ブラックホール時空を運動する粒子からの重力波

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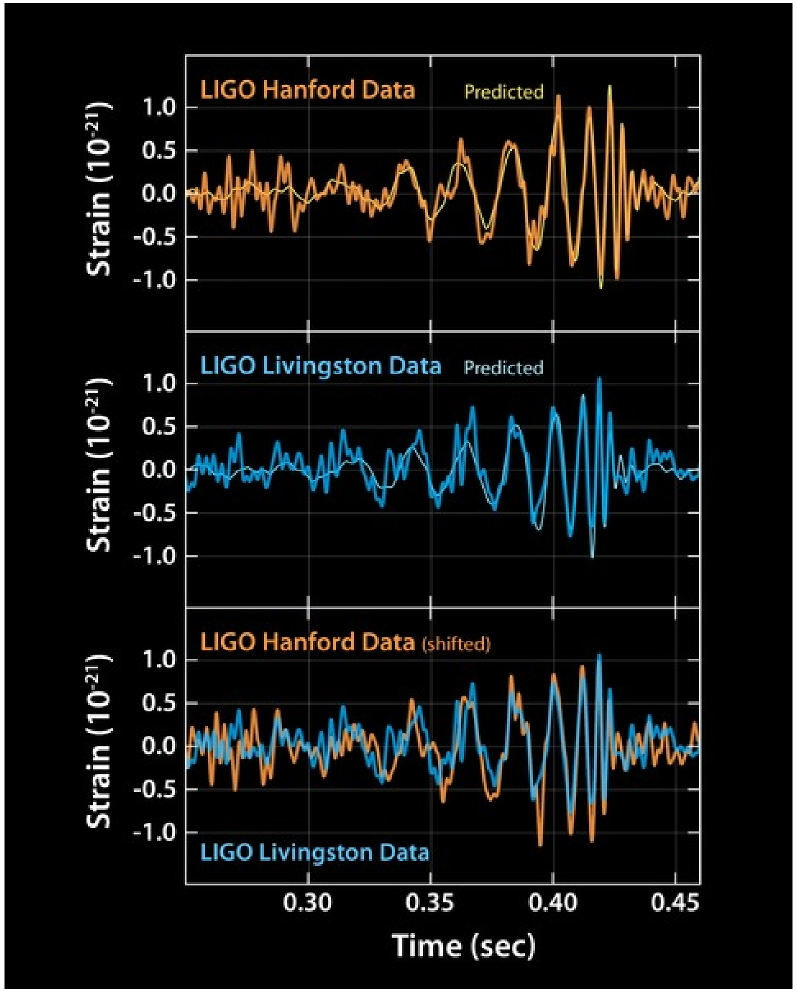
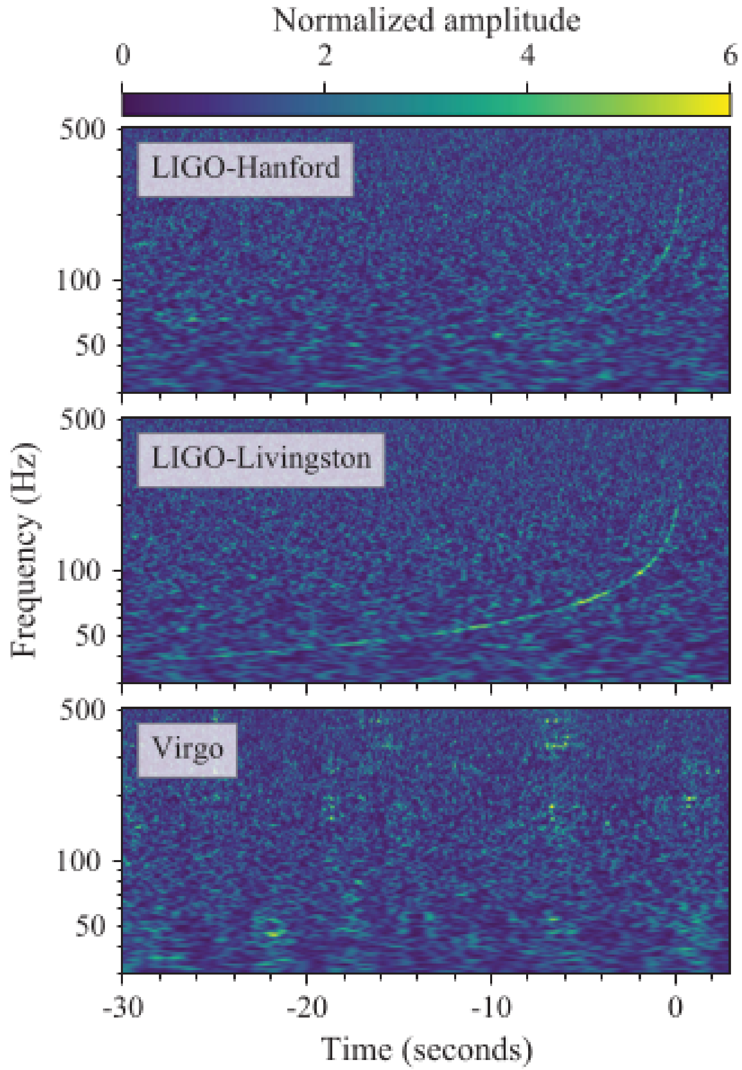
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Gravitational waves (GWs) astronomy began

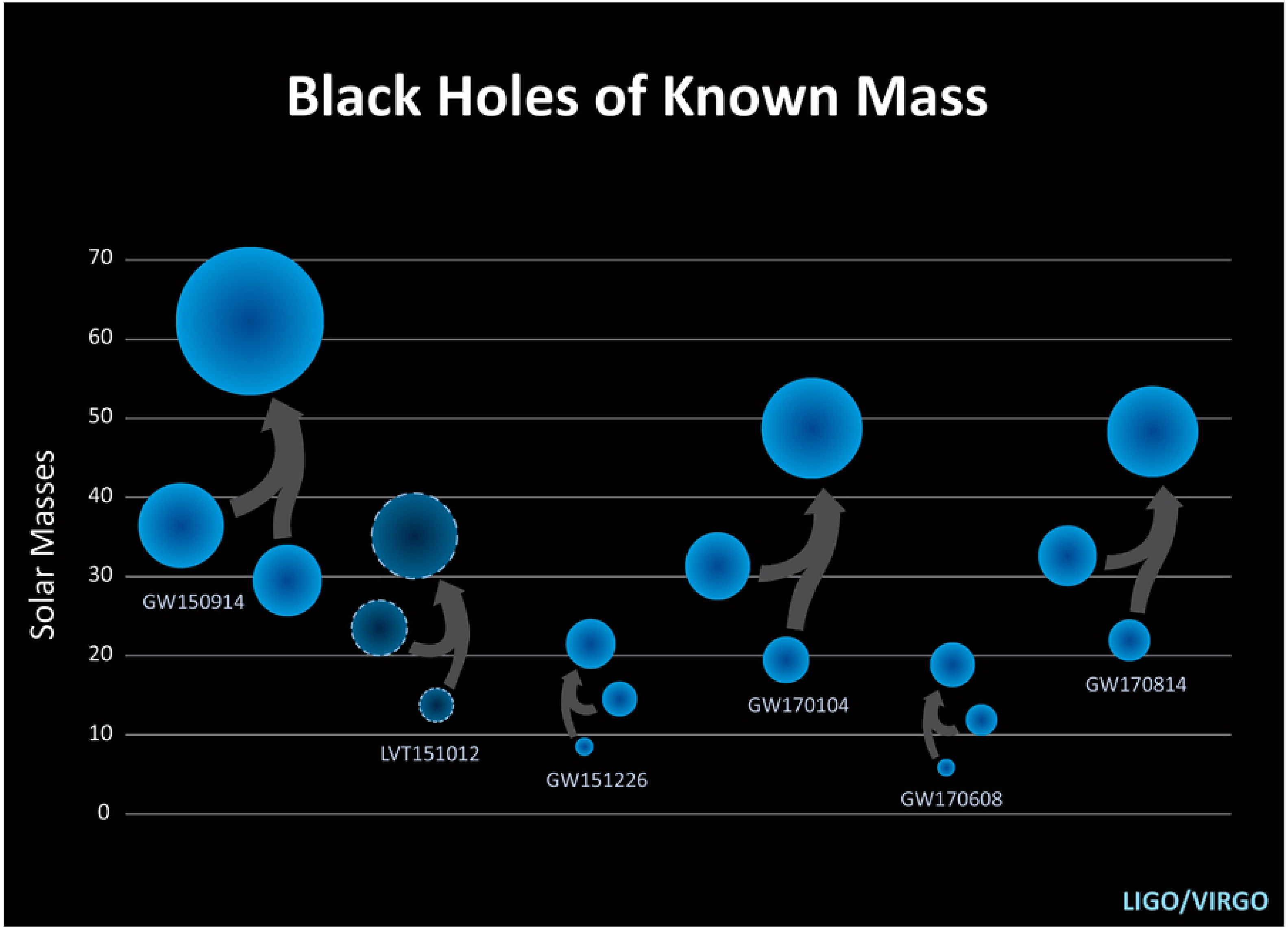
- GW150914 (BBH)



- GW170817 (BNS)



