rho + P)\,A^4 + \dot{\phi}^2\right] + m^2\xi(c-1)J(\xi,\phi)\,,$$

$$2\dot{H}_f = m^2\frac{1-c}{\kappa\xi^2}J(\xi,\phi)\,,$$

Scalar equations

$$\ddot{\phi} + 3H\dot{\phi} = -\alpha A^4\,(\rho - 3P) + M_g^2 m^2 Q_{,\phi}(\xi,\phi)\,,$$

$$R \equiv U - \xi U_{,\xi}/4\,, \quad J \equiv R_{,\xi}/3\,, \quad Q \equiv (c-1)R - cU\,, \quad U \equiv -\left(\beta_4\xi^4 + 4\beta_3\xi^3 + 6\beta_2\xi^2 + 4\beta_1\xi + \beta_0\right)\,,$$

Scaling solutions

- Exact **radiation** dominated and **Λ**-dominated solutions

$$\boxed{\begin{array}{c}\dfrac{d\varphi}{dN_e} = \dfrac{n}{\lambda}, \quad \dfrac{1}{h}\dfrac{dh}{dN_e} = -\dfrac{2}{n}, \\ \xi = const\,, \quad c = const\end{array}}$$

$$\varphi \equiv \phi/M_g\,, \quad h \equiv \frac{H}{m}$$

$$N_e = \ln\left(a(t)/a_i\right)$$

n = 4 (rad.)
n = 3 (dust)
(n = 0) (Λ)

- **Dust**-dominated, under condition

$$\beta\left(\lambda^2 - \frac{3c}{c+\kappa\xi^2}\right) = 0$$

- When $\beta \ll 1$ yields an **approximate scaling solution**.

Scaling solutions

The scaling solutions under homogeneous perturbations

$$\begin{cases} \ln h = \ln h_0 - \frac{n}{2} N_e + \epsilon h^{(1)} \,, \\ \varphi = \frac{n N_e}{\lambda} (1 + \epsilon \varphi^{(1)}) \,, \\ \xi = \bar{\xi} + \epsilon \xi^{(1)} \,, \\ c = c^{(0)} + \epsilon c^{(1)} \,, \end{cases}$$

yield

$$\varphi^{(1)\prime\prime} + \left(1 + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_r \varphi^{(1)} = 0 \,, \qquad {}^\prime \equiv \frac{d}{dN_e}$$

$$\varphi^{(1)\prime\prime} + \left(\frac{3}{2} + \frac{2}{N_e}\right) \varphi^{(1)\prime} + \mathcal{A}_m \varphi^{(1)} = 0$$

$$\mathcal{A}_i > 0$$

Inhomogeneous perturbations

ADM splitting

$$ds_g^2 = -\mathcal{N}^2 dt^2 + \gamma_{ij}(\mathcal{N}^i dt + dx^i)(\mathcal{N}^j dt + dx^j)\,, \quad ds_f^2 = -\tilde{\mathcal{N}}^2 dt^2 + \tilde{\gamma}_{ij}(\tilde{\mathcal{N}}^i dt + dx^i)(\tilde{\mathcal{N}}^j dt + dx^j)$$

Perturbations

$$\phi = \bar{\phi} + \delta\phi \qquad \psi_\alpha = \bar{\psi}_\alpha + \delta\psi_\alpha$$

$$\mathcal{N} = N(1+\Phi)\,, \quad \mathcal{N}_i = N_i + \delta N_i\,, \quad \gamma_{ij} = a^2\delta_{ij} + \delta\gamma_{ij}\,,$$
$$\tilde{\mathcal{N}} = \tilde{N}(1+\tilde{\Phi})\,, \quad \tilde{\mathcal{N}}_i = \tilde{N}_i + \delta\tilde{N}_i\,, \quad \tilde{\gamma}_{ij} = \tilde{a}^2\delta_{ij} + \delta\tilde{\gamma}_{ij}$$