Known black hole binaries



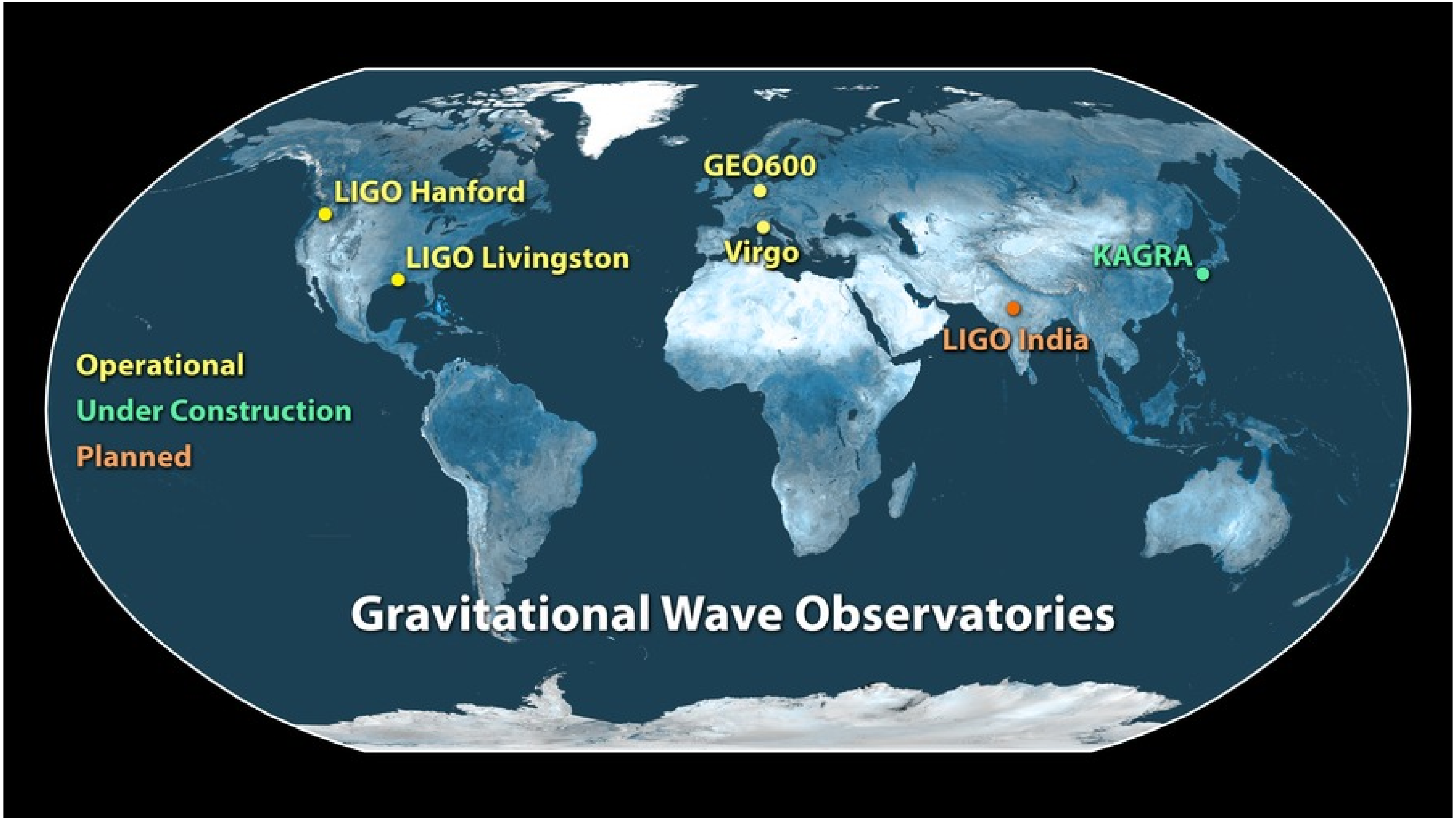
List of gravitational wave events

- 5(+0.8) BH-BH and 1 NS-NS

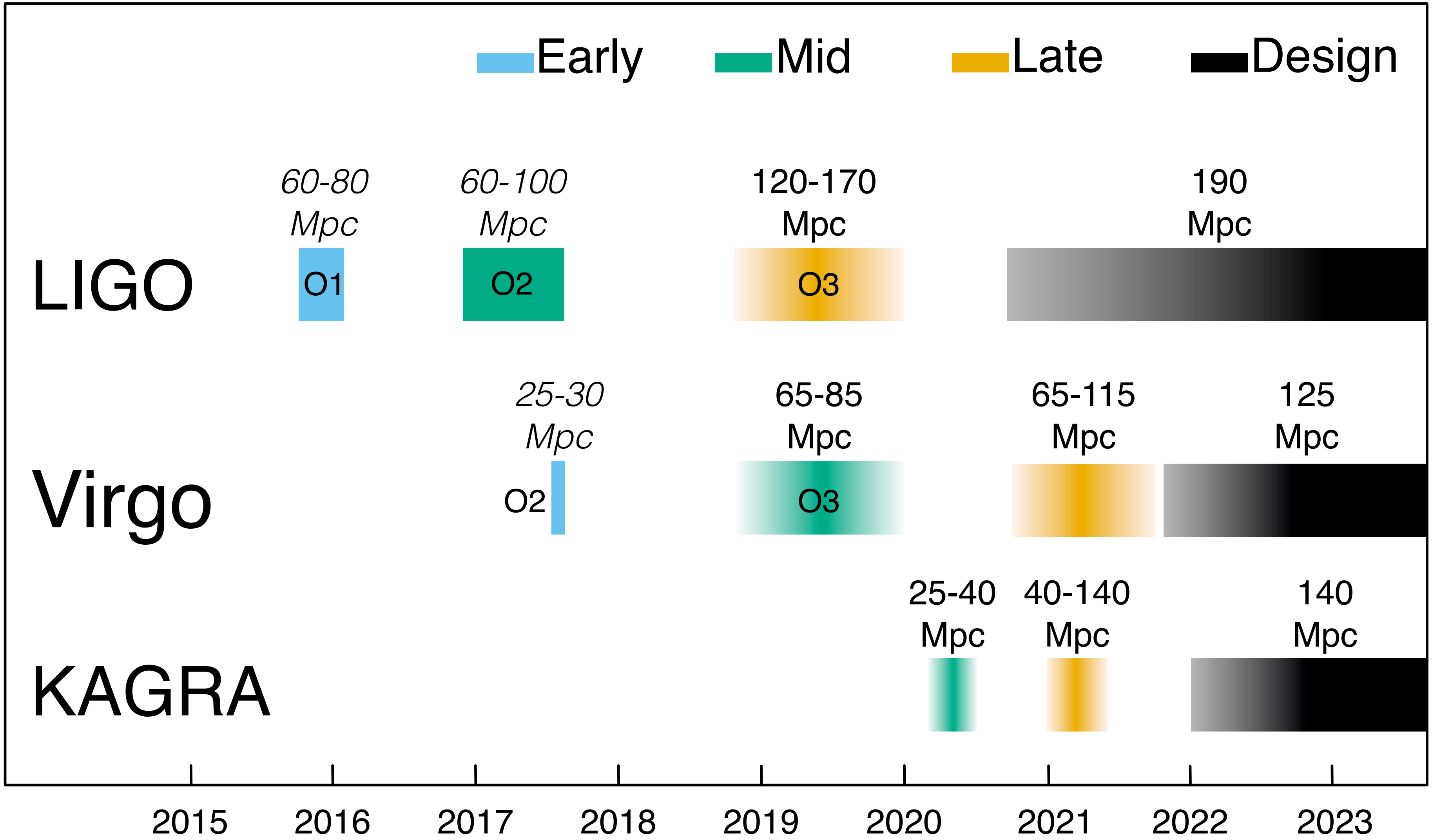
List of binary merger events

GW event	Detection time (UTC)	Date published	Location area[n 1] (deg^2)	Luminosity distance (Mpc)[n 2]	Energy radiated ($c^2 M_\odot$)[n 3]	Chirp mass ($M_\odot$)[n 4]	Primary		Secondary		Remnant			Notes
							Type	Mass ($M_\odot$)	Type	Mass ($M_\odot$)	Type	Mass ($M_\odot$)	Spin[n 5]	
GW150914	2015-09-14 09:50:45	2016-02-11	600; mostly to the south	440^{+160}_{-180}	$3.0^{+0.5}_{-0.5}$	$28.2^{+1.8}_{-1.7}$	BH[n 6]	$35.4^{+5.0}_{-3.4}$	BH[n 7]	$29.8^{+3.3}_{-4.3}$	BH	$62.2^{+3.7}_{-3.4}$	$0.68^{+0.05}_{-0.06}$	First GW detection; first BH merger observed; largest progenitor masses to date
LVT151012 (fr)	2015-10-12 09:54:43	2016-06-15	1600	1000^{+500}_{-500}	$1.5^{+0.3}_{-0.4}$	$15.1^{+1.4}_{-1.1}$	BH	23^{+18}_{-6}	BH	13^{+4}_{-5}	BH	35^{+14}_{-4}	$0.66^{+0.09}_{-0.10}$	Not significant enough to confirm (~13% chance of being noise)
GW151226	2015-12-26 03:38:53	2016-06-15	850	440^{+180}_{-190}	$1.0^{+0.1}_{-0.2}$	$8.9^{+0.3}_{-0.3}$	BH	$14.2^{+8.3}_{-3.7}$	BH	$7.5^{+2.3}_{-2.3}$	BH	$20.8^{+6.1}_{-1.7}$	$0.74^{+0.06}_{-0.06}$	
GW170104	2017-01-04 10:11:58	2017-06-01	1200	880^{+450}_{-390}	$2.0^{+0.6}_{-0.7}$	$21.1^{+2.4}_{-2.7}$	BH	$31.2^{+8.4}_{-6.0}$	BH	$19.4^{+5.3}_{-5.9}$	BH	$48.7^{+5.7}_{-4.6}$	$0.64^{+0.09}_{-0.20}$	Farthest confirmed event to date
GW170608	2017-06-08 02:01:16	2017-11-16	520; to the north	340^{+140}_{-140}	$0.85^{+0.07}_{-0.17}$	$7.9^{+0.2}_{-0.2}$	BH	12^{+7}_{-2}	BH	7^{+2}_{-2}	BH	$18.0^{+4.8}_{-0.9}$	$0.69^{+0.04}_{-0.05}$	Smallest BH progenitor masses to date
GW170814	2017-08-14 10:30:43	2017-09-27	60; towards Eridanus	540^{+130}_{-210}	$2.7^{+0.4}_{-0.3}$	$24.1^{+1.4}_{-1.1}$	BH	$30.5^{+5.7}_{-3.0}$	BH	$25.3^{+2.8}_{-4.2}$	BH	$53.2^{+3.2}_{-2.5}$	$0.70^{+0.07}_{-0.05}$	First detection by three observatories; first measurement of polarization
GW170817	2017-08-17 12:41:04	2017-10-16	28; NGC 4993	40^{+8}_{-14}	> 0.025	$1.188^{+0.004}_{-0.002}$	NS	1.36 - 1.60[n 8]	NS	1.17 - 1.36[n 9]	BH[n 10]	$< 2.74^{+0.04}_{-0.01}$ [n 11]		First NS merger observed in GW; first detection of EM counterpart (GRB 170817A; AT 2017gfo); nearest event to date

GW detectors on the ground



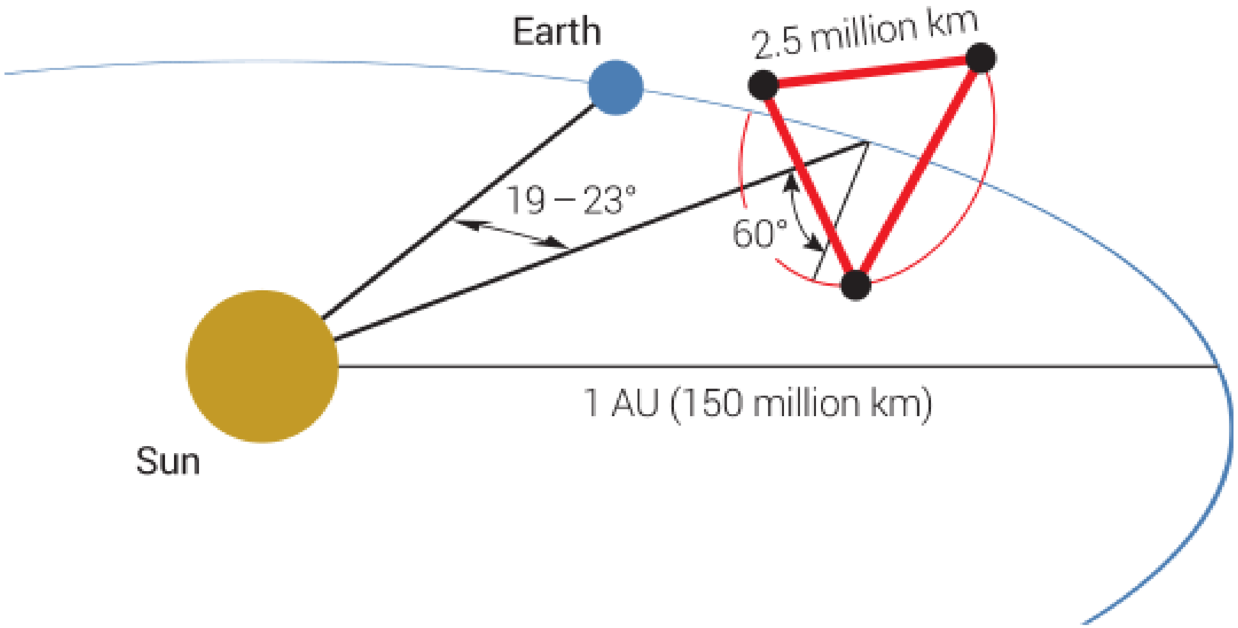
Expected schedule of LIGO, Virgo and KAGRA