tensor vector scalar

Decomposition in SO(3) representations

$$\delta N_i = Na(\partial_i B + B_i)\,, \quad \delta\gamma_{ij} = a^2\left[2\delta_{ij}\Psi + \left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\Delta\right)E + \partial_{(i}E_{j)} + h_{ij}\right]\,,$$
$$\delta\tilde{N}_i = \tilde{N}\tilde{a}(\partial_i\tilde{B} + \tilde{B}_i)\,, \quad \delta\tilde{\gamma}_{ij} = \tilde{a}^2\left[2\delta_{ij}\tilde{\Psi} + \left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\Delta\right)\tilde{E} + \partial_{(i}\tilde{E}_{j)} + \tilde{h}_{ij}\right]$$

Inhomogeneous perturbations

2x2 tensor modes

- $c_{T1} = 1,\ c_{T2} = c$
- 2 massive modes

$$m_T^2 = m^2\Gamma\frac{c + \kappa\xi^2}{\kappa\xi}$$

& 2 massless modes

Non trivial no-ghost condition:

$$c > 0$$

1x2 vector modes

- $c_V = m^2\Gamma\frac{c+1}{2\xi J}$
- 2 massive modes

$$m_V^2 = m_T^2$$

Non trivial no-ghost condition:

$$J > 0$$

Non trivial no-gradient instability condition:

$$\Gamma > 0$$

2 scalar modes

- massive modes
- non trivial sound speeds

+ matter modes

Non trivial no-ghost condition (large expression)

Non trivial no-gradient instability condition (large expression)

5. Numerics

Equations for numerics

Set of equations to integrate

$$\begin{cases} h' = h'(h, \xi, \varphi, \varphi')\,, \\ \varphi'' = \varphi''(h, \xi, \varphi, \varphi')\,, \\ \xi' = \xi'(h, \xi, \varphi, \varphi')\,, \end{cases}$$

Initial conditions : quasi-radiation dominated scaling solution

$$h_i' \approx -2h_i\,, \quad \varphi_i' \approx \varphi_{\rm sc}' = \frac{4}{\lambda}\,, \quad \varphi_i'' \approx 0\,, \quad \xi_i' \approx 0$$

Parameters for numerics

New choice of parameters so that $J > 0$ is always satisfied

$$c_3 c_1 - c_2^2 = \mathfrak{A}\,, \quad c_1 + 2c_2 + c_3 = \mathfrak{B}$$

Finally we chose the parameters

$$c_{\rm in} = \frac{101}{100}\,, \quad c^2_{\rm V,in} = 1\,, \quad \mathfrak{A} = 1\,, \quad \mathfrak{B} = 1\,,$$

$$\Omega^{\rm EF}_{\Lambda i} = 1 \times 10^{-30}\,, \quad \Omega^{\rm EF}_{di} = 1 \times 10^{-5}\,, \quad \Omega^{\rm EF}_{ki} = \frac{3}{200}\,, \quad \Omega^{\rm EF}_{Vi} = \frac{1}{200}\,,$$