[LVK collaboration, arXiv:1304.0670 (Updated on Sept. 8, 2017)]

Laser Interferometer Space Antenna (LISA)

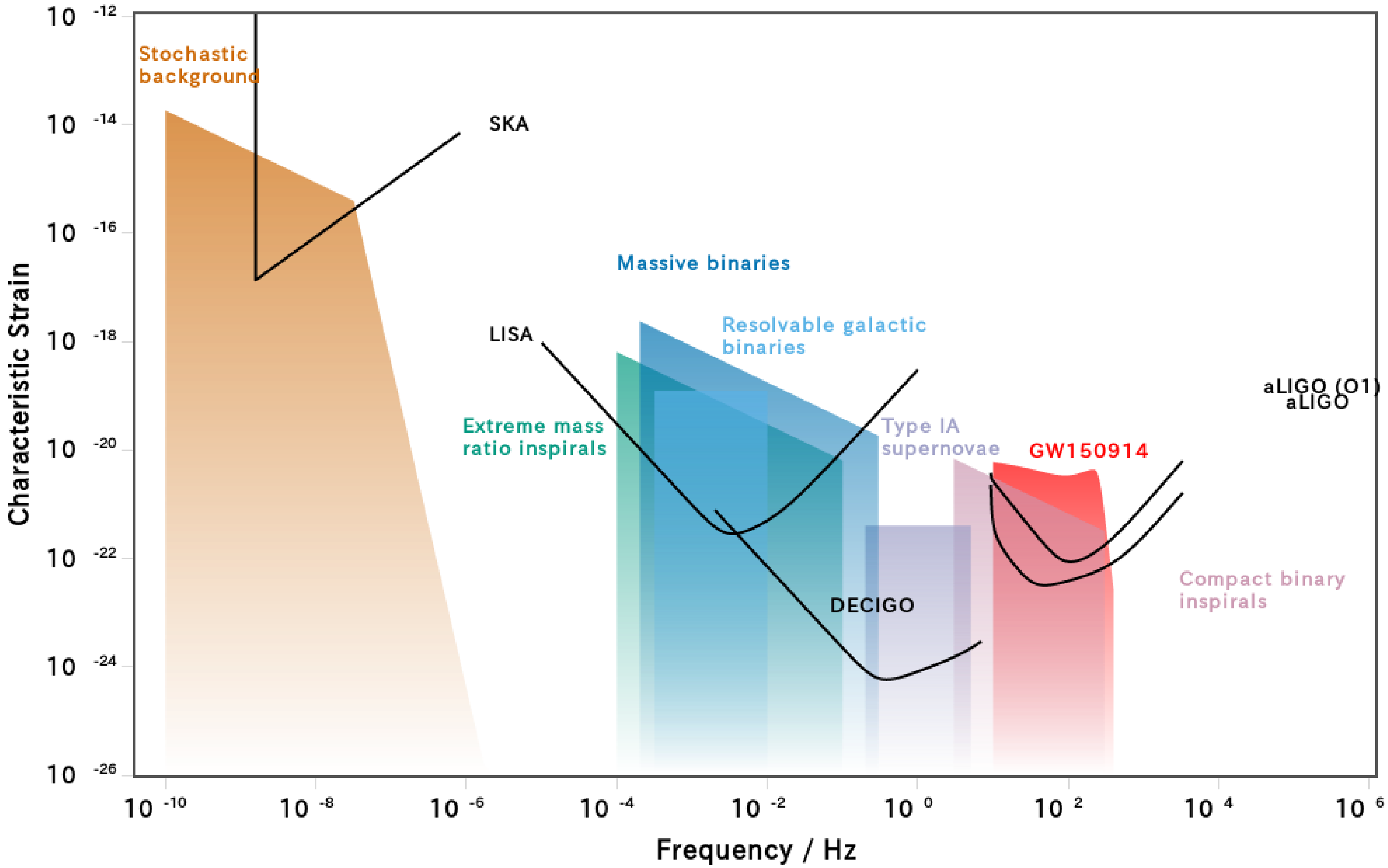
- The third large-class (L3) mission in ESA's Science program
  - ⋆ Launch ∼ 2034
  - ⋆ Three spacecrafts 20° behind the earth
  - ⋆ Arm length: 2.5 million km (cf. 4 km for Advanced LIGO)



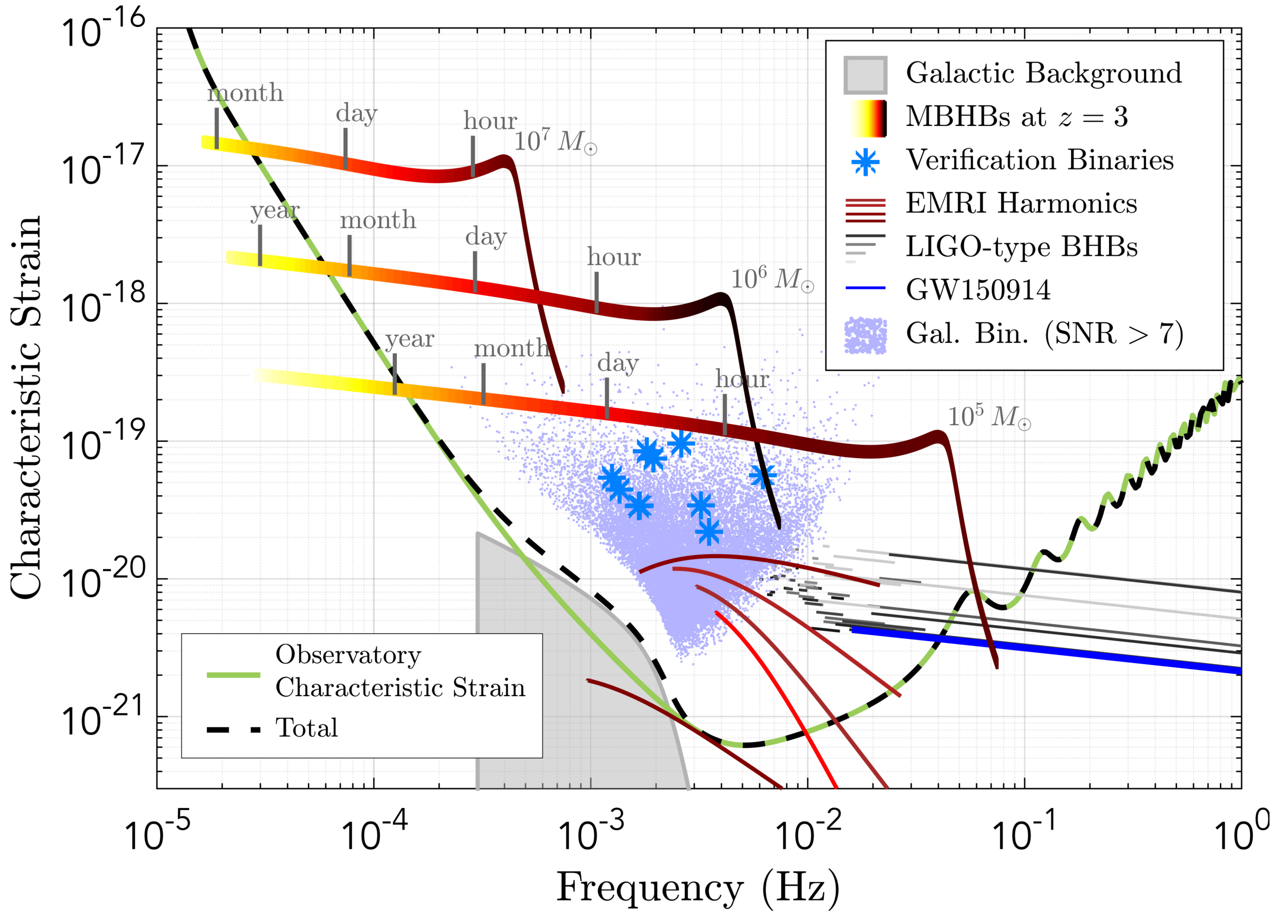
Laser Interferometer Space Antenna (LISA)

- The third large-class (L3) mission in ESA’ s Science program
  - ⋆ Sensitivity: $\sim 10^{-3}$ Hz (cf. $\sim 10^{2}$ Hz for Advanced LIGO)

$$f_{\rm GW} \sim \sqrt{G\rho} \sim 10^2 \left(\frac{10 M_\odot}{M}\right) [\mathrm{Hz}] \sim 10^{-3} \left(\frac{10^6 M_\odot}{M}\right) [\mathrm{Hz}]$$



Binary inspirals in the LISA band



- Galactic binaries: WD, NS, stellar mass BH
- Massive black hole binaries
- EMRIs(SMBH+CO)/IMRIs(SMBH+IMBH) [Fig. Babak et al.,arXiv:1702.00786]