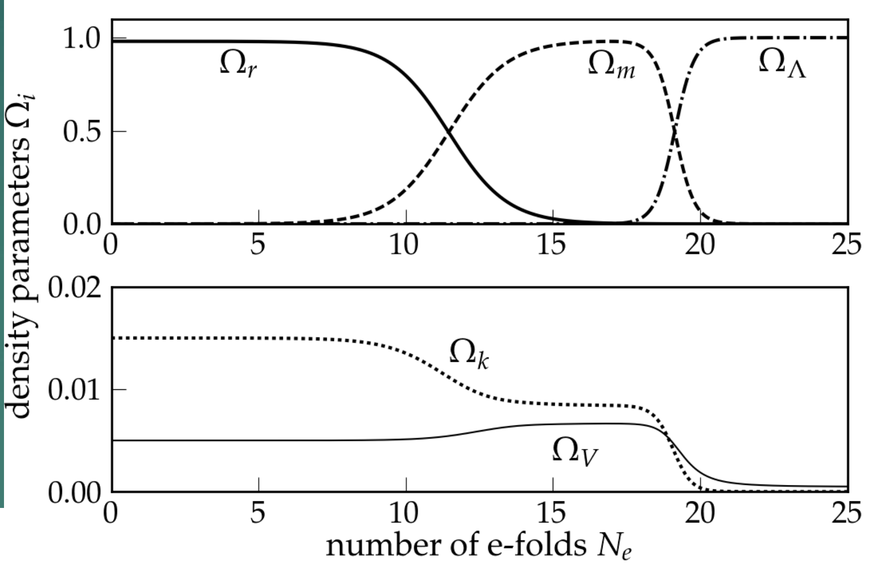
$$\beta = 1 \times 10^{-2}\,, \quad \lambda = \frac{40}{3}$$

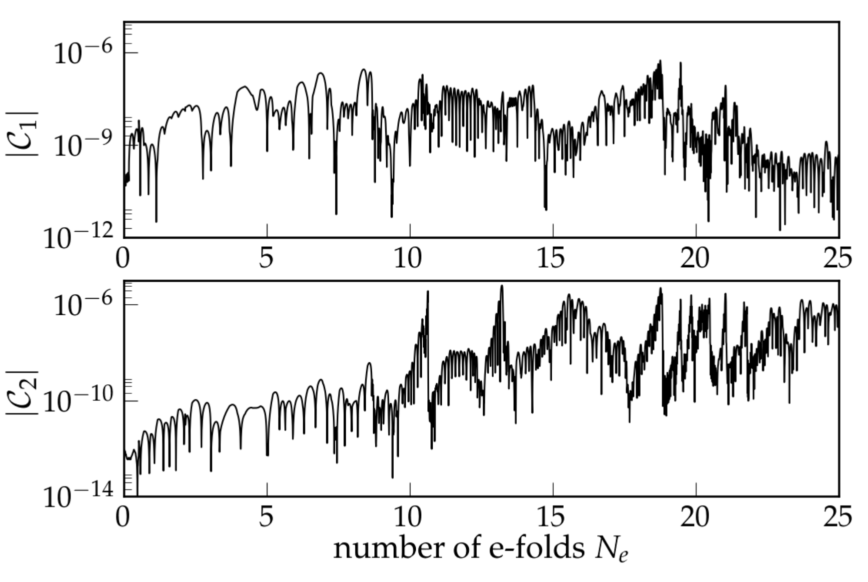
NB : these are non unique…

Numerical results

Evolution as planned !

- Radiation – dust – Λ domination
- Stable scaling solutions
- Small numerical errors