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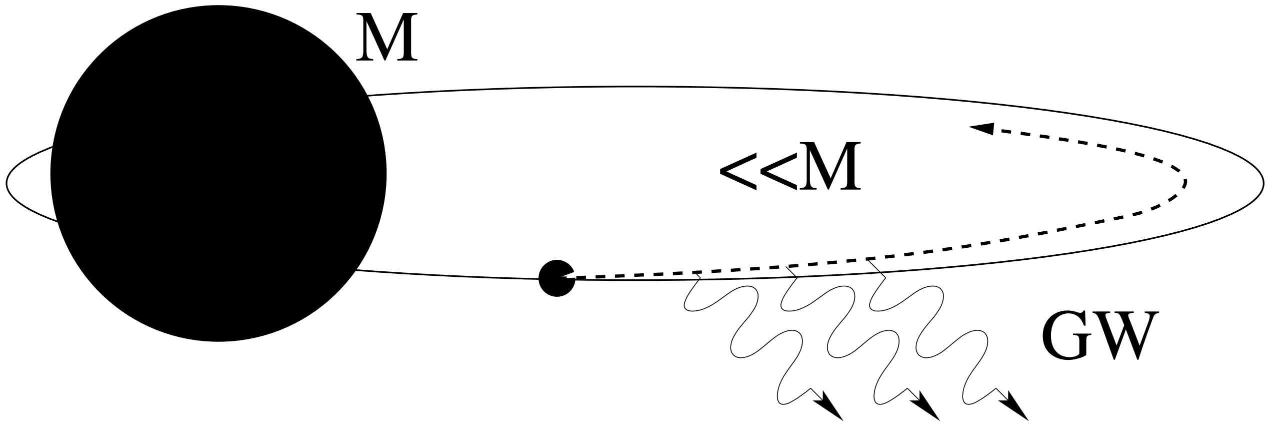
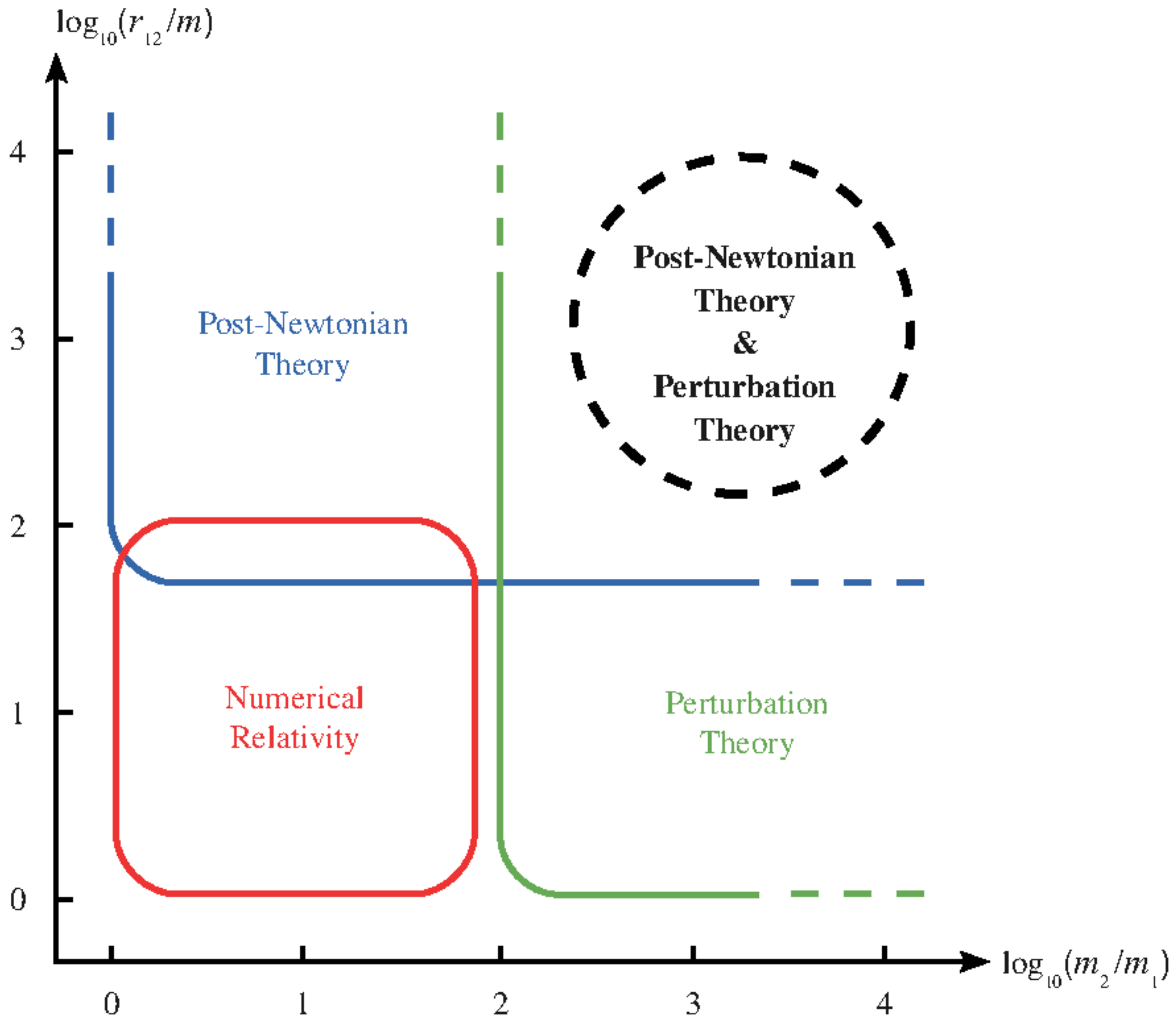
Extreme mass ratio inspirals (EMRIs)



- A compact star orbiting a supermassive black hole in the center of a galaxy
  - ⋆ $\mu/M \sim 10^{-5} - 10^{-7}$
  - ⋆ One of main targets of a space detector (LISA)
    $\sim$ 1-1000 events/yr (Gair et al.,2004)

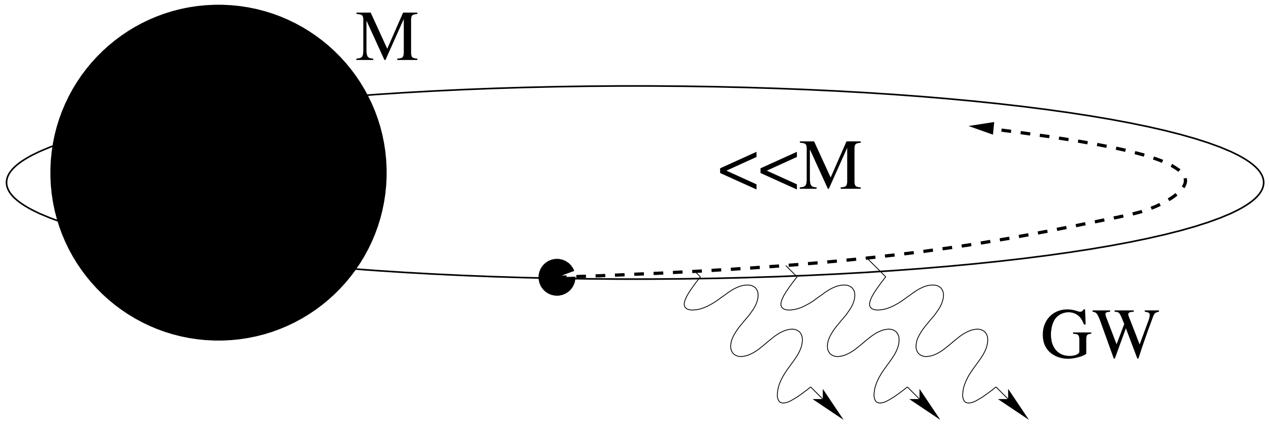
PN, BH perturbation and Numerical Relativity

- $m_1/m_2 \ll 1 \Rightarrow$ Linear perturbation of black hole is applicable



[Blanchet et al.,arXiv:0910.0207]

Black hole perturbation theory



- At the zeroth order in the mass ratio
  - $\frac{D^2 z^\mu}{d\tau^2} = 0$
  - Constants of motion for geodesics: (E, L_z, C)
- At the first order in the mass ratio
  - $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h^{(1)}_{\alpha\beta}, \qquad h^{(1)}_{\alpha\beta} \sim \frac{\mu}{M}$
  - $\frac{D^2 z^\mu}{d\tau^2} = 0 + F^\mu_1$, ($F^\mu_1$: gravitational self-force $\sim \frac{\mu}{M}$)

We want to know how the gravitational self-force affects the particle's motion

Inspiral waveforms

- Inspiral waveform $\tilde{h}(f) \propto f^{-7/6} e^{i\Phi(f)}$

where waveform cycles $\Phi \sim \int (df/dt) dt = \int (dE/df)^{-1} (dE/dt) dt$ for the case of a circular orbit, and

$$\frac{dE}{dt} = 0 + \underbrace{O[\mu/M]}_{\text{1st order BHP}} + \underbrace{O[(\mu/M)^2]}_{\text{2nd order BHP}}, \quad \text{dissipative}$$

$$\frac{dE}{df} = (\text{geodesic}) + \underbrace{O[\mu/M]}_{\text{1st order BHP}} + \underbrace{O[(\mu/M)^2]}_{\text{2nd order BHP}}, \quad \text{conservative}$$

- Dissipative part: $dE/dt = -(dE/dt)_{\rm GW}$, computed by energy balance argument
- Conservative part: dE/df, computed via $E = (1 - 2v^2 + av^3)/\sqrt{1 - 3v^2 + 2av^3}$ and $f = v^3/(1 + av^3)$

Waveform cycles during the last year of insiral

- Inspiral waveform $\tilde{h}(f) \propto f^{-7/6} e^{i\Phi(f)}$
- Waveform cycles $\Phi \sim \int (df/dt) dt = \int (dE/df)^{-1} (dE/dt) dt$ for circular orbits around a Schwarzschild black hole, $M = 10^6 M_\odot$
  - ⋆ Higher order corrections may by necessary for EMRIs
  - ⋆ Higher order corrections may even be larger for IMRIs

$\mu/M_\odot$	0.6	1.4	10
1st order dissipative	133312.6	127503.0	98642.5
1st order dissipative+ 1st order conservative	133311.9	127502.1	98645.8
+2nd order SF	133312.2	127502.6	98643.3

[Huerta and Gair, (2009)]

We need error in waveforms $\lesssim 10^{-5}$ for LISA data analysis of EMRIs

Secular evolution of the orbital parameters

- "Constants of motion": $E = -u^\alpha \xi^{(t)}_\alpha$, $L_z = u^\alpha \xi^{(\phi)}_\alpha$ and $C \sim K_{\alpha\beta} u^\alpha u^\beta$
- $T_{\mathrm{orbit}} \ll T_{\mathrm{radiation}}$ $[T_{\mathrm{orbit}} = O(M),\ T_{\mathrm{radiation}} = O(M^2/\mu)]$

$$\left\langle \frac{dE}{dt} \right\rangle_t = -\mu^2 \sum_{\ell m n_r n_\theta} \frac{1}{4\pi\omega^2_{mn_r n_\theta}} \Bigg(\underbrace{\left| Z^\infty_{\ell m n_r n_\theta} \right|^2}_{\text{Infinity part}} + \underbrace{\alpha_{\ell m n_r n_\theta} \left| Z^{\mathrm{H}}_{\ell m n_r n_\theta} \right|^2}_{\text{Horizon part}} \Bigg),$$

$$\left\langle \frac{dL_z}{dt} \right\rangle_t = -\mu^2 \sum_{\ell m n_r n_\theta} \frac{m}{4\pi\omega^3_{mn_r n_\theta}} \left(\left| Z^\infty_{\ell m n_r n_\theta} \right|^2 + \alpha_{\ell m n_r n_\theta} \left| Z^{\mathrm{H}}_{\ell m n_r n_\theta} \right|^2 \right),$$

$$\left\langle \frac{dC}{dt} \right\rangle_t = -2 <a^2 E \cos^2\theta>_\lambda \left\langle \frac{dE}{dt} \right\rangle_t + 2 <L_z \cot^2\theta>_\lambda \left\langle \frac{dL_z}{dt} \right\rangle_t - \mu^3 \sum_{\ell m n_r n_\theta} \frac{n_\theta \Omega_\theta}{2\pi\omega^3_{mn_r n_\theta}} \left(\left| Z^\infty_{\ell m n_r n_\theta} \right|^2 + \alpha_{\ell m n_r n_\theta} \left| Z^{\mathrm{H}}_{\ell m n_r n_\theta} \right|^2 \right)$$

[Mino (2003), Sago et al. (2005)]

where $\alpha_{\ell m n_r n_\theta} \propto \omega^3(\omega - mq/(2r_+))$ and

$$Z^{\infty,\mathrm{H}}_{\ell m\omega} \sim \int d\tau R^{\mathrm{in/up}}_{\ell m\omega}(r) T_{\ell m\omega}(r),$$

$R^{\mathrm{in/up}}_{\ell m\omega}(r)$: Homogeneous solutions of the radial Teukolsky equation

$T_{\ell m\omega}(r)$: Source term constructed from energy-momuntum tensor of the particle

Teukolsky equation I

- Perturbation equation for Weyl scalar $\Psi \sim C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$

$$\begin{aligned}\Psi &= \frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times) \ \text{ for } \ r \to \infty, \\ &= \sum_{\ell m} \int d\omega e^{-i\omega t + im\varphi} \ {}_{-2}S_{\ell m}(\theta) Z^\infty_{\ell m\omega}(r),\end{aligned}$$

- Teukolsky equation in the frequency domain [Teukolsky (1973)]

$$\Delta^2 \frac{d}{dr}\left(\frac{1}{\Delta}\frac{dZ_{\ell m\omega}}{dr}\right) - V(r) Z_{\ell m\omega} = T_{\ell m\omega},$$

$$\begin{cases} \quad V(r) = -\frac{K^2 + 4i(r-M)K}{\Delta} + 8i\omega r + \lambda : \text{Long range potential} \\ \Delta = r^2 - 2Mr + q^2, \ K = (r^2 + q^2)\omega - m\,q, \\ r = r_\pm : \text{Regular singular points,} \\ r = \infty : \text{Irregular singular point.} \end{cases}$$

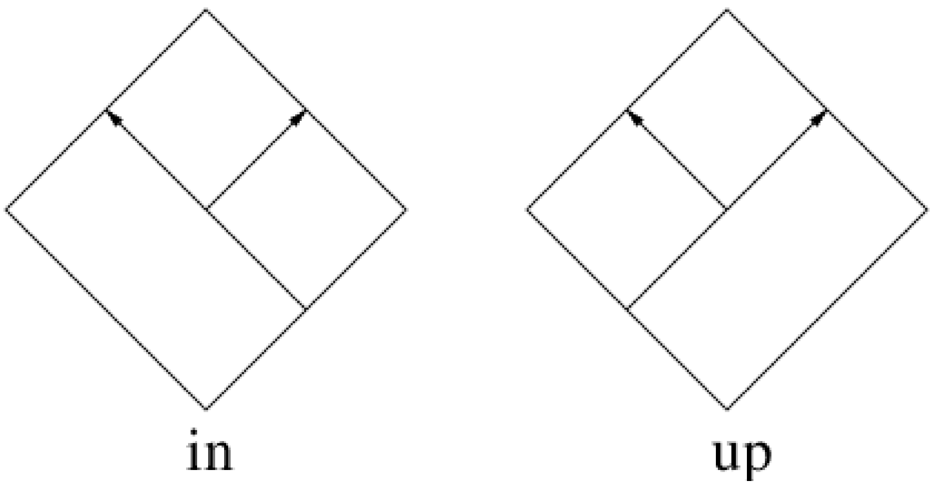
We solve the Teukolsky equation by Green function method

Teukolsky equation II

- Boundary conditions for two kinds of homogeneous solutions

$$R^{\rm in}_{lm\omega} \to \begin{cases} B^{\rm trans}_{lm\omega}\Delta^2 e^{-ikr*} & \text{for } r \to r_+, \\ r^3 B^{\rm ref}_{lm\omega} e^{i\omega r*} + r^{-1} B^{\rm inc}_{lm\omega} e^{-i\omega r*} & \text{for } r \to +\infty, \end{cases}$$

$$R^{\rm up}_{lm\omega} \to \begin{cases} C^{\rm up}_{lm\omega} e^{ikr*} + \Delta^2 C^{\rm ref}_{lm\omega} e^{-ikr*} & \text{for } r \to r_+, \\ r^3 C^{\rm trans}_{lm\omega} e^{i\omega r*} & \text{for } r \to +\infty. \end{cases}$$

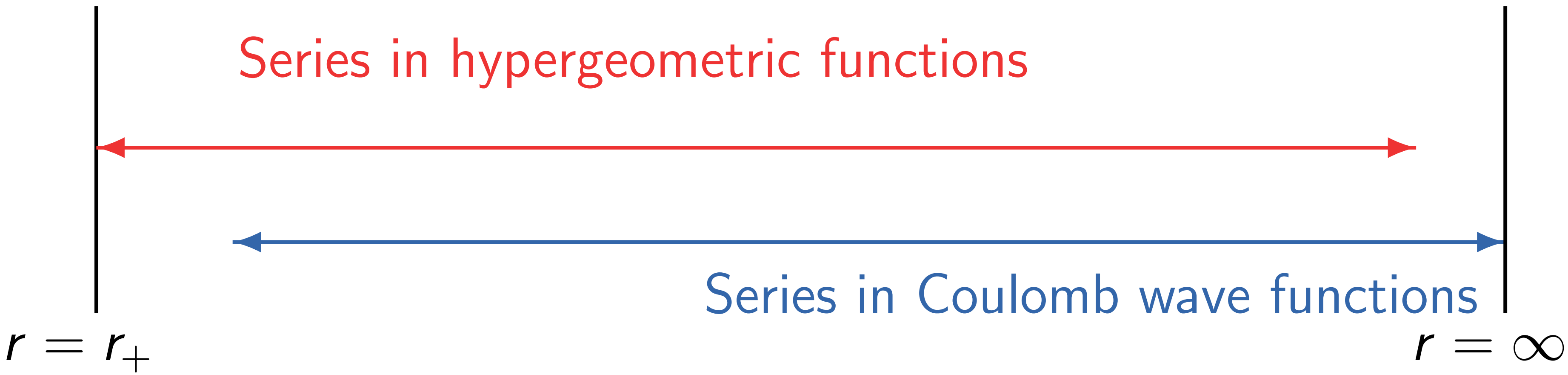


Mano-Suzuki-Takasugi method

- Mano-Suzuki-Takasugi (MST) method (1996)

$$R_{\ell m\omega}(r) \sim \sum_n a_n F_n(r), \; F_n(r) = \begin{cases} \text{Hypergeometric fn.}(r \sim r_+) \\ \text{Coulomb wave fn.}(r \sim \infty) \end{cases}$$

- Region of convergence for series expansions



Mano-Suzuki-Takasugi method

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⇒ Teukolsky equation is reduced to recurrence relation for a_n

- Post-Newtonian expansion of a_n up to 3PN, $O(\omega^2) = O(v^6)$

$$\begin{aligned} a_0 &= 1, \\ a_1 &= i\omega \frac{(\ell+3)^2}{(\ell+1)(2\ell+1)} + \omega^2 \frac{2(\ell+3)^2}{(\ell+1)^2(2\ell+1)}, \\ a_2 &= -\omega^2 \frac{(\ell+3)^2(\ell+4)^2}{(\ell+1)(2\ell+1)(2\ell+3)^2}, \\ a_{-1} &= a_1(\ell \leftrightarrow -\ell-1),\ a_{-2} = a_2(\ell \leftrightarrow -\ell-1). \end{aligned}$$

⇒ MST is powerful method for post-Newtonian expansion

Summary to compute GWs in BH perturbation

- $\mu/M << 1$ but v/c can be $O(1)$
- Kerr geodesic with E, L_z and C
- Homogeneous solutions by Mano-Suzuki-Takasugi (1996)

$$R_{\ell m\omega}(r) \sim \sum_n a_n F_n(r),$$

$$(\alpha_n^\nu a_{n+1}^\nu + \beta_n^\nu a_n^\nu + \gamma_n^\nu a_{n-1}^\nu = 0, \quad \text{where} \ \ a_n \sim O(\omega^{|n|}) = O(v^{3\,|n|}))$$

- Weyl scalar Ψ and the energy flux:

$$\Psi = -\frac{2}{r}\sum_{\ell m} e^{-i\omega t + im\varphi} \ {}_{-2}S_{\ell m}(\theta) Z_{\ell m\omega}^{\infty}(r) \to \frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times),$$

$$\frac{dE}{dt} = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} \frac{|Z_{\ell m\omega}^{\infty}|^2}{4\pi\omega^2} + O(\mathrm{post-Teukolsky\ \ corrections}).$$

Post-Newtonian works

- Circular orbits in Schwarzschild spacetime
  - ⋆ 22PN waveforms and energy flux [RF(2012)] $\rightarrow \Delta N_{\mathrm{GW}} \lesssim 10^{-2}$
- Circular orbits in Kerr spacetime
  - ⋆ 20PN energy flux (numerical fitting) [Shah(2014)]
  - ⋆ 11PN waveforms and flux [RF(2015)] $\rightarrow \Delta N_{\mathrm{GW}} \lesssim 1 \ (q \lesssim 0.3)$
- Eccentric and inclined orbits in Kerr spacetime

	Sago et al. (2006)	Ganz et al. (2007)	Sago,RF (2015)
PN order	2.5PN	2.5PN	4PN
Eccentricity	$O(e^2)$	$O(e^2)$	$O(e^6)$
Inclination	$O(\theta_{\mathrm{inc}})$	No assumption	No assumption
BH absorption	Neglected	Neglected	Included

nPN means $(v^2/c^2)^n$ correction to leading order
4PN means (v^8/c^8) correction to leading order

Orbital parameters

We show results using orbital parameters defined as:

⋆ semi-latus rectum, p, eccentricity, e, inclination angle, ι

$$r = \frac{p}{1 + e\cos\psi}, \quad Y \equiv \cos\iota = \frac{L_z}{\sqrt{L_z^2 + C}},$$

⋆ $q = a/M$ is the normalized spin parameter

⋆ $v = \sqrt{M/p}$ is the post-Newtonian parameter

⋆ $-1 \leq Y \leq 1$ ($Y = \pm 1$: equatorial orbit, $Y = 0$: polar orbit)

Infinity part of $\left\langle \frac{dE}{dt} \right\rangle_t^{\rm gw}$ at $O(v^5, e^6)$

$$
\begin{aligned}
\left\langle \frac{dE}{dt} \right\rangle_t^{\infty} = &-\frac{32}{5}\left(\frac{\mu}{M}\right)^2 (1-e^2)^{3/2} v^{10} \Bigg[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 + \left\{-\frac{1247}{336} - \frac{9181}{672}e^2 + \frac{809}{128}e^4 + \frac{8609}{5376}e^6\right\} v^2 \\
&+ \left\{4\pi - \frac{73}{12}qY + \left(\frac{1375}{48}\pi - \frac{823}{24}qY\right)e^2 + \left(\frac{3935}{192}\pi - \frac{949}{32}qY\right)e^4 + \left(\frac{10007}{9216}\pi - \frac{491}{192}qY\right)e^6\right\} v^3 \\
&+ \Bigg\{-\frac{44711}{9072} + \frac{527}{96}q^2Y^2 - \frac{329}{96}q^2 + \left(-\frac{172157}{2592} - \frac{4379}{192}q^2 + \frac{6533}{192}q^2Y^2\right)e^2 \\
&\quad + \left(-\frac{3823}{256}q^2 - \frac{2764345}{24192} + \frac{6753}{256}q^2Y^2\right)e^4 + \left(\frac{3743}{2304} - \frac{363}{512}q^2 + \frac{2855}{1536}q^2Y^2\right)e^6\Bigg\} v^4 \\
&+ \Bigg\{\frac{3665}{336}qY - \frac{8191}{672}\pi - \frac{9}{32}q^3Y - \frac{15}{32}Y^3q^3 \\
&\quad + \left(\frac{1759}{56}qY - \frac{44531}{336}\pi - \frac{135}{64}q^3Y - \frac{225}{64}Y^3q^3 - \frac{15}{8}qY\right)e^2 \\
&\quad + \left(-\frac{111203}{1344}qY - \frac{4311389}{43008}\pi - \frac{405}{256}q^3Y - \frac{675}{256}Y^3q^3 - \frac{45}{32}qY\right)e^4 \\
&\quad + \left(-\frac{49685}{448}qY + \frac{15670391}{387072}\pi - \frac{45}{512}q^3Y - \frac{75}{512}Y^3q^3 - \frac{5}{64}qY\right)e^6\Bigg\} v^5 + \cdots\Bigg],
\end{aligned}
$$

where $-1 \leq Y \leq 1$ ($Y = 1$: equatorial orbit, $Y = 0$: polar orbit).