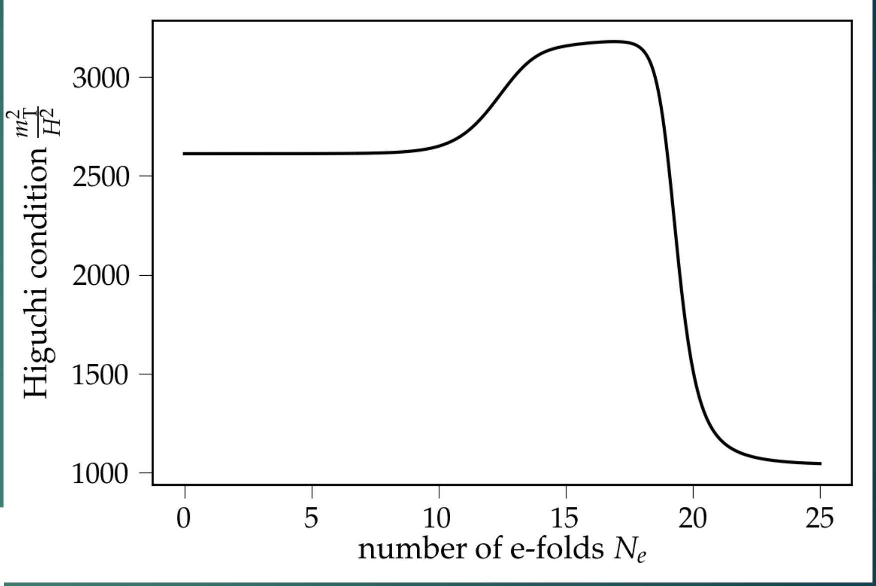


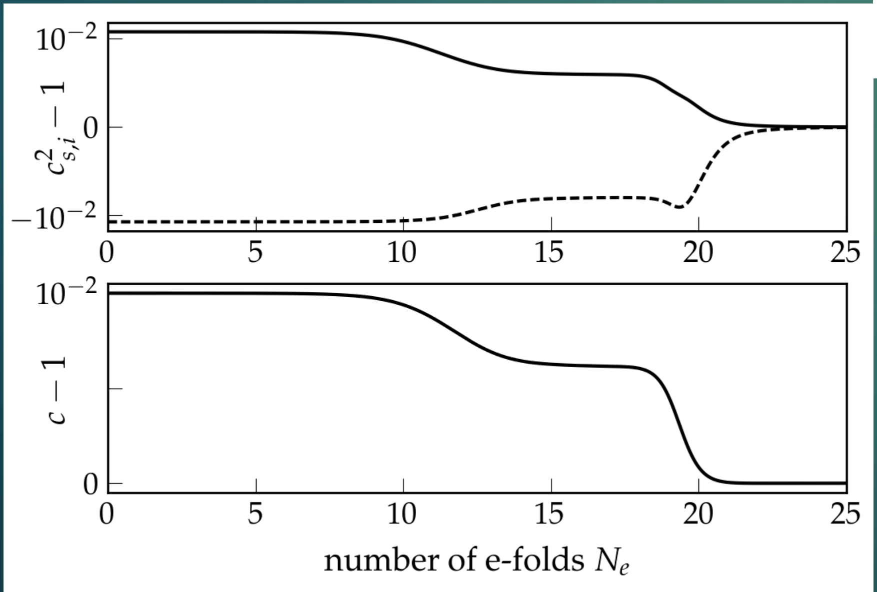
What about the Higuchi bound and the sound-speeds ?

Numerical results

Again just as planned !

- $m_T^2 \gg H^2$ at all times
- Positive sound-speeds, close to 1
- No-ghost conditions are satisfied





Promising !

Proof of existence for a stable cosmology in chameleon bigravity !

Summary

i. **Chameleon bigravity** solves the fine-tuning problems of bigravity and extends its reach

ii. Scaling solutions were described

iii. Stability conditions under homogeneous and inhomogeneous perturbations were found

iv. The model propagates 2x2 tensor, 1x2 vector, 2 scalar + matter modes

v. Numerical integration and example background cosmology were achieved

Future outlook

A promising model, with avenues for further study !

E.g. constraints from:

i. More precise background cosmology
ii. Evolution of perturbations
iii. Solar-system tests
iv. GW wave-forms modified due to graviton oscillations

Merci beaucoup !

((c) DC)

Chameleon boy

heavy

light

((c) marvel)