[Sago and RF (2015)]

Horizon part of $\left\langle \frac{dE}{dt} \right\rangle_t^{\rm gw}$ at $O(v^7, e^6)$

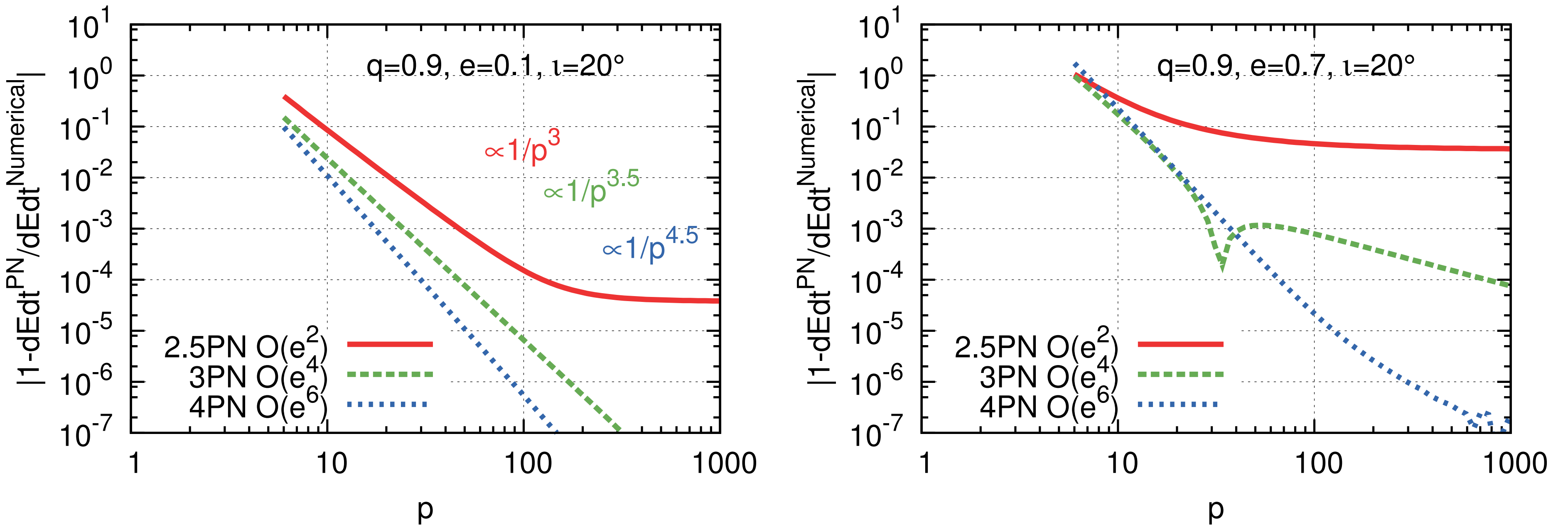
$$
\begin{aligned}
\left\langle \frac{dE}{dt} \right\rangle_t^{\rm H} = &- \frac{32}{5} \left(\frac{\mu}{M}\right)^2 (1-e^2)^{3/2} v^{10} \\
&\times \Bigg[-\frac{1}{512} \left(16 + 120\, e^2 + 90\, e^4 + 5\, e^6\right) \left(8 + 9\, q^2 + 15\, Y^2 q^2\right) qYv^5 \\
&\quad - \Bigg\{ 1 + \frac{81}{32} q^2 - \frac{15}{32} Y^2 q^2 + \left(\frac{57}{4} + \frac{1143}{32} q^2 - \frac{195}{32} Y^2 q^2\right) e^2 \\
&\qquad + \left(\frac{465}{16} + \frac{4455}{64} q^2 - \frac{225}{32} Y^2 q^2\right) e^4 \\
&\qquad + \left(\frac{355}{32} + \frac{6345}{256} q^2 + \frac{75}{256} Y^2 q^2\right) e^6 \Bigg\} qYv^7 + \cdots \Bigg],
\end{aligned}
$$

$\Rightarrow$ Superradiance would be possible, but smaller for inclined orbits (Superradiance terms, $\propto q\,Y = q \cos\iota$, disappear for $Y = 0$)

[Sago and RF (2015)]

Comparison with numerical results

⋆ $|1 - dEdt^{\rm PN}/dEdt^{\rm Num}|$ for $q = 0.9, e = 0.1,$ and $0.7, \iota = 20°$



[Numerical results : RF, Hikida and Tagoshi (2009)]

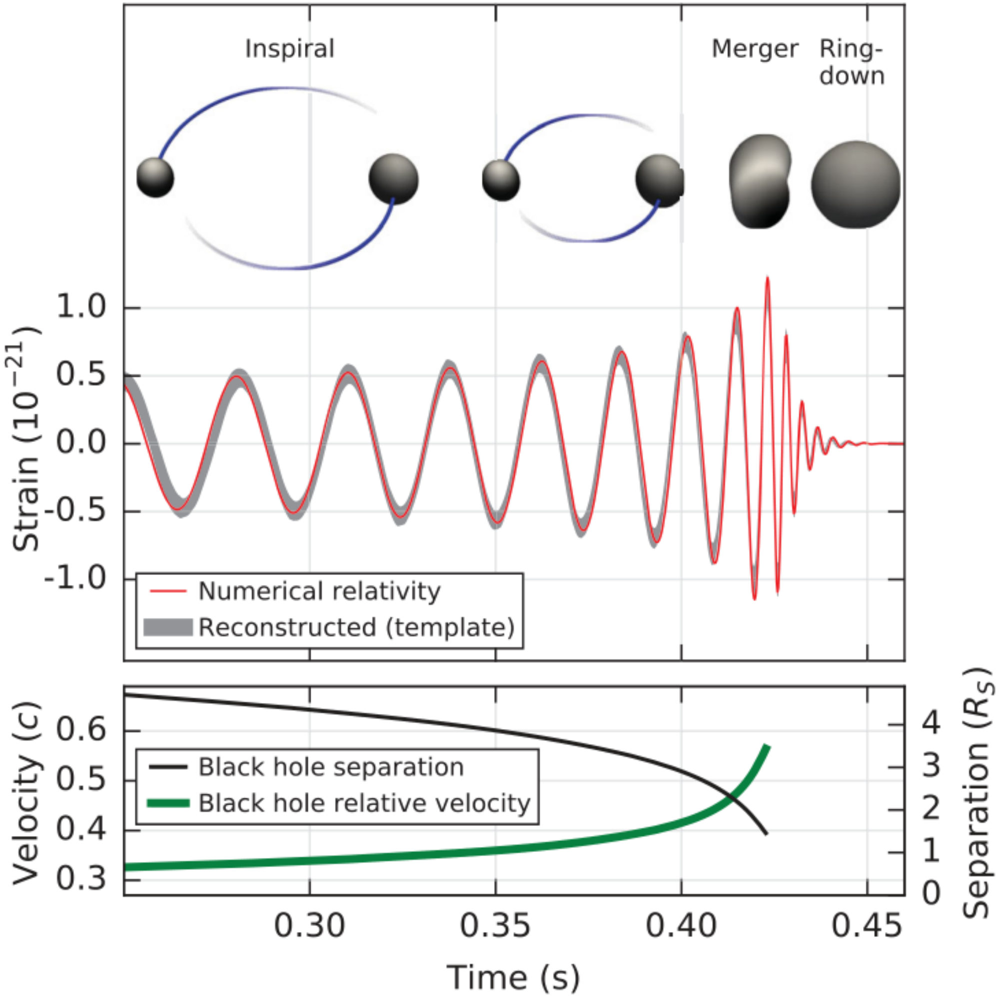
⇒ Relative error at 4PN is $\sim 1/p^{9/2}$ ($\gtrsim 1/p^{9/2}$) when $e = 0.1$ ($e \gtrsim 0.5$)

⇒ Error exists in lower PN order because of expansion in orbital eccentricity

Extreme mass ratio inspirals in Brans-Dicke theory

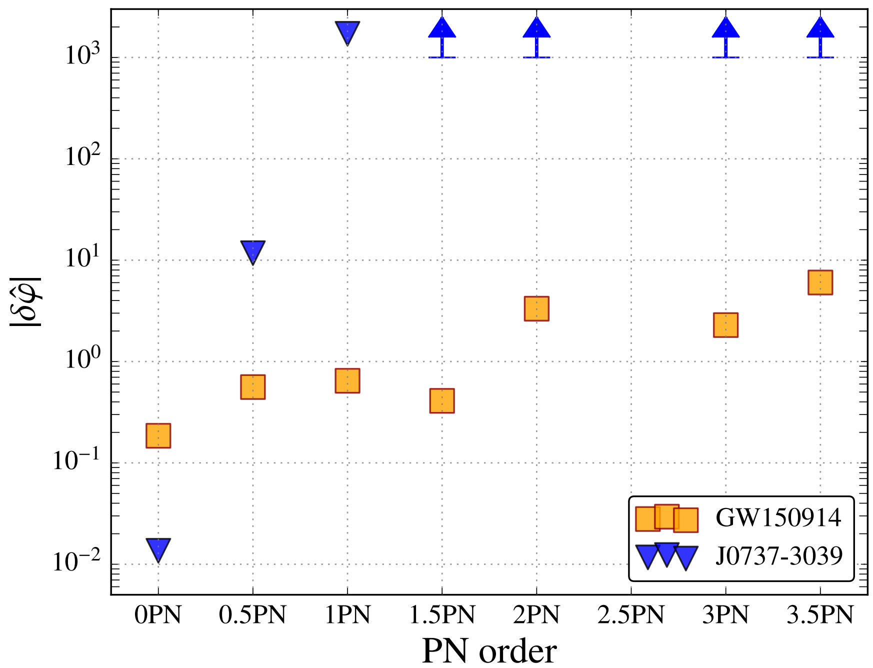
Motivation

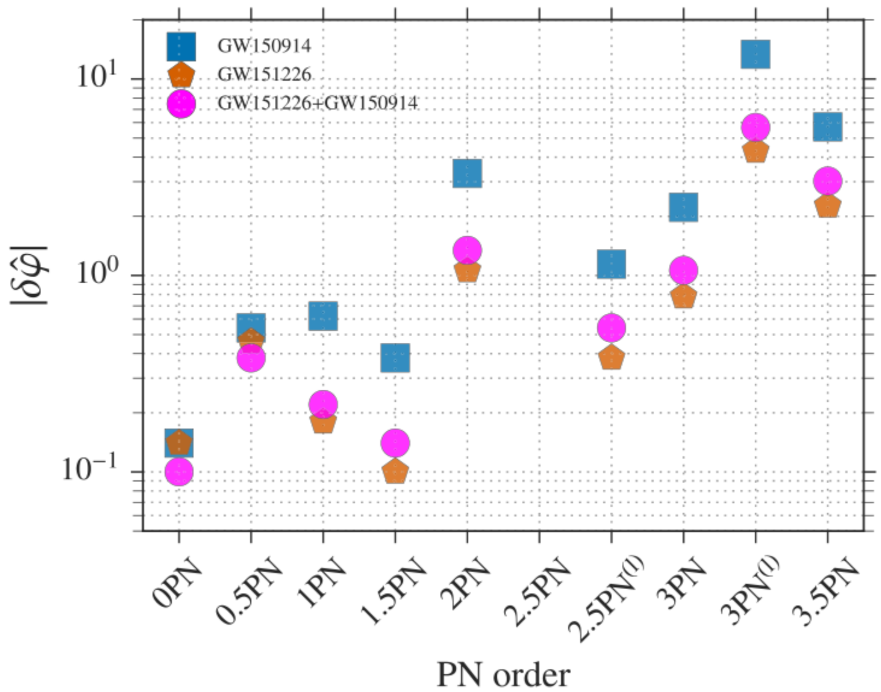
- To test General Relativity using gravitational waves (GWs)
  - ⋆ Gravitational waves from GW150914 [Abbott+(2016)]



Motivation

- To test General Relativity using gravitational waves (GWs)
  - ⋆ Deviations from GR in post-Newtonian waveforms with O1 data

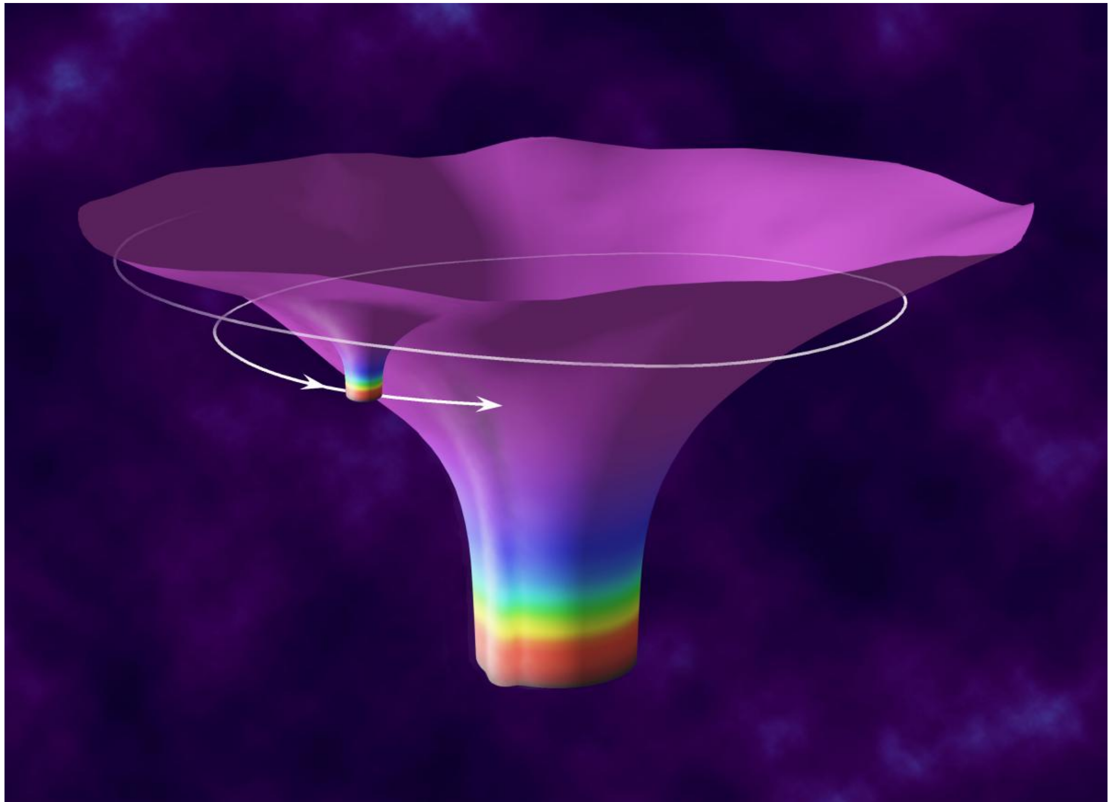




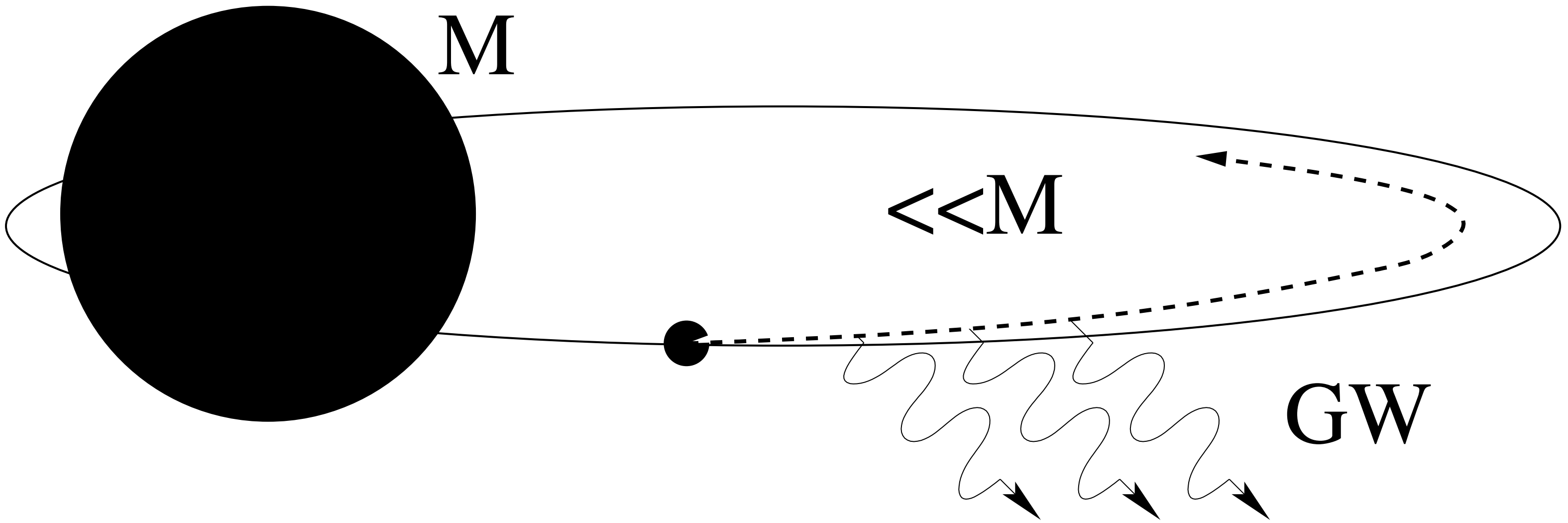
  - ⋆ 1.7s delay of GRB170817A from GW170817 through 40Mpc
    - ∗ $|1 - v_{\mathrm{GW}}/c| \sim 10^{-15}$
    - ∗ Test of Lorentz violation, Equivalence Principle

Motivation

- Extreme-mass ratio inspirals (EMRIs) in Brans-Dicke theory
  - ⋆ EMRIs are one of the main targets for LISA
  - ⋆ The mass ratio μ/M can be used as an expansion parameter
    - ∗ At the first order in the mass ratio:
    - ⇒ Scalar-Tensor theories are reduced to BD theory [Yunes+(2012)]
    - ⇒ Post-Newtonian approximation can be applied [Ohashi+(1996)]



EMRIs in Brans-Dicke theory



- (Quadrupole radiation in GW) + (Dipole radiation in φ)
  - ∗ Deviation from GR: $dE/dt = dE^{\mathrm{gw}}/dt + dE^{s}/dt$
  - ∗ $N_\phi^{\mathrm{gw}+s} \gtrsim 10^5$ and $N_\phi^s < 1$ for $e = 0$ [Yunes+(2012)]
    [$N_\phi = \int \Omega_\phi(t)dt$: Orbital cycles]

⇒ We want to know how orbital eccentricity changes results

EMRIs in Brans-Dicke theory II

- Brans-Dicke theory in Jordan frame

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\varphi R - \omega_{\rm BD}\frac{\phi_{,\mu}\phi^{,\mu}}{\phi}\right] - \int d\tau m(\phi),$$

where $\omega_{\rm BD} > 10^4$ and $\omega_{\rm BD} \to \infty$ in GR.

- Brans-Dicke theory in Einstein frame

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-\bar{g}}\left[\frac{\bar{R}}{16\pi} - \frac{1}{2}\Phi_{,\mu}\Phi^{,\mu}\right] - \int d\bar{\tau}e^{-\beta\Phi/2}m(\phi),$$

where $\bar{g}_{\mu\nu} = \phi g_{\mu\nu}$, $\Phi = \ln\phi/\beta$ and $\beta = \sqrt{16\pi/(2\omega_{\rm BD}+3)}$.

- Field equations in Einstein frame: Teukolsky/Klein-Gordon eqs.

$$G^{(1,\mathrm{E})}_{\mu\nu} = 8\pi T^{(1,\mathrm{E})}_{\mu\nu}, \quad \Box^{(0,\mathrm{E})}\phi = \alpha T^{(1,\mathrm{E})} \ \text{with} \ \varphi = \phi - \phi^{(0)} \ll 1,$$

where $\alpha = \sqrt{16\pi/(2\omega_{\rm BD}+3)}\,(s-1/2)$, $s = -\frac{\ln m}{\ln G}$ and $s_{\rm BH} = 1/2$.

We solve the field equations using PN approximation.

Secular evolution of the orbital parameters

- "Constants of motion": $E = -u^\alpha \xi_\alpha^{(t)}$, $L_z = u^\alpha \xi_\alpha^{(\phi)}$
- $T_{\rm orbit} \ll T_{\rm radiation}$ $[T_{\rm orbit} = O(M),\ T_{\rm radiation} = O(M^2/\mu)]$
  - ⋆ Energy balance argument

$$\left\langle \frac{dE}{dt} \right\rangle_t^{\rm gw} = -\mu^2 \sum_{\ell m n_r n_\theta} \frac{1}{4\pi\omega_{mn_rn_\theta}^2} \Big(\underbrace{\left| Z_{\ell m n_r n_\theta}^{{\rm gw},\infty} \right|^2}_{\text{Infinity part}} + \underbrace{\alpha_{\ell m n_r n_\theta} \left| Z_{\ell m n_r n_\theta}^{{\rm gw},{\rm H}} \right|^2}_{\text{Horizon part}} \Big),$$

$$\left\langle \frac{dL_z}{dt} \right\rangle_t^{\rm gw} = -\mu^2 \sum_{\ell m n_r n_\theta} \frac{m}{4\pi\omega_{mn_rn_\theta}^3} \left(\left| Z_{\ell m n_r n_\theta}^{{\rm gw},\infty} \right|^2 + \alpha_{\ell m n_r n_\theta} \left| Z_{\ell m n_r n_\theta}^{{\rm gw},{\rm H}} \right|^2 \right),$$

$$\left\langle \frac{dE}{dt} \right\rangle_t^{s} = -\mu^2 \sum_{\ell m n_r n_\theta} \omega_{mn_rn_\theta}^2 \Big(\underbrace{\left| Z_{\ell m n_r n_\theta}^{s,\infty} \right|^2}_{\text{Infinity part}} + \underbrace{\alpha_{\ell m n_r n_\theta}^s \left| Z_{\ell m n_r n_\theta}^{s,{\rm H}} \right|^2}_{\text{Horizon part}} \Big),$$

$$\left\langle \frac{dL_z}{dt} \right\rangle_t^{s} = -\mu^2 \sum_{\ell m n_r n_\theta} m\omega_{mn_rn_\theta} \left(\left| Z_{\ell m n_r n_\theta}^{s,\infty} \right|^2 + \alpha_{\ell m n_r n_\theta}^s \left| Z_{\ell m n_r n_\theta}^{s,{\rm H}} \right|^2 \right),$$

where $Z_{\ell m n_r n_\theta}^{{\rm gw}/s,\infty/{\rm H}}$ are the asymptotic amplitudes of the waves and $\alpha_{\ell m n_r n_\theta} \propto (\omega - mq/(2r_+))$