GRAVITON TWINS

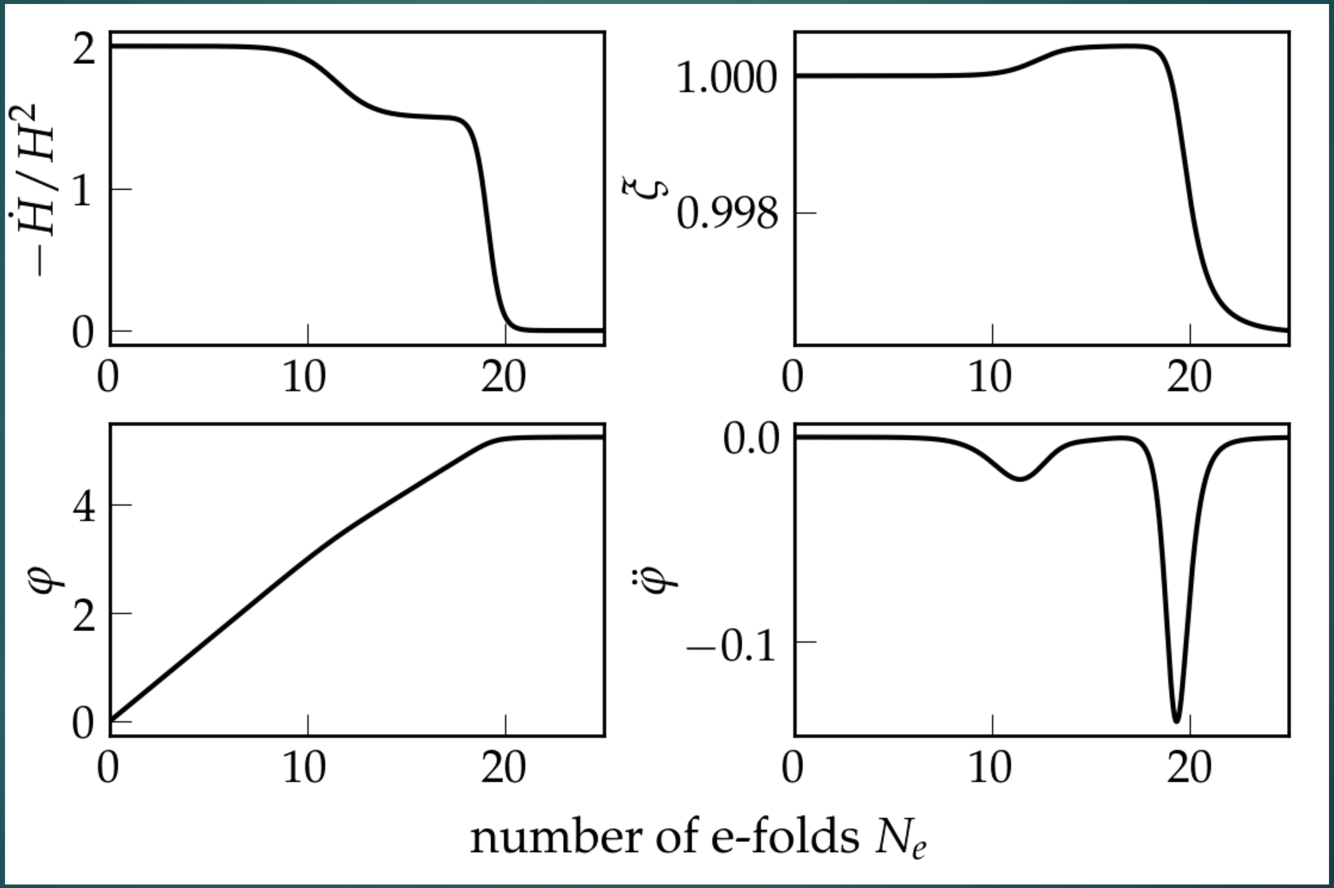
Back-up 1 : density dependence

Compare the density at late times and cosmological distances ρ_∞ with the local density m_{loc}

$$\frac{m^2_{\mathrm{T,loc}}}{m^2_{\mathrm{T},\infty}} = \left(\frac{\rho_{\mathrm{loc}}}{\rho_\infty}\right)^{\frac{\lambda}{\lambda+4\beta}}$$

If β is small enough…

Back-up 2 : other graphs



Backup 3 : Higuchi condition and strong coupling scale