Calculate the energy flux

- Solve Kerr geodesics: Eccentric orbits in the equatorial plane
- Construct energy-momentum tensor
  - ⋆ The source terms of the Teukolsky/Klein-Gordon equations
- Solve the Field equations
  - ⋆ The semi-analytic method by Mano et al. (1995)
    - ∗ $R^{\mathrm{in/up}}_{\ell m\omega}(r) \sim \sum a^{\nu}_{n} F_{n+\nu}(r)$, $F_{n+\nu}(r)$ is hypergeometric function

      $\Rightarrow \; a^{\nu}_{n+1}\alpha^{\nu}_{n} + a^{\nu}_{n}\beta^{\nu}_{n} + a^{\nu}_{n-1}\gamma^{\nu}_{n} = 0$
- Calculate the amplitude of each mode $Z^{\mathrm{gw}/s,\infty/\mathrm{H}}_{\ell m\omega}$
- Sum over all modes in $\left\langle \frac{dE}{dt} \right\rangle^{\mathrm{gw},s}$ and $\left\langle \frac{dL_z}{dt} \right\rangle^{\mathrm{gw},s}$

Infinity part of $\left\langle \frac{dE}{dt} \right\rangle_t^s$ at $O(v^5, e^6)$

$$\left\langle \frac{dE}{dt} \right\rangle_t^{\mathrm{s},\infty} = -\frac{\alpha^2}{12\pi}\left(\frac{\mu}{M}\right)^2 (1-e^2)^{3/2} v^8$$
$$\times\Bigg[1+\frac{1}{2}e^2 - 2\left\{1-4e^2-\frac{11}{8}e^4\right\}v^2$$
$$+\left\{-4q\left(1+4e^2+\frac{5}{8}e^4\right)+2\pi\left(1+3e^2+\frac{13}{32}e^4+\frac{1}{144}e^6\right)\right\}v^3$$
$$+\left\{-10\left(1+\frac{43}{10}e^2-\frac{47}{16}e^4-\frac{7}{8}e^6\right)+q^2\left(1+5e^2+\frac{7}{8}e^4\right)\right\}v^4$$
$$+\Bigg\{4q\left(1-18e^2-\frac{321}{8}e^4-5e^6\right)$$
$$+\frac{12\pi}{5}\left(1+\frac{101}{12}e^2+\frac{707}{48}e^4+\frac{4969}{2304}e^6\right)\Bigg\}v^5+\cdots\Bigg],$$

[cf. $\left\langle \frac{dE}{dt} \right\rangle_N^{\mathrm{gw}} = -\frac{32}{5}\left(\frac{\mu}{M}\right)^2 (1-e^2)^{3/2}\, v^{10}$]

- ⋆ Agrees with 2.5PN and $e = 0$ formula by Ohashi+(1996)
- ⋆ Agrees with leading formula for $\ell = 0, 1$ by Will+(1989)

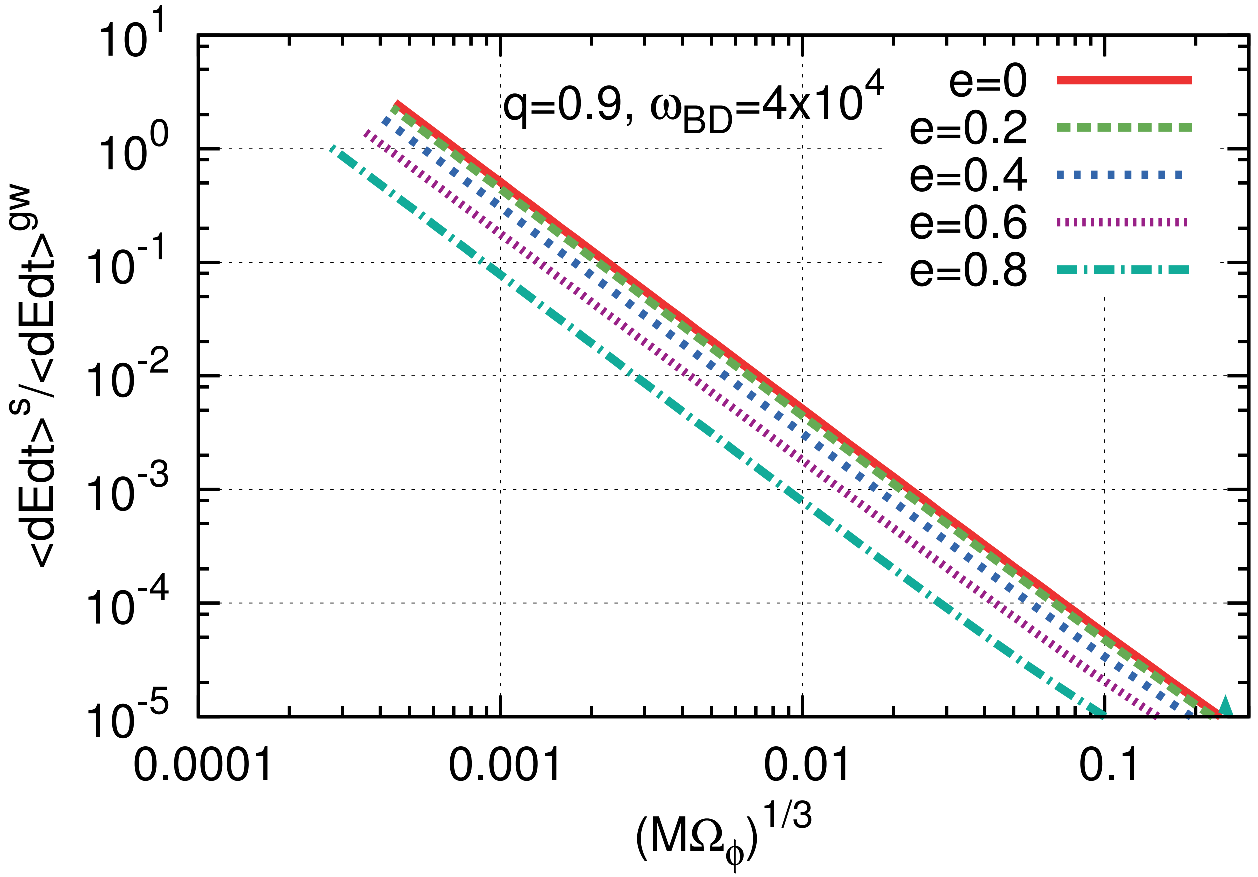
Horizon part of $\left\langle \frac{dE}{dt} \right\rangle_t^s$ at $O(v^5, e^6)$

$$
\begin{aligned}
\left\langle \frac{dE}{dt} \right\rangle_t^{\mathrm{s,H}} = & -\frac{\alpha^2}{12\pi}\left(\frac{\mu}{M}\right)^2 (1-e^2)^{3/2} v^8 \\
& \times \Bigg[3\,(1+\kappa)\,e^2 \left(1+\frac{e^2}{4}\right) v^2 - q\left(1+3\,e^2+\frac{3}{8}\,e^4\right) v^3 \\
& \quad - 3\,(1+\kappa)\,e^2 \left(8+\frac{3}{4}\,e^2-\frac{7}{8}\,e^4\right) v^4 \\
& \quad - q\left(2-\frac{3}{2}\,e^2+21\,e^4+\frac{69}{16}\,e^6+\kappa\,e^2\left(-18+\frac{9}{2}\,e^2+3\,e^4\right)\right) v^5 + \cdots \Bigg],
\end{aligned}
$$

where $\kappa = \sqrt{1-q^2}$.

- ⋆ All the terms are new terms
- ⋆ Appears at 1PN when $e \neq 0$
- ⋆ Appears at 1.5PN when $q \neq 0$ and $e = 0$
- ⋆ Appears at 3PN when $q = 0$ and $e = 0$

Modification due to scalar field energy flux



⋆ cf. EMRIs in LISA band (1yr obs.):

- ∗ Early inspiral: $0.2 \lesssim (M\Omega_\phi)^{1/3} \lesssim 0.25$ for $\mu/M = 10^{-4}$
- ∗ Late inspiral: $0.3 \lesssim (M\Omega_\phi)^{1/3} \lesssim 0.4$ for $\mu/M = 10^{-5}$

$\Rightarrow N_\phi^{\mathrm{gw}+s} \gtrsim 10^5$ and $N_\phi^s < 1$ for $e = 0$ [Yunes+(2012)]

$\Rightarrow$ May not put stronger constraint than current Solar-System ones

Cycles $N_\phi = \int \Omega_\phi[p(t), e(t)]dt$ w/o $\langle dE/dt \rangle_t^s$

$$N_\phi = N_\phi^{(0)} + N_\phi^{\rm gw} + N_\phi^{\rm s},$$

$$N_\phi^{\rm gw} = -\frac{(M\Omega_\phi)^{-5/3}}{32(\mu/M)}\left[1 + \frac{3715}{1008}(M\Omega_\phi)^{2/3} + \left(\frac{565}{24}q - 10\,\pi\right)(M\Omega_\phi) + \cdots + \delta N_\phi^{{\rm gw},(e)}\right],$$

$$\delta N_\phi^{{\rm gw},(e)} = e_0^2 (M\Omega_\phi)^{-19/9}\left[-\frac{785}{272} - \frac{2045665}{225792}(M\Omega_\phi)^{2/3} + \left(\frac{65561}{2880}\pi - \frac{3059}{108}q\right)(M\Omega_\phi) + \cdots\right],$$

$$N_\phi^{\rm s} = -\frac{\alpha^2 (M\Omega_\phi)^{-1}}{4(\mu/M)}\left[1 + \frac{3}{2}(M\Omega_\phi)^{2/3} + (-9\,q + 2\,\pi)(M\Omega_\phi)\ \ln(M\Omega_\phi) + \cdots + \delta N_\phi^{{\rm s},(e)}\right],$$

$$\delta N_\phi^{{\rm s},(e)} = e_0^2 (M\Omega_\phi)^{-2}\left[-\frac{3}{2} + \frac{9}{8}\,\frac{9\,\kappa - 4\,q^2 + 4}{\kappa}(M\Omega_\phi)^{2/3} + (6\pi - 15\,q)(M\Omega_\phi) + \cdots\right],$$

where e_0 is the initial eccentricity.

- ⋆ $N_\phi^{\rm gw}$ agrees with 2.5PN formula by Ganz+(2007)
- ⋆ $N_\phi^{\rm s}$ for $e \neq 0$ would be new formula
- ⋆ Leading term for $\delta N_\phi^{{\rm s},(e)}$ is negative

$\Rightarrow$ May not put stronger constraint than current Solar-System ones

Summary

- GWs from EMRIs using black hole perturbation theory (BHP)
  - ⋆ Dissipative part of the gravitational self-force
    - ∗ Consistent with PN theory for comparable mass binaries
    - ∗ New PN terms in BHP will improve the accuracy of templates for comparable mass binaries
  - ⋆ Conservative part of the gravitational self-force
    - ∗ Comparisons of invariants with those of PN and NR are active Redshift invariant, periastron advance, geodetic spin-precession,